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VIBRATION ABSORPTION PERFORMANCE OF METAMATERIAL LATTICES CONSISTING OF IMPACT DAMPERS

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Abstract. *In recent years, metamaterials are increasingly used in the field of vibration mitigation due to the unique potential these offer. Periodic placement of appropriately designed resonators, the so-called unit cells, has been shown to arrest the propagation of vibration within a specific frequency range, thus resulting in formation of a bandgap. In rendering metastructures efficient, the breadth of the bandgap should be maximized, while for application in structures, it is also necessary to shift the lower limit of the gap to lower frequencies. One potential solution to this direction is the use of nonlinearity at the unit cell scale. In this direction, the current study investigates finite lattice configurations, which consist of impact damper unit cells. The effectiveness of impact dampers in vibration attenuation has been successfully illustrated in limited degree of freedom systems, and is here extended for use in the metastructure sense, i.e., for multiple degree of freedom systems. For this purpose, a one dimensional lattice of a finite number of unit cells is considered. In order to study and better asses the behaviour of the configuration, variable parameters are taken into account, including the number of unit cells, stiffness and mass ratio, while a comparison to a conventional linear oscillator is further offered. For the evaluation of the system performance, several criteria are utilized, including the oscillation amplitude as well as the energy absorption from the primary system as a result of the individual impacts. The obtained results clearly indicate the ability of impact-based meta-structures to successfully contribute to vibration mitigation under appropriate design.*

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1 INTRODUCTION

In recent years, attention has turned to resilience of large-scale civil structures under dynamic loading, and in particular under seismic excitation. In addressing this challenge, the research community has focused on the development of vibration mitigation systems, which are able to ensure structural safety, by minimizing the effects of the excitation onto the structure.

Mitigation of engineering vibration has been widely dealt using linear, passive or active methods. As for the first, several concepts have been developed, among which some take advantage of the favorable effects that increased damping can offer [1, 2, 4], while others influence the nature of the system's dynamic response by introducing secondary devices, which are attached to the main system. Some of the common practices of such methods include oil dampers [5], friction dissipaters [3], tuned mass dampers [8]-[11], as well as tuned liquid dampers [12]. A major disadvantage of such devices, however, lies in the narrow range of frequencies within which their contribution is beneficial. Indeed, there exist frequency regions where the existence of passive devices can even affect the dynamic response in a negative way, as for example in the case of tuned mass dampers. On the other hand, the active control schemes overcome the former difficulties by offering the ability to adapt to diverse loading situations [6, 7, 13, 14]. Nonetheless, considerable disadvantages to such an active approach pertain to the significantly higher costs and the continuous need for maintenance.

A novel idea, which has recently been exploited for civil engineering applications, is based on the concept of metamaterials [17]-[26]. These are configurations, characterized by extraordinary filtering properties, attained as a result of diversified mechanisms, such as their microscopic geometry, periodic arrangement, etc [27]. Appropriately designed resonators, the so-called unit cells, have been shown to arrest the propagation of vibration within a specific frequency range [15]-[16], this resulting in formation of a "bandgap". In rendering metastructures efficient, the breadth of the bandgap should be maximized, while for application in structures, it is also necessary to shift the lower limit of the gap to lower frequencies. One potential solution to this direction is the use of nonlinearity at the unit cell scale. In this direction, following the work of Dertimanis et al. [28], the current study investigates finite lattice configurations, which consist of impact damper unit cells. The effectiveness of impact dampers in vibration attenuation has been successfully illustrated in limited degree of freedom systems [29], and is here extended for use in the metastructure sense, i.e., for multiple degree of freedom systems. The general term used for nonlinear attachments to the primary system is nonlinear energy sinks (NES). Extended research has been carried out regarding several types of NES, including impact dampers [29, 34, 35]. The dynamics of impact phenomena has been investigated in multiple studies [36]-[38]. Significant contribution in the subject of impact dampers has been made in the work of Masri and colleagues on impact dampers, regarding their general motion [30], their stability analysis [31], as well as their dynamic response to random excitation [32]. There is also high relevance between impact dampers and particle dampers, the latter being investigated in the work of Lu et al. [33].

2 META-DAMPER

2.1 Unit cell

The idea of the meta-impactor is inspired by the beneficial effect that a single impact damper can have on a structure, under dynamic loading [29]. The unit cell consists of a rigid container of mass m, containing a laterally unconstrained mass μ, free to impact on the respective bounds (Figure 1).

The nonlinearity in the response results from the contact between the container and the inner mass at the time instances where the relative displacement of the two equals to the free clearance $D/2$. This leads to the outcome of impact realizations, which introduce a highly nonlinear behaviour. Furthermore, since the interaction between the two masses results in sudden changes in their velocity, the phenomenon of impact can be classified as non-smooth.

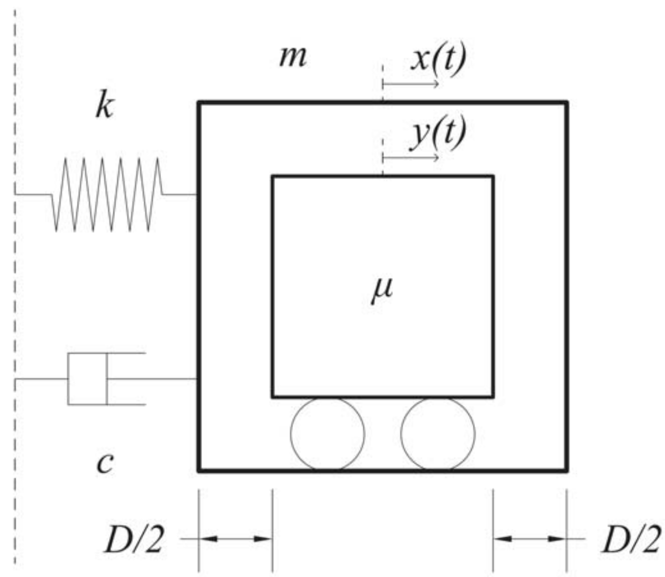

Figure 1: Impact damper unit cell.

2.2 Nonlinear lattice

The goals lies in shielding a target structure via coupling with the metamaterial configuration. The initial oscillator was a single degree of freedom system of mass M, stiffness K and damping C. The concept of the current work is based on the consecutive arrangement of impact damper unit cells. The resonators are connected elastically to each other with stiffness k, as well as to the protected mass on one end, and to a fixed support on the other, while damping c is also applied (Figure 2). In order to better assess the response of the system, excluding misleading effects from complicated input time histories, a harmonic excitation force $F(t)=A\sin(2\pi ft)$ was selected.

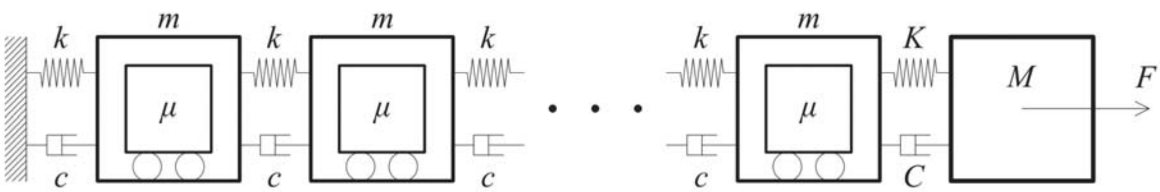

Figure 2: Nonlinear lattice.

2.3 Governing equations

The system is considered to comprise a linear behavior outside the impact events, so it can be described as a conventional, linear multiple degree of freedom system during those instances. It should be mentioned that there is no assumption regarding the number of impacts per cycle at the steady state.

The equations of motion for the first unit cell can be formulated as:

$$\begin{aligned} &m\ddot{x}_1(t) + c\dot{x}_1(t) + kx_1(t) + c[\dot{x}_1(t) - \dot{x}_2(t)] + k[x_1(t) - x_2(t)] = 0 \\ &\mu\ddot{y}_1(t) = 0, \ \ |x_1(t) - y_1(t)| < D/2 \end{aligned} \tag{1}$$

In the same pattern, the equations of motion for the i-th unit cell, $i = 2, ..., N-1$, are:

$$\begin{aligned} &m\ddot{x}_i(t) + c[\dot{x}_i(t) - \dot{x}_{i-1}(t)] + k[x_i(t) - x_{i-1}(t)] + c[\dot{x}_i(t) - \dot{x}_{i+1}(t)] + k[x_i(t) - x_{i+1}(t)] = 0 \\ &\mu\ddot{y}_i(t) = 0, \ \ |x_i(t) - y_i(t)| < D/2 \end{aligned} \tag{2}$$

For the last (N-th) unit cell the equations of motion are:

$$\begin{aligned} &m\ddot{x}_N(t) + c[\dot{x}_N(t) - \dot{x}_{N-1}(t)] + k[x_N(t) - x_{N-1}(t)] + C[\dot{x}_N(t) - \dot{X}(t)] + K[x_N(t) - X(t)] = 0 \\ &\mu\ddot{y}_N(t) = 0, \ \ |x_N(t) - y_N(t)| < D/2 \end{aligned} \tag{3}$$

whereas, for the primary structure they are formed as:

$$M\ddot{X}(t) + C[\dot{X}(t) - \dot{x}_N(t)] + K[X(t) - x_N(t)] = F(t) \tag{4}$$

Under the assumption of a sufficiently small duration of impact, it is further reasonable to consider that the post-impact displacement remains unaffected, opposite to the post impact velocities, which are altered discontinuously. In order to determine the latter, the coefficient of restitution r, related to the energy loss at every impact, is introduced [29]. Therefore, the additional equations describing the individual impacts can be defined as:

$$\begin{aligned} x_i^+ &= x_i^- \\ y_i^+ &= y_i^- \end{aligned} \tag{5}$$

$$r = -\frac{\dot{x}_i^+ - \dot{y}_i^+}{\dot{x}_i^- - \dot{y}_i^-} \tag{6}$$

where the coefficient of restitution ranges between $0 < r < 1$, zero representing purely plastic and unity perfectly elastic impacts [29]. Moreover, the conservation of momentum applies, which indicates:

$$m\dot{x}_i^+ + \mu\dot{y}_i^+ = m\dot{x}_i^- + \mu\dot{y}_i^- \tag{7}$$

Use of Eqs. 6, 7, results in the calculation of post impact velocities for the i-th unit cell.

$$\begin{bmatrix} m & \mu \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \dot{x}_i^+ \\ \dot{y}_i^+ \end{bmatrix} = \begin{bmatrix} m\dot{x}_i^- + \mu\dot{y}_i^- \\ -r\dot{x}_i^- + r\dot{y}_i^- \end{bmatrix} \tag{8}$$

2.4 Simulation

In order to solve the system of differential equations, the system is brought into a nonlinear state space form as:

$$\begin{aligned} &\dot{z}(t) = g(z(t), F(t)), \text{ where} \\ &z = \left[x_1, \dot{x}_1, y_1, \dot{y}_1 \mid \ \dots \ \mid x_N, \dot{x}_N, y_N, \dot{y}_N \mid X, \dot{X} \right]^T \end{aligned} \tag{9}$$

Subsequently, the derivatives of z for the first unit cell are calculated as:

$$
\begin{aligned}
&\dot{z}_1(t) = z_2(t)\\
&\dot{z}_2(t) = -\frac{k}{m}z_1(t) - \frac{c}{m}z_2(t) - \frac{k}{m}[z_1(t) - z_5(t)] - \frac{c}{m}[z_2(t) - z_6(t)]\\
&\dot{z}_3(t) = z_4(t)\\
&\dot{z}_4(t) = 0, \quad |z_1 - z_3| < D/2
\end{aligned}
\tag{10}
$$

Similarly, for the the *i*-th unit cell, $i = 2, ..., N-1$, these are:

$$
\begin{aligned}
&\dot{z}_{4(i-1)+1}(t) = z_{4(i-1)+2}(t)\\
&\dot{z}_{4(i-1)+2}(t) = -\frac{k}{m}[z_{4(i-1)+1}(t) - z_{4(i-2)+1}(t)] - \frac{c}{m}[z_{4(i-1)+2}(t) - z_{4(i-2)+2}(t)]\\
&\qquad\qquad\qquad -\frac{k}{m}[z_{4(i-1)+1}(t) - z_{4i+1}(t)] - \frac{c}{m}[z_{4(i-1)+2}(t) - z_{4i+2}(t)]\\
&\dot{z}_{4(i-1)+3}(t) = z_{4(i-1)+4}(t)\\
&\dot{z}_{4(i-1)+4}(t) = 0, \quad |z_{4(i-1)+1} - z_{4(i-1)+3}| < D/2
\end{aligned}
\tag{11}
$$

and for the last (*N*-th) unit cell, these are respectively obtained as:

$$
\begin{aligned}
&\dot{z}_{4N-3}(t) = z_{4N-2}(t)\\
&\dot{z}_{4N-2}(t) = -\frac{k}{m}[z_{4N-3}(t) - z_{4N-7}(t)] - \frac{c}{m}[z_{4N-2}(t) - z_{4N-6}(t)]\\
&\qquad\qquad -\frac{K}{m}[z_{4N-3}(t) - z_{4N+1}(t)] - \frac{C}{m}[z_{4N-2}(t) - z_{4N+2}(t)]\\
&\dot{z}_{4N-1}(t) = z_{4N}(t)\\
&\dot{z}_{4N}(t) = 0, \quad |z_{4N-3} - z_{4N-1}| < D/2
\end{aligned}
\tag{12}
$$

while for the linear oscillator are:

$$
\begin{aligned}
&\dot{z}_{4N+1}(t) = z_{4N+2}(t)\\
&\dot{z}_{4N+2}(t) = \frac{K}{M}[z_{4N-3}(t) - z_{4N+1}(t)] + \frac{C}{M}[z_{4N-2}(t) - z_{4N+2}(t)] + \frac{1}{M}F(t)
\end{aligned}
\tag{13}
$$

The post impact velocities are computed as:

$$
\begin{bmatrix} m & \mu \\ 1 & -1 \end{bmatrix}
\begin{bmatrix} z^{+}_{4(i-1)+1} \\ z^{+}_{4(i-1)+2} \end{bmatrix}
=
\begin{bmatrix} mz^{-}_{4(i-1)+1} + \mu z^{-}_{4(i-1)+2} \\ -rz^{-}_{4(i-1)+1} + rz^{-}_{4(i-1)+2} \end{bmatrix}
\tag{14}
$$

3 ANALYSIS

The behaviour of the system was studied parametrically, calculating its response for each set of parameters. The analysis was performed using a MATLAB® environment, with integration carried out by means of the *ode45* function (AbsTol=eps, ReTol=eps$^{(2/3)}$, Refine=4 and MaxStep=0.001/sqrt(K/M), eps corresponding to the machine precision epsilon). Moreover, an event function is used, checking the condition $|x_i(t) - y_i(t)| = D/2$ for all unit cells. If at least one impact is detected, Eqns. 5, 8 are deployed for the calculation of post impact displacements and velocities.

The properties of the linear oscillator were chosen as M=1kg, f_n=1Hz and ζ_n=2%, while the harmonic excitation was chosen as $F(t)=\sin(2\pi t)$, with a unit amplitude and an excitation frequency matching the natural frequency of the linear oscillator. Furthermore, the coefficient of restitution is set constant at r=0.6, the clearance is set at D=0.1m and the damping between unit cells is set at c=0, throughout all parametric studies.

In order to better asses the effect of each parameter given to the system, parametric analyses were performed. Varying mass (m/M, μ/m) and stiffness (K/k) ratios were taken into consideration, in the range of $0.01\leq m/M \leq 1.00$, $0.10\leq \mu/m \leq 1.00$ and $10\leq K/k \leq 100$ respectively.

The response of each system is calculated for the first 10 seconds. For the evaluation of the system's performance, several criteria are utilized, including the oscillation amplitude, maximum absolute acceleration, as well as the energy absorption from the primary system as a result of the individual impacts. The latter is calculated as the ratio between the total energy dissipation, resulting from all individual impacts, and the total energy inserted to the system by the external excitation [29].

$$E_{dis}(t) = \frac{\sum_{i=1}^{N-1} \int_0^t c[\dot{x}_i(\tau) - \dot{x}_{i+1}(\tau)]^2 d\tau + \sum_{p=1}^{P_t} \Delta p}{\int_0^t F(\tau)\dot{\mathrm{X}}(\tau) dt} \tag{15}$$

where Δp is the amount of energy dissipation at the p-th impact, p=1,…, P_t and P_t is the number of impacts at any unit cell up to time t. Since in the current investigation c=0, the first part of the numerator is excluded.

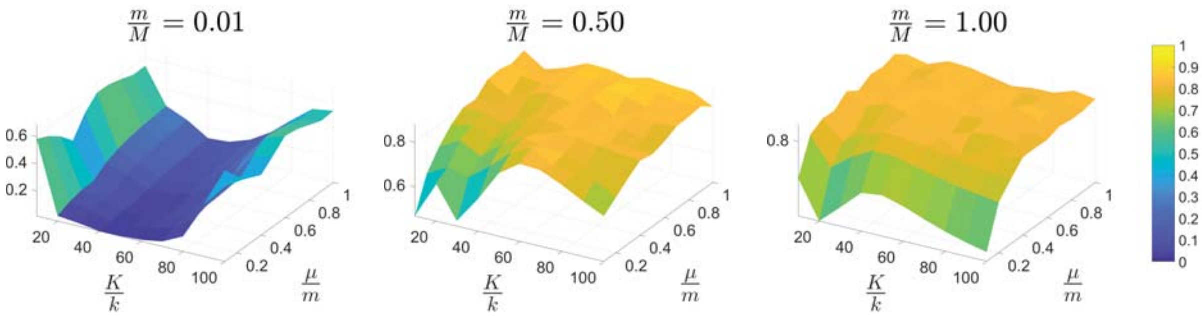

Figure 3: Energy absorption at 1 Hz for N=1 for varying mass and stiffness.

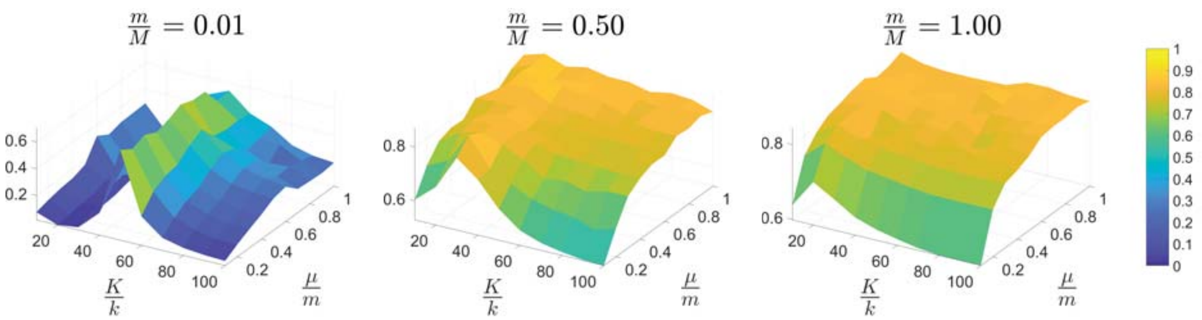

Figure 4: Energy absorption at 1 Hz for N=2 for varying mass and stiffness.

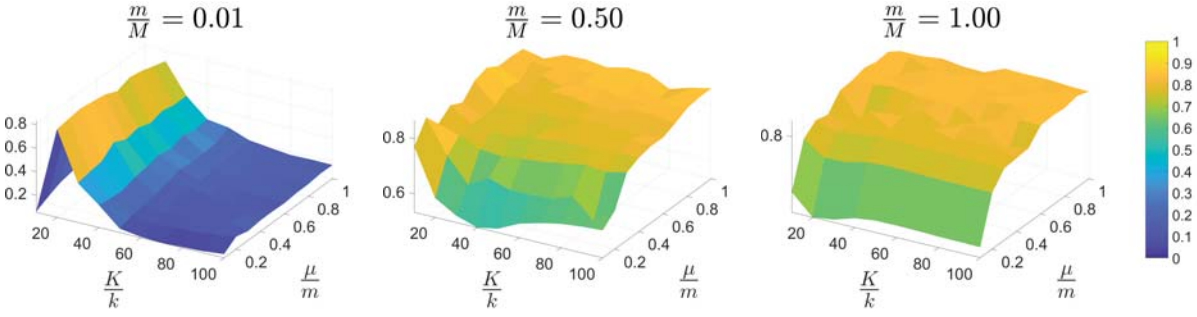

Figure 5: Energy absorption at 1 Hz for N=5 for varying mass and stiffness.

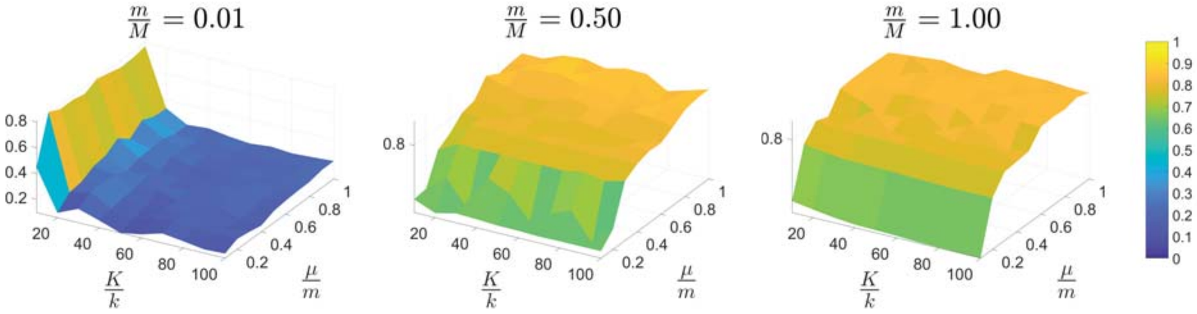

Figure 6: Energy absorption at 1 Hz for N=10 for varying mass and stiffness.

In Figs. 3-6 the results of energy absorption are presented. It is evident that for lower values of external mass ratio m/M the behavior of the system is dominated by the stiffness ratio. In those cases, the variation of the internal mass ratio μ/m does not significantly affect the energy absorption. As the external mass ratio increases, the importance is shifted towards the internal mass ratio as well, while for m/M=1, the internal mass ratio has the principal influence. As the number of unit cells increases, in combination with high values of external mass ratio, the amount of dissipated energy tends to become uniform and unaffected from the variation of the number of unit cells, as it can be inferred from Figs. 5, 6 for m/M=1.

Additional parametric analyses were carried out investigating the influence of the variation in the excitation frequency. For this purpose, the system's parameters were set to m/M=1, μ/m=0.1 and K/k=10. The simulation time is increased to 50s and the maximum absolute displacement and acceleration of the last 40s were recorded for the primary structure. Throughout these analyses, the excitation frequency varies between $0.1 \leq f\,/\,f_n \leq 4.0$.

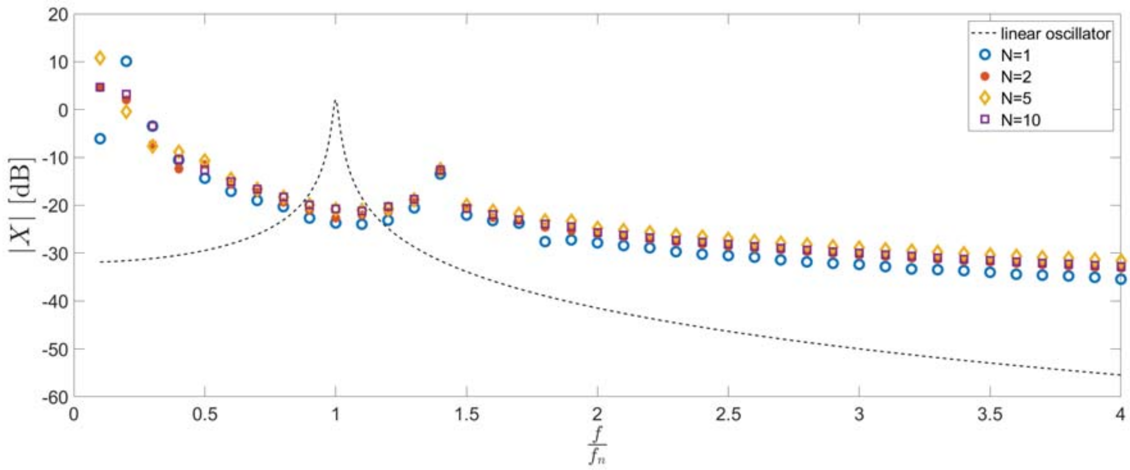

Figure 7: Maximum absolute displacement of the primary structure as a function of the applied frequency ratio.

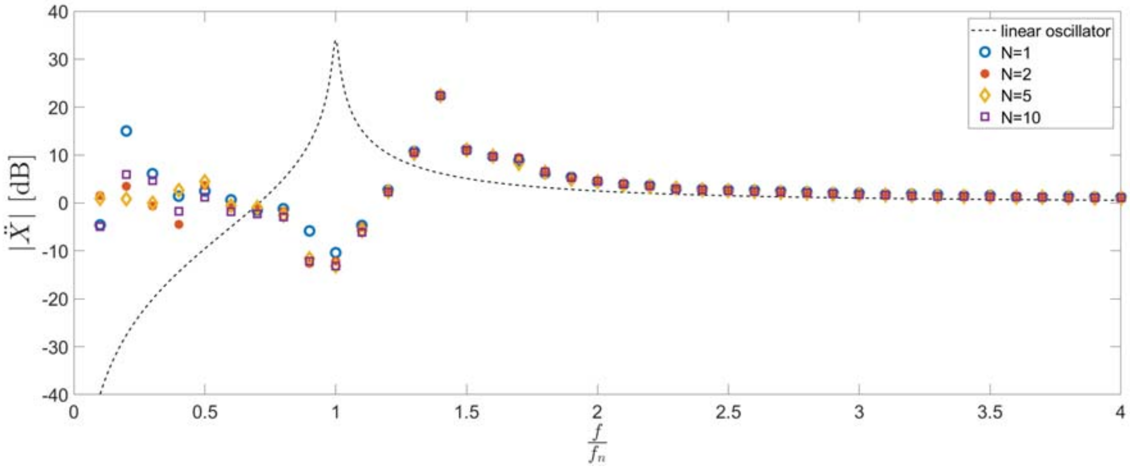

Figure 8: Maximum absolute acceleration of the primary structure as a function of the applied frequency ratio.

As inferred from Figs. 7, 8, a substantial mitigation of the dynamic load effect on the structure is observed for frequency ratios between 0.7 and 1.2. Considering that the damping between unit cells is set to zero, there is an amplification of the system's response in regions away from the resonance frequency. Moreover, it is observed that an increase in the number of unit cells does not have a notable effect on the attenuation properties of the configuration, especially in higher excitation frequencies. However, a distinctive peak at f / f_n=0.2 is indicated for N=1, which is not transferred to lattices with higher unit cell number.

4 CONCLUSIONS

The current study investigates the behavior of a highly nonlinear solution for attenuation of mechanical vibrations. The influence of critical parameters of the system was inspected, including mass and stiffness, number of unit cells, as well as excitation frequency. It is observed that high values of the external mass ratio m/M, as well as of the internal mass ratio μ/m, affect positively the system's vibration mitigation capabilities. Regarding the stiffness ratio, its influence is significant for lower values of the external mass ratio, while for higher values of the latter the amount of energy absorption is less affected by stiffness. The number of unit cells consisting the lattice does not seem to affect the behaviour of the system for high excitation frequencies, while for frequency ratio below unity there are some discrepancies in the behaviour of the system, the most notable being that of the single unit cell attachment. Further investigation is being performed by the authors in exploring this observation, which additionally requires consideration of the free clearance dimension. Moreover, the configuration is able to reduce the dynamic load effect within a frequency range centered around the resonance of the primary system, while some unfavourable effects were observed at adjacent frequencies. The results clearly indicate the effectiveness of the system under specific conditions, while it is also implied that proper adjustment of the metamaterial to specific structures can be challenging.

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REFERENCES

[1] Z. Liang, G.C. Lee, G.F. Dargush, and J. Song. Structural damping: Applications in Seismic Response Modification. *CRC Press*, Boca Raton, FL, USA, 2012.

[2] M.J.N. Priestley and D.N. Grant. Viscous damping in seismic design and analysis. *Journal of Earthquake Engineering*, 9(2):229–255, 2005.

[3] F. L´opez-Almansa, S.T. de la Cruz, and C. Taylor. Experimental study of friction dissipators for seismic protection of building structures. *Earthquake Engineering and Engineering Vibration*, 10(4):475–486, 2011.

[4] H.P. Zhu, D.D. Ge, and X. Huang. Optimum connecting dampers to reduce the seismic responses of parallel structures. *Journal of Sound and Vibration*, 330(9):1931–1949, 2011.

[5] Y.L. Xu and Z. Yu. Mitigation of three-dimensional vibration of inclined sag cable using discrete oil dampers - II. Application. *Journal of Sound and Vibration*, 214(4):675–693, 1998.

[6] L. Venanzi. A Review on Adaptive Methods for Structural Control. *The Open Civil Engineering Journal*, 10:653–667, 2016.

[7] W. Yu and S. Thenozhi. Active Structural Control with Stable Fuzzy PID Techniques. *Springer International Publishing*, Switzerland, 2016.

[8] R. Rana and T.T. Soong. Parametric study and simplified design of tuned mass dampers. *Engineering Structures*, 20(3):193–204, 1998.

[9] F. Sadek, B. Mohraz, A.W. Taylor, and R.M. Chung. A method of estimating the parameters of tuned mass dampers for seismic applications. *Earthquake Engineering and Structural Dynamics*, 26(6):617–635, 1997.

[10] J. Salvi and E. Rizzi. Optimum tuning of tuned mass dampers for frame structures under earthquake excitation. *Structural Control and Health Monitoring*, 22(4):707–725, 2015.

[11] S. Elias and V. Matsagar. Research developments in vibration control of structures using passive tuned mass dampers. *Annual Reviews in Control*, 44:129–156, 2017.

[12] R.O. Ruiz, D. Lopez-Garcia, and A.A. Taflanidis. Modeling and experimental validation of a new type of tuned liquid damper. *Acta Mechanica*, 227(11):3275–3294, 2016.

[13] M.S. Miah, E.N. Chatzi, V.K. Dertimanis, and F. Weber. Realtime experimental validation of a novel semiactive control scheme for vibration mitigation. *Structural Control and Health Monitoring*, 24(3):# e1878, 2017.

[14] M.S. Miah, E.N. Chatzi, V.K. Dertimanis, and F. Weber. Nonlinear modeling of a rotational MR damper via an enhanced BoucWen model. *Smart Materials and Structures*, 24(10):# 105020, 2015.

[15] M. Sigalas and E. Economou. Band structure of elastic waves in two dimensional systems. *Solid State Communications*, 86(3):141–143, 1993.

[16] M.S. Kushwaha, P. Halevi, G. Martinez, L. Dobrzynski, and B. Djafari-Rouhani. Theory of acoustic band structure of periodic elastic composites. *Physical Review B*, 49(4):2313–2322, 1994.

[17] V. La Salandra, M. Wenzel, O.S. Bursi, G. Carta and A.B. Movchan Conception of a 3D Metamaterial-Based Foundation for Static and Seismic Protection of Fuel Storage Tanks. *Frontiers in Materials*. 4:1-13, 2017.

[18] F. Basone, M. Wenze, O.S. Bursi, M. Fossetti. Finite locally resonant Metafoundations for the seismic protection of fuel storage tanks. *Earthquake Engng Struct Dyn*, 48: 232–252, 2019.

[19] I. Antoniadis, A. Paradeisiotis. Acoustic Meta-materials Incorporating the KDamper Concept for Low Frequency Acoustic Isolation. *Acta acustica united with acustica*, 104: 636-646, 2018.

[20] H. J. Xiang, Z. F. Shi, S. J. Wang, and Y. L. Mo. Periodic materials-based vibration attenuation in layered foundations: Experimental validation. *Smart Materials and Structures*, 21(11), 2012.

[21] Z. Shi, Z. Cheng, and H. Xiang. Seismic isolation foundations with effective attenuation zones. *Soil Dynamics and Earthquake Engineering*, 57(5-12):143–151, 2014.

[22] P. Roux, D. Bindi, T. Boxberger, A. Colombi, F. Cotton, I. Douste‐Bacque, S. Garambois, P. Gueguen, G. Hillers, D. Hollis, T. Lecocq, I. Pondaven. Toward Seismic Metamaterials: The METAFORET Project. *Seismological Research Letters*, 89 (2A): 582–593, 2018.

[23] P.-R. Wagner, V.K. Dertimanis, I.A. Antoniadis, and E.N. Chatzi. On the feasibility of structural metamaterials for seismic-induced vibration mitigation. *International Journal of Earthquake and Impact Engineering*, 1(1/2):20–56, 2016.

[24] V.K. Dertimanis, I.A. Antoniadis, and E.N. Chatzi. Feasibility Analysis on the Attenuation of Strong Ground Motions Using Finite Periodic Lattices of Mass in Mass Barriers. *Journal of Engineering Mechanics*, 142(9):# 04016060, 2016.

[25] A. Palermo, S. Krödel, K.H. Matlack, R. Zaccherini, V.K. Dertimanis, E.N. Chatzi, A. Marzani, and C. Daraio. Hybridization of Guided Surface Acoustic Modes in Unconsolidated Granular Media by a Resonant Metasurface. *Physical Review Applied*, 9(5):# 54026, 2018.

[26] R. Zaccherini, V.K. Dertimanis, A. Palermo, S. Krödel, A. Marzani, C. Daraio, and E.N. Chatzi. Resonant Metabarriers as Seismic Attenuators in Granular Media. *In Proceedings of the ISMA2018 conference on Noise and Vibration Engineering*, Leuven, Belgium, 2018.

[27] T. J. Cui, D.R. Smith, and R. Liu. Metamaterials, *Springer*, ISBN: 978-1-4419-0573-4, 2010.

[28] V.K. Dertimanis, S.F. Masri and E.N. Chatzi. On the vibration attenuation properties of finite periodic lattices of impact dampers. *IX ECCOMAS Thematic Conference on Smart Structures and Materials, SMART 2019*, Paris, France, 2019.

[29] A.F. Vakakis, O.V. Gendelman, L.A Bergman, D.M. McFarland, G. Kerschen and Y.S. Lee. Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems, Vol. I & II, *Springer*, ISBN: 978-1-4020-9130-8, 2008.

[30] S.F. Masri. General Motion of Impact Dampers. *The Journal of the Acoustical Society of America*, 47:229–237, 1970.

[31] S.F. Masri and T.K. Caughey. On the Stability of the Impact Damper. *Journal of Applied Mechanics*, 33(3):586–592, 1967.

[32] S.F. Masri and A.M. Ibrahim. Response of the Impact Damper to Random Excitation. *The Journal of the Acoustical Society of America*, 53(1):200–211, 1973.

[33] Z. Lu, S.F. Masri and X. Lu. Studies of the performance of particle dampers attached to a two-degree-of-freedom system under random excitation. *Journal of Vibration and Control*. 17: 1454-1471, 2011.

[34] M.R. Duncan, C.R. Wassgren, and C.M. Krousgrill. The damping performance of a single particle impact damper. *Journal of Sound and Vibration*, 286(1–2):123–144, 2005.

[35] O.V. Gendelman. Analytic treatment of a system with a vibro-impact nonlinear energy sink. *Journal of Sound and Vibration*, 331:4599–4608, 2012.

[36] R.A. Ibrahim. Vibro-Impact dynamics: Modelling, Mapping and Applications, *Springer*, ISBN: 978-3-642-10156-4, 2009.

[37] M.F. Dimentberg and D.V. Iourtchenko. Random Vibrations With Impacts: A Review. *Nonlinear Dynamics*, 36(2–4):229–254, 1973.

[38] V.N. Philipchuk. Impact modes in discrete vibrating systems with rigid barriers. *International Journal of Non-Linear Mechanics*, 36(6):999–1012, 2001.