



Expanding brackets with surds

Getting ready for A-Level Maths...

*"The most important investment you can make is in **yourself**."*

Expanding brackets with surds

Important rules

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Be careful...

$$\sqrt{a} + \sqrt{b} \neq \sqrt{a + b}$$

$$\sqrt{a} - \sqrt{b} \neq \sqrt{a - b}$$

Learn by heart...

$$1^2 = 1 \times 1 = \mathbf{1}$$

$$2^2 = 2 \times 2 = \mathbf{4}$$

$$3^2 = 3 \times 3 = \mathbf{9}$$

$$4^2 = 4 \times 4 = \mathbf{16}$$

$$5^2 = 5 \times 5 = \mathbf{25}$$

$$6^2 = 6 \times 6 = \mathbf{36}$$

$$7^2 = 7 \times 7 = \mathbf{49}$$

$$8^2 = 8 \times 8 = \mathbf{64}$$

$$9^2 = 9 \times 9 = \mathbf{81}$$

$$10^2 = 10 \times 10 = \mathbf{100}$$

$$11^2 = 11 \times 11 = \mathbf{121}$$

$$12^2 = 12 \times 12 = \mathbf{144}$$

$$13^2 = 13 \times 13 = \mathbf{169}$$

$$14^2 = 14 \times 14 = \mathbf{196}$$

$$15^2 = 15 \times 15 = \mathbf{225}$$

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Question

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

What's the answer

$$\sqrt{2} \times \sqrt{2} =$$

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Question

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

What's the answer

$$\sqrt{7} \times \sqrt{7} =$$

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Question

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

What's the answer

$$\sqrt{919} \times \sqrt{919} =$$

Expanding brackets with surds

Expanding brackets

Expand and simplify

$$2(x + 4)$$

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Expanding brackets

Expand and simplify

$$(x + 2)(x + 4)$$

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My turn

Evaluate without a calculator

$$\sqrt{2}(\sqrt{8} + \sqrt{50})$$

$$\begin{aligned}\sqrt{2}(\sqrt{8} + \sqrt{50}) &= \sqrt{2}\sqrt{8} + \sqrt{2}\sqrt{50} \\ &= \sqrt{2 \times 8} + \sqrt{2 \times 50} \\ &= \sqrt{16} + \sqrt{100} \\ &= 4 + 10 \\ &= \underline{\underline{14}}\end{aligned}$$

Your turn

Evaluate without a calculator

$$\sqrt{3}(\sqrt{12} + \sqrt{27})$$

$$\begin{aligned}\sqrt{3}(\sqrt{12} + \sqrt{27}) &= \sqrt{3}\sqrt{12} + \sqrt{3}\sqrt{27} \\ &= \sqrt{3 \times 12} + \sqrt{3 \times 27} \quad \omega 1 \\ &= \sqrt{36} + \sqrt{81} \\ &= 6 + 9 \\ &= \underline{\underline{15}}\end{aligned}$$

$$\begin{array}{r} \omega 1 \quad 27 \\ \times 3 \\ \hline 21 \quad 3 \times 7 \\ 60 \quad 3 \times 20 \\ \hline 81 \end{array}$$

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My turn

Express in the form $a + b\sqrt{3}$

$$(4 + \sqrt{3})(1 + 2\sqrt{3})$$

State the values of a and b .

$$\begin{aligned}(4 + \sqrt{3})(1 + 2\sqrt{3}) &= 4(1 + 2\sqrt{3}) + \sqrt{3}(1 + 2\sqrt{3}) \\ &= 4 + 8\sqrt{3} + \sqrt{3} + 2\sqrt{3}\sqrt{3} \\ &= 4 + 9\sqrt{3} + 6 \\ &= \underline{\underline{10 + 9\sqrt{3}}}\end{aligned}$$

$$\begin{aligned}a &= 10 \\ b &= 9\end{aligned}$$

Your turn

Express in the form $a + b\sqrt{5}$

$$(2 + \sqrt{5})(3 + 4\sqrt{5})$$

State the values of a and b .

$$\begin{aligned}(2 + \sqrt{5})(3 + 4\sqrt{5}) &= 2(3 + 4\sqrt{5}) + \sqrt{5}(3 + 4\sqrt{5}) \\ &= 6 + 8\sqrt{5} + 3\sqrt{5} + 4\sqrt{5}\sqrt{5} \\ &= 6 + 11\sqrt{5} + 20 \\ &= \underline{\underline{26 + 11\sqrt{5}}}\end{aligned}$$

$$\begin{aligned}a &= 26 \\ b &= 11\end{aligned}$$

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My turn

Express in the form $a + b\sqrt{3}$

$$(5 + 3\sqrt{3})^2$$

State the values of a and b .

$$\begin{aligned}(5+3\sqrt{3})(5+3\sqrt{3}) &= 5(5+3\sqrt{3}) + 3\sqrt{3}(5+3\sqrt{3}) \\ &= 25+15\sqrt{3} + 15\sqrt{3} + 3\sqrt{3} \times 3\sqrt{3} \\ &= 25 + 30\sqrt{3} + 27 \\ &= \underline{\underline{52 + 30\sqrt{3}}}\end{aligned}$$

$$a = 52$$

$$b = 30$$

Your turn

Express in the form $a + b\sqrt{5}$

$$(7 + 2\sqrt{5})^2$$

State the values of a and b .

$$\begin{aligned}(7+2\sqrt{5})(7+2\sqrt{5}) &= 7(7+2\sqrt{5}) + 2\sqrt{5}(7+2\sqrt{5}) \\ &= 49 + 14\sqrt{5} + 14\sqrt{5} + 2\sqrt{5} \times 2\sqrt{5} \\ &= 49 + 28\sqrt{5} + 20 \\ &= \underline{\underline{69 + 28\sqrt{5}}}\end{aligned}$$

$$a = 69$$

$$b = 28$$

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My turn

Express in the form $a + b\sqrt{3}$

$$(2\sqrt{3} - 1)^2$$

State the values of a and b .

$$(2\sqrt{3} - 1)(2\sqrt{3} - 1) = 2\sqrt{3}(2\sqrt{3} - 1) - 1(2\sqrt{3} - 1)$$

$$= 2\sqrt{3} \times 2\sqrt{3} - 2\sqrt{3} - 2\sqrt{3} + 1$$

$$= 12 - 4\sqrt{3} + 1$$

$$= \underline{\underline{13 - 4\sqrt{3}}}$$

$$\begin{aligned} a &= 13 \\ b &= -4 \end{aligned}$$

Your turn

Express in the form $a + b\sqrt{5}$

$$(3\sqrt{5} - 2)^2$$

State the values of a and b .

$$(3\sqrt{5} - 2)(3\sqrt{5} - 2) = 3\sqrt{5}(3\sqrt{5} - 2) - 2(3\sqrt{5} - 2)$$

$$= 3\sqrt{5} \times 3\sqrt{5} - 6\sqrt{5} - 6\sqrt{5} + 4$$

$$= 45 - 12\sqrt{5} + 4$$

$$= \underline{\underline{49 - 12\sqrt{5}}}$$

$$\begin{aligned} a &= 49 \\ b &= -12 \end{aligned}$$

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My turn

Simplify

$$(\sqrt{7} - 1)(\sqrt{7} + 1)$$

$$\begin{aligned}(\sqrt{7} - 1)(\sqrt{7} + 1) &= \sqrt{7}(\sqrt{7} + 1) - 1(\sqrt{7} + 1) \\ &= (\sqrt{7})^2 + \sqrt{7} - \sqrt{7} - 1^2 \\ &= (\sqrt{7})^2 - (1)^2 \\ &= 7 - 1 \\ &= \underline{\underline{6}}\end{aligned}$$

Your turn

Simplify

$$(\sqrt{11} - 2)(\sqrt{11} + 2)$$

$$\begin{aligned}(\sqrt{11} - 2)(\sqrt{11} + 2) &= \sqrt{11}(\sqrt{11} + 2) - 2(\sqrt{11} + 2) \\ &= (\sqrt{11})^2 + 2\sqrt{11} - 2\sqrt{11} - 2^2 \\ &= (\sqrt{11})^2 - 2^2 \\ &= 11 - 4 \\ &= \underline{\underline{7}}\end{aligned}$$

Expanding brackets with surds

Difference of two squares

$$(a + b)(a - b) = a^2 - b^2$$

$$(a - b)(a + b) = a^2 - b^2$$

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My turn

Simplify

$$(a + b)(a - b)$$
$$(2 + \sqrt{5})(2 - \sqrt{5})$$

$$(2 + \sqrt{5})(2 - \sqrt{5}) = 2^2 - (\sqrt{5})^2$$
$$= 4 - 5$$
$$= \underline{\underline{-1}}$$

Your turn

Simplify

$$(a + b)(a - b)$$
$$(8 + \sqrt{7})(8 - \sqrt{7})$$

$$(8 + \sqrt{7})(8 - \sqrt{7}) = 8^2 - (\sqrt{7})^2$$
$$= 64 - 7$$
$$= \underline{\underline{57}}$$

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My turn

Simplify $(a - b)(a + b)$
 $(6 - 4\sqrt{2})(6 + 4\sqrt{2})$

$$\begin{aligned}(6 - 4\sqrt{2})(6 + 4\sqrt{2}) &= 6^2 - (4\sqrt{2})^2 \quad \text{w1} \\ &= 36 - 32 \\ &= \underline{\underline{4}}\end{aligned}$$

$$\begin{aligned}\text{w1: } (4\sqrt{2})^2 &= 4\sqrt{2} \times 4\sqrt{2} \\ &= 16 \times 2 \\ &= 32\end{aligned}$$

Your turn

Simplify $(a - b)(a + b)$
 $(10 - 3\sqrt{5})(10 + 3\sqrt{5})$

$$\begin{aligned}(10 - 3\sqrt{5})(10 + 3\sqrt{5}) &= 10^2 - (3\sqrt{5})^2 \quad \text{w1:} \\ &= 100 - 45 \\ &= \underline{\underline{55}}\end{aligned}$$

$$\begin{aligned}\text{w1: } (3\sqrt{5})^2 &= 3\sqrt{5} \times 3\sqrt{5} \\ &= 9 \times 5 \\ &= 45\end{aligned}$$

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Review Exercise

1. Evaluate without a calculator.

$$\sqrt{5}(\sqrt{20} + \sqrt{45})$$

2. Express in the form $a + b\sqrt{2}$.

$$(5 + \sqrt{2})(6 + 3\sqrt{2})$$

State the values of a and b .

3. Express in the form $a + b\sqrt{6}$.

$$(4 + 2\sqrt{6})^2$$

State the values of a and b .

4. Express in the form $a + b\sqrt{7}$.

$$(2\sqrt{7} - 3)^2$$

State the values of a and b .

5. Simplify $(\sqrt{13} - 2)(\sqrt{13} + 2)$

6. Simplify $(7 + \sqrt{6})(7 - \sqrt{6})$

7. Simplify $(9 - 5\sqrt{3})(9 + 5\sqrt{3})$

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Review Exercise (Answers)

- Evaluate without a calculator. $\sqrt{5}(\sqrt{20} + \sqrt{45})$ **25**
- Express in the form $a + b\sqrt{2}$. $(5 + \sqrt{2})(6 + 3\sqrt{2})$ **$36 + 21\sqrt{2}$**
State the values of a and b . **$a = 36$**
 $b = 21$
- Express in the form $a + b\sqrt{6}$. $(4 + 2\sqrt{6})^2$ **$40 + 16\sqrt{6}$**
State the values of a and b . **$a = 40$**
 $b = 16$
- Express in the form $a + b\sqrt{7}$. $(2\sqrt{7} - 3)^2$ **$37 - 12\sqrt{7}$**
State the values of a and b . **$a = 37$**
 $b = -12$
- Simplify $(\sqrt{13} - 2)(\sqrt{13} + 2)$ **9**
- Simplify $(7 + \sqrt{6})(7 - \sqrt{6})$ **43**
- Simplify $(9 - 5\sqrt{3})(9 + 5\sqrt{3})$ **6**