

On Schur 2-groups

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A finite group G is called a Schur group, if any Schur ring over G is the transitivity module of a point stabilizer in a subgroup of $\text{Sym}(G)$ that contains all right translations. We complete a classification of abelian 2-groups by proving that the group $Z_2 \times Z_{2^n}$ is Schur. We also prove that any non-abelian Schur 2-group of order larger than 32 is dihedral (the Schur 2-groups of smaller orders are known). Finally, in the dihedral case, we study Schur rings of rank at most 5, and show that the unique obstacle here is a hypothetical S-ring of rank 5 associated with a divisible difference set.