

DISPLACEMENT BASED ANALYSIS AND DESIGN OF ROCKING BRIDGES

Michalis F. Vassiliou¹, Natalia Reggiani Manzo²

¹ Assistant Professor, Chair of Seismic Design and Analysis, IBK, ETH Zürich
Stefano-Francini-Platz 5, CH-8093 Zürich
e-mail: vassiliou@ibk.baug.ethz.ch

² Ph.D. Candidate, Chair of Seismic Design and Analysis, IBK, ETH Zürich
Stefano-Francini-Platz 5, CH-8093 Zürich
reggianimanzo@ibk.baug.ethz.ch

Keywords: rocking, uplifting structures, dimensional analysis, dimensionality reduction, displacement based design

Abstract. *The response of a rigid rocking block is traditionally described by its tilt angle. This is a correct description, but this paper suggests that describing rocking via displacements is more meaningful, because it uncovers that two geometrically similar blocks of different size will experience the same top displacement, provided that they are not close to overturn. The above is illustrated for both analytical pulse excitations and for recorded ground motions. Thus the displacement demand of a ground motion on a rocking block is only a function of its slenderness; not of its size. This reduces the dimensionality of the problem and allows for the construction of size-independent rocking demand spectra.*

1 INTRODUCTION

The systematic study of the rocking oscillator started with Housner's seminal paper in 1963 [1]. Motivated by the surprising stability that tall slender "golf-ball-on-a-tee" structures presented in the 1960 Chilean earthquake, he showed that (a) out of 2 geometrically similar planar rigid objects, the larger one is harder to overturn dynamically, and (b) the overturning potential of a ground motion increases with its dominant period.

The interest on the rocking oscillator [2-6] sources from its ability to describe systems that cannot be described adequately by the classical elastic oscillator [7]. Indeed, the rocking oscillator can be used to understand the behavior of masonry structures [8-14], the seismic behavior of non-anchored equipment [15-22], as well as to explain the stability of ancient Greco-Roman and Chinese temples that have been standing for more than 2,500 years in earthquake prone regions [23-27]. Rocking motion has also inspired researchers to use inerters as seismic protection devices [28-29]. What is not widely known in the western world, is that rocking has been used since more than 40 years as a seismic isolation method in the USSR (and now in former USSR countries) [30, 31]. The Soviet system comprises an intentionally designed soft rocking story. The uplift of the rocking columns works as a mechanical fuse and limits the forces transmitted to the superstructure.

Rocking walls have been suggested as a resilient design approach for buildings [32, 33 and references therein].

Moreover, a 33-m-tall chimney at the Christchurch airport has been designed to uplift [34], and three 30 to 38-m-tall chimneys in Piraeus, Greece, have been retrofitted by allowing them to uplift in case of an earthquake.

Rocking systems are perfectly compatible with Accelerated Bridge Construction as they comprise prefabricated elements with dry connections. In fact, restrained rocking systems [35-42] have already found their way to practice, with the Wigram-Magdala restrained rocking bridge in New Zealand [43] being the first restrained rocking bridge constructed.

However, several researchers have suggested that the restraining tendon in rocking bridges is not only obsolete, but might unnecessarily decrease the design forces of both the superstructure and the foundation, maybe requiring a pile foundation when it could have been avoided [44-60]. Makris and Vassiliou [61] and Vassiliou and Makris [62] have suggested that as the size of a rocking bridge increases, the restraining system can become obsolete and merely increases the design forces of both the superstructure and the foundation.

A main drawback of unrestrained rocking bridges stems from their response being absolutely uncorrelated to any elastic system. Therefore, the elastic-based research results are not applicable: e.g. intensity measures, response spectra, motion-to-motion variability, design ground motions need to be re-determined. To this end, the rocking oscillator should be described with the minimum parameters needed.

This paper suggests that the current state of the art of using the tilt angle θ as the DOF of a rocking system is, of course, correct, but it is not the optimal. Using the top displacement of the oscillator, u , reduces the dimensionality of the problem. Then, the displacement *demand* on a rocking block becomes only slightly dependent on its size and is a function only of its slenderness [63].

2 ROTATION BASED DIMENSIONAL ANALYSIS OF THE ROCKING OSCILLATOR

The equation of in-plane motion for a rigid rectangular rocking column (Figure 1) with slenderness α and a semi-diagonal of length R (Figure 1) is:

$$\ddot{\theta} = -p^2 \cdot \left(\sin(\pm\alpha - \theta) + \frac{\ddot{u}_g}{g} \cos(\pm\alpha - \theta) \right) \quad (1)$$

where

$$p = \sqrt{(3g)/(4R)} \quad (2)$$

is the frequency parameter of the rocking column. The upper sign in front of α corresponds to a positive, and the lower to a negative rocking angle θ with respect to the defined coordinate system (Figure 1).

It is assumed that energy is only dissipated during impact. Housner [1] assumed that (a) the impact is instantaneous and (b) that the impact forces are concentrated on the impacting corner. Under these assumptions the ratio of post to pre impact rotational velocities is

$$r = \frac{\dot{\theta}_{after}}{\dot{\theta}_{before}} = 1 - \frac{3}{2} \sin^2 \alpha \quad (3)$$

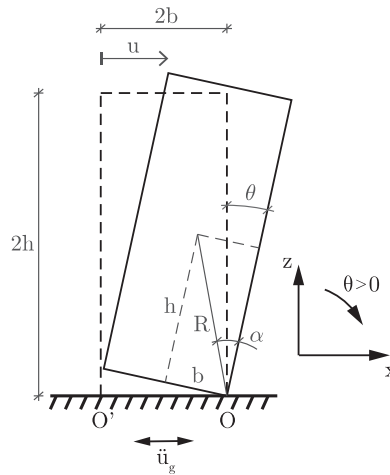


Figure 1: Geometric characteristics of the rigid rocking block.

Researchers (including the senior author of this paper) have critically evaluated the Housner model - especially its damping assumptions [64-68]. Indeed, while assuming the impact to be instantaneous seems a reasonable assumption, there is no evident reason to assume that the impact forces act on the impacting corner. Given the large sensitivity of the time history response of the rocking oscillator to all the parameters that define it, Housner's model might seem simplistic. However, experimental testing shows that even though it cannot predict the response to an individual ground motion, it can predict the statistics of the response to a set of ground motions [69]. Therefore, we consider it adequate within the scope of earthquake engineering [70].

By inspecting Equation (1) and (2) one can conclude that the rotational response of a rocking block to a ground motion is a function of

$$\theta_{max} = f_1(R, \alpha, g, \ddot{u}_g(t)) \quad (4)$$

As the gravity acceleration, g , is constant, the rotational response to a given ground motion is a function of 2 parameters α and R , similarly to the elastic oscillator, in which the response is a function of the eigenperiod, T , and damping ratio ζ . Therefore, by keeping one parameter constant (R or α) one can construct rotational spectra for rocking structures. However, unlike

the elastic oscillator, where, for usual structures, one parameter (T) is more influential than the other (ζ), in the case of rocking structures, both R and α strongly influence the rotational response.

Since ground motions containing distinguishable acceleration and/or velocity pulses are particularly destructive [71 and references therein], Zhang and Makris [72] have studied the response of a planar rocking block to acceleration pulses given by analytical expressions. A pulse of a given waveform can be described by two parameters. Zhang and Makris [69] chose the acceleration amplitude a_p and the dominant cyclic frequency ω_p . Then, the response will be a function of

$$\theta_{\max} = f_2(R, \alpha, g, a_p, \omega_p) \quad (5)$$

Equation (5) involves 6 quantities with 2 reference dimensions (Time and Length). Therefore, according to Vaschy - Buckingham's Π -Theorem of Dimensional Analysis ([73, 74]), the number of dimensionless parameters describing the problem is $6 - 2 = 4$. There is not a unique solution for choosing these four parameters. Zhang and Makris [72] suggested describing the problem as

$$\theta_{\max} = \varphi_1\left(\alpha, \frac{\omega_p}{p}, \frac{a_p}{g \tan \alpha}\right) \quad (6)$$

ω_p/p is often called size-frequency parameter and depends on the frequency of the excitation and on the size of the block. $a_p/(g \tan \alpha)$ is usually called non-dimensional acceleration but it can also be perceived as a non-dimensional strength parameter, since $mgR \sin \alpha$ is the moment that withstands uplift ("strength") and is $ma_p R \cos \alpha$ the overturning moment.

Therefore, dimensional analysis reduces the dimensionality of the problem from 6 to 4. Hence, by keeping the slenderness parameter α constant, one can produce contour plots of the maximum tilt angle θ as a function of ω_p/p and $a_p/(g \tan \alpha)$, the so called "rocking spectra". It is worth mentioning that Dimitrakopoulos and DeJong [75] have shown that for small values of α one can drop it as an independent parameter from Equation (6) as long as the coefficient of restitution, r , is treated as an extra independent parameter – however in this section r is not treated independently.

Figure 2 shows the rocking spectra of symmetric and antisymmetric Ricker wavelets. Ricker wavelets are defined as the 2nd and 3rd derivative of the Gaussian:

$$\ddot{u}_g = a_p \left(1 - \frac{2\pi^2 t^2}{T_p^2}\right) e^{-\frac{1}{2} \frac{2\pi^2 t^2}{T_p^2}} \quad (7)$$

$$\ddot{u}_g = \frac{a_p}{\beta_r} \left(\frac{4\pi^2 t^2}{3T_p^2} - 3\right) \frac{2\pi t}{\sqrt{3}T_p} e^{-\frac{1}{2} \frac{4\pi^2 t^2}{3T_p^2}} \quad (8)$$

where

$$T_p = \frac{2\pi}{\omega_p} \quad (9)$$

and $\beta_r = 1.3801$ to enforce that the function maximum is equal to a_p .

The spectra confirm the remarkable observation that larger structures are harder to overturn dynamically and that higher frequency pulses have a lower overturning potential. Interestingly, they show a heavy dependence of the response on both ω_p/p and $a_p/g \tan \alpha$.

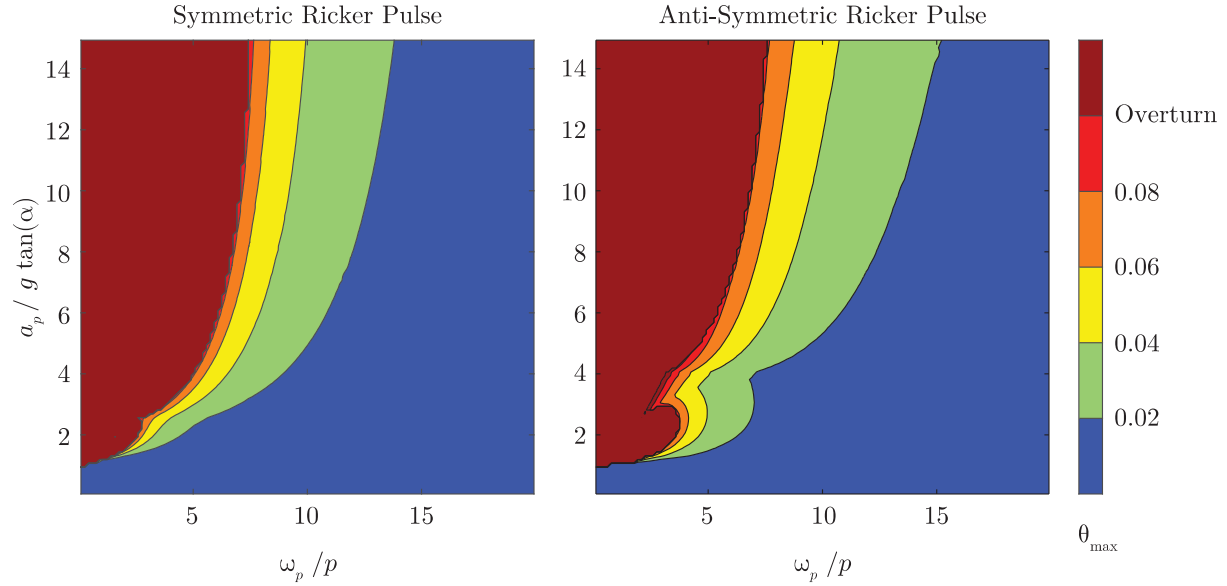


Figure 2: Non-dimensional rocking spectra based on rotations. $\alpha = 0.1$

3 DISPLACEMENT BASED DIMENSIONAL ANALYSIS OF A ROCKING OSCILLATOR EXCITED BY ANALYTICAL PULSES

3.1 Analysis based on the frequency parameter of the block p

The dimensional analysis of the previous section is one of the many correct solutions to describe the problem. It is based on rotations. This section, however, suggests that there is another, displacement based basis of describing the problem, which is also mathematically correct and more convenient. The convenience does not lie only on the fact that earthquake engineers are more used to displacements than rotations: A displacement based analysis further reduces the dimensionality of the problem allowing the construction of 2D rocking spectra.

Indeed, the rotation based analysis of the problem is based on the “recipe for similarity analysis” described in Chapter 5 of the well-known Dimensional Analysis textbook of Barenblatt [76]: “If the problem has an explicit mathematical formulation, the independent variables in the problem and the constant parameters that appear in the equations, boundary conditions and initial conditions, etc., are adopted as the governing parameters.” As this section shows, choosing the parameters that appear in the analytical equation might not be the most convenient way of describing this particular problem.

The top displacement of the rocking block can be obtained by a one-to-one mapping on the rotations:

$$u = 2R \sin(\pm \alpha) - 2R \sin(\pm \alpha - \theta) \quad (10)$$

The upper sign in front of α corresponds to a positive, and the lower sign to a negative tilt angle θ with respect to the defined coordinate system. If we use the top displacement as the single DOF of the problem, then the maximum response can be described as:

$$u_{\max} = f_3(R, \alpha, g, a_p, \omega_p) \quad (11)$$

To numerically compute the response of the block, we will resort to Equation (1), which is given in terms of rotation θ . Then, using Equation (10) we compute the displacement response.

Applying Buckingham's Π -theorem on Equation (11), one possible non-dimensionalization is

$$\frac{u_{\max} \omega_p^2}{a_p} = \varphi_2 \left(\alpha, \frac{\omega_p}{p}, \frac{a_p}{g \tan \alpha} \right) \quad (12)$$

Figure 3 shows the contour plots of $\frac{u_{\max} \omega_p^2}{a_p}$ as a function of ω_p/p and $a_p/(g \tan \alpha)$ for a given $\alpha = 0.1$. The remarkable observation is that within the non-overturning region the non-dimensional displacement depends heavily (and strongly non-linear) on the non-dimensional strength parameter $a_p/(g \tan \alpha)$ but only loosely on the size-frequency parameter ω_p/p . When the block is not close to overturning, the influence of ω_p/p is practically negligible.

Figure 4 plots $\frac{u_{\max} \omega_p^2}{a_p}$ as a function of $a_p/(g \tan \alpha)$ for different values of ω_p/p (and a constant slenderness $\alpha = 0.1$). For reasons of figure clarity, only non-overturning values of $\frac{u_{\max} \omega_p^2}{a_p}$ are plot, i.e. not plotting $\frac{u_{\max} \omega_p^2}{a_p}$ means that the block has overturned. Figure 5 plots $\frac{u_{\max} \omega_p^2}{a_p}$ as a function of ω_p/p for different values of $a_p/(g \tan \alpha)$ (and $\alpha = 0.1$).

Again, it is observed that, as long as the system is away from overturning, the dominant factor that influences $\frac{u_{\max} \omega_p^2}{a_p}$ is $a_p/(g \tan \alpha)$; not ω_p/p . In fact for small values of non-dimensional acceleration $a_p/(g \tan \alpha)$, the response for all values of size-frequency parameter ω_p/p is practically the same. The response starts to deviate only when the system is close to overturning – or has overturned.

In other words, a small and a large block, geometrically similar to each other and excited by analytical pulses, will have roughly equal top displacement, provided that the displacement is not enough to bring them close to overturn. A given pulse will induce the same *displacement demand*. The larger block is more stable simply because its displacement capacity (i.e. the displacement needed to cause overturn, i.e. its width) is larger.

Therefore, using a displacement basis to describe the problem further decreases the number of parameters needed to define it. Practically, the displacement demand on a rocking oscillator excited by a pulse is only a function of its non-dimensional strength parameter $a_p/(g \tan \alpha)$; not of its size.

The strongly nonlinear nature of rocking motion is also evident in Figures 4 and 5. $\frac{u_{\max} \omega_p^2}{a_p}$, which expresses the relation of the rocking displacement to the ground motion

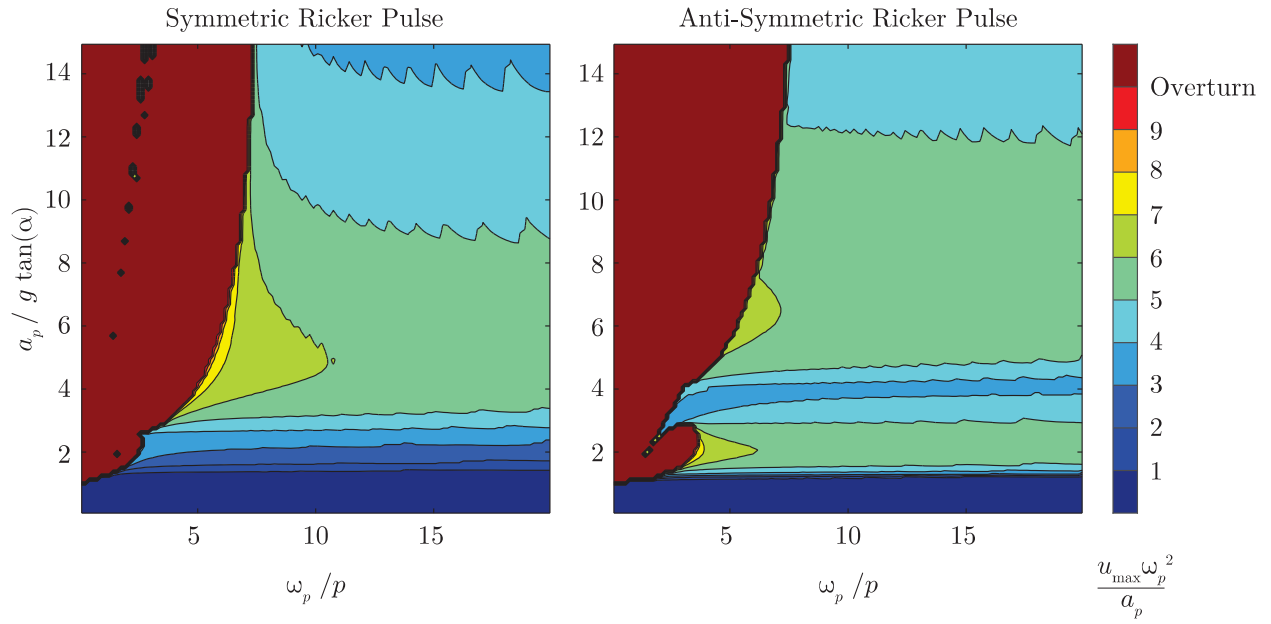


Figure 3: Non-dimensional rocking spectra based on displacements.

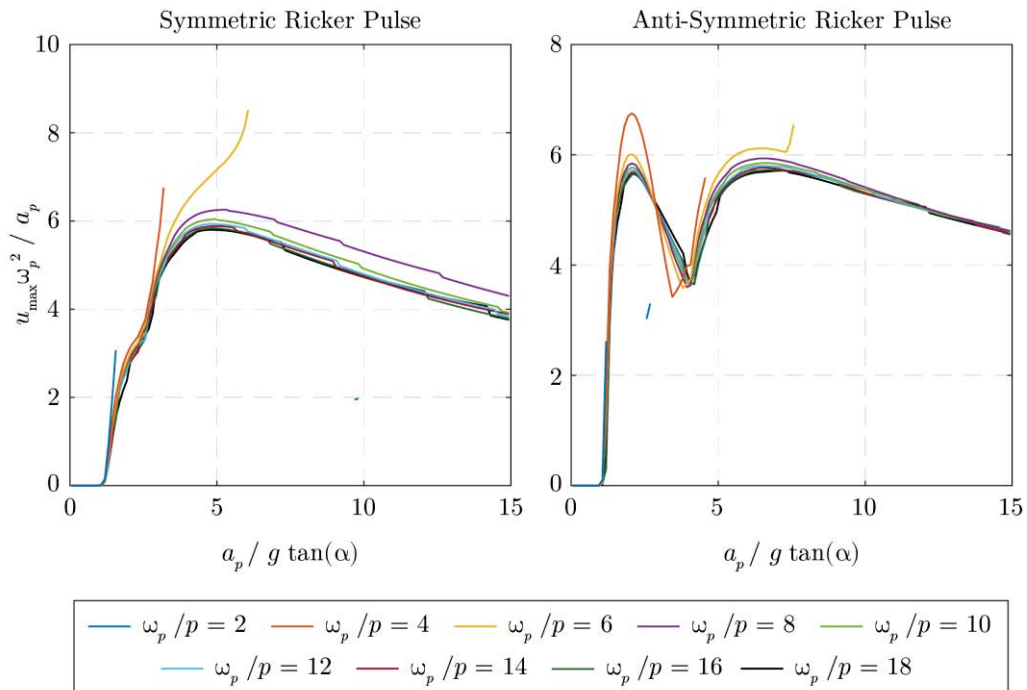


Figure 4: $\frac{u_{\max} \omega_p^2}{a_p}$ vs $a_p / (g \tan \alpha)$ plots for constant ω_p / p . $\alpha = 0.1$

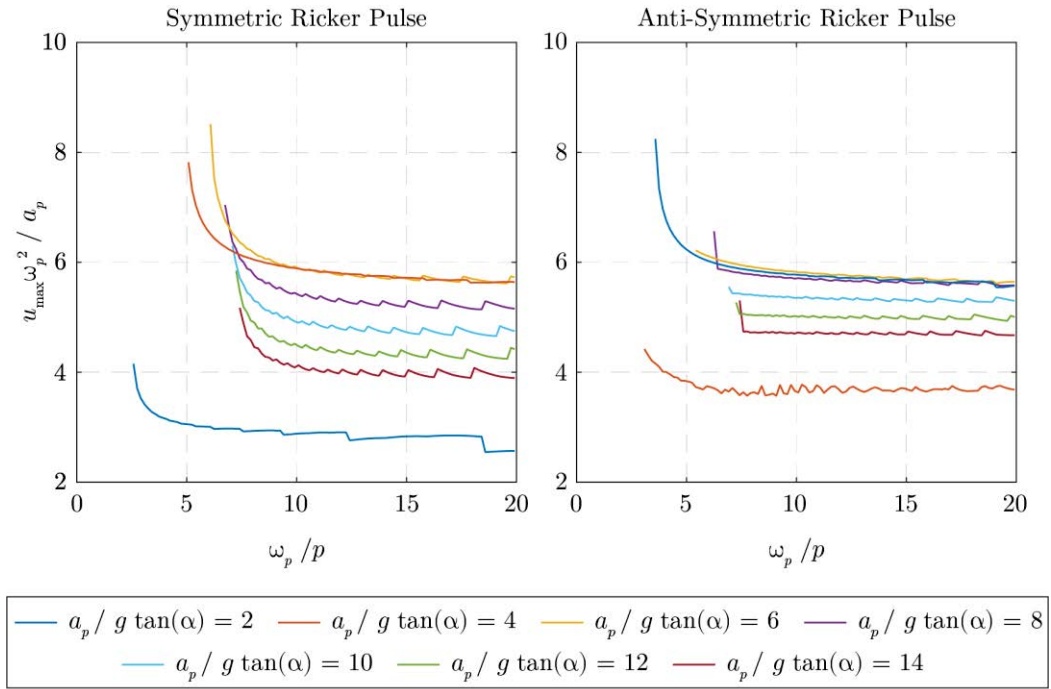


Figure 5: $\frac{u_{\max} \omega_p^2}{a_p}$ vs ω_p / p plots for constant $a_p / (g \tan \alpha)$ and $\alpha = 0.1$

displacement, does not depend monotonically on the strength parameter $a_p / (g \tan \alpha)$. In fact, the discontinuities of the $\omega_p / p = 2$ line of Figure 4 convey that a block can survive a stronger pulse and overturn in a weaker one.

Going back to dimensional quantities, Figure 6 plots the displacement response to a symmetric Ricker pulse with $a_p = 1g$ and $T_p = 0.5s$ and to an antisymmetric Ricker pulse with $a_p = 1g$ and $T_p = 1s$. The plots confirm that the displacement demand only loosely depends on the size, if the block is not close to overturning. The dominant factor is the slenderness. Therefore we can define the displacement demand rocking spectrum of a ground motion as a unary function

$$u_{\text{demand}} = f(\alpha) \text{ if } u_{\text{demand}} \leq 2b \quad (13)$$

that is computed via Equations (1) and (10) for a large enough block size. To check the stability of a block, one has to compute the maximum displacement demand via Equation (13) and compare it with the displacement capacity (i.e. the block width).

Therefore the reduction of the dimension of the problem follows two steps: a) Applying Buckingham's theorem and b) Observing that the displacement demand is roughly independent of the size. The first step is exact and follows from dimensional analysis. The second step is approximate and in this section illustrated for analytical pulses. Blöchliger [77] gave a first indication that the approximation also works for recorded ground motions. Further evidence supporting this approximation and highlighting its limitations are given in a next section of this paper.

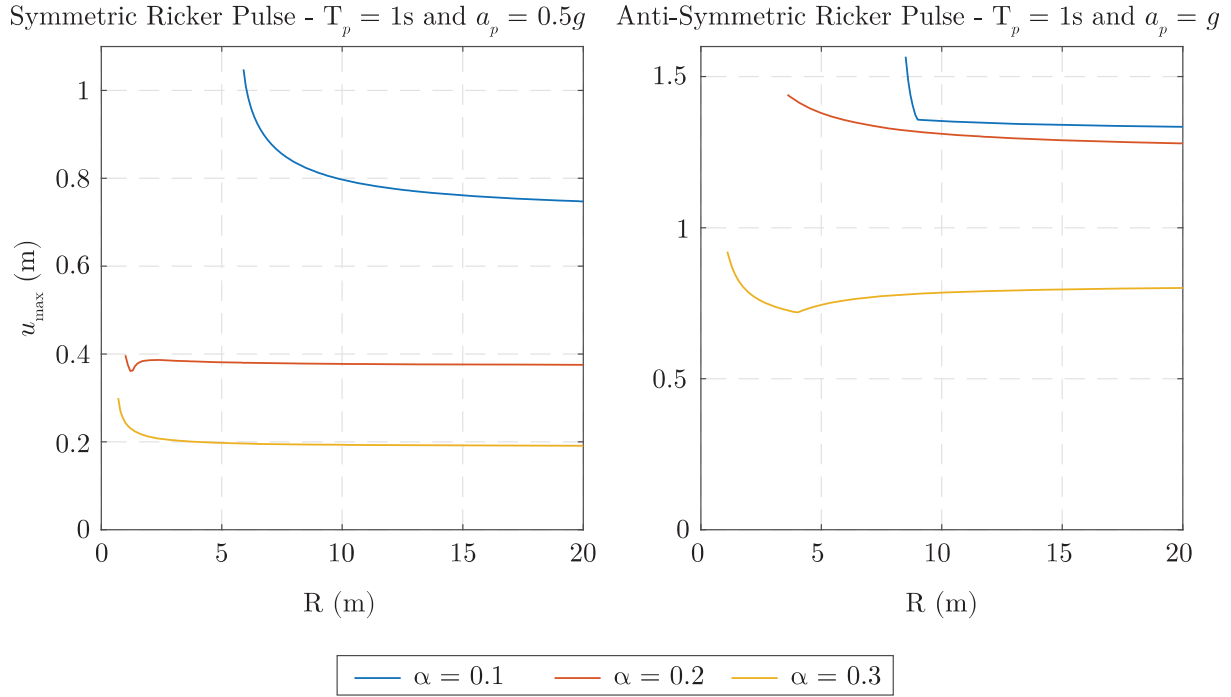


Figure 6: Maximum block displacement as a function of block size for Symmetric and Anti-Symmetric Ricker excitation.

3.2 Analysis based on the base width of the block b

The previous section chooses the frequency parameter p and the slenderness of the block, α , as the two parameters to define it. However, p has a physical meaning that is totally unrelated to rocking. It is the natural frequency that the block would have had *if* it was hanging from its corner [75]. But this is merely a coincidence, rocking blocks have no natural frequency [1], and the use of p often creates misunderstandings. In this section we propose to describe the block with two physical parameters that have a clear physical meaning, directly related to the rocking problem. The slenderness α is retained, as it controls the uplift of the structure (and could be parallelized with the strength of a system), but p is replaced by b , which is the half-width of the base and exactly equal to one half of its displacement capacity. Then the displacement response will be:

$$u_{\max} = f_4(b, \alpha, g, a_p, \omega_p) \quad (14)$$

Using Buckingham's Π theorem we get:

$$\frac{u_{\max} \omega_p^2}{\alpha_p} = \varphi_3 \left(\frac{a_p}{g \tan \alpha}, \frac{b \omega_p^2}{a_p}, \alpha \right) \quad (15)$$

The term $a_p/(g \tan \alpha)$ would be the reciprocal of the non-dimensional strength, $(b \omega_p^2)/a_p$ would be the non-dimensional displacement capacity, and α (taken as an independent parameter) controls damping, because it controls the coefficient of restitution.

Figure 7 plots displacement spectra according to the suggested non-dimensionalization. One can observe that for both pulses a base (i.e a displacement capacity = $2b$) of roughly 9

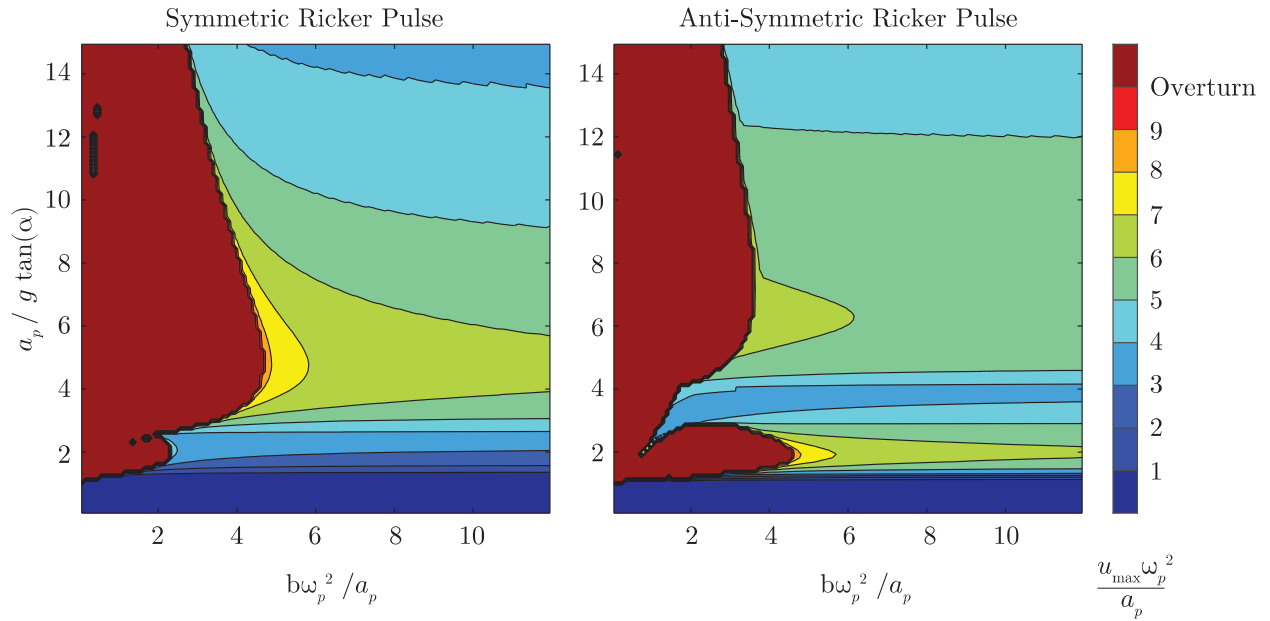


Figure 7: Displacement based non-dimensional rocking spectra using the width of the block to characterize its size.

times the length scale of the pulse $L_e = a_p / \omega_p^2$ is enough to keep the block stable, no matter what the non-dimensional strength parameter is.

4 DISPLACEMENT BASED ANALYSIS OF A ROCKING OSCILLATOR EXCITED BY RECORDED GROUND MOTIONS

Analytical pulses can be used to qualitatively study the rocking oscillator. However, as the rocking problem is very sensitive to all of its parameters, pulses would not suffice to prove that the displacement demand on a rocking structure depends only on its slenderness and not on its size. Therefore, this section examines the displacement response of a rocking block excited by recorded ground motions.

4.1 FEMA P695 Ground motions

There is no consensus in the engineering community on what ground motions should be used in time history analysis. Several approaches exist including using recorded (scaled or unscaled), artificial, or synthetic ground motions. In this paper we choose to use the 3 sets of ground motions proposed by FEMA P695 [79] (far field, near field pulse-like, and near field non-pulse-like) only as a means to illustrate our rocking-related argument, without taking stance on the debate around ground motions. It is evident that any ground motion selection method based on the response of an elastic system is in principle not applicable in the case of the rocking oscillator, as the elastic and rocking oscillator are uncorrelated. More information on the FEMA P695 ground motions can be found in FEMA [79].

4.2 Equal displacement rule for rocking structures and displacements demand spectra

Vassiliou et al. [80] have proven that rigid rocking oscillators of equal height attached to massless foundations of the same size behave identically, no matter what their actual column width is (Figure 8). Therefore, the design question of a rocking structure would be: Find the size, $2B'$, of the foundation for a given oscillator height $2H$. Hence, it is more meaningful to

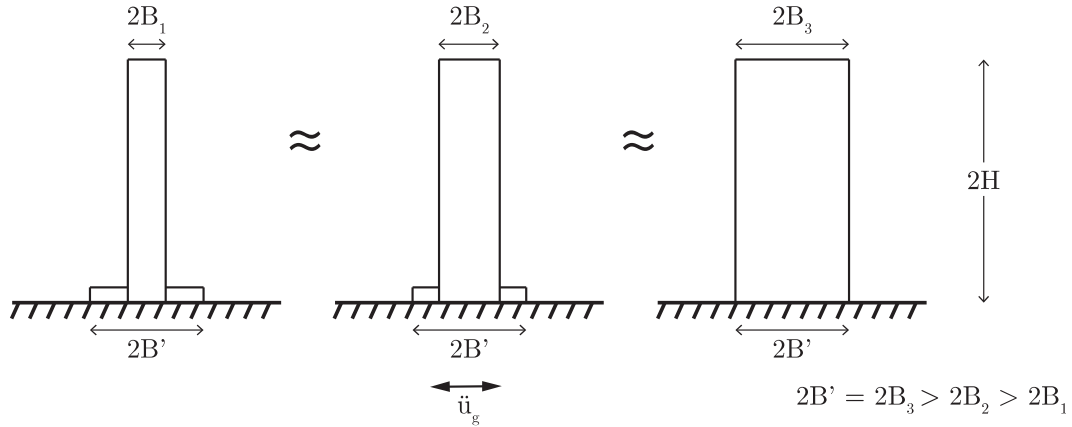


Figure 8: Rocking oscillators of equal height.

use H as a size parameter instead of R , even if the former does not explicitly appear in the equation of motion.

Figure 9 offers the displacement of a rocking oscillator as function of its slenderness α , and for $2H=2, 4, 10, 20, 80$, and $1000m$, for a selection of the FEMA P695 ground motions. The $2H=1000m$ is offered only for reasons of mathematical completeness, to study the limit case of $H \rightarrow \infty$. For reasons of plot clarity, each line is plotted only for $\alpha > \alpha_{crit}$, where α_{crit} is the minimum slenderness angle for which the block overturns. We observe that all blocks of same slenderness angle present roughly the same displacement, as long as they are not close to overturning. The same observation holds for all the ground motions tested.

As analysis and design of a rocking structure would not involve a single ground motion, but a set of design motions, it makes sense to study the problem by applying sets of multiple excitations and comparing the statistics of the results (e.g. the median displacement among all the ground motions of the excitation set). Figure 10 plots displacement spectra of the median of the displacement for 7 variations of the near-field pulse-like FEMA P695 set: a) Unscaled ground motions, b) scaled so that their $\overline{PGA}$ is equal to $0.5\overline{PGA}$, or $\overline{PGA}$, or $2\overline{PGA}$, c) scaled so that their $\overline{PGV}$ is equal to $0.5\overline{PGV}$, or $\overline{PGV}$, or $2\overline{PGV}$, where $\overline{PGA}$, $\overline{PGV}$ are defined as

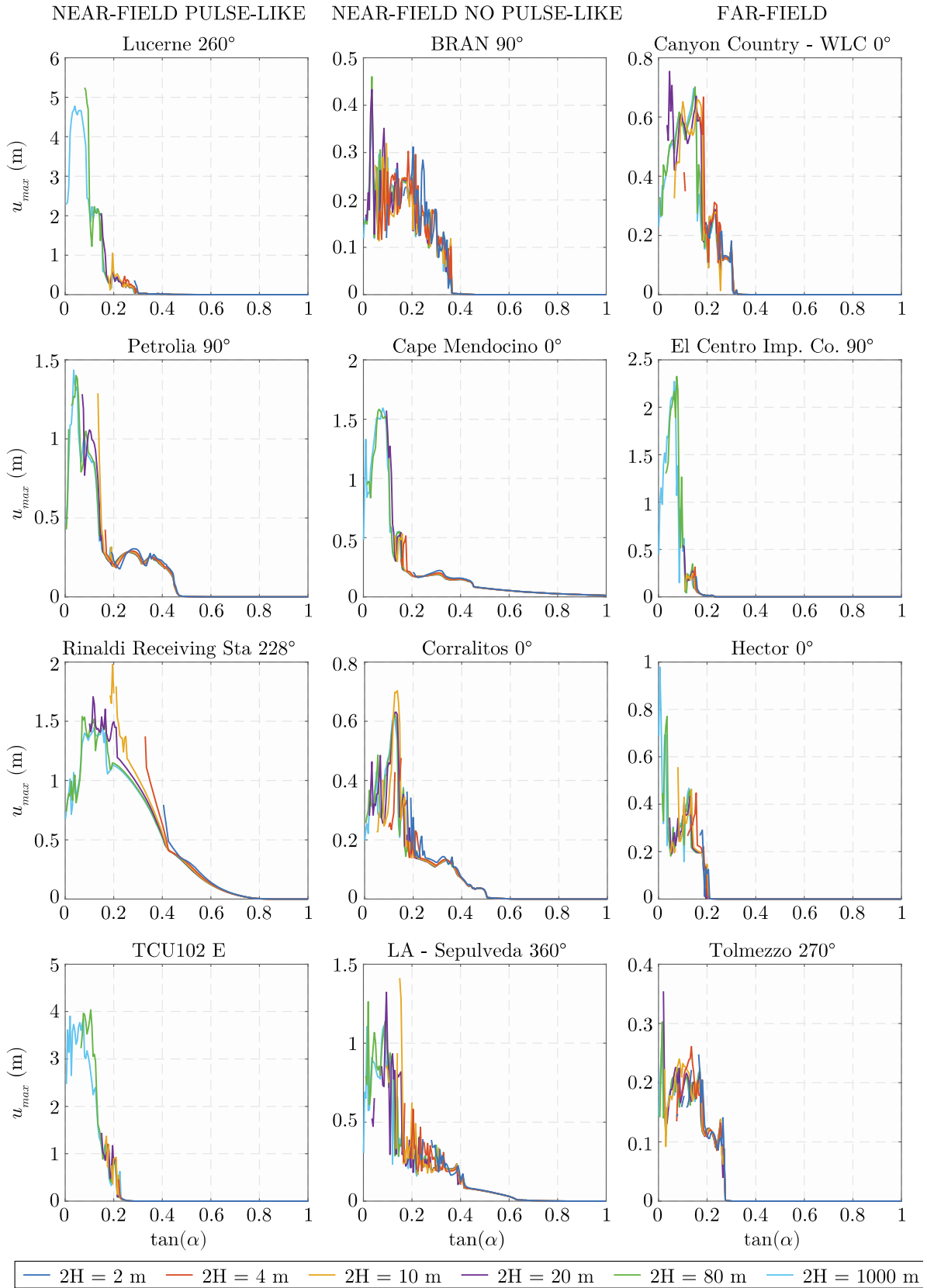
$$\overline{PGA} = \text{median}_{i=1 \dots N} \left(\sqrt{PGA_{i_x} \times PGA_{i_y}} \right) \quad (16)$$

$$\overline{PGV} = \text{median}_{i=1 \dots N} \left(\sqrt{PGV_{i_x} \times PGV_{i_y}} \right) \quad (17)$$

where N is the number of the ground motions and x and y are the two components of each ground motion. Note that each horizontal component of each ground motion is treated as an independent motion. Figures 11 and 12 plot the same spectra for the far field and near field non-pulse-like ground motions.

The following observations can be made:

- The median spectra are smoother, likewise design elastic spectra that were derived by statistical processing of elastic spectra of single ground motions are smoother than single ground motion spectra.
- As long as the system is not close to overturning, the displacement does not depend on the size of the block. For this part of the spectrum, instead of computing a different spectrum for each block size, one can compute the design spectrum for $2H \rightarrow \infty$ ($2H =$


 Figure 9: Displacement of a rocking oscillator as function of its slenderness α .

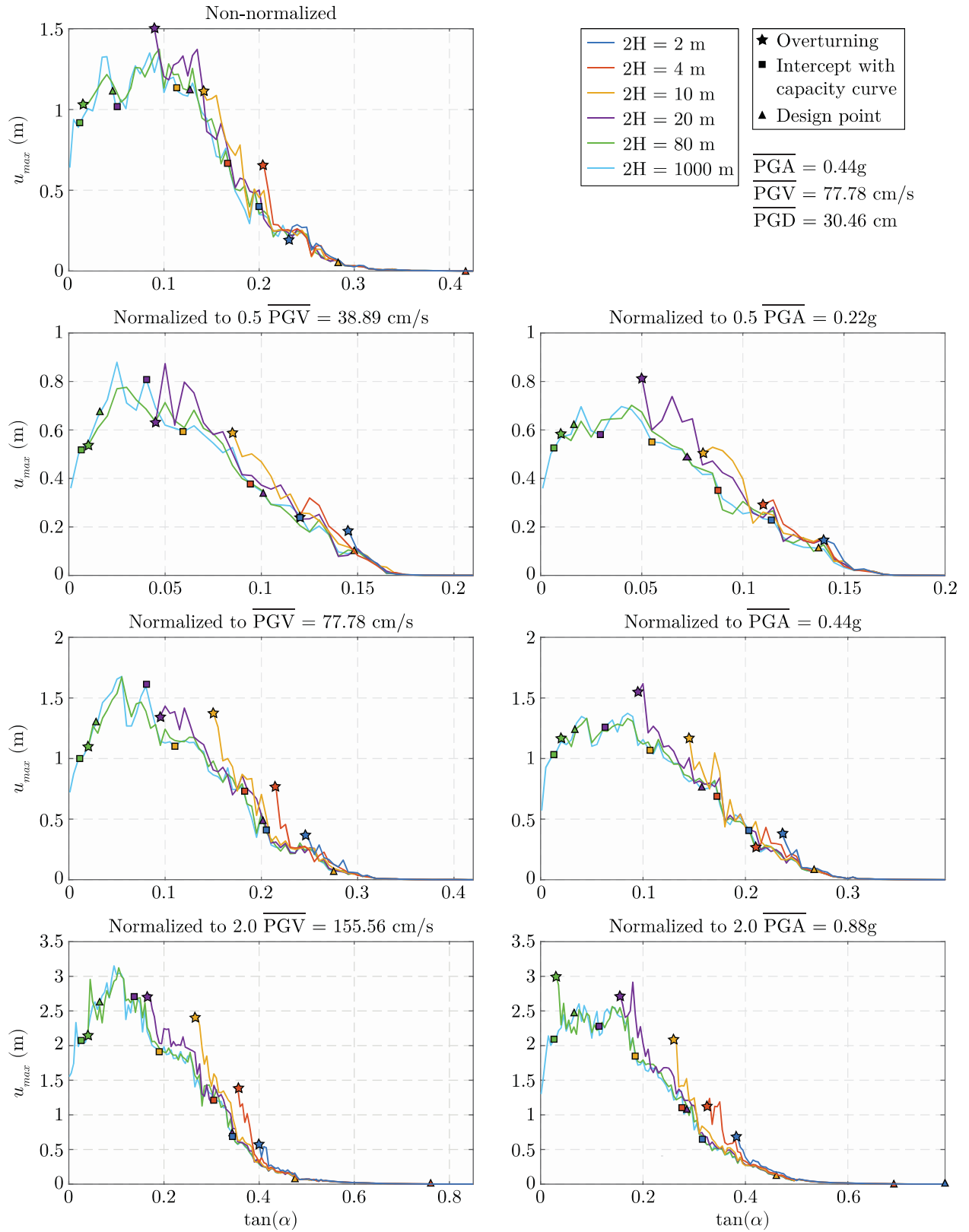


Figure 10: Median Displacement Spectra for Near-Field Pulse-Like Record Set.

1000m seems an adequate value) and use it to calculate the displacement *demand* on any rocking structure (i.e. $u_{\max}=f(\alpha)$). We name the above finding “equal displacement rule” for rocking structures.

- c) As the system gets closer to overturning the equal displacement rule does not apply: smaller systems present larger displacements than larger ones. Moreover, as the system approaches overturning, the slope of the spectrum increases dramatically i.e. a small decrease in $\tan\alpha$ leads to very large increase in displacement. This trend dictates that a rational design of a rocking structure would require that this steep part of the spectrum be avoided. because an earthquake slightly stronger than the design one would cause a tremendous increase in displacement. Therefore, the equal displacement rule applies to the rational design region.
- d) The form of the spectrum for all 3 sets of ground motions presents some repetitive pattern:
 - i. As α tends to zero, $u_{\max}$ tends to a finite value. For spectra of individual ground motions, this value is $\frac{3}{2}PGD$. An explanation for this is offered in the next section.
 - ii. As α increases from zero, the displacement demand amplifies 2-2.5 times and reaches a plateau.
 - iii. Further increase of α leads to a monotonic decrease of the displacement demand.
 - iv. Naturally, when $\tan\alpha$ reaches PGA/g , the displacement demand becomes zero, as there is no uplift.

4.3 Preliminary design based on the equal displacement rule

If not for a final design, the equal displacement rule can be used for preliminary calculations. Indeed it is not an exact method, but a preliminary design method that does not aim at being exact, but at providing a tool for initial calculations, that for certain cases and required degree of accuracy can be enough. The same holds for yielding structures, where the “equal displacement rule” is used for many structural systems, while for more complicated systems it is used only for preliminary design and then more refined methods are applied. It could be stated that the findings of this paper constitute the generalization of equal displacement rule from yielding to rocking systems. This section proposes a methodology to design a rocking structure based on the equal displacement rule:

- a) On the $u_{\max} - \tan\alpha$ curve, we plot the capacity line $u_C = 2H\tan\alpha$.
- b) We determine the intersection of the capacity line and the $2H = \infty$ line. We define the abscissa of this point as $\tan\alpha_k$.
- c) We use a multiplier of 2.5 to determine the design slenderness: $\tan\alpha_D = 2.5\tan\alpha_k$. The multiplier serves as a safety factor to move the design point away from the steep part of the spectrum.

Figure 13 outlines the design procedure applied for a rocking bridge with columns of 6.7m height ($2H = 6.7m$). Based on Makris and Vassiliou [46] the response of the frame is equal to the response of a solitary block of $2H = 10m$. For this bridge, twenty one design scenarios are explored, corresponding to the 21 spectra of Figures 10-12. Tables 1-3 and Figure 14 summarize the findings for the 21 design scenarios and compare the displacement predicted

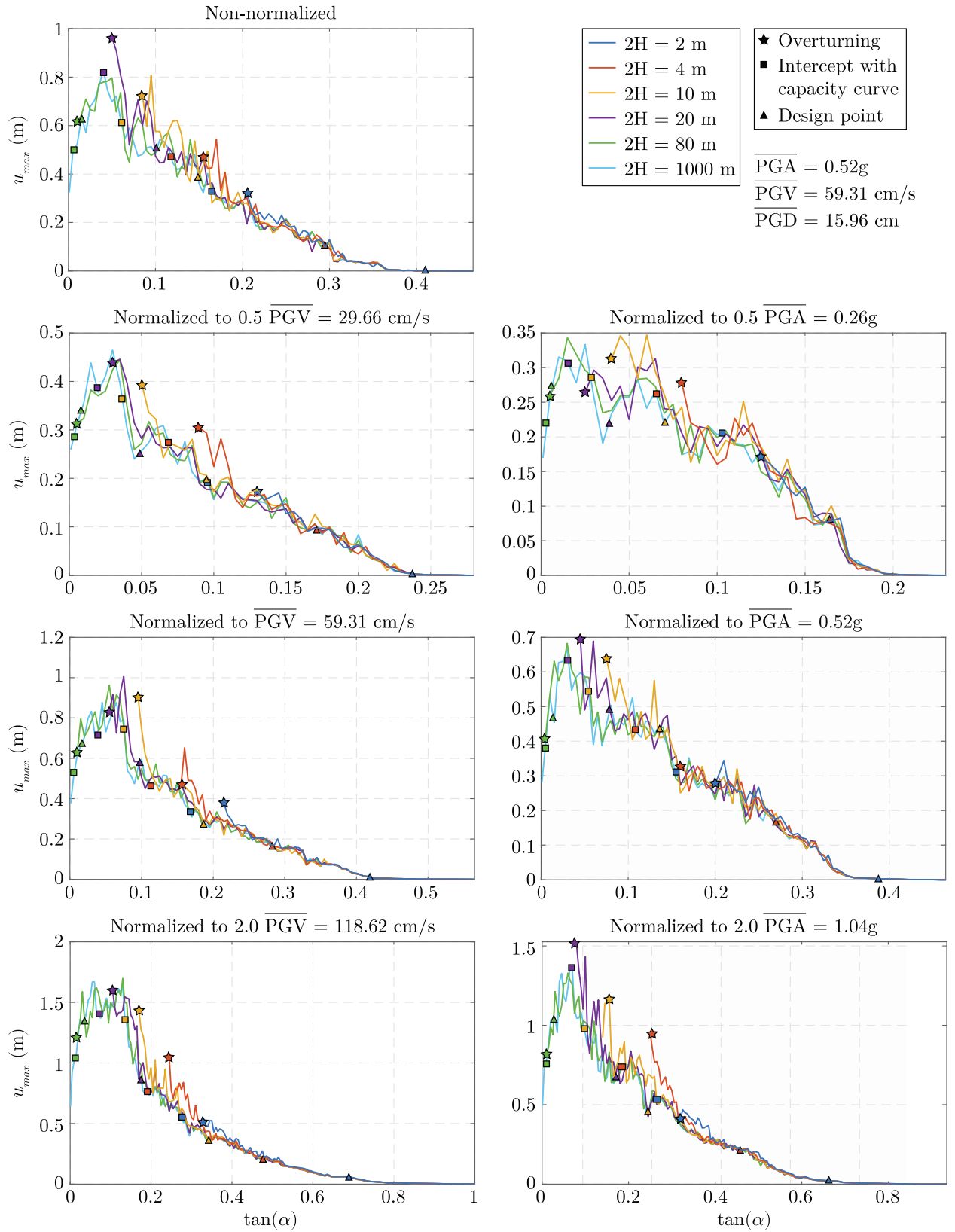


Figure 11: Median Displacement Spectra for Near-Field No Pulse-Like Record Set.

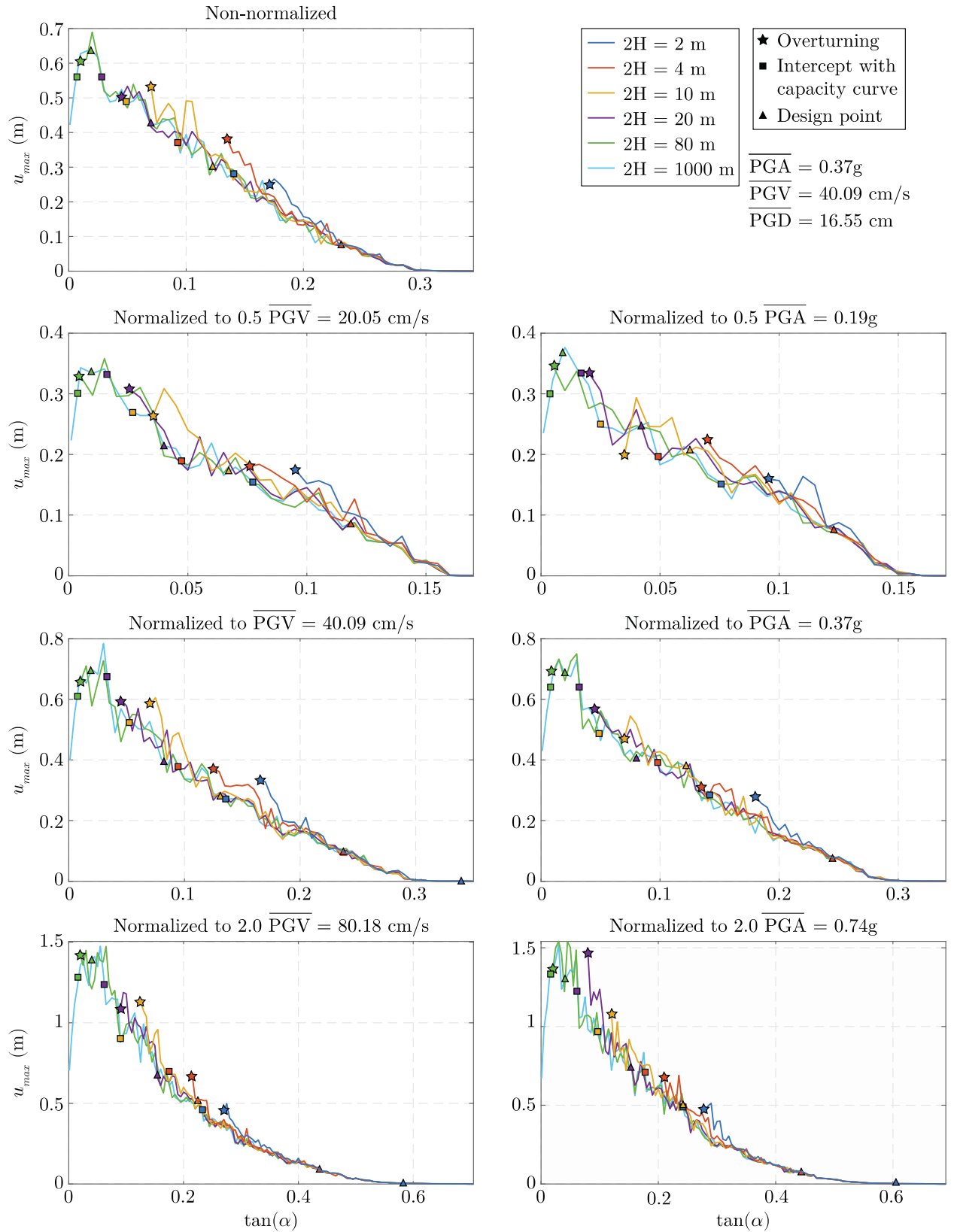


Figure 12: Median Displacement Spectra for Far-Field Record Set.

by the demand spectrum ($2H = 1000\text{m}$) to the displacement predicted by the $2H=10\text{m}$ spectrum. We observe that in all but two cases (near fault pulse-like scaled to $0.5\overline{PGA}$ and near fault non-pulse-like scaled to $2\overline{PGA}$) the error in predicting the median displacement is less than 20%. In all cases, the error is smaller than 40%, and no system overturned.

	Unscaled	$0.5\overline{PGA}$	$\overline{PGA}$	$2\overline{PGA}$	$0.5\overline{PGV}$	$\overline{PGV}$	$2\overline{PGV}$
$\tan(\alpha_D)$	0.2839	0.1378	0.2671	0.4618	0.0596	0.1094	0.1909
$2H = 1000\text{m}$	0.05	0.11	0.08	0.12	0.10	0.07	0.08
$2H = 10\text{m}$	0.06	0.15	0.07	0.13	0.10	0.10	0.08

Table 1: Near field pulse like FS=2.5.

	Unscaled	$0.5\overline{PGA}$	$\overline{PGA}$	$2\overline{PGA}$	$0.5\overline{PGV}$	$\overline{PGV}$	$2\overline{PGV}$
$\tan(\alpha_D)$	0.1524	0.0708	0.1356	0.2446	0.0916	0.1892	0.3405
$2H = 1000\text{m}$	0.34	0.22	0.44	0.44	0.19	0.27	0.36
$2H = 10\text{m}$	0.41	0.26	0.41	0.69	0.21	0.30	0.38

Table 2: Near field non pulse like FS=2.5.

	Unscaled	$0.5\overline{PGA}$	$\overline{PGA}$	$2\overline{PGA}$	$0.5\overline{PGV}$	$\overline{PGV}$	$2\overline{PGV}$
$\tan(\alpha_D)$	0.1228	0.0620	0.1215	0.2411	0.0675	0.1320	0.2249
$2H = 1000\text{m}$	0.30	0.21	0.38	0.52	0.17	0.28	0.52
$2H = 10\text{m}$	0.33	0.20	0.37	0.50	0.19	0.29	0.49

Table 3: Far field FS=2.5.

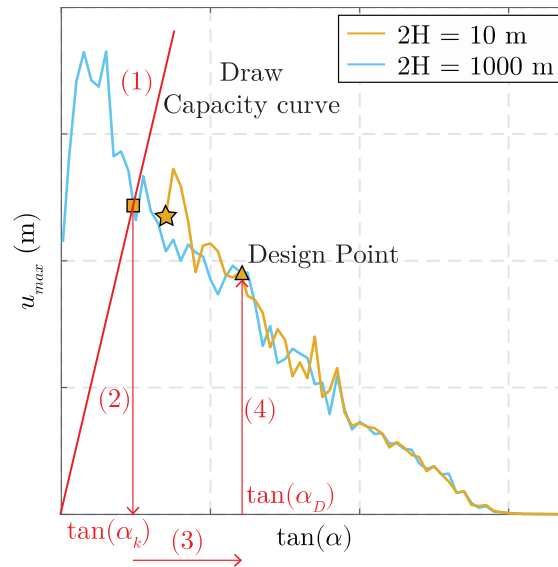


Figure 13: Design procedure.

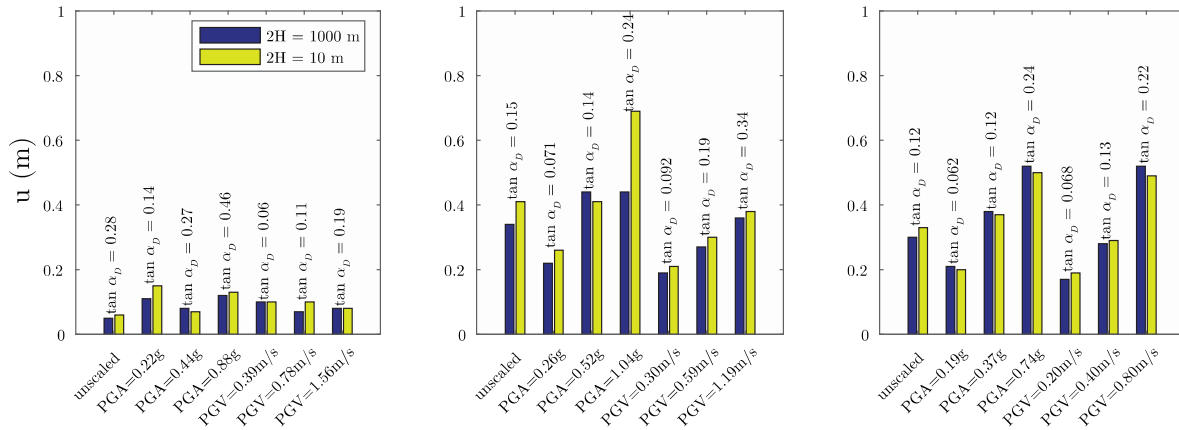


Figure 14: Comparison of the displacement response at the design point. Predictions based on the $2H=1000m$ and on the $2H=10m$ spectra.

5 INTERPRETATION OF THE EQUAL DISPLACEMENT RULE BASED ON THE EQUATION OF MOTION

The equal displacement rule can be interpreted by properly manipulating the equation of motion. Assuming small rotation angles ($\sin x \approx x$ and $\cos x \approx 1$), Equation (1) gives

$$\ddot{\theta} = -\frac{3g}{4H} \cdot \left(\pm \alpha - \theta + \frac{\ddot{u}_g}{g} \right) \quad (18)$$

For small angles, $u = 2H\theta$. Then

$$\ddot{u} = -\frac{3g\alpha}{2} \cdot \left(\pm 1 - \frac{u}{2b} + \frac{\ddot{u}_g}{g\alpha} \right) \quad (19)$$

When $u/2b$ is small (i.e. the block is not close to overturning, the other terms dominate the response and u becomes a function only of α . Furthermore, when $\ddot{u}_g/(g\alpha) \ll 1$, then $\ddot{u} = -3\ddot{u}_g/2$. Therefore, as $\alpha \rightarrow 0$ (which can only happen for blocks with $H \rightarrow \infty$), $u_{\max} \rightarrow 3/2 \text{ PGD}$.

6 CONCLUSIONS

The widely used description of the rocking block via its rotation is correct, but not optimal. It reveals that larger blocks are more stable and that higher frequency pulses present less overturning potential. However, it does not reveal the “equal displacement rule of rocking structures”, namely that a large and a small block of the same aspect ratio will present the same top displacement, if they both are not close to overturning. Not being close to overturning is a design necessity anyway, therefore, for the scope of design, we can claim that the displacement demand is the same and it only depends on the slenderness, not on the size of the block. The above is illustrated for both analytical pulse excitations and for sets of recorded ground motions. As the response of a rocking block away from overturning essentially becomes a unary function, rocking spectra that plot the displacement demand as a function of the slenderness of the block can be developed. The shape of these spectra seem to follow a repetitive pattern, starting for $\tan \alpha = 0$ from $u = 3/2 \times \text{PGD}$, reaching a plateau of $2.25 \times 3/2 \times \text{PGD}$, and then gradually decreasing to zero. More research with more ground motions is needed to better understand the shape of the spectra. Based on the above, a design method that uses a size-

independent rocking spectrum is suggested. This should be taken into account when intensity measures for rocking structures [81-84] designed not to get close to overturning are explored.

ACKNOWLEDGEMENTS

This work was supported by the ETH Zurich under Grant ETH-10 18-1. The methods, results, opinions, findings and conclusions presented in this report are those of the authors and do not necessarily reflect the views of the funding agency.

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