
Periodic resonator-based optimization of an acoustic package made of glass wool

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Nowadays, modern urbanization and traffic increase could cause severe noise-induced health damages, such as annoyance, sleep disturbance, or even ischemic heart disease, and thus the interest on environment noise control is quickly growing.

Unfortunately, a sound absorbing material that well performs in overall the frequency range is not present in nature. For instance, acoustic solutions such as Helmholtz resonators have good performances at low frequencies, while porous media, whose foam cavities dissipate the energy by viscous and thermal losses, perform better at high frequencies.

The scope of this work is to investigate the sound transmission loss of an acoustic package made of glass wool, as well as to propose innovative solutions based on the inclusion of a periodic pattern of Helmholtz resonators inside its core, aiming at passively improving the acoustic performance in a chosen range of frequencies. In order to reach this goal, a numerical model is studied, and the effect of Helmholtz resonators tuned at several frequencies are simulated. For each of these layouts, also the mass increase of the so-called meta-core, compared to that of its classical homogeneous counterpart, is estimated. Results obtained herein may constitute a good basis in order to perform preliminary design considerations that could be interesting for further generalizations.

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1. Introduction

Acoustic meta-materials offer great flexibility for manipulating sound waves and promise unprecedented functionality, ranging from transformation acoustics, super-resolution imaging to acoustic cloaking. However, the design of acoustic meta-materials with exciting functionality remains challenging with traditional approaches using classic acoustic elements such as Helmholtz resonators (HR). Helmholtz resonators are used in a variety of applications to reduce the transmission of unwanted sound.

Alster [2] derived the classical formula for calculation of resonant frequencies of Helmholtz resonators, under the assumptions that all mass significant for oscillation of a resonator is concentrated in the neck of the resonator and that the spring constant is given by the volume of the resonator. Tang *et al.* [13] developed the theory of a generalized Helmholtz resonator, based on the jet-flow model which is manifested in the non-linearity of the orifice flow upon the passage of a high intensity wave. Fahy *et al.* [6] coupled a single Helmholtz resonator to an enclosure and tuned it to the natural frequency of one of its low order acoustic modes, also analyzing the effect on the free, and forced, vibrations of the fluid in the enclosure. Chanaud [3] developed an equation for the resonance frequency

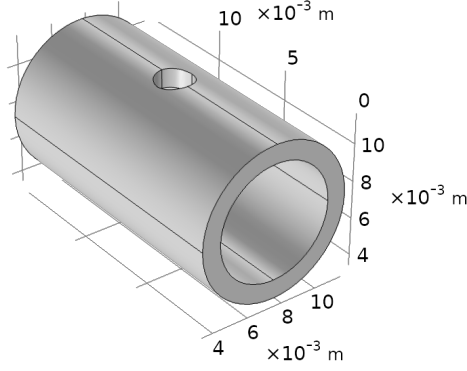


Figure 1: Geometry of Helmholtz resonator tuned at 3000 [Hz].

of a Helmholtz resonator from the wave equation for the case of a cavity volume that has the shape of a rectangular parallelepiped and orifices of different geometries. de Bedout *et al.* [5] presented a tunable Helmholtz resonator and a feedback-based control law that achieves optimal resonator tuning for time-varying tonal noise control applications. Lei *et al.* [8] described a strategy to characterize power and ground-plane structures using a full cavity-mode frequency-domain resonator model. Griffin *et al.* [7] demonstrated that mechanically coupled resonators can be used to design a particular transmission loss response, provide a wider bandwidth of attenuation, and adapt the transmission loss characteristics of a structure to attenuate disturbances of varying frequency. Tang [14] experimentally and theoretically investigated the acoustical properties of Helmholtz resonators with necks having cross-section dimensions decreasing away from the entry of the resonator cavities. Park [12] introduced micro-perforated panel absorbers backed by Helmholtz resonators in order to improve sound absorption in the low-frequency region, where conventional micro-perforated panel absorbers cannot provide sufficient absorption. Cheng *et al.* [4] demonstrated an ultraslow-fluid-like particle with intense artificial resonances for low-frequency airborne sound.

In the present work, a new concept of acoustic package is numerically studied and investigated, with the target of taking advantage from the exploitation of the combined effects of periodicity-induced and Helmholtz resonances; this may result in a meaningful increase of acoustic performance in two desired frequency ranges at the same time without any increase in terms of package thickness, thus allowing to pursue the target of resource efficiency.

In Section 2, the properties of the studied acoustic package are introduced, together with the 3-dimensional finite element (FE) geometries that are investigated in the following sections. Then, in Section 3, some possible meta-core solutions based on a glass wool layer are proposed and discussed. Furthermore, a numerical test campaign is performed for five setups, each with Helmholtz resonators tuned at different frequencies; for each of these configurations, also the mass increase of the meta-core, compared to that of its classical homogeneous counterpart, is estimated. In this context, the obtained results may constitute a good basis in order to perform preliminary design considerations that could be interesting for further generalizations. In conclusions, in Section 4, the results of this investigation are commented, and some possible future expansions of the present research are evaluated.

2. Definition of the problem

In this section, an innovative configuration based on an acoustic package made of commonly-used glass wool with embedded periodic Helmholtz resonators made of steel is numerically modeled and studied. Helmholtz resonators are considered in the shape of a hollow cylinder, with a hole acting as the neck, as shown in Figure 1.

Color	Yellow
Material	Glass wool
Porosity	0.99
Tortuosity	1
Airflow resistivity [Pa*s/m²]	9000
Viscous characteristic length [mm]	0.192
Thermal characteristic length [mm]	0.384

Table 1: Properties of JCA-modeled glass wool.

Color	Green
Material	Steel
ρ [kg/m ³]	7800
E [Pa]	210E+09
ν	0.30

Table 2: Properties of solid layers.

2.1 Geometrical and material properties

The physical properties of the studied acoustic package are reported in Tables 1 and 2, together with a legend of colors. Properties of air (indicated with pink-colored layers) are: density $\rho_0 = 1.213 \left[\frac{\text{kg}}{\text{m}^3} \right]$, speed of sound $c_0 = 342 \left[\frac{\text{m}}{\text{s}} \right]$. The total thickness of the different studied packages $T = 75[\text{mm}]$ is kept constant.

The glass wool is modeled as an equivalent fluid through the Johnson-Champoux-Allard (JCA) approach; the FE implementation and analysis of this kind of materials has already been validated by the authors in their previous works [11], as well as for more complicated di-phasic behaviors through the use of Biot theory of poro-elasticity [9]. Unit cells with Helmholtz resonators tuned at different frequencies are shown in Figure 2.

2.2 Finite Element implementation

For what concerns the FE implementation, the module “Pressure Acoustics and Frequency Domain” of COMSOL MultiPhysics is used both as modeling environment and numerical solver. For all configurations presented in this work, the acoustic package consists of a repetition of five unit cells, meshed with tetrahedral elements generated through physics-controlled algorithms that are pre-implemented in the software. Since the analyses are carried out considering an excitation consisting of a normal incidence plane wave acting along z -axis, the only fundamental boundary condition to apply is the so-called Perfectly Matched Layer (PML) on the very bottom face of the models: indeed, this represents an artificial absorbing layer for wave equations, commonly used to truncate computational regions in numerical methods in order to simulate problems with open boundaries; this allows

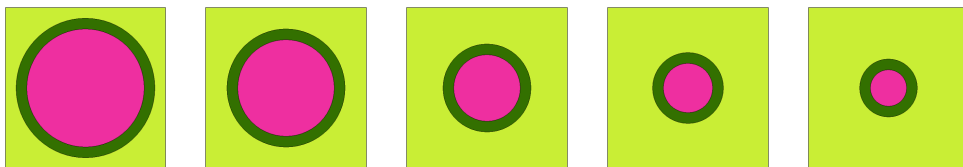


Figure 2: From left to right, meta-core unit cells with Helmholtz resonators tuned at 1000, 2000, 3000, 4000 and 5000 [Hz].

Configuration	HR tuning freq. [Hz]	HR resonance freq. [Hz]	Domain elem.	Boundary elem.	Edge elem.
1	1000	980	15950	7870	1272
2	2000	2010	16059	7084	1186
3	3000	2890	26816	8232	1296
4	4000	3850	29266	8400	1331
5	5000	5100	22587	7622	1261

Table 3: Number of FE mesh elements for each of the studied configurations.

the PML to strongly absorb outgoing waves from the interior of a computational region, without reflecting them back into the interior. Such property essentially simulates the interior of the aircraft cabin. On the contrary, boundary conditions applied on faces normal to x - and y - axes are not relevant for analyses with the above-mentioned kind of excitation, since under this condition the waves do not have propagating components along those directions; thus, for the sake of computational simplicity, a Sound Hard Boundary Wall (SHBW) boundary condition is applied herein. Eventually, for different angles of excitation, proper periodic conditions should be used instead.

In Table 3 the number of FE mesh elements for each of the studied configurations are reported; it should be pointed out that the difference between the values reported in “HR tuning frequency” and “HR resonance frequency” columns is that the first ones represent the ideal resonance frequency targets, while the second ones are the actual resonance frequencies that the authors were able to achieve. The sound transmission loss may be estimated through different approaches, such as the Transfer Matrix Method (TMM) [1], the Finite Element Method (FEM) [10], the Wave Finite Element Method (WFEM) [16] and the Statistical Energy Analysis (SEA) [15, 18], all having their own advantages and limitations [17]. In the context of this research, TMM and FEM are used. In particular, for a plane wave configuration at normal incidence $\theta = \phi = 0$, and thus oriented towards the negative direction of z -axis, the transmission loss is computed as reported in Eq. 1.

$$TL = 10\log_{10} \frac{\Pi_{incident}}{\Pi_{transmitted}}. \quad (1)$$

In the context of this work, the TLs related to the homogeneous (which has no inclusions) and the meta-core models are estimated through a FE implementation of Eq. 1. It should be highlighted that homogeneous results are compared with those obtained through TMM, as defined in Eq. 2 and Eq. 3.

$$TL = 10\log_{10} \left(\frac{1}{4} |T_{11} + \frac{T_{12}}{\rho_0 c_0} + \rho_0 c_0 T_{21} + T_{22}|^2 \right), \quad (2)$$

$$\text{with } \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \cos(kd) & j \sin(kd) Z_c \\ \frac{j \sin(kd)}{Z_c} & \cos(kd) \end{bmatrix}. \quad (3)$$

3. Analyses and results

The well-known analytical formula for predicting the resonance frequency f_{HR} of a Helmholtz resonator is:

$$f_{HR} = \frac{c_0}{2\pi} \sqrt{\frac{S_n}{V_c (l_n + d_n \cdot CF)}}, \quad (4)$$

where S_n is the wetted surface of the neck, V_c is the cavity volume, l_n is length of the neck, d_n is the equivalent diameter of the neck and CF is a correction factor, which may need to be tuned according to the case of interest. In the context of the configurations analyzed herein, the aforementioned quantities are reported in Table 4.

Configuration	$l_n[\text{m}]$	$d_n[\text{m}]$	$S_n[\text{m}^2]$	$V_c[\text{m}^3]$	CF
1	0.001	0.001	7.85E-07	1.43E-06	0.71
2	0.001	0.002	3.14E-06	9.54E-07	0.71
3	0.001	0.002	3.14E-06	4.53E-07	0.74
4	0.001	0.002	3.14E-06	2.49E-07	0.77
5	0.001	0.002	3.14E-06	1.36E-07	0.82

Table 4: Geometrical characteristics of Helmholtz resonators, with reference to Configurations 1-5 described in Table 3.

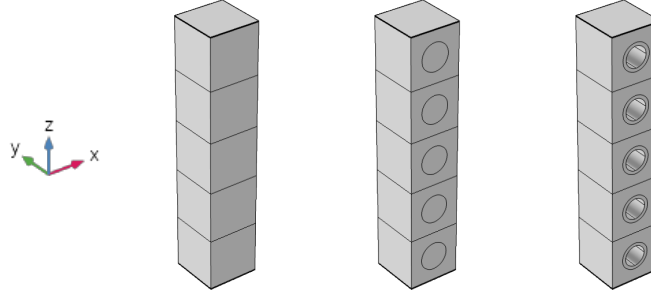


Figure 3: From left to right, acoustic packages made of glass wool: homogeneous, with periodic inclusions and with HRs (the latter being studied either filled by foam or air).

3.1 Investigation about several meta-core configurations

In this subsection, some possible meta-core solutions based on a glass wool layer are proposed and discussed. With reference to Figure 3, TL performance are estimated for:

- A homogeneous layer of glass wool (both through TMM and FEM);
- A layer of glass wool with a periodic pattern of five steel plain inclusions;
- A layer of glass wool with a periodic pattern of five Helmholtz resonators (Configuration 3 of Table 3), with steel walls and filled by glass wool;
- A layer of glass wool with a periodic pattern of five Helmholtz resonators (Configuration 3 of Table 3), with steel walls and filled of air.

All the aforementioned models have the same total thickness $T = 75[\text{mm}]$. Looking at Figure 4, it is evident how TL results related to the model having homogeneous layers show an almost perfect agreement with those analytically computed through the TMM; therefore, this comparison plays the role of an analytical validation of numerical results related to the multi-layered configuration without inclusions. Unfortunately, it is not conceptually possible to directly perform the same kind of validation also for the meta-configurations. As expected, through the adoption of a periodic pattern of five steel plain inclusions, the meta-core shows a performance peak related to periodicity effects, when half of the wavelength λ is equal to periodicity dimension (i.e. to the unit cell thickness along z -axis); it should be noted that, since this kind of acoustic resonance still relates on the thickness of the unit cell, it may be challenging to obtain performance peaks at low frequencies, when only a limited volume is available. Going further, the plain inclusions are replaced with specifically tuned Helmholtz resonators, thus awakening the correspondent resonance behavior: clearly, the related TL peak is significantly lower and slightly wider when the HR is filled by glass wool rather than air, due to the higher dissipation in the first system; anyway, solutions based on air-filled HRs may be more interesting with reference to the mass variation of the whole acoustic package (this aspect is thoroughly faced in Subsection 3.2).

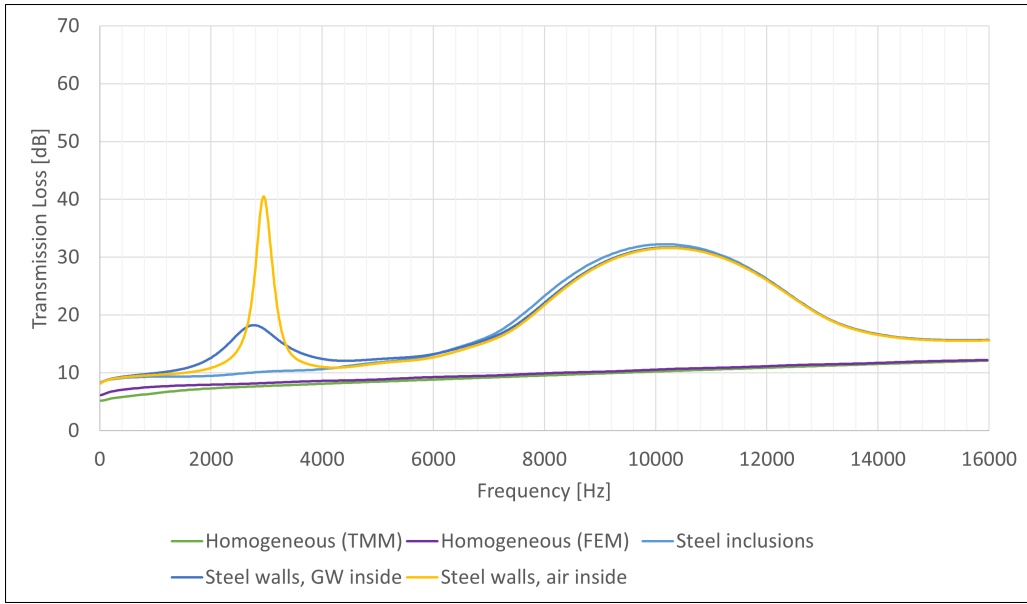


Figure 4: Transmission loss study performed on several meta-core solutions.

Configuration	Mass [kg]	Surface density $\left[\frac{\text{kg}}{\text{m}^2}\right]$	Mass increase [%]
1	2.21E-02	2.95E-01	8036
2	1.84E-02	2.46E-01	6721
3	1.33E-02	1.78E-01	4833
4	1.04E-02	1.39E-01	3730
5	8.21E-03	1.09E-01	2940

Table 5: Mass data of Configurations 1-5 described in Table 3, with air-filled HRs.

3.2 Numerical study on Helmholtz resonators tuned at different frequencies

In this subsection, a numerical test campaign is performed for five geometrical setups, each with Helmholtz resonators tuned at different frequencies; results are presented in Figure 5. In addition, considering that the homogeneous configuration has mass $=2.70\text{E-}04$ [kg] and surface density $=3.60\text{E-}03$ $\left[\frac{\text{kg}}{\text{m}^2}\right]$, also the mass increases of the meta-cores are estimated and reported in Table 5. Concentrating on Figure 5, first of all one may notice that, at increasing HR resonance frequencies, the periodicity peak does not shift in frequency, but decreases its amplitude: this is explained by the fact that, in order to work at higher frequencies, the HR needs to have a smaller cavity; clearly, this has a non-negligible effect also on the amplitude of the periodicity-induced resonance peak. Furthermore, it should be noted how the resonance peaks computed in the (purely academic) cases of Helmholtz resonator walls modeled as SHBW almost perfectly agree with those analytically predicted through Eq. 4; instead, in general, when using a real material (e.g.: steel) to model such walls, HR resonance frequencies present a slight shift in their precise value, compared to those analytically calculated. The authors empirically verified that such a shift meaningfully increases for decreasing HR mass, thus representing a non-trivial obstacle to overcome in order to find an acceptable compromise between accuracy in resonance designability and mass increase. The latter consideration is useful to justify the very high values in terms of mass increase reported in Table 5, which therefore classify this solution concept, in its present form, as mainly viable for civil applications.

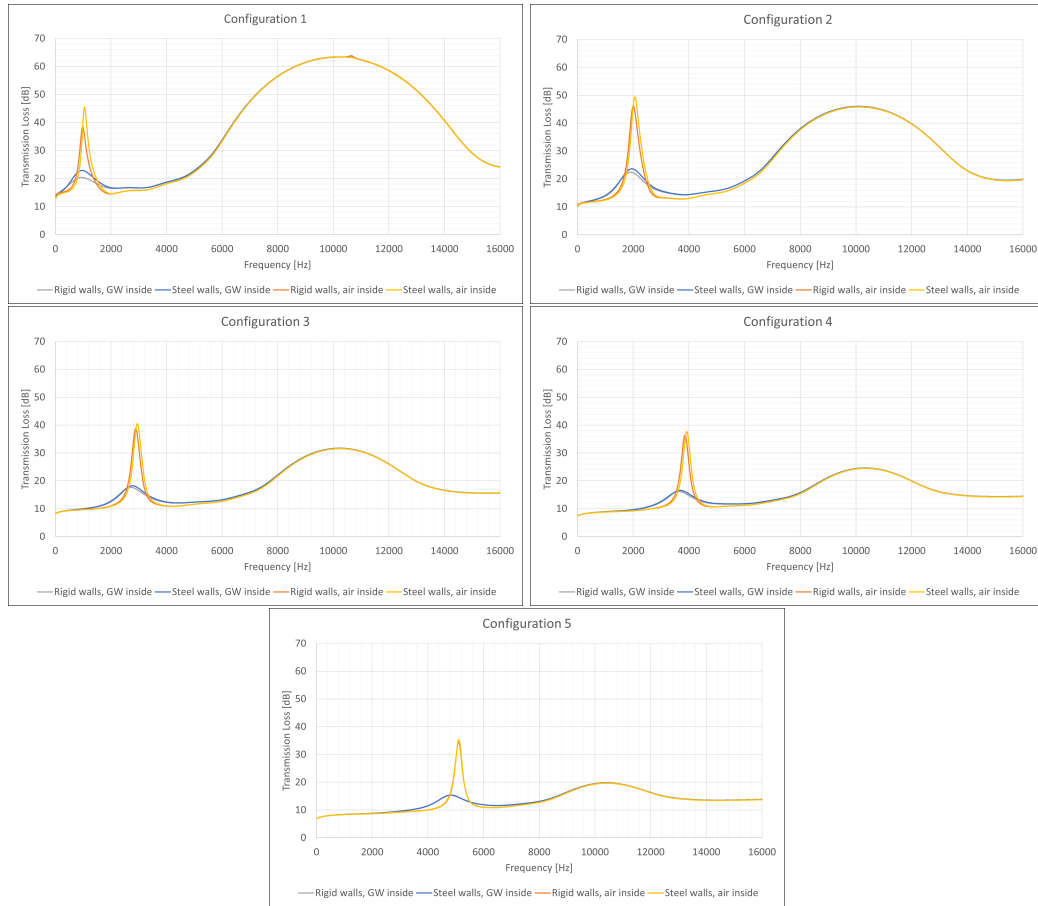


Figure 5: Transmission loss comparisons for Configurations 1-5 of Table 3.

4. Conclusions

In the present work, a new concept of acoustic package is numerically studied and investigated, with the target of taking advantage from the exploitation of the combined effects of periodicity-induced and Helmholtz resonances; this results in a meaningful increase of acoustic performance in two desired frequency ranges at the same time without any increase in terms of package thickness, thus allowing to pursue the target of resource efficiency. After having introduced the properties of the studied acoustic package, some possible meta-core solutions based on a glass wool layer are proposed and discussed. Then, a numerical test campaign is performed for five setups, each with Helmholtz resonators tuned at different frequencies; for each of these configurations, also the mass increase of the meta-core, compared to that of its classical homogeneous counterpart, is estimated. Future works could focus on the estimation of the correction factor of the analytical formulation of Helmholtz resonance frequency for different cavity geometries, and possibly for different cavity/neck ratios. Also a parametric analyses on different resonator patterns could represent an interesting extension of the concept presented herein. Most of all, it would be fundamental to investigate a way to further reduce the mass increase related to the use of meta-cores, thus making this solution viable also for applications in transportation engineering and increasing its overall resource efficiency.

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