

## Imbedding Types for Linear Families, Markov Chains, and Smoothing

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The genus polynomial  $g_G(z) = \sum a_i z^i$  of graph  $G$  is the generating function where  $a_i$  is the number of imbeddings of  $G$  in the surface of genus  $i$ . A genus polynomial is always *interpolating*, that is the nonzero coefficients form an interval. Little else is known; for example the number of imbeddings of  $K_7$  is  $(6!)^7$  and its genus polynomial has not been computed. On the other hand, for any “linear family” of graphs  $G_n$ , one can partition into imbedding types and compute a “production” (recursion) matrix  $M(z)$  with polynomial entries (with non-negative integer coefficients) such that the vector  $v_n(z)$  of genus polynomials partitioned by imbedding types satisfies  $v_n(z) = M^n(z)v_0(z)$ . The matrix  $M(1)$  has constant column sum  $s$ , so  $(1/s)M(1)$  is the matrix of a Markov chain. We conjecture that it is regular, that is  $M^n(1)$  has all positive entries for some  $n$ . We also conjecture that multiplication by  $M^n(z)$  is a form of smoothing analogous to multiplication of any polynomial (with nonnegative coefficients) by  $(1+z)^n$ , which is eventually interpolating and log-concave.