

Order congruence lattices are EL-shellable

Russ Woodroffe

Mississippi State University

`rw1003@msstate.edu`

Jay Schweig

Oklahoma State University

`jayschweig@gmail.com`

The lattice of order congruences of a poset P is the subposet $O(P)$ of the lattice of partitions of P , consisting of all partitions that satisfy a certain order-convexity property. The partition lattice is both semi-modular and supersolvable, but Krtesi, Radelecki and Szilgyi gave examples of order congruence lattices that are not semi-modular, and we find examples that are not supersolvable.

We show that the order congruence lattices satisfy a recursive condition on the existence of modular elements, which we call *lm-decomposability*. *Lm-decomposability* generalizes both supersolvability and semimodularity. We show that any *lm-decomposable* lattice is EL-shellable. This result simultaneously improves a weaker result of Jena and Sarkoci, and also exposes inclusion-exclusion information (Mbius numbers).

This project is joint with Jay Schweig.