

INTERVAL SERVICEABILITY ASSESSMENT OF FOOTBRIDGES

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Abstract. *This paper studies serviceability assessment of footbridges through a non-deterministic approach. The parameters defining pedestrian-induced loading and the structural dynamic properties are characterized through possible ranges of variation. Starting from analytical expressions for the spectral moments of the structural response, the improved interval analysis is applied together with an optimization strategy that allows us to obtain the bounds of the standard deviation of the footbridge acceleration and of the mean value and cumulative distribution function of its maximum value. Based on this approach, a possible interval of variation of the structural response is evaluated, rather than a single deterministic value. Thus, an interval level of comfort can be defined.*

1 INTRODUCTION

Serviceability assessment of footbridges is commonly carried out, based on current guidelines (e.g., [1]), by comparing a deterministic value of the expected maximum acceleration and suitable thresholds defining different comfort classes. Actually, the problem should be tackled through a non-deterministic approach due to the uncertainties characterizing both pedestrian-induced loading (i.e. dynamic loading factor, pedestrian weight and step frequency) and the structural dynamic properties (i.e. modal damping ratios, natural frequencies and modal masses). An approach accounting only for the errors connected with the estimate of structural parameters has been introduced through a fuzzy analysis [2]. Based on the characterization of the coefficient of variation of both the loading and the structural uncertain parameters, the propagation of uncertainties on the serviceability assessment of footbridges has been studied adopting the Taylor series expansion technique [3]. Furthermore, Monte Carlo simulations have been performed assuming plausible probability distributions of the uncertain parameters, and the probability distribution of the mean value of the maximum acceleration has been estimated numerically [3].

Actually, probability distributions of the loading and structural parameters are not available, but the literature analysis in [3] provides possible ranges of variation of these parameters. Thus, a suitable tool for taking into account uncertainties in serviceability assessment of footbridges is interval analysis (e.g., [4], [5]). In this paper, starting from the analytical expression for the spectral moments of the structural response [6], the *improved interval analysis* [4] is applied together with an optimization strategy (see e.g., [7]) that allows us to obtain the bounds of the maximum footbridge acceleration. Based on this approach, serviceability assessment is carried out within a non-probabilistic framework, and an interval level of comfort is estimated.

2 ANALYTICAL FORMULATION

The force exerted on footbridges in multi-pedestrian traffic condition is commonly schematized in the time domain as the sum of moving harmonic loads with random amplitude, frequency and velocity. Based on this force model, serviceability assessment has to be carried out through Monte Carlo simulations.

According to the Equivalent Spectral Model (ESM) [6], multi-pedestrian loading can be modelled as an equivalent Gaussian stationary random process. Considering the first mode shape of a simply-supported beam of length L , $\varphi_j(x)=\sin(\pi x/L)$, the power spectral density function (PSDF) of the modal force $F_j(t)$ is given by:

$$S_{F_j}(\omega) = \frac{N_p}{4} (\alpha G)^2 p_\Omega(\omega) \quad (1)$$

where N_p is the number of pedestrians, α is the mean value of the dynamic loading factor, G is the mean value of the weight of pedestrians, $p_\Omega(\omega)$ is the probability density function (PDF) of the circular step frequency of pedestrians, assumed as normally-distributed with mean value μ_{op} and standard deviation σ_{op} . The spectral moments of zero- and second-order of the acceleration of the j -th principal coordinate $p_j(t)$ can be approximated as follows:

$$\lambda_{0,\ddot{p}_j} \approx \frac{\pi \omega_j S_{F_j}(\omega_j)}{4 \xi_j M_j^2} = \sigma_{\ddot{p}_j}^2; \quad \lambda_{2,\ddot{p}_j} \approx \frac{\pi \omega_j^3 S_{F_j}(\omega_j)}{4 \xi_j M_j^2} \quad (2)$$

where M_j , ω_j and ξ_j are, respectively, the j -th modal mass, natural circular frequency and damping ratio of the structure. The maximum acceleration of the j -th principal coordinate in the time interval T is then defined as the following random process:

$$\ddot{p}_{j,\max}(T) = \max_{0 \leq t \leq T} |\ddot{p}_j(t)| \quad (3)$$

where the time interval T can be conventionally defined as a multiple of the average crossing time of pedestrians ($T = NL/c_m$, $N=10$, $c_m=0.9 \mu_{ap}/2\pi$).

Under the Poisson's assumption of independent up-crossings of a prescribed threshold, the cumulative distribution function (CDF) of the maximum acceleration is defined as follows:

$$L_{\ddot{p}_{j,\max}}(b, T) = \Pr[\ddot{p}_{j,\max}(T) \leq b] \approx \exp\left[-2 \nu_{\ddot{p}_j} T \exp\left(-\frac{b^2}{2\sigma_{\ddot{p}_j}^2}\right)\right] \quad (4)$$

where $\nu_{\ddot{p}_j}$ is the expected frequency of the acceleration of the j -th principal coordinate, given by:

$$\nu_{\ddot{p}_j} = \frac{1}{2\pi} \sqrt{\frac{\lambda_{2,\ddot{p}_j}}{\lambda_{0,\ddot{p}_j}}} = \frac{\omega_j}{2\pi}. \quad (5)$$

The mean value of the maximum mid-span acceleration can be approximated according to Davenport's formulation ([8], [9]) as follows:

$$\ddot{q}_{\max} = E[\ddot{p}_{j,\max}] = g_{\ddot{p}_j} \sigma_{\ddot{p}_j}; \quad g_{\ddot{p}_j} = \sqrt{2 \ln(2\nu_{\ddot{p}_j} T)} + \frac{0.5772}{\sqrt{2 \ln(2\nu_{\ddot{p}_j} T)}} \quad (6)$$

3 INTERVAL SERVICEABILITY ASSESSMENT

Interval serviceability assessment is based on the assumption of uncertain loading and structural parameters, represented as interval variables [10] according to the interval model of uncertainty:

$$\mathbf{x}^I = \{\mu_{\omega p}^I \quad \sigma_{\omega p}^I \quad \alpha^I \quad G^I \quad \omega_j^I \quad \xi_j^I \quad M_j^I\}^T \quad (7)$$

where the apex denotes interval variables. Each component of the vector \mathbf{x}^I , i.e. the k -th interval variable x_k^I , is defined as:

$$x_k^I = x_{0,k} (1 + \Delta x_k \hat{e}_k^I) \quad (8)$$

where $\hat{e}_k^I = [-1, 1]$ is the so-called *extra unitary interval* [4], $x_{0,k}$ and Δx_k are the midpoint and normalized deviation amplitude, defined as follows:

$$x_{0,k} = \frac{\bar{x}_k + \underline{x}_k}{2}; \quad \Delta x_k = \frac{\bar{x}_k - \underline{x}_k}{2x_{0,k}} \quad (9)$$

being \underline{x}_k and \bar{x}_k the lower bound (LB) and upper bound (UB) of the interval variable x_k^I .

Based on the interval representation of the involved quantities, the PSDF of the modal force is an interval function:

$$S_{F_j}^I(\omega) = \frac{N_p}{4} (\alpha^I G^I)^2 p_\Omega^I(\omega). \quad (10)$$

As a consequence, also the spectral moments of zero- and second-order of the acceleration of the j -th principal coordinate are interval variables:

$$\lambda_{0,\ddot{p}_j}^I = \frac{\pi \omega_j^I S_{F_j}^I(\omega_j^I)}{4 \xi_j^I (M_j^I)^2} = (\sigma_{\ddot{p}_j}^I)^2; \quad \lambda_{2,\ddot{p}_j}^I = \frac{\pi (\omega_j^I)^3 S_{F_j}^I(\omega_j^I)}{4 \xi_j^I (M_j^I)^2}. \quad (11)$$

The interval CDF is defined as:

$$L_{\ddot{p}_j,\max}^I(b, T) = \exp \left[-2 \nu_{\ddot{p}_j}^I T^I \exp \left(-\frac{b^2}{2 (\sigma_{\ddot{p}_j}^I)^2} \right) \right] \quad (12)$$

where the interval expected frequency is:

$$\nu_{\ddot{p}_j}^I = \frac{1}{2\pi} \sqrt{\frac{\lambda_{2,\ddot{p}_j}^I}{\lambda_{0,\ddot{p}_j}^I}} = \frac{\omega_j^I}{2\pi}. \quad (13)$$

Finally, the interval mean value of the maximum acceleration process is defined as follows:

$$\ddot{q}_{\max}^I = g_{\ddot{p}_j}^I \sigma_{\ddot{p}_j}^I; \quad g_{\ddot{p}_j}^I = \sqrt{2 \ln(2 \nu_{\ddot{p}_j}^I T^I)} + \frac{0.5772}{\sqrt{2 \ln(2 \nu_{\ddot{p}_j}^I T^I)}}. \quad (14)$$

The LB and UB of the standard deviation of the acceleration are given by:

$$\underline{\sigma}_{\ddot{p}_j} = \min_{\mathbf{x} \in \mathbf{x}^I} \left\{ \sqrt{\frac{\pi \omega_j S_{F_j}(\omega_j)}{4 \xi_j M_j^2}} \right\}; \quad \bar{\sigma}_{\ddot{p}_j} = \max_{\mathbf{x} \in \mathbf{x}^I} \left\{ \sqrt{\frac{\pi \omega_j S_{F_j}(\omega_j)}{4 \xi_j M_j^2}} \right\}. \quad (15)$$

Analogously, the LB and UB of the CDF of the maximum acceleration are:

$$\begin{aligned} \underline{L}_{\ddot{p}_j,\max}(b, T) &= \min_{\mathbf{x} \in \mathbf{x}^I} \left\{ \exp \left[-2 \nu_{\ddot{p}_j} T \exp \left(-\frac{b^2}{2 \sigma_{\ddot{p}_j}^2} \right) \right] \right\}; \\ \bar{L}_{\ddot{p}_j,\max}(b, T) &= \max_{\mathbf{x} \in \mathbf{x}^I} \left\{ \exp \left[-2 \nu_{\ddot{p}_j} T \exp \left(-\frac{b^2}{2 \sigma_{\ddot{p}_j}^2} \right) \right] \right\}. \end{aligned} \quad (16)$$

The LB and UB of the maximum mid-span acceleration are given by:

$$\begin{aligned} \underline{\ddot{q}}_{\max} &= \min_{\mathbf{x} \in \mathbf{x}^I} \left\{ \left[\sqrt{2 \ln(2 \nu_{\ddot{p}_j} T)} + \frac{0.5772}{\sqrt{2 \ln(2 \nu_{\ddot{p}_j} T)}} \right] \sigma_{\ddot{p}_j} \right\}; \\ \bar{\ddot{q}}_{\max} &= \max_{\mathbf{x} \in \mathbf{x}^I} \left\{ \left[\sqrt{2 \ln(2 \nu_{\ddot{p}_j} T)} + \frac{0.5772}{\sqrt{2 \ln(2 \nu_{\ddot{p}_j} T)}} \right] \sigma_{\ddot{p}_j} \right\}. \end{aligned} \quad (17)$$

Finally, the *coefficients of interval uncertainty (c.i.u.)* for the two response quantities of interest can be defined as the ratio between their deviation amplitude and their midpoint as follows:

$$c.i.u.[\sigma_{\ddot{p}_j}^I] = \frac{\bar{\sigma}_{\ddot{p}_j} - \underline{\sigma}_{\ddot{p}_j}}{\bar{\sigma}_{\ddot{p}_j} + \underline{\sigma}_{\ddot{p}_j}}; \quad c.i.u.[\ddot{q}_{\max}^I] = \frac{\bar{\ddot{q}}_{\max} - \underline{\ddot{q}}_{\max}}{\bar{\ddot{q}}_{\max} + \underline{\ddot{q}}_{\max}}. \quad (18)$$

4 NUMERICAL APPLICATION

Current guidelines require to perform serviceability assessment of footbridges by comparing a deterministic value of the expected maximum acceleration and suitable thresholds defining different comfort classes. Table 1 reports the threshold values as defined by SETRA [1].

Comfort class	Min [m/s ²]	Max [m/s ²]
Maximum	0	0.5
Medium	0.5	1
Minimum	1	2.5
Unacceptable	2.5	

Table 1: Comfort classes (SETRA).

In this Section, serviceability assessment of an ideal footbridge is carried out. Results of a conventional evaluation are compared with those deriving from interval analysis. In particular, at first interval analysis is carried out assuming fixed normalized deviation amplitudes of the parameters based on [3]. Then, an increasing degree of uncertainty is considered according to the following definition of the interval variables:

$$x_k^I(\beta) = x_{0,k} (1 + \beta \Delta x_k \hat{e}_k^I), \quad 0 \leq \beta \leq 1. \quad (19)$$

Furthermore, three different interval analyses are carried out: considering all the parameters as uncertain, i.e. full uncertainty analysis (FU), taking into account only the uncertainties in the structural parameters (SU), and taking into account only uncertainties in the loading parameters (LU).

An ideal, steel, simply-supported beam with span length $L=50$ m, a deck width $b=2$ m is analyzed [3]. Considering load category III, sparse crowd [1], a pedestrian density $\rho=0.5$ pedestrians/m² is assumed, i.e. $N_p=50$, and the predicted maximum acceleration applying SETRA procedure is $\ddot{q}_{\max}=2.15$ m/s², falling into a minimum comfort class according to Table 1. Interval analysis is carried out assuming the midpoint and the normalized deviation amplitude of the uncertain parameters in Table 2.

Parameter	$x_{0,k}$	Δx_k
μ_{ap} [rad/s]	$2 \pi 1.88$	0.03
σ_{ap} [rad/s]	$2 \pi 0.17$	0.17
α	0.35	0.20
G [N]	700	0.10
ω_j [rad/s]	$2 \pi 1.88$	0.10
ξ_j [%]	0.42	0.60
M_j [kg]	50000	0.10

Table 2: Midpoint and normalized deviation amplitude of interval variables (Example 1).

The nominal solutions for the standard deviation and the mean value of the maximum acceleration are $\sigma_{\ddot{p}_j} = 0.50 \text{ m/s}^2$ (Eq.(2)), $\ddot{q}_{\max} = 1.94 \text{ m/s}^2$ (Eq. (6)). Table 3 reports the LB and UB of $\sigma_{\ddot{p}_j}^I$ (see Eq.(15)) and \ddot{q}_{\max}^I (see Eq. (17)) together with their *c.i.u.* (see Eq. (18)), obtained in the three analyses. The significant deviation amplitudes of the uncertain parameters (Table 2) lead to very large *c.i.u.*, especially if all the parameters (structural and loading) are considered as uncertain (FU). However, the uncertainty in the dynamic response is mainly governed by the uncertainties in the structural parameters. When only loading parameters are assumed as uncertain (LU), the deviation amplitude of the response is significantly reduced. It should be remarked that the maximum acceleration obtained applying SETRA procedure is higher than the nominal value and it is close to the UB provided by interval analysis if only loading uncertainties are accounted for (LU). In all the other cases (FU and SU), SETRA prediction is well below the UB provided by interval analysis, and it can be non-conservative.

Analysis	$\underline{\sigma}_{\ddot{p}_j}$	$\overline{\sigma}_{\ddot{p}_j}$	$c.i.u.[\sigma_{\ddot{p}_j}^I]$	\ddot{q}_{\max}	$\overline{\ddot{q}}_{\max}$	$c.i.u.[\ddot{q}_{\max}^I]$
FU	0.13	1.29	0.82	0.49	5.02	0.82
SU	0.25	0.88	0.56	0.97	3.42	0.56
LU	0.32	0.72	0.38	1.26	2.81	0.38

Table 3: LB, UB and *c.i.u.* of the standard deviation of the response and of the maximum acceleration (Example 1).

Figure 1 provides the UB and LB of $\sigma_{\ddot{p}_j}^I$ (a) and its *c.i.u.* (b) versus the coefficient β which measures the degree of uncertainty according to Eq. (19): results obtained for the three different analyses (FU, SU, LU) are reported. Fig. 1(a) shows that the interval of variation of the standard deviation of the response is very wide, especially if both loading and structural uncertainties are taken into account (FU). From Fig. 1(b) it can be deduced that the *c.i.u.* is almost proportional to the degree of uncertainty β . Furthermore, the *c.i.u.* takes values lower than 10% only if a small degree of uncertainty of the parameters is assumed, or at least structural parameters are assumed as deterministic. When all the parameters are assumed as uncertain, the *c.i.u.* is very large.

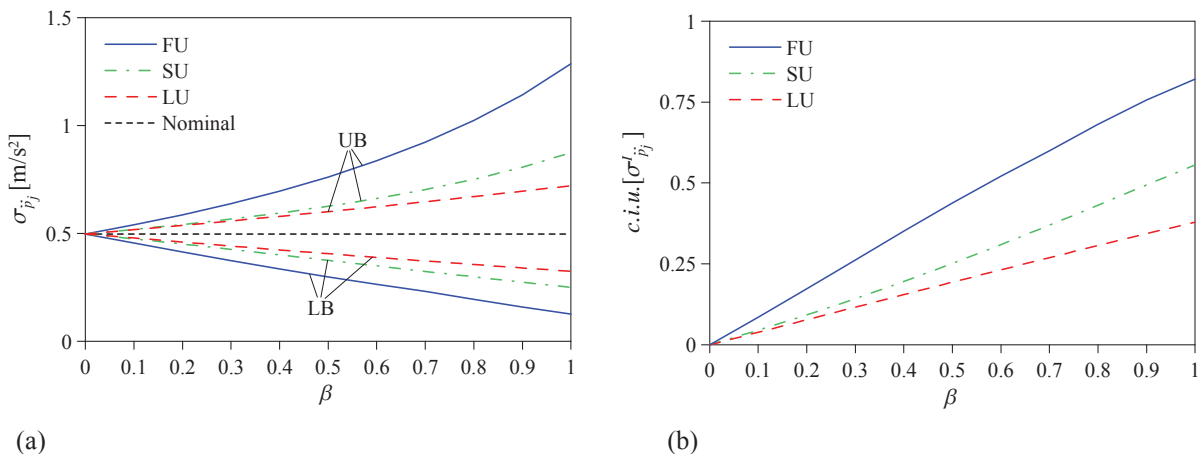


Figure 1: UB and LB of standard deviation of the acceleration (a) and its *coefficient of the interval uncertainty* (b) for increasing degree of uncertainty (Example 1).

Figure 2 provides the LB and UB of the CDF of the maximum acceleration for $\beta=1$, for the full uncertainty analysis (FU), taking into account only the uncertainties in the structural parameters (SU), and taking into account only uncertainties in the loading parameters (LU). From Fig. 2, the LB and UB of the probability of falling within each comfort class can be estimated. It can be deduced that, if the nominal values of the parameters are assumed, the footbridge will surely fall within a minimum comfort class. When all the parameters are considered as uncertain, the UB and LB of the CDF are extremely far from each other: considering the LB of the CDF, the maximum footbridge acceleration is surely higher than 4 m/s^2 , and thus the comfort level is unacceptable, while considering the UB of the CDF, the maximum acceleration is almost surely lower than 0.5 m/s^2 and the maximum comfort level is assured. When only the structural or the loading parameters are considered as uncertain, the interval of variation of the CDF becomes narrower. For example, if only loading parameters are considered as uncertain, the LB of the CDF falls within unacceptable comfort class and the UB of the CDF falls within a minimum comfort class.

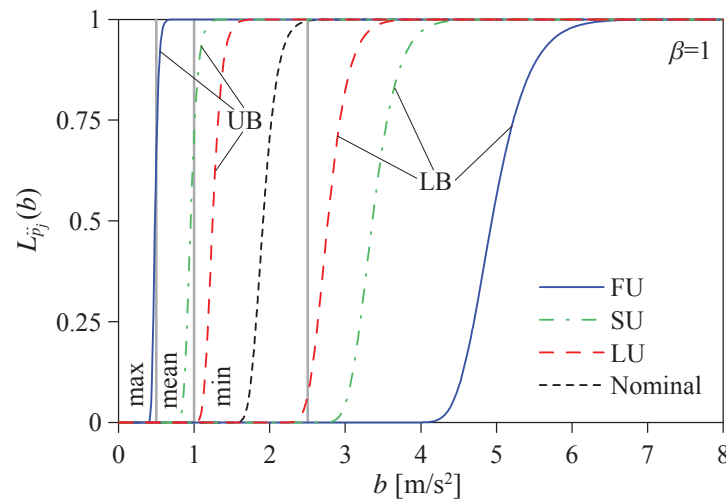


Figure 2 UB and LB of the CDF of the maximum acceleration ($\beta=1$) (Example 1).

Figure 3 provides the UB and LB of \ddot{q}_{\max}^I compared with the comfort limits provided by SETRA (a) and its *c.i.u.* (b) for increasing degree of uncertainty. Figure 3(a) shows that, if all the parameters are considered as uncertain and the maximum degree of uncertainty is assumed, then the interval of variation of the response is so wide that the footbridge could fall in any comfort class (from unacceptable to maximum comfort). If only the uncertainties of the loading are considered, the footbridge is almost surely in a minimum comfort class, as predicted by SETRA guideline. As regards structural uncertainties, if the degree of uncertainty is small then interval serviceability assessment provides a minimum comfort class, while for large degree of uncertainty a comfort level ranging from minimum to unacceptable can be obtained. From Fig. 3(b), it can be deduced that the *c.i.u.* of the maximum acceleration has the same trend as the one of the standard deviation of the response (Fig. 1(b)). Also from a quantitative point of view, the two *c.i.u.* are almost coincident: the uncertainty in the maximum acceleration is then governed by the uncertainty in the standard deviation.

In order to assess the influence of uncertainties on serviceability assessment of footbridges less sensitive to human-induced vibrations, a second footbridge is analyzed. The same parameters listed in Table 2 are considered, except for the natural circular frequency ω_j : its nominal value is not coincident with the nominal value of the mean step circular frequency,

but it is assumed $2\pi \cdot 2.17$ rad/s. Figure 4 is analogous to Figure 3, but it is referred to this second example. Results are similar to the ones obtained for Example 1 from a qualitative point of view. From a quantitative point of view, Figure 4(a) shows that the comfort level of this footbridge is surely higher than the one of Example 1. In particular, if the degree of uncertainty is small ($\beta < 0.25$) the footbridge comfort class can range between mean and minimum. However, if all the parameters are considered as uncertain and the maximum degree of uncertainty is assumed, then the interval of variation of the response is so wide that also this footbridge could fall in any comfort class (from unacceptable to maximum comfort). Figure 4(b) shows that the coefficient of interval uncertainty is even larger than in Example 1. In particular, if all the uncertainties are taken into account, for high degree of uncertainty the *c.i.u.* tends to an asymptotic value.

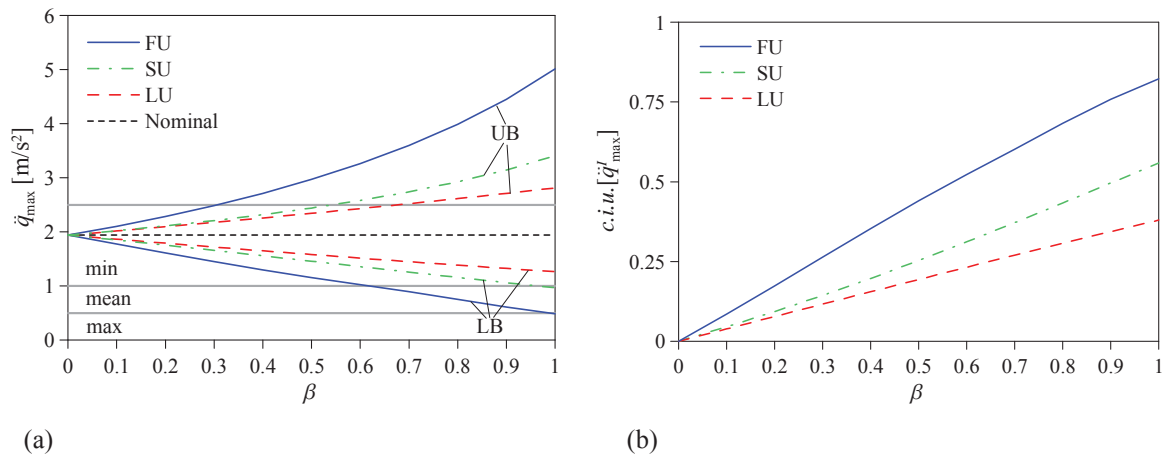


Figure 3: UB and LB of mean value of the maximum acceleration (a) and its *coefficient of the interval uncertainty* (b) for increasing degree of uncertainty (Example 1).

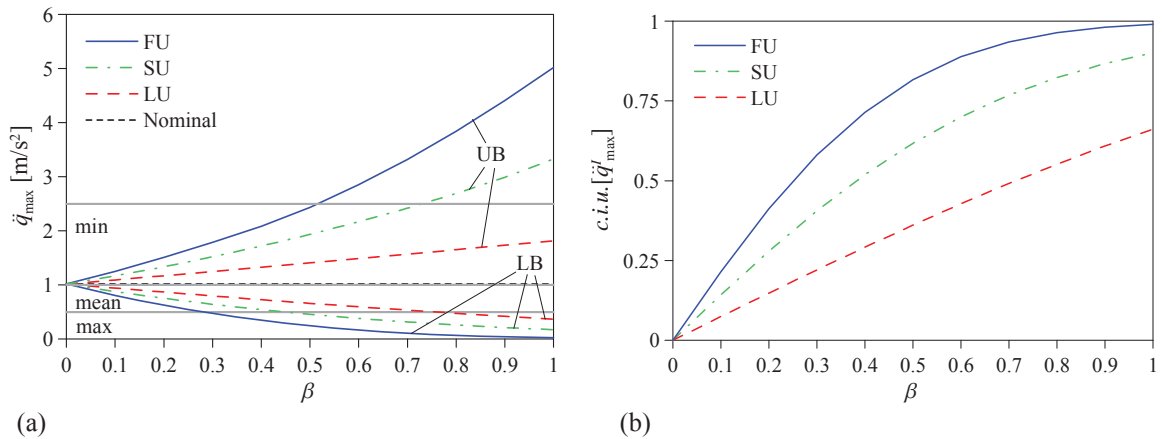


Figure 4: Non resonant case: UB and LB of mean value of the maximum acceleration (a) and its *coefficient of the interval uncertainty* (b) for increasing degree of uncertainty (Example 2).

5 CONCLUSIONS

Serviceability assessment of footbridges is studied through a non-deterministic approach, modelling structural and loading parameters as interval variables. Based on this approach, the standard deviation of the footbridge acceleration and the mean value of the maximum accel-

eration are also interval variables, while the cumulative distribution function of the maximum acceleration is an interval function.

The application to an ideal case study has shown that the high degree of uncertainty in the loading and structural parameters leads to very large intervals of variation of the response. If all the parameters are considered as uncertain and the maximum degree of uncertainty is assumed, then the interval of variation of the response is so wide that the footbridge could fall in any comfort class (from unacceptable to maximum comfort). The separate analysis of the role of the structural and loading parameters has shown that uncertainty in the structural parameters provides a determinant contribution to the variation of the response. In order to obtain narrow intervals of variation of the response, a reliable characterization of the loading and of the structural properties is mandatory.

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