

## Harmonious and achromatic colorings of fragmentable hypergraphs

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A *harmonious* coloring of a  $k$ -uniform hypergraph  $H$  is a rainbow vertex coloring such that each  $k$ -set of colors appears on at most one edge. A rainbow coloring of  $H$  is *achromatic* if each  $k$ -set of colors appears on at least one edge. The *harmonious number* (resp. *achromatic number*) number of  $H$ , denoted by  $h(H)$  (resp.  $\psi(H)$ ) is the minimum (resp. maximum) possible number of colors in a harmonious (resp. achromatic) coloring of  $H$ . A class  $\mathcal{H}$  of hypergraphs is *fragmentable* if for every  $H \in \mathcal{H}$ ,  $H$  can be fragmented to components of a bounded size by removing a "small" fraction of vertices.

We show that for every fragmentable class  $\mathcal{H}$  of bounded degree hypergraphs, for every  $\epsilon > 0$  and for every hypergraph  $H \in \mathcal{H}$  with  $m \geq m_0(\mathcal{H}, \epsilon)$  edges we have  $h(H) \leq (1 + \epsilon) \sqrt[k]{k!m}$  and  $\psi(H) \geq (1 - \epsilon) \sqrt[k]{k!m}$ .

As corollaries, we answer a question posed by Blackburn (concerning the maximum length of  $t$ -subset packing sequences of constant radius) and derive an asymptotically tight bound on the minimum number of colors in a vertex-distinguishing edge coloring of cubic planar graphs (which is a step towards confirming a conjecture of Burriss and Schelp).