

NUMERICAL EVALUATION OF THE SOUND TRANSMISSION OF A STRUCTURAL SYSTEM WITH ATTACHED NOISE CONTROL TREATMENTS USING STATISTICAL ENERGY ANALYSIS

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Abstract. *Acoustical and structural systems are often found in engineering, whose importance of dynamic responses to random excitation is extended to high frequencies. On vibroacoustics, the prediction of the sound transmission loss and damping of a structural element in full spectral range is important for design purposes in many areas, like the automotive and aerospace industry. In the context of noise control, Statistical Energy Analysis (SEA) coupled with the Transfer Matrix Method (TMM) is widely used in high-frequency modeling of multilayered systems. Two mechanisms are involved in the application of a noise control treatment: the sound insertion loss (IL) and the damping loss factors of the equivalent systems. The first one is related to the transmission loss (TL) of the layers and, conceptually, is defined as the ratio between the acoustic TL of an treated panel over the TL of the panel without the attached treatment. The second one is related to the dissipative part that imposes acoustic or structural damping to the medium in contact with the treatment surfaces. This work presents a theoretical review involving the SEA approach coupled with the TMM for modeling the Sound Transmission Loss of treated subsystems. Additionally, implementing strategies are presented for considering this kind of wave approach in a SEA model. The results obtained from the SEA-Toolbox, a new open-source code developed in this project, are compared with experimental data.*

1 INTRODUCTION

Acoustical and structural systems are often found in engineering, whose importance of dynamic responses to random excitation is extended to high frequencies. Examples of such situations include:

- The structural response of an aerospace payload in flight;
- The vibration result of a structure under diffuse field conditions;
- The calculation of sound pressure into a car;
- The dynamical analysis of an aircraft structure (fuselage) due to jet noise and other dynamical sources;
- The sound transmission between partitions in a building;
- The knowledge of marine structures excited by intense vibration.

Statistical Energy Analysis (SEA) can be used for modeling a given system as an interaction of modes associated with isolated parts of the system (subsystems), considering each mode group as a single-degree-of-freedom oscillator described by the average energy response. The responses are obtained from the analysis of the energy distribution through the system, excited by a power source. This technique is based on the power balance equations for each mode group [1].

Some hypotheses are assumed in the determination of a model in SEA. The most important is that the vibration energy varies from one subsystem to another with a rate proportional to the difference between the modal energy and the proportionally dissipated energy within each subsystem. Another important hypothesis assumes weak coupling between subsystems. The weak coupling condition is valid if the internal dissipation of the subsystem is more important than the coupling path in the total loss of energy. It makes the ratio between the coupling factors and the internal dissipation factors considerably less than 1. Thus, the energy flow between the subsystems may be based on the difference between the energies of the decoupled subsystems [2, 3]. Additionally, it is important to ensure that the input powers are uncorrelated, and they can be simply summed. It guarantees the linearity of the equations that govern the SEA problem.

This paper concerns on the presentation and validation of the SEA Toolbox, an open source code that has been developed in the present project. In this version, the main connection types are implemented between beams, plates and acoustic subsystems. From the coupling between SEA and the Transfer Matrix Method (TMM) [4, 5], it is also possible to model plates and cavities with attached noise control treatments. An overview concerning the implementation of the method is presented and the Transmission Loss of treated subsystems obtained from the in-house code is compared with experimental data collected from the bibliography [6].

2 FUNDAMENTALS

In SEA the system is divided into physical elements in which the vibroacoustic characteristics, such as damping, excitation and coupling properties, can be considered similar. Subsystems directly excited by external sources will transmit a portion of energy to the others by means of coupling characteristics. The main mechanism in SEA is based on the relationship between power inputs and outputs from each subsystem. Figure 1 shows a simple problem of sound transmission loss in the SEA point of view.

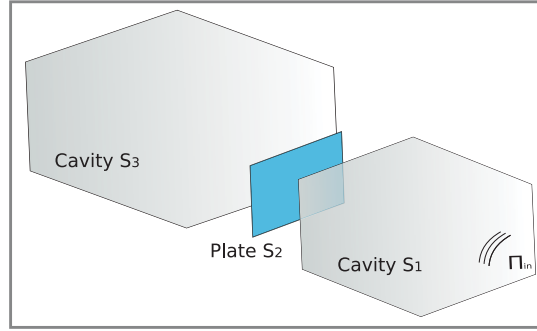


Figure 1: Example of a sound transmission loss problem.

Figure 1 shows the coupling of 3 SEA subsystems. However, it is common to have more robust models with a high number of elements. Considering a subsystem i coupled to a number of n subsystems, the power flow is [1, 2]

$$\Pi_i^{in} = \Pi_i^{absorbed} + \sum_{j \neq i}^n \Pi_{i;j}^{transmitted} . \quad (1)$$

Equation (1) shows that there are a dissipated portion of the input power into a subsystem (controlled by the dissipation loss factor) and a transmitted portion of this power, controlled by the coupling loss factors. The transmitted powers by the coupling between subsystems will depend on the radiation characteristics and differences between the environments.

Considering a 3-subsystems problem, where the subsystem 1 is excited by an input power, as shown in Figure 1, the power flow can be illustrated by Figure 2.

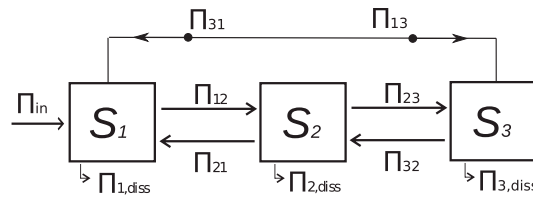


Figure 2: SEA flow for the cavity-plate-cavity problem.

It is possible to obtain the equations of the power flow illustrated in Figure 2, for each subsystem, as

$$\begin{aligned} \text{Subsystem 1: } & \Pi_{in} + \Pi_{21,trans} + \Pi_{31,trans} = \Pi_{1,diss} + \Pi_{12,trans} + \Pi_{13,trans} \\ \text{Subsystem 2: } & \Pi_{12,trans} + \Pi_{13,trans} = \Pi_{2,diss} + \Pi_{21,trans} + \Pi_{23,trans} \\ \text{Subsystem 3: } & \Pi_{23,trans} + \Pi_{13,trans} = \Pi_{3,diss} + \Pi_{31,trans} + \Pi_{32,trans} . \end{aligned} \quad (2)$$

The power Π is related to the energy from

$$\Pi_i = \omega \eta_i E_i , \quad (3)$$

where ω is the angular frequency, η is the loss factor and E is the energy of a subsystem. Then, the dissipated power and the transmitted power will be given by

$$\Pi_{i,diss} = \omega \eta_{i,diss} E_i \quad [i = 1, 2, 3] ; \quad (4)$$

$$\Pi_{ij,trans} = \omega \eta_{ij} E_i \quad [i \neq j]. \quad (5)$$

being η_{ij} the coupling factor between the subsystems i and j .

Substituting Equation (4) and (5) into (2), the following linear system is obtained

$$\omega \begin{bmatrix} \eta_{1,diss} + \eta_{12} + \eta_{13} & -\eta_{12} & -\eta_{13} \\ -\eta_{21} & \eta_{2,diss} + \eta_{21} + \eta_{23} & -\eta_{23} \\ -\eta_{31} & -\eta_{32} & \eta_{3,diss} + \eta_{31} + \eta_{32} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} \Pi_{in} \\ 0 \\ 0 \end{Bmatrix} . \quad (6)$$

It can be observed that the SEA method is based on the resolution of linear systems for calculating the energy for each subsystem. The determination of damping variables for more complex systems is based on theoretical models or from direct experimental data. However, if the modal density of the subsystems involved in the problem are known, the coupling loss factor in one direction is required, and the following consistency ratio is valid to complement the damping matrix

$$n_i \eta_{ij} = n_j \eta_{ji} , \quad (7)$$

where n_i e n_j are the modal densities of the subsystems i and j , respectively. Theoretical models for the modal densities of flat plates and rectangular cavities can be found in [1, 3].

For the problem illustrated in Figure 1, the mean square value of the pressure inside the cavities $p_i^2 = E_i \rho_i c_i^2 / V_i$ can be recovered from the solution, with ρ_i being the fluid density, c_i the sound velocity and V_i the volume of the cavity $i = 1, 3$. The transmission loss of the partition can be calculated as [7]

$$TL = 10 \log_{10} \left(\frac{p_1^2}{p_3^2} \right) + 10 \log_{10} \left(\frac{S}{A_{eq}} \right) , \quad (8)$$

where $A_{eq} = 4\omega V_3 \eta_{3,diss} / c_3$ is the equivalent absorption area of the reception cavity and S is the area of the panel.

2.1 The cavity-plate-cavity connection

The coupling between the three subsystems showed in Figure 1 is controlled by different propagation paths. The most important ones are controlled by the resonant and non-resonant sound transmission. The first one is affected by the radiation capacity of the structural system and is important in frequencies above the coincidence spectral region of the plate. On the other hand, the non-resonant path is controlled by the mass of the structure and it is important for global responses below the coincidence frequency [8]. The radiation loss factor that models the connection between plate s and cavity a is given by

$$\eta_{sa} = \eta_{rad} = \frac{\rho_a c_a S_s}{\omega M_s} \sigma_{rad} , \quad (9)$$

with ρ_a being the fluid density, c_a the fluid sound velocity, S_s the area of the partition, M_s the structural mass and σ_{rad} the radiation efficiency of the structural system that can be recovered in theoretical models as that one proposed by Leppington [9]. The non-ressonant loss factor that models the coupling between two cavities i and j is

$$\eta_{ij} = \eta_{mass} = \frac{c_i A_i \tau_m}{4 \omega V_i}, \quad (10)$$

where A is the total surface area, V is the volume of the cavity and τ_m is the mass controlled transmission coefficient of the system. In this work, the field incidence transmission coefficient presented in [8] was used.

2.2 Noise control treatments modeling using transfer matrices

Multilayer panels are often used in the context of noise control. The mechanisms involved in the application of a noise control treatment (NCT) on a face of a structural system are: the sound insertion loss (IL) that can change the acoustic radiation and the mass law transmission of the structure, the sound absorption seen by the acoustic medium in contact with the surface of the treatment and the added structural damping. The methodology of the Transfer Matrix Method (TMM) [4] is based on the theory of plane wave propagation in a given layer. The material of each layer is assumed homogeneous, transversely isotropic and the surface of the treatment is considered infinite in this formulation. Figure 3 shows an incident plane acoustic wave upon a media with thickness h , at an incident angle θ .

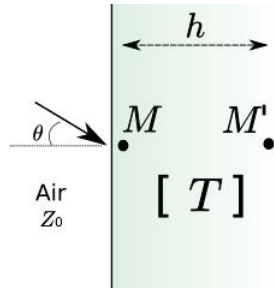


Figure 3: Illustration of an infinite extent layer on the Tranfer Matrix Method.

The sound propagation inside the layer is represented by a transfer matrix T such that

$$V(M) = [T]V(M'), \quad (11)$$

where V is a vector containig the response for each degree of freedom of the media at the points M and M' set close to the foward and the backward of the layer [4]. The matrix T depends on the physical proprieties as well as the thickness of the medium. If considering the layer illustrated in Figure 3 as an equivalent fluid, the vector $V = \{p, v\}^T$ is composed by the pressure and the particle velocity. The transfer matrix T^f , in this case, is given by

$$[T^f] = \begin{bmatrix} \cos(k_x h) & j \frac{\omega \rho_0}{k_x} \sin(k_x h) \\ j \frac{k_x}{\omega \rho_0} \sin(k_x h) & \cos(k_x h) \end{bmatrix}, \quad (12)$$

where $k_x = \sqrt{k_c^2 - k_0^2 \sin^2(\theta)}$ is the longitudinal component of the wavenumber into the layer, k_c being the propagating wavenumber of the material, k_0 and ρ_0 the wavenumber and density

of the air. The continuity between adjacent layers is applied and a global transfer matrix $[T_g]$ is constructed according to the references [4] and [5]. The acoustic properties depend on the surface impedance computed from the ratio between pressure and velocity $p(M)/v(M)$, obtained from the solution of a linear system $V = [T_g]^{-1}\{F\}$.

2.2.1 Insertion Loss (IL) from the TMM

The Insertion Loss is defined as the ratio between the acoustic TL of a panel with the attached treatment over the TL of the untreated panel. This parameter can be calculated from the TMM simulation if considering a semi-infinite fluid as the boundary condition. The insertion coefficient IC will be given by

$$IC(\omega) = \frac{\int_0^{\theta_f} |T_{NCT}(\omega, \theta)|^2 \sin(\theta) \cos(\theta) d\theta}{\int_0^{\theta_f} |T_{bare}(\omega, \theta)|^2 \sin(\theta) \cos(\theta) d\theta}, \quad (13)$$

where θ_f is the final incidence angle, T_{NCT} and T_{bare} are the transmission coefficients of the treated and untreated infinite extent systems, respectively. In order to solve this problem, the bare panel is considered as the first solid layer in the TMM simulation. The insertion loss $IL(\omega) = -10 \log_{10}(IC(\omega))$ is recovered from the insertion coefficient. Finally, the insertion loss from TMM is applied as a scaling factor to the radiation and the mass law coupling loss factors in the SEA problem [5], then

$$\eta_{rad} = \eta_{rad} IC(\omega) = \eta_{rad} (10^{-IL(\omega)/10}); \quad (14)$$

$$\eta_{mass} = \eta_{mass} IC(\omega) = \eta_{mass} (10^{-IL(\omega)/10}). \quad (15)$$

With *SEA-Toolbox*, it is possible to model equivalent fluids, solids and poroelastic materials, over the incidence range from 0 to $\pi/2$.

3 METHODOLOGY

The present study consists of testing the functionality of Statistical Energy Analysis coupled with the Transfer Matrix Method for Transmission Loss problems. The SEA simulation of two acoustic cavities with a multilayered partition was done and the obtained results were compared with experimental data extracted from reference [6]. The experimental study was developed at the Laboratory of Vibration and Acoustics of the Federal University of Santa Catarina - Brazil and followed the guidelines of the ISO 10140-2 standard [7]. Kulakauskas [6] presented a numerical and experimental evaluation of the sound transmission loss for single and double-wall setups using different porous samples.

In this work, the single and double-wall setups were reproduced in the new open-source code (*SEA-Toolbox*) using two different porous samples, considering the SEA implicit approach. It means that the multi-layer structures were represented by applying the obtained Insertion Loss from the TMM analysis to the SEA coupling factors (see Equations 9 and 10), as illustrated in Figure 4.

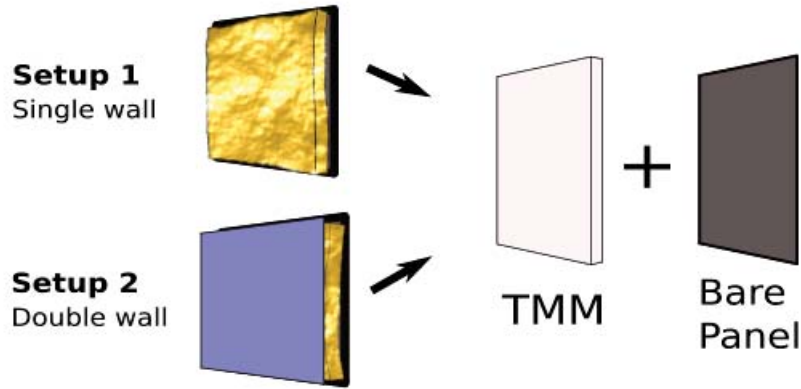


Figure 4: Illustration of the simulated systems.

Based on the experimental procedure, the porous samples were modeled as simply supported limp fibers. The Johnson-Champoux-Allard model [10, 11] was used for the characterization of the fluid part in addition to the equivalent density calculated from the simplification of the Biot's Theory for low-stiffness framed materials [4]. This approach avoids artificial clamping between the plate and the porous material, but in case of using the full Biot's Theory (poroelastic material), an air gap should be applied between the structure and the foam to simulate the cited mounting condition. The properties for the solid structures and fibers considered in this work are presented in Table 3.

Table 1: Material properties of the setups considered in this work.

| | Solid | | Porous | |
|--|----------------|----------------|-------------------|-------------------|
| | Plate 1 | Plate 2 | Material A | Material B |
| Thickness h [mm] | 1.0 | 0.5 | 50 | 58 |
| Young's Modulus E [Pa] | 71E9 | | - | - |
| Poisson's ratio ν [-] | 0.33 | | - | - |
| Density ρ [kg/m ³] | 2700 | | 27 | 9.6 |
| Flow Resistivity σ [kN/m ⁴] | - | | 5 | 38.72 |
| Porosity ϕ [-] | - | | 0.89 | 0.92 |
| Tortuosity α_∞ [-] | - | | 1.37 | 1.06 |
| Viscous charac. length Δ [μ m] | - | | 213 | 37 |
| Thermal charac. length Δ' [μ m] | - | | 288 | 148 |

As already mentioned, the SEA simulation consists of 3 subsystems illustrated by Figure 1. The emission (S1) and reception (S3) cavities have a volume of 147.54 m³ and 199.0 m³, based on the dimensions of the experimental reverberant chambers [6]. The total surface area A_1 and perimeter P_1 of the cavity S1 are 190 m² and 70.44 respectively. For the cavity S3, these geometrical dimensions are $A_3 = 210$ m² and $P_3 = 72.0$ m. The total surface area of the bare panel is $S_2 = 1.0$ m². An acoustic power source of 1 Watt was applied in the S_1 subsystem. The total mean absorption of the non-treated surfaces of the cavities was considered 1%, as well as the dissipation loss factor of the solid plate.

A propagation way that can be important in experimental procedures is the leakage due to the mounting conditions. According to Beranek [8], this phenomena causes a reduction in the

Insertion Loss that can be simply modeled as

$$\Delta IL_L = IL_s - IL_L \approx 10 \log_{10}(1 + \beta 10^{TL_s/10}) \quad (16)$$

where IL_s and IL_L are the insertion losses of the sealed and leaky systems, TL_s the transmission loss of the system and β is the leak ratio factor. Experimentally, the influence of this phenomena becomes more important in high frequencies [6]. In this paper, the application of the model presented in Equation 16 was evaluated in the Transmission Loss of the double wall systems, considering $\beta = S_{leak}/S_{plate} = 2.0 \text{ E} - 6$. The determination of S_{leak} was done by adjusting experimental data. The analyses are presented in Section 4.

4 RESULTS

The results were obtained in one third-octave band, from 250 Hz to 8000 Hz. Although the damping of the subsystems in contact with the surfaces of the NCT can be changed when applying the noise control treatment, these parameters are not important for this work. Since the treated acoustic surface is too small if compared to the total surface area of the reception cavity, the mean absorption of the subsystem 3 would not be significantly altered in this scenario. On the other hand, the loss factor of the structural system can be important for the sound transmission on frequencies above the critical frequency ($\approx 12 \text{ kHz}$) for this panel. Therefore, the Insertion Loss (IL) of the treatment is the only parameter that contributes significantly to the numerical transmission loss of the complete system. Figure 5 shows the infinite size insertion loss for the single and double-wall setups, considering the Material A and Material B, obtained from the Transfer Matrix Method.

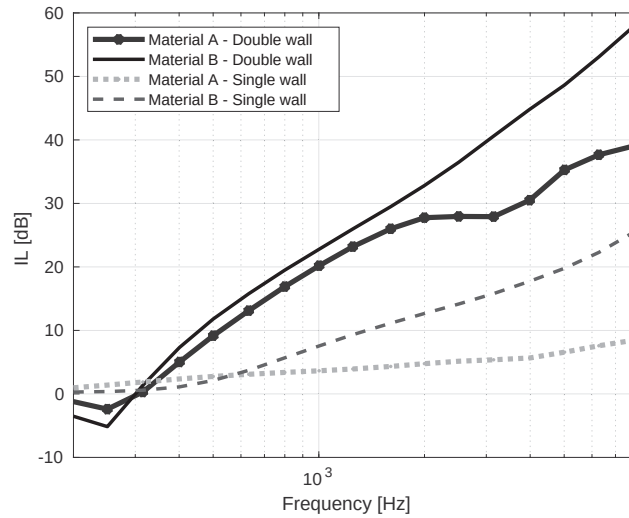


Figure 5: Insertion Loss obtained from the TMM for the studied systems.

It can be noticed in Figure 5 that the formulation of transfer matrices is capable of capturing the poor performance of the double-wall configuration in frequencies below the resonance of the spring-mass system ($f \approx 300 \text{ Hz}$), represented by a negative value of the IL. The results also show the importance of the flow resistivity parameter for improving the Insertion Loss of the system, which is expected since this parameter controls the barrier capacity of the porous material. The Insertion Loss was applied to the coupling factors, as presented in Equations 10

and 9. The Transmission Loss results calculated for the single-wall configurations (plate + porous) are presented in Figure 6.

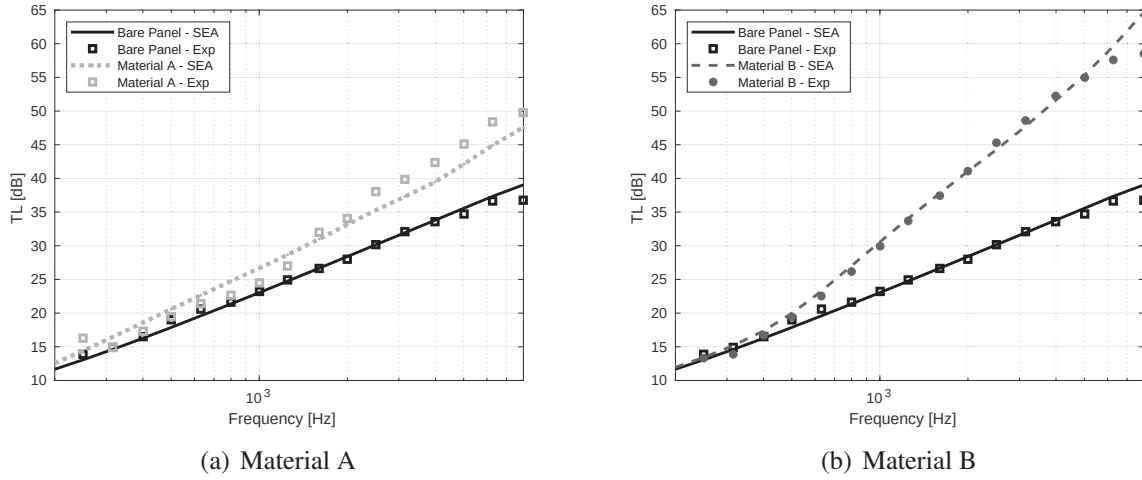


Figure 6: Sound Transmission Loss obtained for the single-wall configurations.

Figure 6 shows good agreement between numerical and experimental data for the considered single-wall systems, which indicates the efficiency of the presented modeling approach for flat structures. It can be noticed in the obtained result for Material B the drop of TL in high frequency that probably occurs due to the leakage of the sound wave in the experimental condition. This characteristic is more evident in Figure 7, that brings the collected Transmission Loss for the double-wall setups (plate - porous - plate) with and without the leakage correction applied to the numerical data.

Figure 7 shows that the greater the complexity of the multilayers, the more difficult it becomes to obtain a perfectly sealed condition from an experimental point of view. It can be seen in Figures 7 (a) and 7 (c) that the experimental TL of the different systems converges in high frequencies even though it does not occur when the same samples were used in the single-panel setups. It suggests greater importance of leaks in the double wall configurations. Although adjusting the parameter β imposes uncertainties, Figures 7 (b) and 7 (d) shows that the use of a model for considering this propagation path is necessary when the presence of leaks becomes evident in the experimental results. Nevertheless, the method of SEA/TMM can be a useful tool when due care is taken.

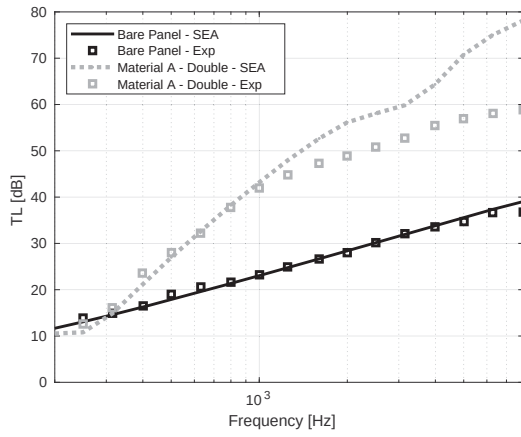
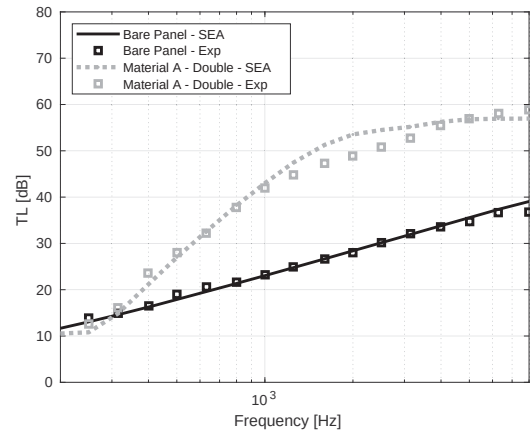
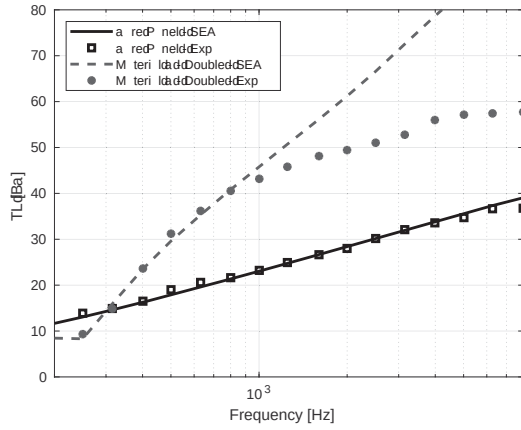
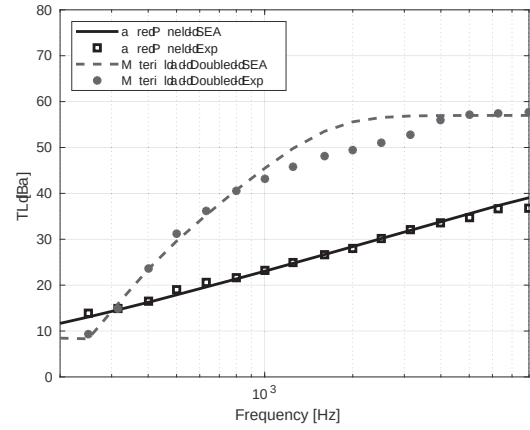
(a) Material A - without correction β (b) Material A - with correction β (c) Material B - without correction β (d) Material B - with correction β

Figure 7: Sound Transmission Loss obtained for the double-wall setups.

5 CONCLUSIONS

This paper presented an overview of the transmission loss characterization of multilayered structures using Statistical Energy Analysis and the Transfer Matrix Method. The Insertion Loss of the treated systems was calculated by using the TMM and applied to the coupling loss factors of the SEA matrix. Additionally, a model for leaks was presented and applied to the TL results for simulating experimental conditions of a double-wall system composed by plate-porous-plate. The results obtained from the *SEA – Toolbox*, an open-source code developed in the present project, demonstrated good agreement with experimental data.

6 ACKNOWLEDGMENTS

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