

## On the Hamilton-Waterloo problem with odd cycle lengths

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The Hamilton-Waterloo problem  $\text{HWP}(K_v; m, n; \alpha, \beta)$  asks whether there exists a decomposition of  $K_v$  into  $\alpha$   $m$ -cycle factors and  $\beta$   $n$ -cycle factors, where  $3 \leq m \leq n$ . Necessarily, if such a decomposition exists, then  $m$ ,  $n$  and  $v$  are all odd,  $m$  and  $n$  are divisors of  $v$ , and  $\alpha + \beta = (v - 1)/2$ . We show that these necessary conditions are sufficient whenever  $m \geq 5$ ,  $v$  is a multiple of  $mn$  and  $v > mn$ , except possibly if  $\beta \in \{1, 3\}$ . For  $m \geq 5$  and  $v = mn$ , we solve the problem whenever  $\beta > (n + 5)/2$ , except possibly if  $(v, m, n, \alpha, \beta) = (35, 5, 7, 9, 8)$ . Similar results are obtained when  $m = 3$ , with only a few extra possible exceptions in this case. Our results rely on the existence of factorizations of  $C_m[n]$  (the lexicographic product of an  $m$ -cycle with the complement of  $K_n$ ), which will be further discussed in Tommaso Traetta's talk.