

Minimum difference representations of graphs

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In this talk, we want to represent the vertices of a graph with finite sets in such a way that two vertices are adjacent if and only if their sets are “very different” (this is similar, e.g., to the notion of Kneser graphs). More precisely, a *k-min-difference representation* of G is an assignment of a set A_i to each vertex $i \in V(G)$ so that

$$ij \in E(G) \Leftrightarrow \min\{|A_i \setminus A_j|, |A_j \setminus A_i|\} \geq k.$$

The smallest k such that there exists a k -min-difference representation of G is denoted by $\rho_{\min}(G)$.

After Boros, Gurvitch and Meshulam introduced this notion in 2004, it was not even known whether there are any graphs with $\rho_{\min}(G) \geq 3$. Balogh and Prince proved in 2009 that is indeed so: for every k there is a graph G with $\rho_{\min}(G) \geq k$ (in fact they proved more—for every k there is an n_0 such that whenever $n > n_0$, then for a graph G on n vertices we have $\rho_{\min}(G) \geq k$ with high probability). However, they used a Ramsey-type result in their proof, and as a consequence the resulting bounds are weak.

We improved this, using a different method. The main result of this talk is that there is a constant $c > 0$ such that if n is large enough, there is a bipartite graph G on n vertices such that $\rho_{\min}(G) \geq \frac{cn}{\log n}$.