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VIBRATIONS FOR ESTIMATING BOLTED JOINT INTEGRITY (VEBJI) PROJECT: CHALLENGES AND RESULTS

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Abstract. *Bolted joints are one of the oldest and widely-used means of connecting parts of technical installations. For mission-critical and expensive assets, periodic inspection of bolted joints is the conventional way of ensuring the assets' structural integrity, and hence their safe operation. Currently, the inspections require hours of physically demanding and monotonic work, as in practice it often means re-tightening the bolts. Thus, the asset maintenance units will benefit from a simpler and less demanding technique, which would allow checking the correct bolt tightening and notifying if a bolt is loose.*

For the last three years, a joint research team led by the Technical University of Denmark in collaboration with Brüel & Kjær and Karlsruhe Institute of Technology was conducting a project, where vibrations, artificially induced in a bolted joint, were utilized for estimation of the static axial force acting in the bolt. The presented paper provides an overview of the project, lists the numerous challenges the project had to deal with and describes what was learned and achieved.

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1 INTRODUCTION

Bolted joints are one of the most used means of connecting parts of technical installations. For mission-critical assets, held together by bolted joints, regular inspections of proper bolt tightness are mandatory for certification and safe operation. As the bolt tension is impossible to measure and check in-situ, most of such inspections today are performed using a torque wrench. This procedure could potentially lead to the overtightening of bolts, which is undesirable. With large structures and heavy bolts, using a torque wrench, the bolt inspection procedure becomes a monotonous and physically demanding work, often conducted in tough environments. That is why a simpler and less demanding technique of checking bolt tightness is on high demand in many industries.

The abovementioned became a motivation for establishing the VEBJI project, a collaborative work between Technical University of Denmark (DTU), Brüel & Kjær and Karlsruhe Institute of Technology. VEBJI stands for Vibration for Estimating Bolted Joints Integrity. As it follows from the title, the project aims in utilizing vibrations, artificially induced in a bolt, as the source of information regarding its tension. As a starting point, we used an observation that, when impacted, a well-tightened bolt sounds differently from a loose one [1]. An impact excites different types of vibrations in the bolt, and some of them cause acoustic waves in the audible range, which a human being can perceive and judge. Thus, it was suggested to study how the bolt tension affects its vibrations, and possibly propose a technique for detecting loose bolts, based on the analysis of bolt transient vibrations, excited, for example, by impacting the bolt with a hammer.

There are alternative methods for bolt tightening check, a review can be found in [2]. Besides torque wrench and hydraulic tensioners, which are used for controllable bolt tightening, the most developed technique for controlling bolt tension is by check bolt elongation utilizing ultrasonic transducers. Commercial solutions employing this technology exist, for example, i-Bolt® by ERREKA [3] can be named as one of them. The drawbacks of the technique are the sensitivity to temperature and special requirement to machining of the bolts' butt-ends.

Two pilot studies, conducted by DTU students as master projects [4], [5], demonstrated that indeed the vibration patterns of tightened and loose bolts are different. Comparing a tightened / loose bolt with a differently tightened guitar string, it is natural to expect the difference in the pitch of the radiated sound. However, in the bolt case, audibly, the difference in pitch is hardly distinguishable. But in contrast, the tightened bolt produces a clear ringing sound, whilst the sound from a loose bolt is rather dull, i.e. decaying faster and containing less high-frequency content. The idea of the pilot projects was to find some properties in the measured vibration signal that can be utilized to characterized bolt tension. In structural health monitoring literature, these properties are often called *features*. Farrar and Worden in [6] provide the following definition: "A damage sensitive feature is some quantity extracted from the measured system response data that indicates the presence (or not) of damage in a structure". Alongside with the abovementioned natural frequencies and damping, other vibration features and / or their combinations are possible, for example, the ratio of natural frequencies, modal interaction, attack time, amplitudes of nonlinear harmonics, etc.

However, it was not straightforward to find the features of the measured vibration signals that could in a robust way indicate if the bolt is well-tightened or not. As bolt tightening features, [4] attempted the natural frequencies and damping of the lowest transverse modes, and in [5] the authors attempted psychoacoustic metrics, the vibration energy decay rate, and spectral centroids. Generally, the results of the two projects were not satisfactory, however, the projects indicated that the experimental work needs to be promoted to a much higher quality level to ensure the experiments being more controllable and reproducible. Alongside that, it became clear that a better understanding of bolt vibrations is required to explain what was observed during the experiments. These altogether became the groundwork for the VEBJI project.

The paper is built as follows: Section 2 describes the extensive measurement campaign, followed by Section 3, which introduces a model of a single-bolt joint. Section 4 concerns multi-bolt joints. Section 5 touches practical considerations for real-life scenarios. Finally, section 6 introduces the on-going research.

2 EXPERIMENTAL WORK

It was decided to start the project with an extensive measurement campaign, in an attempt to experimentally study the vibration patterns of differently tightened bolts in well-controlled environments and examine the vibration features that are correlated with the bolt tension. The results of the test campaign were thought to initiate the theoretical studies aimed to understand and explain what was observed during the experiments, and theoretically check the applicability of the found vibration features to different bolt sizes and bolted joint configurations.

During the pilot projects, the bolts were tightened using a torque wrench. It was suspected that the torque wrench was the main source of the observed variability. Indeed, applying the wrench in numerous tightening/untightening circles deforms the threads of the bolt and the nut, and damages the mating surfaces of the bolt, washers and the test rig. To retain the geometric and material properties of the test setup, it was decided to replace the torque wrench with a hydraulic tensioner. During the tightening process, the tensioner is pressurized using a hand pump, causing the bolt elongation. Then the nut is finger tightened. After the pressure release, the bolt contracts and compresses the test rig [7].

It is known that a torque wrench is a highly inaccurate tool with the variation of torque-to-tension up to 30% [8]. In the initial experiments conducted in the frame of the pilot studies, a strain gauge mounted on the bolt body was used to estimate its tension. This arrangement was proved to be impractical as the strain gauge is a very delicate sensor, and it requires a special test rig design to allow cabling. Subsequently, the strain gauge was replaced by a force washer directly measuring the compressing force, which is proportional to the bolt's tension.

Considering bolt excitation, it was decided to employ impact excitation as the most practical one. The pilot studies demonstrated that transverse vibrations can be easily excited by impacting the bolt at any point and in any direction. Even an impact in axial direction could excite transverse vibrations (this was later investigated in [9]). For the experiments, the excitation in the transverse direction was chosen.

To catch a possible nonlinear behavior of the bolt, it is necessary to excite the bolt by a well-controllable excitation force with increasing strength. To achieve this, a special pendulum-hammer was designed, where the impact strength was controllable by the initial inclination of the pendulum, and the impact force was measured by a force transducer.

In contrast to the pilot studies, in the VEBJI test campaign, the vibration signal was measured by two miniature accelerometers mounted on the bolt head. One accelerometer was aligned to measure in the direction of the impact, and another in the direction perpendicularly to the first one. This improvement allowed measuring bolt vibration in both transverse directions.

The experiment (the setup is shown in Figure 1), is described in full detail in [10], [11], [12], here a brief overview is given. It was a rather extensive test campaign, two types of bolts were tested, M12x260 and M12x140; the bolt was tightened in 10 load steps, in the range from approximately 0.07 of the yield stress σ_y to approximately 0.8 σ_y (the nominal bolt tension for this class of bolts is 0.7 σ_y). At each tension level, the bolt was impacted with four increasing impact strengths, and each measurement was conducted three times. To ensure repeatability, the experiment was repeated three times, including the full disassembly of the test rig. During

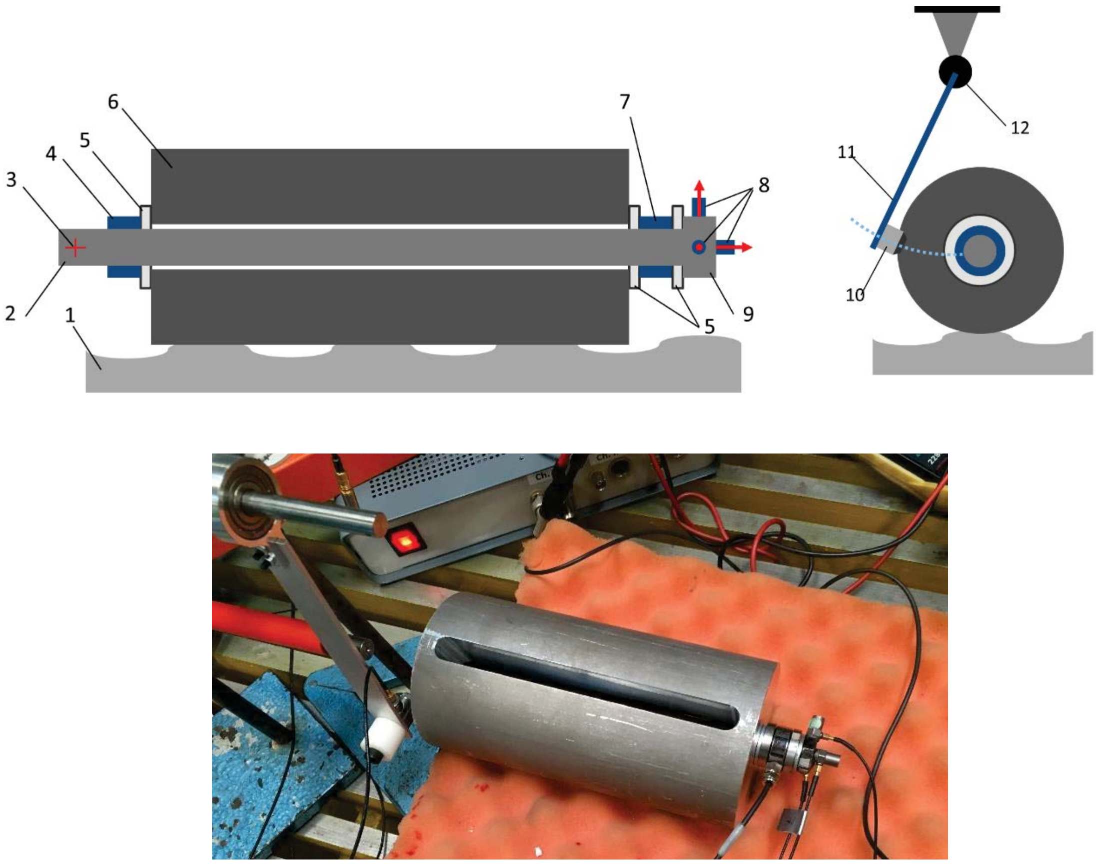


Figure 1. Experimental setup, sketch and photo (from [11]). 1. Foam supporting the structure; 2. Bolt; 3. Impact point; 4; Nut; 5. Washers; 6. Structure; 7. Force washer; 8 – Three monoaxial accelerometers; 9 – Bolt head; 10 – Force transducer; 11 – Pendulum hammer; 12 – Ball bearing holding the hammer.

the experiments, 720 time-histories were acquired with a high sampling rate, each time history consisted of two acceleration signals and the impact force signal.

2.1 Overview of the measurement campaign results

A typical acceleration signal is shown in Figure **2**a. As the bolt is a nearly symmetrical structure, one can expect the pairs of transverse modes of vibration with almost identical natural frequencies. The spectra in Figure **2**b show the peaks corresponding to the first pair of modes (just below 1 kHz) and the second pair of modes (above 2.5 kHz). The double peaks are (sometimes) distinguishable in the zoomed view (as shown). Applying a narrow band-pass filter (BPF) around the natural frequencies of the modes' pairs, it was possible to separate the pairs in the time domain; the resulting filtered signals $a_1(t)$ and $a_2(t)$ measured in the direction of the impact and the perpendicular direction, respectively, are shown in Figure **2**c, together with their envelopes and the envelope of $a_\Sigma(t) = \sqrt{a_1(t)^2 + a_2(t)^2}$.

The amplitudes of the measured signals in Figure **2**c is slowly modulated, which is a characteristic of a beating, the phenomenon which appears when two sinusoidal signals with slightly different frequencies are added together. The frequency of the modulation is then equal to the difference between the frequencies of the sinusoids. In [13], the likely explanation is provided: both modes of the first pair are excited by the hammer impact, and the modal response is the exponentially decaying sinusoidal oscillation at the damped natural frequencies of the mode. The system is effectively linear, and the two modes do not interact but the accelerometers pick up the mixture of both sinusoids, resulting in the observed beating. Another plausible

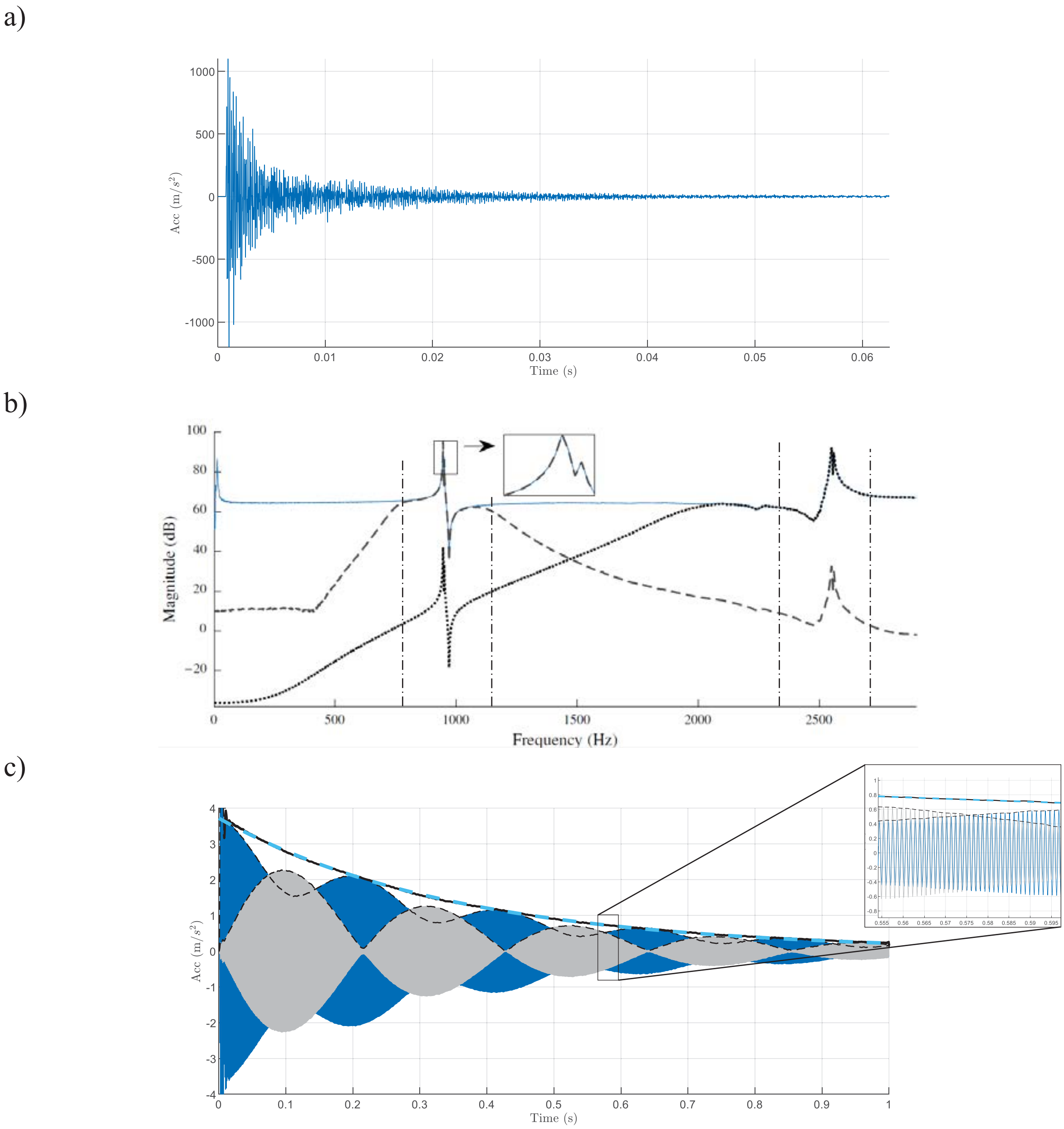


Figure 2. Measure signal and signal processing (from [11]). a) Raw acceleration signal measured in the impact direction. b) Spectra: blue - the spectrum of the raw signal, dash line – after applied BPF around the first natural frequency, dot-line – after applying BPF around the second natural frequency. Vertical dot-dash lines – the cut-off frequencies of the applied BPF. c) Two transverse acceleration signals $a_1(t)$ and $a_2(t)$, after applying BPF around the first transverse resonant frequency. Thin dash lines – their envelopes. Thick dash line – the envelope of $a_\Sigma(t) = \sqrt{a_1(t)^2 + a_2(t)^2}$.

explanation involves a non-linear interaction (i.e. energy exchange) between the two transverse modes, as explained in [14].

Based on the observations during the experiment, numerous vibration characteristics were considered as potential features [12], including natural frequencies, modal damping, modes interaction inside and outside the pairs, variation in time of the instantaneous natural frequencies and damping, input/output linearity, impact duration, etc. These characteristics were correlated with bolt's tension and their reproducibility was checked using available time histories. Based on the analysis, two characteristics were studied in detail, as they demonstrated solid correlation

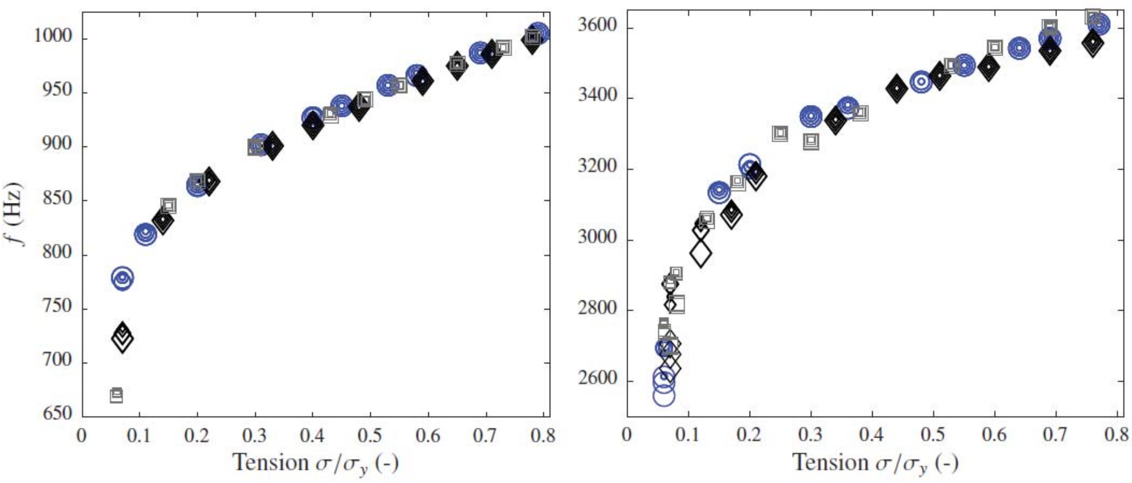


Figure 3. Measured first natural frequency as a function of the bolt tension: long bolt (left) and short bolt (right). The different markers indicate the test series, the marker size denotes the impact strength (from [11], [12]).

with the bolt tension: the natural frequency of the transverse vibrations and the corresponding modal damping. The other attempted potential featues did not demonstrate the sufficient correlation with bolt tension or were not sufficiently reproducible.

2.2 Selected features

Natural frequencies

The natural frequencies obtained by zero-cross counting of the filtered signal are shown in Figure 3. As expected, there is a strong correlation between the natural frequencies and the bolt tension, observed for both long and short bolts.

Damping ratio

The effective linear damping ratio of the first mode was estimated by fitting the exponential decay to the envelope of the filtered signal a_Σ (Figure **2**c). The damping ratio decreases with increasing bolt tension, as shown in Figure **4**, [11] and [12]. In absolute values, the damping ratios

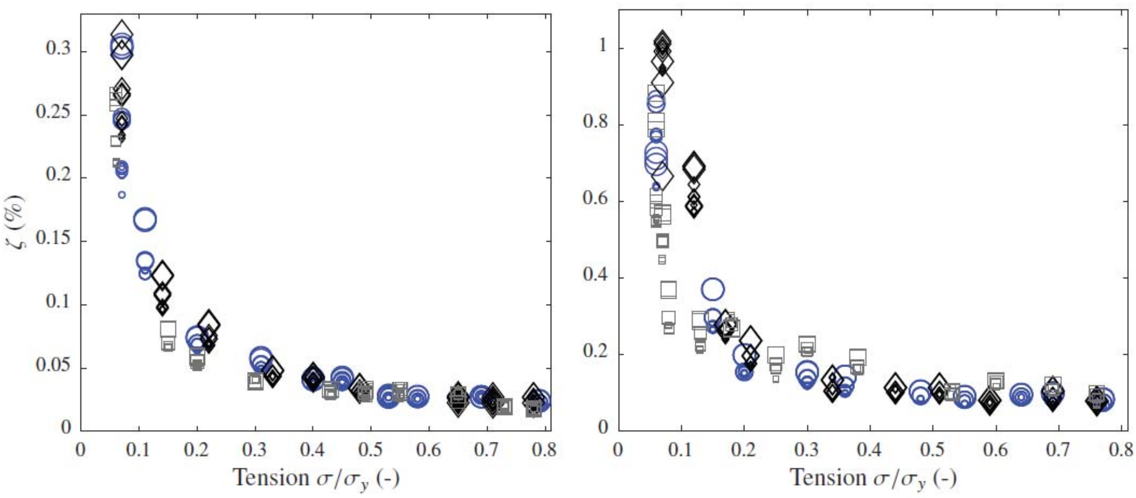


Figure 4. The damping ratio of the first mode as a function of the bolt tension for the long (left) and short (right) bolts (from [11], [12]). Legend is the same as in Figure 3.

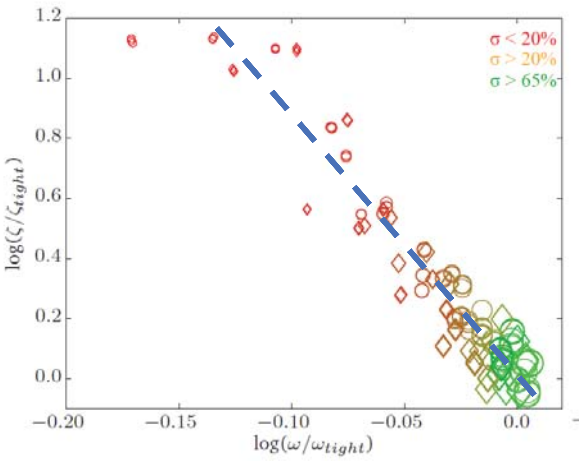


Figure 5. Distribution of the points with coordinates (normalized frequency; normalized damping) (from [12]). The markers' color and size correspond to the tension of the bolt.

differ for the long and short bolt, however, if scaled with the bolt slenderness ratio, the damping ratio becomes invariant to bolt sizes. The slenderness ratio is defined as $s = l/\sqrt{I/A}$, where l, I and A are bolt's length, the moment of inertia of bolt's section and area of bolt's section, respectively. The theoretical explanation of this interesting observation is provided in [12].

Combined frequency and damping feature

The combination of the normalized natural frequency and damping ratio into a single bolt tension feature was suggested in [12] and is shown in Figure 5 for both long and short bolts. The frequency and damping are normalized by the respective characteristics of the nominally tightened bolt. In Figure 5 the points corresponding to the well-tightened bolts are concentrated in the bottom-right corner, whilst those of the relatively loose bolts are spread along the dashed line leading towards the upper-left corner of the plot.

Discussion

During the experiments, it became clear that the frequency and damping, either separately or combined, can be considered as reliable qualitative indicators of bolt tension. The next natural step would be suggesting a model explaining the experimentally observed phenomena and providing a quantitative relation between the measured vibration properties and the unknown bolt tension.

3 MODELLING A SINGLE-BOLT JOINT

From the first glance, it is natural to model a tightened bolt as a pre-stressed clamped-clamped beam. For such a beam the dependence of the i-th transverse natural frequencies from the tension is well-known [15],

$$\omega_i = \frac{\lambda_i}{l\,s}\sqrt{\frac{E}{\rho}\left(1 + \frac{\sigma}{E}\left(\frac{K_i\,s}{\lambda_i^2}\right)^2\right)}, i = 1,2 \ldots \tag{1}$$

where l is the length of the beam, s is the slenderness ratio, σ is the axial stress, E and ρ are beam's material Young's modulus and density, respectively and K_i and λ_i are non-dimensional constants, depending on the i-th mode shape.

As it follows from (1), the square of the natural frequencies is directly proportional to the axial stress σ, and thus to the bolt tension. Figure 6 shows the corresponding line on top of the experimental results (adjusting the length l to fit the high values of σ). Apparently, for the long

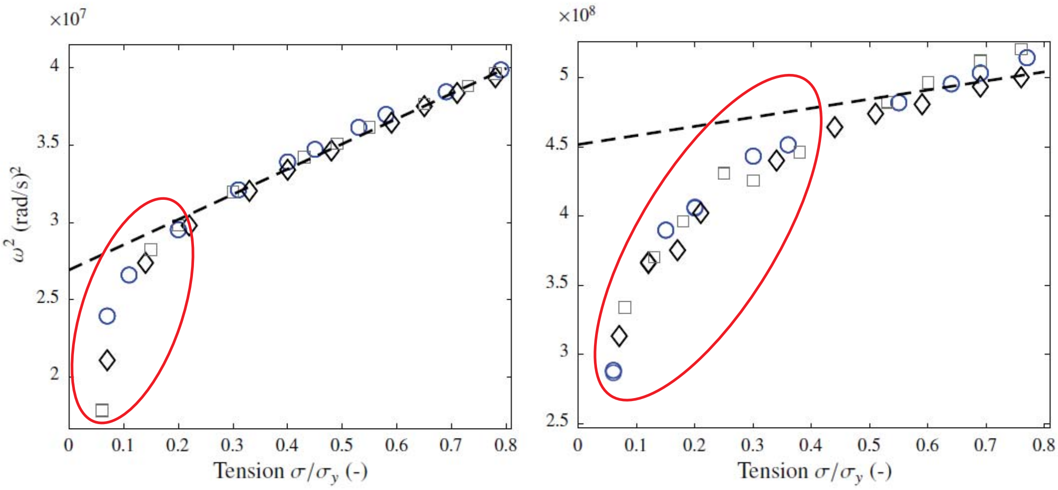


Figure 6. Square of the first natural frequency $\boldsymbol{\omega_1}$ as function of bolt tension $\boldsymbol{\sigma}$. The markers denote experimental results; dashed line is the model according to (1). Left – long bolt, right – short bolt. The red ovals indicate the region where the clamped-clamped beam model is not suitable (from [10]).

bolt, the fit is good for a wide range of σ, and less for the short bolt. Common for both cases, the fit is not good for low tension levels (circled), indicating that the clamped-clamped beam model does not reflect reality at low levels of tension.

The study [10] suggested an extended model that could explain the observed behavior at the full range of the tension levels. The model (Figure **7**) consists of a Bernoulli-Euler beam, subjected to an axial tensile force σA, and supports at both ends, which both have translational and rotational stiffnesses, k_u and k_θ, respectively. To reflect reality, where the bolt tension affects the contact stiffness, in the model, these stiffnesses are functions of the axial force. For high levels of σA, the stiffness values approach infinity, and the beam boundary conditions become clamped-clamped.

With the model in hand, the next task is to obtain the stiffness parameters and their dependence on tension. In [16] it was demonstrated that it is possible to estimate the stiffness parameters from the measured natural frequencies of the beam. Another approach was taken in [10], where the model of the contact between the mating surfaces was introduced. It was noticed that the contact surfaces in real structures are rough, which is manifested as a distribution of tiny asperities at a micro-scale level, whose interactions can be thought of as springs connecting the mating surfaces. It was observed that the normal (interfacial) stiffness increases with the increase of the contact pressure. Using the asperities-based contact model (Figure 8), it was possible to obtain the values of k_u and k_θ, which resulted in quite good agreement with the measured data, for the full range of tension (Figure 9).

The study [17] extended the model suggested in [10] (Figure 8), by the damping elements, which were added to all springs. It was shown that assuming the damping linearly proportional to the rotational and translational stiffnesses, one can get quite a good agreement with the experimental results shown in Figure **4**. Apparently contradicting though, is that the underlying

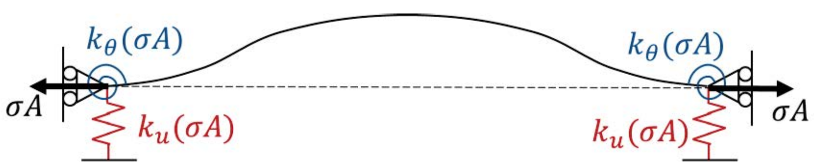


Figure 7. The model suggested in [10].

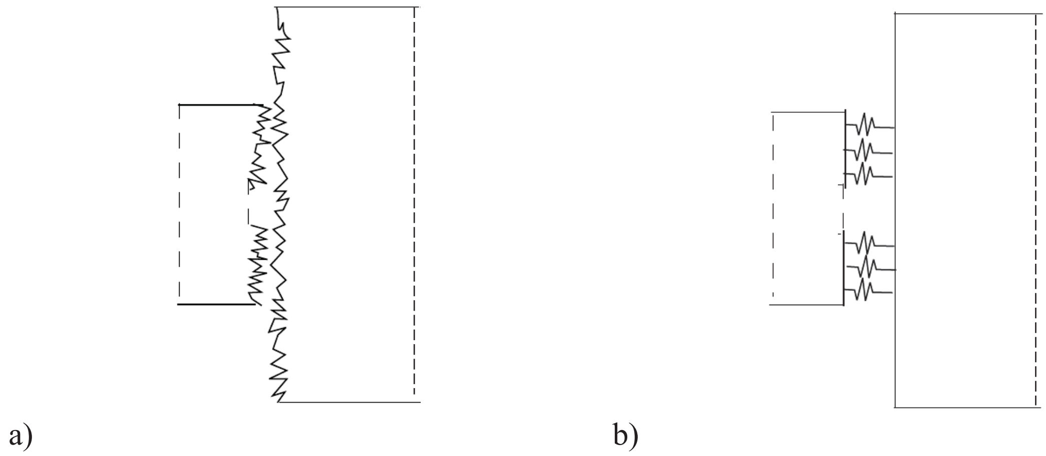


Figure 8. Modeling of the contact between the mating surfaces. a) Micro-level, with the roughness presented as asperities; b) modeling as springs connecting the surfaces (from [10]).

microscale damping mechanism is essentially nonlinear, whilst the collective macroscale behavior is linear. The study [17] suggested a model, which demonstrated that the non-linear multi-degree-of-freedom asperities-based model can be effectively converted to a single-degree-of-freedom model with a cubic nonlinearity, and further showed that the nonlinear term vanishes as the number of asperities increase, thus validating the observed effective linearity in damping.

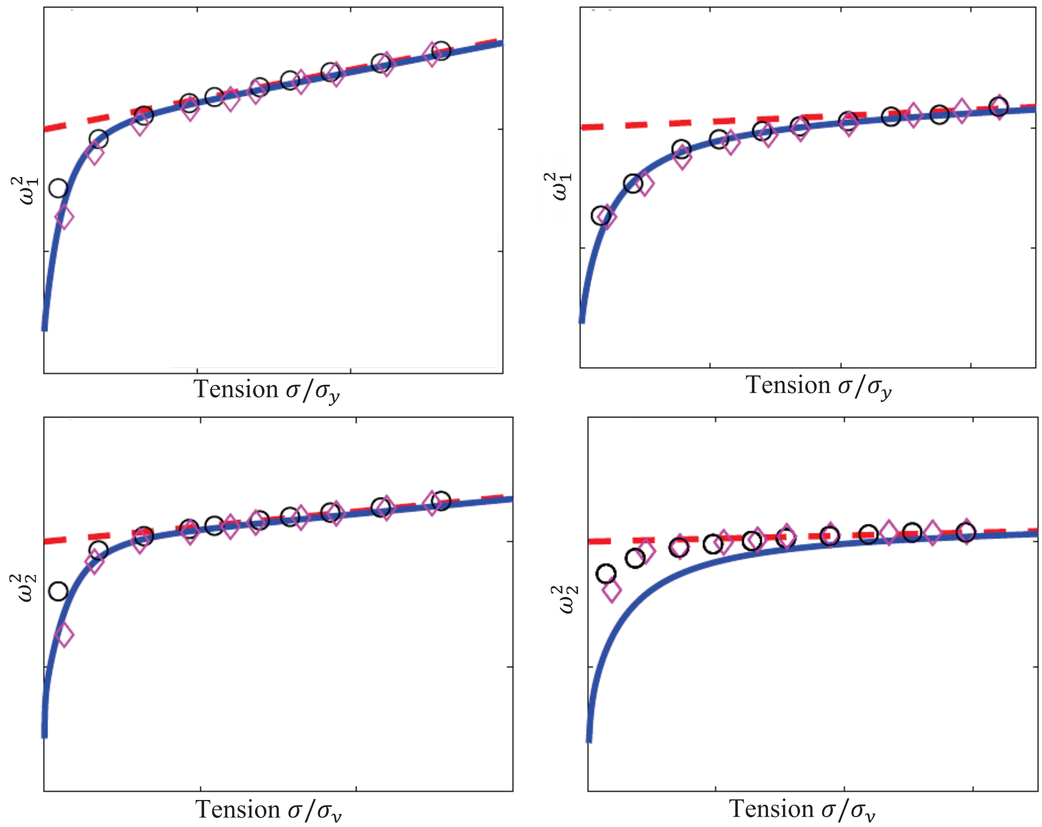


Figure 9. Qualitative comparison of the experimental results (markers), the clamped-clamped model (1) (red dashed line) and the extended model (Figure 7) (blue curve). Left / right columns – long / short bolts; Top / bottom rows – first / second natural frequencies (from [17]).

4 MODELLING MULTI-BOLT JOINT

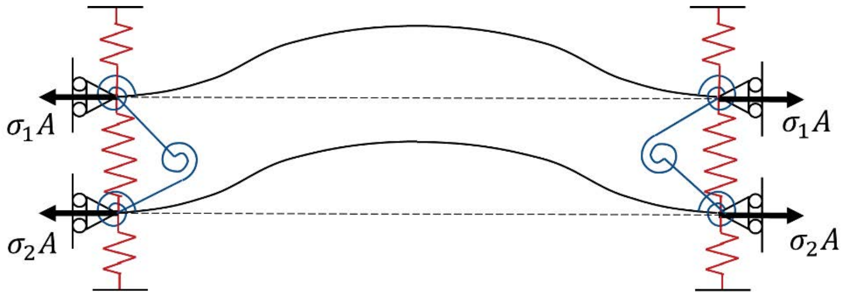
In studies [18], [19], another aspect of the bolt tension estimation problem was investigated. As the most bolted joints consist of many nearly identical bolts tightened to almost identical levels, the bolts will have very similar boundary conditions and thus almost identical natural frequencies. In a likely case of very light damping, a frequency response measured on one bolt after an impact might include the vibrational response of other bolts. So, a question arises, if it is possible to separate which frequencies belong to which bolt. To start with, analytical, numerical and experimental models of a structure with two identical bolts were established (Figure 10).

It was found that, if the bolts have identical or very similar tension, the system behaves as a whole, and it is impossible to assign natural frequencies to a single bolt, as the global in-phase and anti-phase modes arise. However, if the bolts' tensions are different, their modes can be considered as local, the bolts can be considered separately, and the single-bolt model is fully applicable.

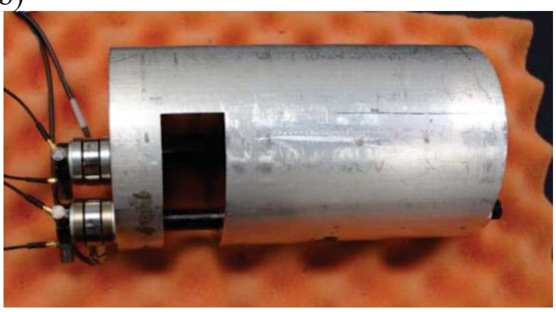
5 PRACTICAL CONSIDERATIONS

One of the aims of the VEBJI project was to explore feasible methods for estimating bolt tension having potential for industrial application. It is apparent (Figure 5), that the studied vibration features are suitable for distinguishing a nominally tightened bolt ($0.7\ \sigma_y$) from a very loose one ($< 0.3\ \sigma_y$). However, the ambition of the project was to be able to distinguish a nominally tightened bolt from the slightly under- and over-tightened ones, for example, to be able to find the tension within $\pm 0.1\sigma_y$ around the nominal tension $0.7\ \sigma_y$. In this region the vibration-based features have a low sensitivity to the tension (Figure 3, Figure **4**), i.e. the value of the feature changes little with the significant change of the tension level. As the vibration features are obtained from noisy measurements, it makes it challenging to develop a robust technique for estimating the tension with the desirable precision.

a)



b)



c)

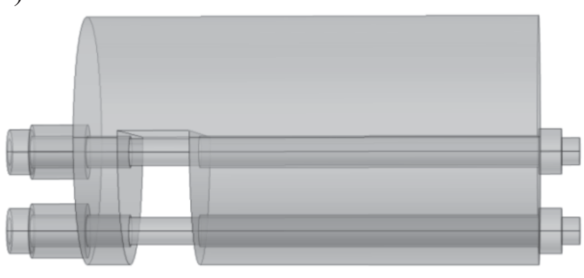


Figure 10. Two bolts structure from [19]: a) Analytical model; b) Experimental setup; c) COMSOL model

The study [20] attempted to suggest and test the algorithm that employs the first transverse natural frequency as the vibration feature, and then takes the next step towards classification, i.e. distinguishing the loose bolts from the tightened ones. The algorithm uses the gradient of the first natural frequency with respect to the tension from the clamp-clamp beam model (1). Also, the algorithm assumes that initially the bolt was tightened to the nominal tension value. The latter is used as the reference for the subsequent tension checks. The propagation of the uncertainty of the measured vibration feature to the uncertainty of the estimated bolt tension was conducted following the Delta method [21]. The variability of the vibration features was estimated by measuring the first transverse natural frequency of a big number of similar bolts holding a wind turbine blade on the test rig (Figure 11). It was concluded that the described above approach is not readily feasible, and it was suggested that the following can potentially improve the technique: 1) Improvements of the excitation/measurement/signal processing procedures; 2) Employing more sensitive features; 3) Combining several vibration features.

6 ON-GOING RESEARCH

The study [9] paved a way to another promising technology developed in the frame of VEBJI, which is currently patent pending [22]. In [9], it was analytically shown and numerically demonstrated that the boundary stiffnesses parameters (Figure 7) can be estimated from measured transverse natural frequencies. However, the method requires identification of a high number of modes, which is difficult to measure in practice (with hammer excitation only the first two transverse modes can be reliably identified). The new method suggests changing the dynamic properties of the bolt in a controllable way, by adding a known mass to the threaded end

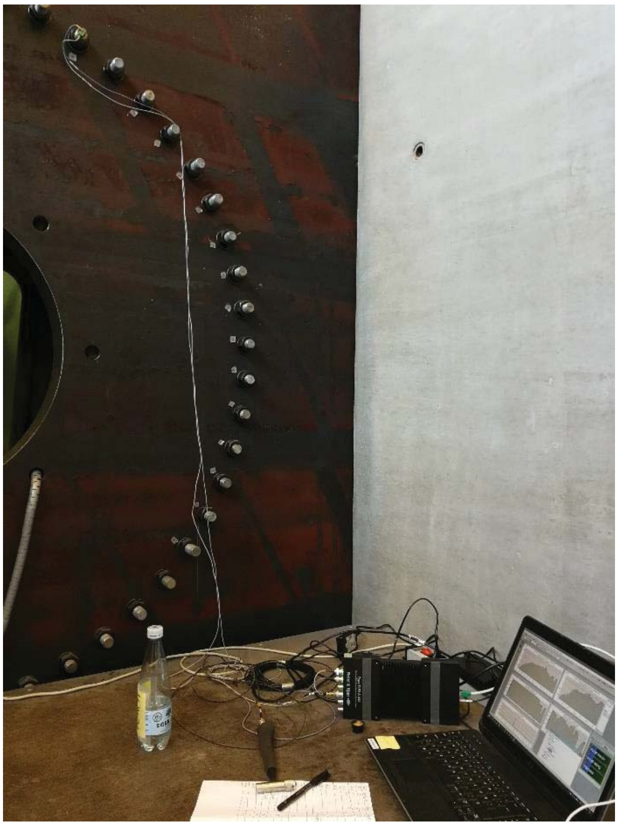


Figure 11. A test rig (wind turbine blade test bench) used to test the classification algorithm suggested in [20].

of the bolt. Thus, adding one or two masses to the bolt and measuring the first few natural frequencies, it is possible to estimate the tensional force acting in the bolt; the details of the method are presented in [23].

In the multi-bolt scenario, it is also anticipated that adding mass to the bolt end will make its dynamics different from the dynamics of the surrounding bolts; this will facilitate distinguishing the response of the tested bolt from the others (see Section 4).

As it was observed during the experiments, with hammer excitation employed, it is only possible to reliably identify the first two transverse modes, mainly due to the lack of excitation energy at the higher frequency range. The attempt to use the method in a real-life scenario, described in Section 5, also called for the improvement of the applied excitation and signal processing procedures. An approach to deal with this problem is to replace the hammer impacts with excitation by piezoelectric elements [24], which are generally more efficient at higher frequency ranges.

CONCLUSION

The paper describes the challenges, learnings, and achievements of the VEBJI project, whose aim was to generate knowledge and potentially develop a technique for estimating bolt tension using vibrations artificially excited in a bolt.

The study demonstrated that it is rather straightforward to distinguish a well-tightened bolt from a loose one, which could be done using the natural frequency and damping of the first transverse mode. However, it is more difficult to distinguish a nominally tightened bolt (characterized by axial stress σ_n) from slightly under- or overtightened ones ($< 0.9\ \sigma_n$ and $> 1.1\ \sigma_n$, respectively). Though the added mass method (Section 6) produces promising results, it still needs to be tested for multi-bolt joints scenarios.

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