Appendix A. Model equations

A.1 TA equations

As mentioned in Section 2.1, TA increases (decreases) due to processes that produce (consume) NH_4^+ or consume (produce) NO_3^- . Therefore, TA in the model increases due to new primary production, remineralization of LDON and RDON, water column denitrification, and sediment fluxes of NH_4^+ ; it decreases due to regenerated primary production, water column nitrification, and sediment fluxes of NO_3^- . The equation for TA follows (note that the factor 14 represents the conversion of grams of nitrogen to moles of nitrogen; i.e., 14 g N = 1 mol N):

$$\frac{\partial TA}{\partial t} = PP_{new} - PP_{reg} - Nitr_{TA} + R_{LDON} + R_{RDON} + Den_{TA} + P_{predTA} + P_{lossTA} + Sed_{TA}$$

where each term represents the following:

$$PP_{new} = New \ primary \ production = \sum_{i=1}^{2} (1 - PNi) * Pi * ANCi * Bi/14$$
$$PP_{reg} = Regenerated \ primary \ production = \sum_{i=1}^{2} PNi * Pi * ANCi * Bi/14$$

where the subscripts i =1,2 represent diatoms and dinoflagellates, respectively. The preferential uptake of NH₄⁺ is $PNi = NH4\left(\frac{NO3}{(KhNi + NH4)*(KhNi + NO3)} + \frac{KhNi}{(NO3 + NH4)*(KhNi + NO3)}\right)$ as in Thomann and Fitzpatrick (1982). *Bi* is the algal biomass in g C m⁻³ and the coefficient *ANCi* is the nitrogen-to-carbon ratio for each type of phytoplankton. *Pi* is the total phytoplankton growth, given by *Pi* = *PMi* * *FTi* * *FIi* * *min* (*NLi*, *PLi*) / *CCHLi*, where the temperature dependence term *FTi* is $e^{-KTg1i*(T-TMPi)^2}$ if temperature < *TMPi* and $e^{-KTg2i*(T-TMPi)^2}$ otherwise. The light limitation term is *FIi* = *Iavg*/ $\sqrt{(Pi/ai)^2 + Iavg^2}$ (Jassby and Platt, 1976), where *Iavg* is the average irradiance in each cell (incoming irradiance undergoes an exponential decay with depth). The nutrient limitation terms for nitrogen and phosphorus are $NLi = \frac{NH4 + DIN - PNi * DIN}{KhNi + NH4 + DIN - PNi * DIN}$ and $PLi = \frac{PO4}{KhPi + PO4}$, respectively, where DIN = NH₄⁺ + NO₃⁻. The values of the coefficients *PMi* (maximum growth rate in d⁻¹), *CCHLi* (carbon-to-chlorophyll ratio in g C g Chl⁻¹), *ANCi*, *KhNi* (half saturation concentration for nitrogen uptake in g N m⁻³), *KhPi* (half saturation concentration for phosphorus uptake in g P m⁻³), *KTg1i*, *KTg2i*, and *ai* (photosynthesis vs. irradiance slope) are inputs to the model and can be found in Table A1.

 $Nitr_{TA} = water \ column \ nitrification = 2 * NT/14$

where $NT = \frac{DO}{DO + KhOnt} * \frac{NH4}{KhNnt + NH4} * FT_{nt} * NTM$ is the nitrification rate in g N m⁻³ d⁻¹, with $FT_{nt} = e^{-KTNT1*(TMNT - T[I,K])^2}$ if temperature < TMNT and $FT_{nt} = e^{-KTNT2*(TMNT - T[I,K])^2}$ otherwise. The factor 2 represents that both the consumption of NH₄⁺ and the production of NO₃⁻during

nitrification decrease TA (Wolf-Gladrow et al., 2007). The coefficients *KhOnt* (half saturation concentration for DO uptake during nitrification in g $O_2 m^{-3}$), *KhNnt* (half saturation concentration of NH₄⁺ required for nitrification in g N m⁻³), *KTNT1*, *KTNT2*, *TMNT*, and *NTM* (maximum nitrification rate in g N m⁻³ d⁻¹) are given in Table A1.

$$R_{LDON} = remineralization of LDON = \frac{DO + AANOX * KhOdoc}{KhOdoc + DO} * KLDON * FTrm * \frac{LDON}{14}$$
$$R_{RDON} = remineralization of RDON = \frac{DO + AANOX * KhOdoc}{KhOdoc + DO} * KRDN * FTrm * \frac{RDON}{14}$$

where $N = KLDN + KDNALG * (\sum_{i=1}^{2} Bi) * \frac{KhNavg}{KhNavg + DIN}$, with KhNavg = (KhN1 + KhN2)/2, and $FTrm = e^{KTMNL*(T-TRMNL)}$. The input coefficients KTMNL, TRMNL, AANOX (ratio of denitrification to oxic carbon respiration), KhOdoc (half saturation concentration of DO required for oxic respiration), KLDN (remineralization rate of LDON in d⁻¹), and KRDN (remineralization rate of RDON in d⁻¹) are found in Table A1.

$$Den_{TA} = water \ column \ denitrification$$
$$= ANDC * KLDOC * FTrm * AANOX * \frac{KhOdoc}{KhOdoc + DO} * \frac{NO3}{KhNdn + NO3} * \frac{LDOC}{14}$$

where the input coefficients ANDC (mass of nitrate-nitrogen reduced per mass of dissolved organic carbon oxidized in g N g C^{-1}) and *KhNdn* (half saturation concentration of NO_3^- required for denitrification in g N m⁻³) are given in Table A1.

$$P_{predTA} = P \ losses \ by \ predation = \frac{1}{14} \sum_{i=1}^{2} FNIP * BPRi * FTpr * ANCi$$

where $FTpr = e^{KTPR(T-TRPR)}$ is the temperature dependence and the input coefficients *KTPR*, *TRPR*, *BPRi* (predation rate at temperature *TRPR*, in d⁻¹), and *FNIP* (fraction of inorganic nitrogen produced by predation), are provided in Table A1.

$$P_{lossTA} = P \text{ losses by basal mentabolism and photorespiration}$$
$$= \frac{1}{14} \sum_{i=1}^{2} FNIi * ANCi * (Pi * PRSPi + BMi) * Bi$$

where $BMi = BMRi * e^{KTBi(T-TRi)}$ and the input coefficients *KTBi*, *TRi*, *BMRi* (metabolic rate at temperature *TRi*, in d⁻¹), *FNIi* (fraction of inorganic nitrogen produced by basal metabolism and photorespiration), and *PRSPi* (fraction of photosynthesis lost to photorespiration) can be found in Table A1.

$$Sed_{TA} = sediment \ flux \ of \ TA \ to \ water = \frac{(BenthicFlux_{NH4} - BenthicFlux_{NO3})}{14} * \frac{1000}{dz_{bott}}$$

where dz_{bott} is the thickness of the bottom layer of the water column and *BenthicFlux_{NH4}* and *BenthicFlux_{NO3}* are the fluxes of NH₄⁺ and NO₃⁻ from the top sediment layer to the water column (units are g N m⁻² d⁻¹, such that the factor 1000/14 converts them to mmol N m⁻² d⁻¹). They are calculated as *BenthicFlux_C* = *SDV* * (*C*_{TopSediment} - *C*_{BottomSeawater}), where *C* stands for the concentrations of either NH₄⁺ or NO₃⁻ and *SDV* represents the surface diffusion velocity (in m d⁻¹): *SDV* = *SOD/DO*_{BottomSeawater}, where *SOD* stands for Sediment Oxygen Demand (g O₂ m⁻² d⁻¹) and is divided by the DO concentration in the bottom layer of the water column (g O₂ m⁻³). The calculation of *SOD*, *S*, and the benthic fluxes follows an iterative approach described in Roberts et al. (2015) and Di Toro (2001). *Sed*_{TA} is only added to the bottom layer of water column.

A.2 DIC equations

The processes that consume DIC are primary production and water column nitrification, while production of DIC occurs through remineralization of LDOC and RDOC, water column denitrification, phytoplankton losses by predation, basal metabolism and photorespiration, and sediment fluxes. Air-sea CO_2 fluxes can either increase or decrease DIC depending on whether the atmospheric pCO₂ is higher or lower than the seawater pCO₂. The equation for the temporal rate of change in DIC is represented by

$$\frac{\partial DIC}{\partial t} = -PP - Nitr_{DIC} + R_{LDOC} + R_{RDOC} + Den_{DIC} + P_{predDIC} + P_{lossDIC} + GasEx + Sed_{DIC}$$

where each term represents the following (please refer to Appendix A.1 for indices, functions, and parameters that were already defined there):

$$PP = primary \ production = \sum_{i=1}^{2} Pi * Bi$$
$$Nitr_{DIC} = water \ column \ nitrification = \frac{NT}{14 * 11}$$

where the factor 14 transforms the nitrification rate NT (in g N m⁻³ d⁻¹) to mol m⁻³ d⁻¹ and the factor 11 is the ratio of mol N to mol C (Wezernak and Gannon, 1967; Billen, 1976).

$$R_{LDOC} = remineralization of LDOC = KLDOC * FTrm * \frac{DO}{KhOdoc + DO} * LDOC$$

$$R_{RDOC} = remineralization of RDOC = KRDOC * FTrm * \frac{DO}{KhOdoc + DO} * RDOC$$

$$Den_{DIC} = water \ column \ denitrification$$
$$= KLDOC * FTrm * AANOX * \frac{DO}{KhOdoc + DO} * \frac{NO3}{KhNdn + NO3} * LDOC$$

$$P_{predDIC} = P \text{ losses by predation} = \sum_{i=1}^{2} FDOP * BPRi * FTpr * \frac{DO}{KhRi + DO}$$

where FDOP (fraction of inorganic carbon produced by predation) and *KhRi* (half saturation concentration for LDOC excretion, in g $O_2 m^{-3}$) are given in Table A1. The main difference between $P_{predDlC}$ and P_{predTA} is the oxygen dependence term in $P_{predDlC}$. When DO becomes scarce (DO close to or lower than *KhRi*), the fraction of inorganic carbon produced by predation decreases while the excretion of LDOC increases by a factor 1 - DO/(KhRi + DO).

$$P_{lossDIC} = P \text{ losses by basal metabolism and photorespiration}$$
$$= \sum_{i=1}^{2} FCIi * (Pi * PRSPi * BMi) * \frac{DO}{KhRi + DO} * Bi$$

where, again, a DO-dependence term means that the generation of inorganic carbon decreases as DO becomes closer to or less than *KhRi*. *FCli* (fraction of inorganic carbon produced by basal metabolism and photorespiration) is not directly given as an input, but calculated as the reminder of all other fractions sourced to LDOC, RDOC, LPOC, and RPOC: FCLi = 1 - FCLDi - FCRDi - FCLPi - FCRPi (where all of the fractions other than *FCLi* are given as inputs, shown in Table A1).

$$GasEx = air - sea CO_2 \ exchange = 0.251 \ * \ WMS^2 \ * \ \sqrt{\frac{600}{Sc} * \frac{24}{100} * \frac{[CO_{2sat}^* - CO_{2surf}^*]}{dz_{surf}}}$$

where 0.251 is the gas transfer velocity in cm h⁻¹ (Wanninkhof, 2014), *WMS* is the wind speed (m s⁻¹) provided as input to the model, *Sc* is the Schmidt number for CO₂ (Wanninkhof, 2014), the factor 24/100 converts the units of the gas transfer velocity from cm hr⁻¹ to m d⁻¹, and *dz_{surf}* is the thickness of the surface layer. When gaseous CO₂ from the atmosphere dissolves in seawater, it first gets hydrated into aqueous form (CO_{2aq}) and immediately reacts with water to form carbonic acid (H₂CO₃) with the equilibrium reaction $CO_{2aq} + H_2O \leftrightarrow H_2CO_3$. As it is difficult to distinguish analytically between CO_{2aq} and H₂CO₃, a hypothetical species called CO₂^{*} expresses the sum of both concentrations. Therefore, the exchange of CO₂ between air and sea depends on the difference between the seawater saturation concentration of CO₂^{*} (CO_{2sat}^*) and the CO₂^{*} at surface (CO_{2surf}^*). The latter is calculated from CO2SYS equations, and CO_{2sat}^* is calculated from the atmospheric pCO₂ (pCO_{2atm}) through Henry's Law: $CO_{2sat}^* = K0 * pCO_{2atm}$, where *KO* is the equilibrium constant of the reaction $CO_{2(gas)} \leftrightarrow CO_2^*$ from Weiss (1974). *GasEx* is only added to the surface layer of the water column:

$$Sed_{DIC} = sediment \ flux \ of \ DIC \ to \ water = \frac{BenthicFlux_{CO2}}{dz_{bott}} * \frac{1000}{12}$$

where *BenthicFlux_{co2}* is the flux of CO₂ from the sediment layer to the water column (in g C m⁻² d⁻¹) and the factor 1000/12 converts it to mmol C m⁻² d⁻¹ (using a mole-to-gram ratio for carbon of 12 g C mol C⁻¹). *BenthicFlux_{co2}* is calculated as the sum of the CO₂ produced by aerobic oxidation of organic matter in the sediments (*CSOD*) and the CO₂ produced by anoxic oxidation of dissolved organic carbon by denitrification (*CDen*): *BenthicFlux_{co2}* = (*CSOD* + *CDen*)/2.667, where *CSOD* and *CDen* have units of g O₂ m⁻² d⁻¹ and the denominator is the oxygen-to-carbon ratio (2.667 g O₂ g C⁻¹). The calculation of *CSOD* and *CDen* follows an iterative approach described in Roberts et al. (2015) and Di Toro (2001). Sed_{DIC} is only added to the bottom layer of water column.

Appendix B. Riverine concentrations of pH and TA

B.1 pH in rivers

Mohamedali et al. (2011) calculated pH as a function of river flow with a multilinear regression based on available data from US and Canadian rivers for the period 1998–2008. Our further analysis of observations from 21 US rivers showed that 99.9% of pH data was above 6.5, and 99.7% of pH data were below 8.5. Therefore, we decided to cap the pH values calculated from the flow regression between 6.5 and 8.5 for US rivers. For Canadian rivers, the previously established pH-flow relationships (Mohamedali et al., 2011) gave constant values for pH. For our study, we used observed pH values for Fraser, Cowichan, English, and Tsolum Rivers. As no pH data were available for Squamish, Clowhorn, or Harris Rivers, we assumed them to be the same as Cowichan River due to their similar flows.

B.2 TA in rivers

TA observations in both US and Canadian rivers were separated according to their flows in five categories: little (flow < $1 \text{ m}^3 \text{ s}^{-1}$), small (1–10 m³ s⁻¹), medium (> 10–100 m³ s⁻¹), large (> 100–1000 m³ s⁻¹), and extra-large (> 1000 m³ s⁻¹). Only the Fraser River fell in the last category. The median TA in each category was used as the TA input in all rivers with flows corresponding to that category. From little to extra-large flows, the following were the median TA values: 1420, 740, 540, 480, and 1060 mmol m⁻³. One of the exceptions was the discharge from North Vancouver Island, which is made up of two small rivers but had low alkalinity: its TA median value was 200 mmol m⁻³ for all months. When more than one river flowed into the same model node, the rivers were combined such that the resulting TA was the flow-weighted average of TA in each river.

B.3 pH and TA in point sources

Data were obtained from 13 plants: Shelton, Tamoshan, Gig Harbor, Manchester, Port Orchard, Redondo, Lakota, Central Kitsap, Midway, Tacoma central, Chambers, West Point, and South King. From all of the available data, we calculated monthly-mean time series for pH and a median value for TA (2000 mmol m^{-3}) which were used for point sources.

Parameter	Description	Value ^b
AANOX	Ratio of denitrification to oxic carbon respiration	0.5
ANCi	Nitrogen-to-carbon ratio of algae group i	$0.175, 0.175 \text{ g N g C}^{-1}$
ANDC	Mass of nitrate-nitrogen reduced per mass of	0.933 g N g C ⁻¹
	dissolved organic carbon oxidized	
BMRi	Metabolic rate at temperature <i>TRi</i> of algae group <i>i</i>	0.1, 0.1 d ⁻¹

Table A1. Biological model parameters from equations in Appendix A^a.

BPRi	Predation rate on algae group <i>i</i> at temperature <i>TRPR</i>	1.0, 0.5 d ⁻¹
CCHLi	Carbon-to-chlorophyll ratio of algae group i	$37.0, 50.0 \text{ g C g Chl}^{-1}$
FCLDi	Fraction of LDOC produced by predation of algae group <i>i</i>	0, 0
FCLPi	Fraction of LPOC produced by predation of algae group <i>i</i>	0, 0
FCRDi	Fraction of RDOC produced by predation of algae group <i>i</i>	0, 0
FCRPi	Fraction of RPOC produced by predation of algae group <i>i</i>	0, 0
FDOP	Fraction of inorganic carbon produced by predation	0
FNIi	Fraction of inorganic nitrogen produced by basal metabolism and photorespiration of algae group <i>i</i>	0.55, 0.55
FNIP	Fraction of inorganic nitrogen produced by predation	0.4
KhNdn	Half saturation concentration of NO ₃ ⁻ required for denitrification	0.1 g N m^{-3}
KhNi	Half saturation concentration for nitrogen uptake by algae group <i>i</i>	0.06, 0.06 g N m ⁻³
KhNnt	Half saturation concentration of NH4 ⁺ required for nitrification	0.5 g N m ⁻³
KhOdoc	Half saturation concentration of DO for oxic respiration	$0.5 \text{ g O}_2 \text{ m}^{-3}$
KhOnt	Half saturation concentration of DO required for nitrification	$3.0 \text{ g O}_2 \text{ m}^{-3}$
KhPi	Half saturation concentration for phosphorus uptake by algae group <i>i</i>	0.02, 0.02 g P m ⁻³
KhRi	Half saturation concentration for LDOC excretion by algae group <i>i</i>	0.5, 0.5 g $O_2 m^{-3}$
KLDN	Minimum remineralization rate of LDON	0.05 d ⁻¹
KRDN	Remineralization rate of RDON	$0.0025 d^{-1}$
КТВі	Effect of temperature on basal metabolism of algae group i	0.032, 0.032 °C ⁻¹
KTG1i	Effect of temperature below TMPi on growth of algae group <i>i</i>	0.008, 0.2 °C ⁻¹
KTg2i	Effect of temperature above TMPi on growth of algae group <i>i</i>	0.05, 0.0008 °C ⁻¹
KTMNL	Effect of temperature deviations from TRMNL on remineralization	0.092 °C ⁻¹
KTNT1,2	Effect of temperature below, above TMNT on	0.0045, 0.0045 °C ⁻¹

	nitrification	
KTPR	Effect of temperature in predation	0.069 °C ⁻¹
NTM	Maximum nitrification rate at optimal temperature	$0.4 \text{ g N m}^{-3} \text{ d}^{-1}$
PMi	Maximum growth rate of algae group <i>i</i>	300–525, 357–525 d ⁻
PRSPi	Fraction of photosynthesis lost to photorespiration by algae group <i>i</i>	0.25, 0.25
TMNT	Optimal temperature for nitrification	30.0 °C
TMPi	Optimal temperature for growth of algae group <i>i</i>	12.0, 18.0 °C
TRi	Reference temperature for metabolism by algae	20.0, 20.0 °C
	group i	
TRMNL	Reference temperature for remineralization	20.0 °C
TRPR	Reference temperature for predation	20.0 °C
αί	Photosynthesis vs. irradiance slope for algae group <i>i</i>	6, $6 \frac{g C g C h^{-1} d^{-1}}{\mu E m^{-2} s^{-1}}$

^aCoefficients that end with "i" represent two algal groups: diatoms (i = 1) and dinoflagellates (i = 2).

^bWhere two values are provided, the first corresponds to diatoms (i = 1) and the second to dinoflagellates (i = 2)

^cTwo values were set for each algal group, with the smaller one in the Puget Sound Central Basin.

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