

Facial colourings of plane graphs

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Let $G = (V, E, F)$ be a connected loopless and bridgeless plane graph, with vertex set V , edge set E , and face set F . Let $X \in \{V, E, F, V \cup E, V \cup F, E \cup F, V \cup E \cup F\}$: Two elements x and y of X are *facially adjacent* in G if they are incident, or they are adjacent vertices, or adjacent faces, or facially adjacent edges (i.e. edges that are consecutive on the boundary walk of a face of G). A k -colouring is facial with respect to X if there is a k -colouring of elements of X such that facially adjacent elements of X receive different colours. It is known that:

1. G has a facial 4-colouring with respect to $X \in \{V, F\}$. The bound 4 is tight. (The Four Colour Theorem, Appel and Haken 1976)
2. G has a facial 6-colouring with respect to $X = V \cup F$. The bound 6 is tight. (The Six Colour Theorem, Borodin 1984)

We prove that:

3. G has a facial 4-colouring with respect to $X = E$. The bound 4 is tight.
4. G has a facial 6-colouring with respect to $X \in \{V \cup E, E \cup F\}$. There are graphs required 5 colours in such a colouring.
5. G has a facial 8-colouring with respect to $X = V \cup E \cup F$. There is a graph requiring 7 colours in such a colouring.