

Near 1-factors with prescribed edge-lengths in the complete graph

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Let K_v be the complete graph with vertex-set $\{0, 1, \dots, v-1\}$. The length of any edge $[x, y]$ of K_v is defined by $d(x, y) = \min(|x - y|, v - |x - y|)$. In 2007 Buratti conjectured that if v is a prime, then there exists a Hamiltonian path of K_v whose list of edge-lengths is ANY prescribed multiset L of $v - 1$ integers in $\{1, 2, \dots, \lfloor \frac{v}{2} \rfloor\}$. Partial results about this conjecture can be found in [1-5].

Recently, Meszka proposed the following similar conjecture: “If v is an odd prime, then there exists a near 1-factor of K_v whose list of edge-lengths is ANY prescribed multiset L of $\frac{v-1}{2}$ integers in $\{1, 2, \dots, \lfloor \frac{v}{2} \rfloor\}$ ”. Rosa [7] proved that the conjecture is true when the elements of the list are $\{1, 2, 3\}$ or $\{1, 2, 3, 4\}$ or $\{1, 2, 3, 4, 5\}$. Also he proved that the conjecture is true when the list L contains a “sufficient” number of 1’s.

In this talk we propose a generalization of this problem to the case in which v is an odd integer not necessarily prime [6]. In particular, we give a necessary condition for the existence of such a near 1-factor and we conjecture that such a condition is also sufficient. If v is a prime our conjecture reduces to Meszka’s one. Then we show that our conjecture is true for any list L whose underlying set S has size 1, 2, or $\lfloor \frac{v}{2} \rfloor$, or when $S = \{1, 2, t\}$ for any positive integer t not coprime with v . Also, we give partial results when t and v are coprime. Finally, we present a complete solution for $t \leq 11$.

References:

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