CHAPTER 37 ANGLES AND TRIANGLES

EXERCISE 152 Page 412

1. Evaluate 52° 39′ + 29° 48′

$$\begin{array}{r} 52°\ 39' \\ +\ \underline{29°\ 48'} \\ \underline{82°\ 27'} \\ 1° \end{array}$$

(i) 39′ + 48′ = 87′

(ii) Since 60′ = 1°, 87′ = 1°27′

(iii) The 27′ is placed in the minutes column and 1° is carried in the degrees column

(iv) 52° + 29° + 1°(carried) = 82°. Place 82° in the degrees column.

This answer can be obtained using the calculator as follows:

1. Enter 52 2. Press ° ’ ’ ’ 3. Enter 39 4. Press ° ’ ’ ’ 5. Press +

6. Enter 20 7. Press ° ’ ’ ’ 8. Enter 48 9. Press ° ’ ’ ’

10. Press = Answer = 82° 27′

Thus, 52°39′ + 29°48′ = 82°27′

2. Evaluate 76° 31′ – 48° 37′

$$\begin{array}{r} 76°\ 31' \\ -\ \underline{48°\ 37'} \\ \underline{27°\ 54'} \end{array}$$

(i) 31′ – 37′ cannot be done

(ii) 1° or 60′ is ‘borrowed’ from the degrees column, which leaves 75° in that column

(iii) (60′ + 31′) – 37′ = 54′ which is placed in the minutes column

(iv) 75° – 48° = 27° which is placed in the degrees column.

This answer can be obtained using the calculator as follows:

1. Enter 76 2. Press ° ’ ’ ’ 3. Enter 31 4. Press ° ’ ’ ’ 5. Press –

6. Enter 48 7. Press ° ’ ’ ’ 8. Enter 37 9. Press ° ’ ’ ’

10. Press = Answer = 27° 54′

Thus, $76° 31' – 48° 37' = 27° 54'$

3. Evaluate $77° 22' + 41° 36' – 67° 47'$

Using a calculator as explained in Problems 1 and 2 gives:

$$77° 22' + 41° 36' – 67° 47' = 51° 11'$$

4. Evaluate $41° 37' 16'' + 58° 29' 36''$

Using a calculator as explained in Problems 1 and 2 gives:

$$41° 37' 16'' + 58° 29' 36''= 100° 6' 52''$$

5. Evaluate $54° 37' 42'' – 38° 53' 25''$

Using a calculator: $54° 37' 42'' – 38° 53' 25'' = 15° 44' 17''$

6. Evaluate $79° 26' 19'' – 45° 58' 56'' + 53° 21' 38''$

Using a calculator: $79° 26' 19'' – 45° 58' 56'' + 53° 21' 38'' = 86° 49' 1''$

7. Convert $72° 33'$ to degrees in decimal form.

$$72° 33' = 72\frac{33}{60}^{\circ}$$

$$\frac{33°}{60} = 0.55° \text{ by calculator}$$

Hence, $72° 33' = 72\frac{33°}{60} = 72.55°$

This answer can be obtained using the calculator as follows:

1. Enter 72 2. Press ° ’ ’ ’ 3. Enter 33 4. Press ° ’ ’ ’ 5. Press =

6. Press ° ’ ’ ’ Answer = 72.55°

8. Convert $27° 45' 15''$ to degrees correct to 3 decimal places.

$$27° \, 45' \, 15'' = 27° \, 45\frac{15}{60}' = 27° \, 45.25'$$

$$27° \, 45.25' = 27\frac{45.25}{60}^{o} = 27.7542°$$

Hence, $27° \, 45' \, 15'' = 27.754°$ correct to 3 decimal places

This answer can be obtained using the calculator as follows:

1. Enter 27 2. Press ° ’ ’ ’ 3. Enter 45 4. Press ° ’ ’ ’ 5. Enter 15

6. Press ° ’ ’ ’ 7. Press = 8. Press ° ’ ’ ’ Answer = 27.754°

9. Convert 37.952° to degrees and minutes.

$0.952° = 0.952 \times 60' = 57.12' = 57'$ to the nearest minute

Hence, $37.952° = 37° \, 57'$

This answer can be obtained using the calculator as follows:

1. Enter 37.952 Press = 3. Press ° ’ ’ ’ Answer = 37° 57′ to the nearest minute

10. Convert 58.381° to degrees, minutes and seconds.

$0.381° = 0.381 \times 60' = 22.86'$

$0.86' = 0.86 \times 60'' = 52''$ to the nearest second

Hence, $58.381° = 58° \, 22' \, 52''$

This answer can be obtained using the calculator as follows:

1. Enter 58.381 2. Press = 3. Press ° ’ ’ ’ Answer = 58° 22′ 51.6″

EXERCISE 153 Page 414

1. State the general name given to an angle of 197°.

197° is an example of a reflex angle

2. State the general name given to an angle of 136°.

136° is an example of a obtuse angle

3. State the general name given to an angle of 49°.

49° is an example of a acute angle

4. State the general name given to an angle of 90°.

90° is a right angle

5. Determine the angles complementary to the following: (a) 69° (b) 27°37′ (c) 4l°3′43″

(a) The angle complementary 69° is: $90° - 69° = 21°$

(b) The angle complementary 27°37′is: $90° - 27° 37' = 62° 23'$

(c) The angle complementary 4l°3′43″ is: $90° - 41° 3' 43'' = 48° 56' 17''$

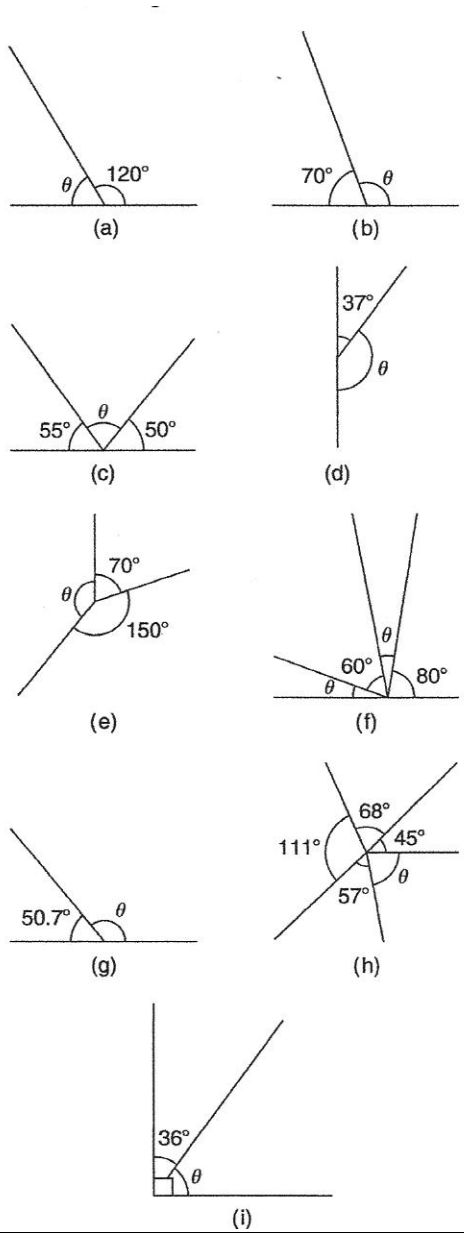
6. Determine the angles supplementary to (a) 78° (b) 15° (c) l69° 41′ 11″

(a) The angle supplementary to 78° is: $180° - 78° = 102°$

(b) The angle supplementary to 15° is: $180° - 15° = 165°$

(c) The angle supplementary to l69° 41′ 11″ is: $180° - 169° 41' 11'' = 10° 18' 49''$

7. Find the values of angle θ in diagrams (a) to (i).



(a) $\theta = 180° - 120° = 60°$

(b) $\theta = 180° - 70° = 110°$

(c) $\theta = 180° - 55° - 50° = 75°$

(d) $\theta = 180° - 37° = 143°$

(e) $\theta = 360° - 150° - 70° = 140°$

(f) $180° = 60° + 80° + 2\theta$ from which, $2\theta = 180° - 60° - 80° = 40°$

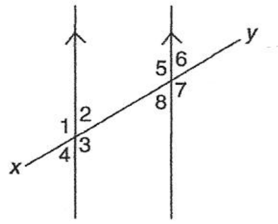
i.e. $\theta = 40°/2 = 20°$

(g) $\theta = 180° - 50.7° = 129.3°$

(h) $\theta = 360° - 45° - 68° - 111° - 57° = 79°$

(i) $\theta = 90° - 36° = 54°$

8. With reference to the diagram below, what is the name given to the line *XY*?



Give examples of each of the following:

(a) vertically opposite angles (b) supplementary angles

(c) corresponding angles (d) alternate angles

The name given to the line *XY* is transversal

(a) 1 & 3, 2 & 4, 5 & 7, 6 & 8

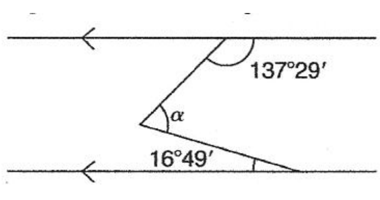
(b) 1 & 2, 2 & 3, 3 & 4, 4 & 1, 5 & 6,

6 & 7, 7 & 8, 8 & 5, 3 & 8, 1 & 6, 4 & 7 or 2 & 5

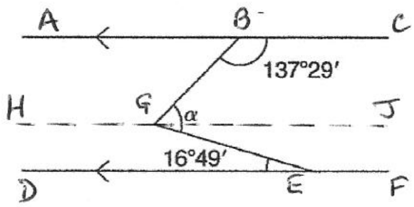
(c) 1 & 5, 2 & 6, 4 & 8, 3 & 7

(d) 3 & 5 or 2 & 8

9. In the diagram, find angle α



Let a straight line *HJ* be drawn through *G* such that *HJ* is parallel to *AC* and *DF*



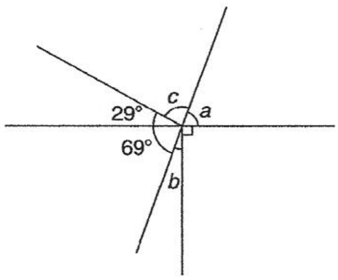
$\angle CBG = \angle HGB$ (alternate angles between parallel lines *AC* and *HJ*) hence $\angle BGJ = 180° - 137° 29' = 42° 31'$

$\angle DEG = \angle JGE$ (alternate angles between parallel lines HJ and DF)

hence $\angle JGE = 16° 49'$

Angle $\alpha = \angle BGJ + \angle JGE = 42° 31' + 16° 49' = 59° 20'$

10. In the diagram, find angles a, b and c.

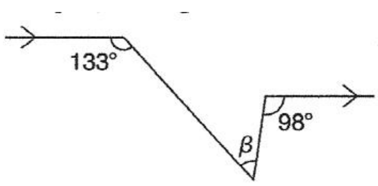


Angle $a = 69°$ (vertically opposite angles)

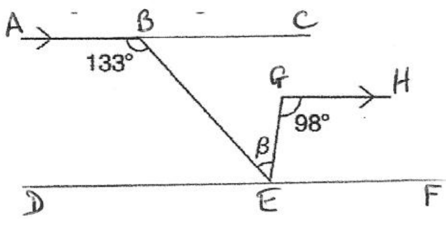
Angle $b = 90° - 69° = 21°$ (angles in a right angle add up to 90°)

Angle $c = 180° - 29° - 69° = 82°$ (angles on a straight line add up to 180°)

11. Find angle β in the diagram.



In the diagram below, DEF is parallel to GH and ABC, and $\angle CBE = 180° - 133° = 47°$



$\angle DEB = 47°$ (alternate angles between parallel lines)

If $\angle HGE = 98°$ then $\angle FEG = 180° - 98° = 82°$

Hence, angle $\beta = 180° - \angle DEB - \angle FEG = 180° - 47° - 82° = 51°$

12. Convert 76° to radians, correct to 3 decimal places.

Since 180° = π rad then 1° = $\dfrac{\pi}{180}$ rad

Hence, 76° = $76\times\dfrac{\pi}{180}$ rad = 1.326 rad

13. Convert 34° 40′ to radians, correct to 3 decimal places.

34° 40′ = $34\dfrac{40}{60}^{o}$ = 34.666666….°

Hence, 34° 40′ = 34.666666…° = 34.666666…× $\dfrac{\pi}{180}$ rad

= 0.605 rad

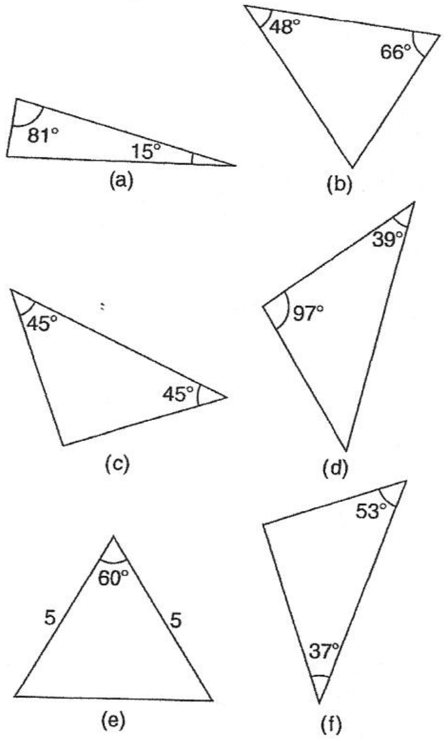
14. Convert 0.714 rad to degrees and minutes.

Since 180° = π rad then 1 rad = $\dfrac{180^{o}}{\pi}$

Hence, 0.714 rad = 0.714 × $\dfrac{180^{o}}{\pi}$ = 40.90918…° = 40° 55′ correct to the nearest minute

EXERCISE 154 Page 418

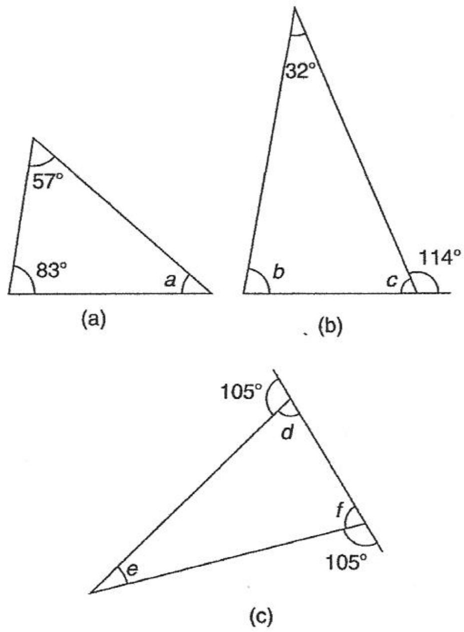
1. Name the types of triangle shown in diagrams (a) to (f).



(a) acute-angled scalene triangle (b) isosceles triangle (c) right-angled triangle

(d) obtuse-angled scalene triangle (e) equilateral triangle (f) right-angled triangle

2. Find the angles *a* to *f* in the triangles below.



In Figure (a), angle $a = 180° – 83° – 57° = 40°$

In Figure (b), angle $c = 180° – 114° = 66°$

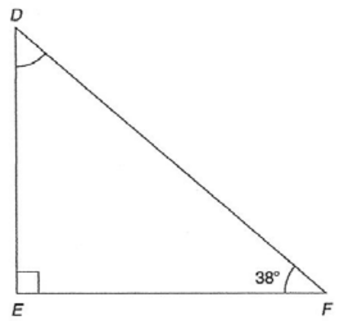
and angle $b = 180° – 32° – 66° = 82°$

In Figure (c), angle $d = 180° – 105° = 75°$

angle $f = 180° – 105° = 75°$

and angle $e = 180° – 75° – 75° = 30°$

3. In the triangle *DEF* which side is the hypotenuse? With reference to angle *D*, which side is the adjacent?



The hypotenuse is the longest side in a right-angled triangle and is the side opposite the right angle.

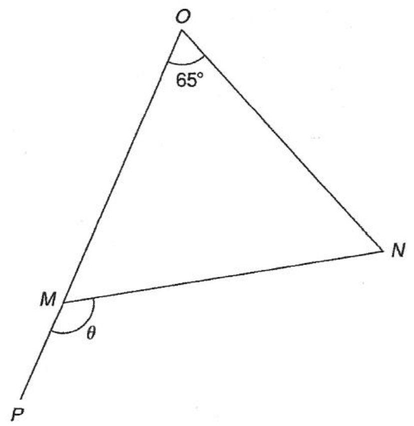
Hence, the hypotenuse is side *DF*

The side opposite angle *D* is *EF*; thus the adjacent side to angle *D* is the side *DE*

4. In triangle *DEF* of Problem 3, determine angle *D*.

Angle $D = 180° – 90° – 38° = 52°$

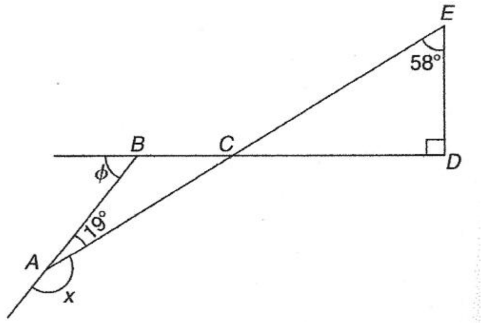
5. *MNO* is an isosceles triangle in which the unequal angle is 65° as shown. Calculate angle θ.



If triangle MNO is isosceles, then $\angle OMN = \angle MNO = \dfrac{180° - 65°}{2} = 57.5°$

Hence, angle θ = 180° – 57.5° = 122.5°

6. Determine $\angle\phi\phi$ and $\angle x$ in the diagram below.



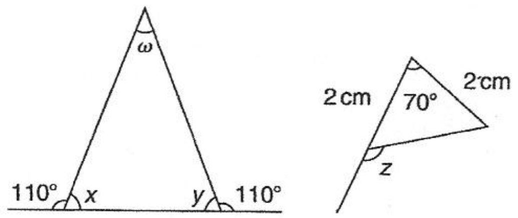
$\angle DCE$ = 90° – 58° = 32°

Hence, $\angle ACB$ = 32° Then $\angle ABC$ = 180° – 19° – 32° = 129°

Thus, $\angle\phi$ = 180° – 129° = 51°

and $\angle x$ = 180° – 19° = 161°

7. In diagrams (a) and (b), find angles *w*, *x*, *y* and *z*. What is the name given to the types of triangle shown in (a) and (b)?



(a) (b)

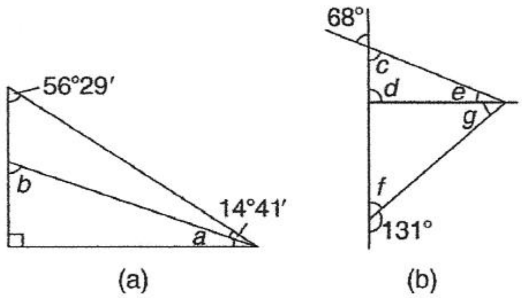
In Figure (a), $\angle x = \angle y$ = 180° – 110° = 70°

and $\angle w$ = 180° – 70° – 70° = 40°

In Figure (b), the triangle is isosceles, with the two equal angles equal to $\dfrac{180° - 70°}{2}$ = 55°

Hence, $\angle z$ = 180° – 55° = 125°

8. Find the values of angles *a* to *g* in diagrams (a) and (b).



In diagram (a), $\angle a$ + 14° 41´ = 180° – 90° – 56° 29´ = 33° 31´

from which, $\angle a$ = 33°31´ – 14° 41´ = 18° 50´

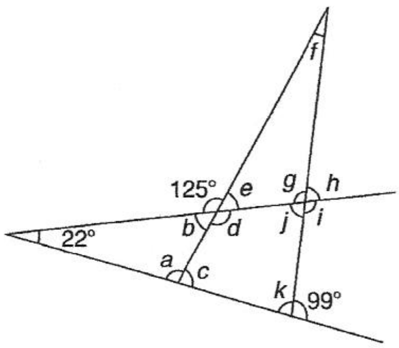
Also, $\angle b$ = 180° – 90° – 18° 50´ = 71° 10´

In diagram (b), $\angle c$ = 68° $\angle d$ = 90° and $\angle e$ = 180° – 90° – 68° = 22°

Also, $\angle f$ = 180° – 131° = 49°

and $\angle g$ = 180° – 90° – 49° = 41°

9. Find the unknown angles *a* to *k*.



$\angle d$ = 125° and $\angle b = \angle e$ = 180° – 125° = 55°

$\angle a$ = 180° – 55° – 22° = 103°

$\angle c$ = 180° – 103° = 77°

$\angle k$ = 180° – 99° = 81°

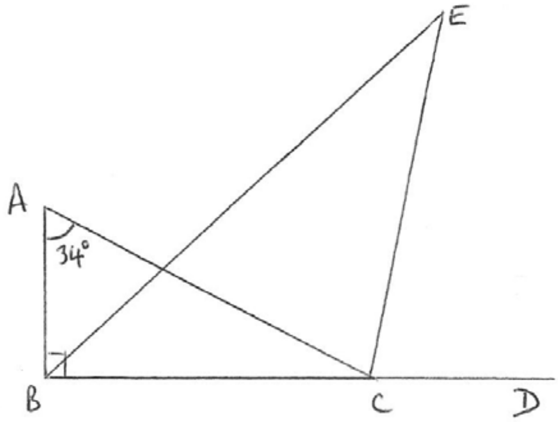
$\angle f$ = 180° – 77° – 81° = 22°

$\angle g$ = 180° – 22° – 55° = 103° = $\angle i$

$\angle j = \angle h$ = 180° – 103° = 77°

10. Triangle ABC has a right-angle at B and $\angle BAC$ is 34°. BC is produced to D. If the bisectors of $\angle ABC$ and $\angle ACD$ meet at E, determine $\angle BEC$.

The diagram is shown sketched below.



$\angle EBC = 90° \div 2 = 45°$

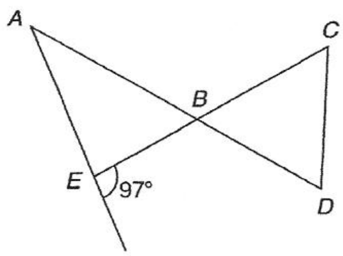
$\angle ACB = 180° - 90° - 34° = 56°$

$\angle ACD = 180° - 56° = 124°$ hence, $\angle ACE = 124° \div 2 = 62°$

Thus, $\angle BCE = \angle ACB + \angle ACE = 56° + 62° = 118°$

Hence, $\angle BEC = 180° - 45° - 118° = 17°$

11. If in the diagram, triangle BCD is equilateral, find the interior angles of triangle ABE.



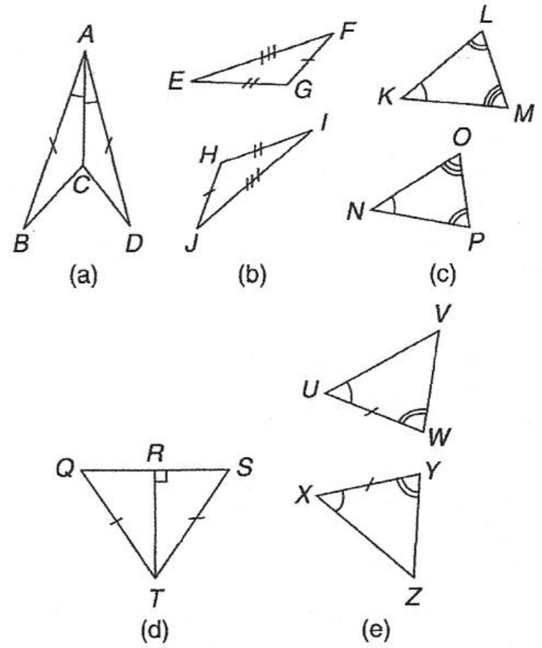
$\angle CBD = \angle ABE = 60°$ since triangle BCD is equilateral

$\angle AEB = 180° - 97° = 83°$

Hence, $\angle BAE = 180° - 83° - 60° = 37°$

EXERCISE 155 Page 420

1. State which of the pairs of triangles below are congruent and name their sequence.



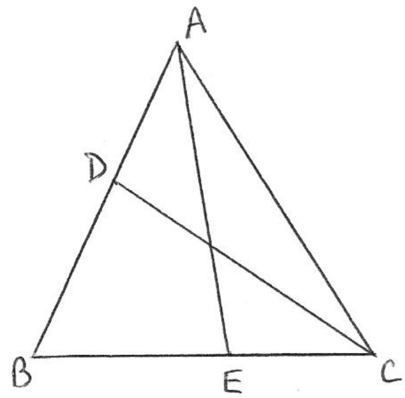
(a) Congruent *BAC*, *DAC* (SAS) (b) Congruent *FGE*, *JHI* (SSS)

(c) Not necessarily congruent (d) Congruent *QRT*, *SRT* (RHS)

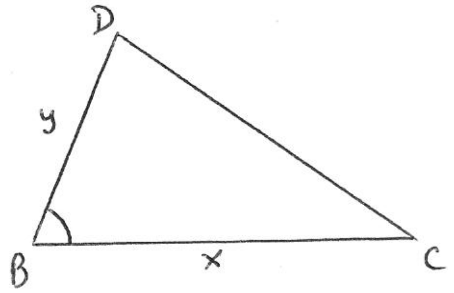
(e) Congruent *UVW*, *XZY* (ASA)

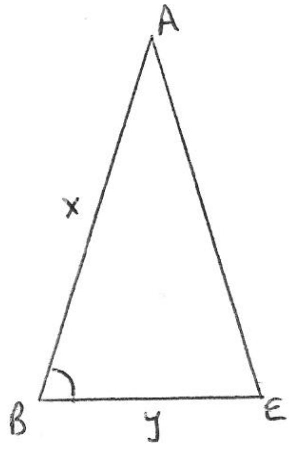
2. In a triangle *ABC*, *AB* = *BC* and *D* and *E* are points on *AB* and *BC*, respectively, such that *AD* = *CE*. Show that triangles *AEB* and *CDB* are congruent.

In the sketch shown below, *AB* = *BC* and *AD* = *CE*



Triangles AEB and CDB are sketched separately as shown below where $BC = AB = x$ and since $AD = EC$ then BD must be equal to BE, and these are labelled as y



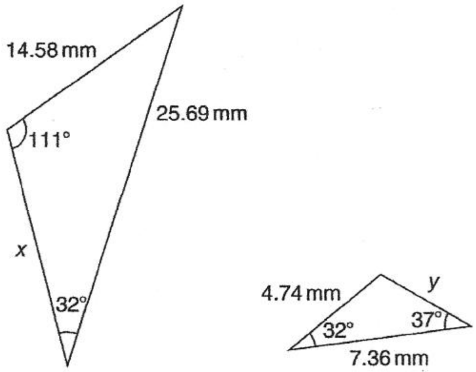


Since angle B in each triangle is the same then DC must equal AE, $\angle D = \angle E$ and $\angle A = \angle C$

Hence, triangles AEB and CDB are congruent

EXERCISE 156 Page 422

1. In the triangles below, find the lengths *x* and *y*.



The third angle in the first triangle = 180° – 111° – 32° = 37°

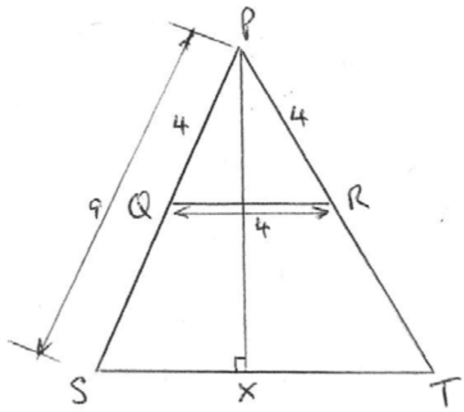
Hence the two triangles are similar since all three angles are the same in each.

Thus, $\frac{25.69}{7.36} = \frac{x}{4.74}$ from which, $x = \frac{25.69 \times 4.74}{7.36}$ = 16.54 mm

Thus, $\frac{25.69}{7.36} = \frac{14.58}{y}$ from which, $y = \frac{14.58 \times 7.36}{25.69}$ = 4.18 mm

2. *PQR* is an equilateral triangle of side 4 cm. When *PQ* and *PR* are produced to *S* and *T*, respectively, *ST* is found to be parallel with *QR*. If *PS* is 9 cm, find the length of *ST*. *X* is a point on *ST* between *S* and *T* such that the line *PX* is the bisector of ∠*SPT*. Find the length of *PX*.

A sketch is shown below where triangle *PQR* is similar to triangle *PST*



Hence, $\frac{PQ}{PS} = \frac{QR}{ST}$ i.e. $\frac{4}{9} = \frac{4}{ST}$ from which, $ST = \frac{4 \times 9}{4}$ = 9 cm

If angle *SPT* is bisected and *QR* and *ST* are parallel, then ∠*PXS* = 90°

i.e. triangle PSX is right-angled, where $SX = 9/2 = 4.5$ cm

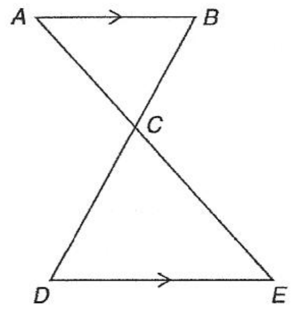
Using Pythagoras, $(PS)^2 = (PX)^2 + (SX)^2$

i.e. $9^2 = (PX)^2 + (4.5)^2$

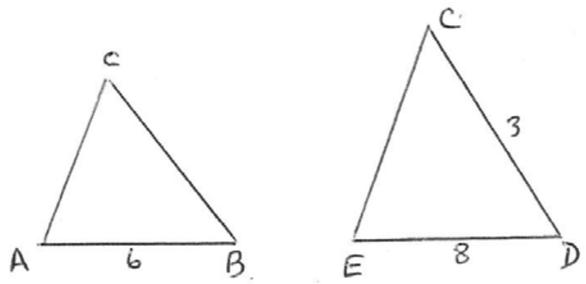
i.e. $PX = \sqrt{9^2 - 4.5^2} = 7.79$ cm

3. In the diagram, find (a) the length of BC when AB = 6 cm, DE = 8 cm and DC = 3 cm,

(b) the length of DE when EC = 2 cm, AC = 5 cm and AB = 10 cm

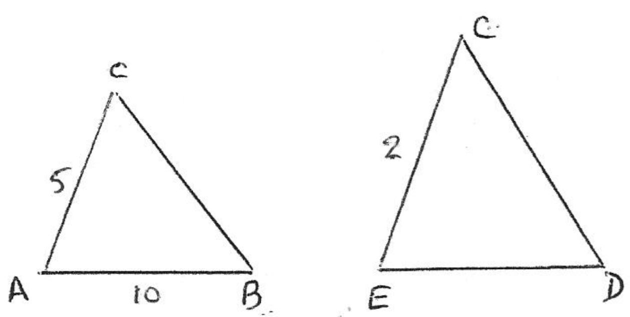


(a) Triangle ABC is similar to triangle EDC. The two triangles are shown below side by side (not to scale)



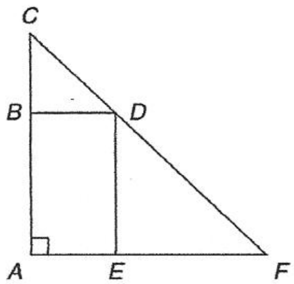
Thus, $\dfrac{AB}{ED} = \dfrac{BC}{DC}$ i.e. $\dfrac{6}{8} = \dfrac{BC}{3}$ from which, $BC = \dfrac{3 \times 6}{8} = 2.25$ cm

(b) The two triangles are again shown below side by side (not to scale)



Thus, $\dfrac{AB}{ED} = \dfrac{AC}{EC}$ i.e. $\dfrac{10}{ED} = \dfrac{5}{2}$ from which, $DE = \dfrac{2\times 10}{5}$ = 4 cm

4. In the diagram, *AF* = 8 m, *AB* = 5 m and *BC* = 3 m. Find the length of *BD*.



Triangles *CBD* and *CAF* are similar.

Thus, $\dfrac{CB}{CA} = \dfrac{BD}{AF}$ i.e. $\dfrac{3}{5+3} = \dfrac{BD}{8}$ from which, $BD = \dfrac{3\times 8}{8}$ = 3 m

EXERCISE 157 Page 424

1. Construct the triangle *ABC* given: a = 8 cm, b = 6 cm and c = 5 cm.

See Worked Problem 30 on page 423 for a similar method of construction

2. Construct the triangle *ABC* given: a = 40 mm, b = 60 mm and C = 60°.

See Worked Problem 31 on page 423 for a similar method of construction

3. Construct the triangle *ABC* given: a = 6 cm, C = 45° and B = 75°.

See Worked Problem 32 on page 424 for a similar method of construction

4. Construct the triangle *ABC* given: c = 4 cm, A = 130° and C = 15°.

See Worked Problem 32 on page 424 for a similar method of construction

5. Construct the triangle *ABC* given: a = 90 mm, B = 90°, hypotenuse = 105 mm.

See Worked Problem 33 on page 424 for a similar method of construction