

## Forbidden subgraphs in the norm graph

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Let  $H$  be a fixed graph. The *Turán number* of  $H$ , denoted  $ex(n, H)$ , is the maximum number of edges that a graph with  $n$  vertices and not containing a copy of  $H$  can have. When  $H$  is bipartite, in many cases, even the question of determining the right order of magnitude for  $ex(n, H)$  is not known. We have the following bounds:

$$cn^{2-1/t} \leq ex(n, K_{t,s}) \leq \frac{1}{2}(s-1)^{\frac{1}{t}}n^{2-\frac{1}{t}} + \frac{1}{2}(t-1)n$$

where the lower one is probabilistic. In [1], the authors construct the so called *norm graph*  $\Gamma(t)$ , proving that the upper bound is asymptotically sharp for  $K_{t,s}$ ,  $s > (t-1)!$ . We prove that  $\Gamma(t)$  also contains no copy of  $K_{t+1,s}$ ,  $s \geq (t-1)! - 1$ , improving the known lower bound for  $t = 4$  and showing that  $\Gamma(t)$  is the best determinist construction for  $(t-1)! - 1 \leq s \leq t!$ ,  $t \geq 4$ .

### References:

- [1] N. Alon, L. Rónyai and T. Szabó, Norm-Graphs: Variations and applications, *J. Combin. Theory Ser. B*, **76** (1999) 280–290.
- [2] S. Ball and V. Pepe, Asymptotic improvements to the lower bound of certain bipartite Turán numbers, *Combin. Probab. Comput.*, **21** (2012) 323–329.
- [3] S. Ball and V. Pepe, New forbidden graphs in the norm graph, arXiv: <http://arxiv.org/abs/1502.01502>.