

## Langford sequences from products of labeled digraphs

Susana-C. López

*Universitat Politècnica de Catalunya*

susana@ma4.upc.edu

Francesc-A. Muntaner-Batle

*The University of Newcastle*

fambles@yahoo.es

For  $m \leq n$ , we denote the set  $\{m, m + 1, \dots, n\}$  by  $[m, n]$ . Let  $d$  be a positive integer. A *Langford sequence* of order  $m$  and defect  $d$  is a sequence  $(l_1, l_2, \dots, l_{2m})$  of  $2m$  numbers such that (i) for every  $k \in [d, d + m - 1]$  there exist exactly two subscripts  $i, j \in [1, 2m]$  with  $l_i = l_j = k$ , (ii) the subscripts  $i$  and  $j$  satisfy the condition  $|i - j| = k$ . A Langford sequence of order  $m$  and defect  $d = 1$  is a Skolem sequence of order  $m$ . Abraham showed that the number of Skolem sequences of order  $m$  is at least  $2^{\lfloor m/3 \rfloor}$  for every  $m \equiv 0$  or  $1 \pmod{4}$ .

We say that a digraph  $D$  admits a labeling  $f$  if its underlying graph admits the labeling  $f$ . For any Skolem sequence of order  $m$ , we define a *Skolem labeling*  $g$  of the directed matching  $m\vec{K}_2$ , up to isomorphism. Let  $f : E(m\vec{K}_2) \rightarrow [1, m]$  be any bijective function such that if  $f(u, v) = k$ , then the Skolem labeling  $g$  of  $m\vec{K}_2$  assigns to  $u$  one of the two positions occupied by  $k$  and to  $v$  the other position occupied by  $k$ , in such a way that the label of  $u$  is strictly smaller than the label of  $v$ . In a similar way, we can define a *Langford labeling* of  $m\vec{K}_2$ , up to isomorphism, for any Langford sequence of order  $m$  and defect  $d$ .

In this talk, we present a study of Skolem and Langford sequences through Skolem and Langford labelings of  $m\vec{K}_2$ . By multiplying a Skolem labeled matching  $m\vec{K}_2$  by a particular family of labeled 1-regular digraphs of order  $n$ , we obtain a Langford labeled matching  $(mn)\vec{K}_2$ . We will show that this procedure can also be applied to obtain Langford sequences from existing ones, and that in this way, we can obtain a lower bound for the number of Langford sequences, for particular values of the defect. We also extend this procedure to hooked and extended Skolem sequences.

*2010 Mathematics subject classification:* 11B99, 05C76 and 05C78.

*Key words:* Skolem sequence, Langford sequence,  $\otimes_h$ -product, super edge-magic labeling.

**Acknowledgements** The research conducted in this document by the first

## GENERAL SESSION TALKS

---

author has been supported by the Spanish Research Council under project MTM2011-28800-C02-01 and symbolically by the Catalan Research Council under grant 2014SGR1147.