

Linear codes from Hermitian curves

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We deal with some problems in Finite geometry arising from current research on AG (algebraic-geometry) codes defined over the Hermitian curve.

In $PG(2, q^2)$, let \mathcal{H} be the Hermitian curve. For a fixed positive integer d , let \mathbf{S}_d be the family of all degree d plane algebraic curves (possibly singular or reducible) defined over \mathbf{F}_{q^2} which do not have \mathcal{H} as a component. A natural question in Finite geometry is to ask for the maximum number $N(d)$ of common points in $PG(2, q^2)$ of \mathcal{H} with \mathcal{S} where \mathcal{S} ranges over \mathbf{S}_d . From Bézout's theorem, $N(d) \leq d(q + 1)$. The problem of finding better upper bounds on $N(d)$ for certain families of curves \mathbf{S} is motivated by the “minimum distance problem” for AG -codes.

A family of curves which play role in the study of Hermitian codes is defined as follows:

Let β be a Baer involution of $PG(2, q^2)$ which preserves \mathcal{H} . The set of points of \mathcal{H} which are fixed by β has size $q + 1$ and it is the complete intersection $\mathcal{H} \cap \mathcal{C}^2$ of \mathcal{H} with an irreducible conic \mathcal{C}^2 . For a positive integer m less than $q + 1$, define D to be the set of all points of \mathcal{H} other than those in U , together with the divisor (formal sum) \mathbf{G} of points on \mathcal{H} :

$$\mathbf{G} := m \sum_{\mathcal{H} \cap \mathcal{C}^2} P.$$

The functional AG -code $C_L(\mathbf{G}, D)$ is obtained taking the rational functions with pole numbers at most m at any point in $\mathcal{H} \cap \mathcal{C}^2$ and evaluating them at the points of D .

For m even, the minimum distance problem for $C_L(D, \mathbf{G})$ is equivalent to the problem of determining the maximum number of points on $\mathcal{H} \cap \mathcal{S}$ with $\mathcal{S} \in \mathbf{S}_m$.

For m odd, let \mathbf{T}_{m+1} be the subfamily of \mathbf{S}_d consisting of all curves \mathcal{T} of degree $m + 1$ which contain all points in D . The minimum distance prob-

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lem for $C_L(D, G)$ is equivalent to the problem of determining the maximum number of points on $\mathcal{H} \cap \mathcal{T}$ with $\mathcal{T} \in \mathbf{T}_{m+1}$.

The automorphism group of $C_L(D, G)$ contains a subgroup $G \cong PGL(2, q)$. Is it true that this subgroup is the full automorphism group?

We address the above problems using tools from Finite geometry.