

## MODAL PARAMETERS IDENTIFICATION ON ENVIRONMENTAL TESTD OF ANCIENT BELL TOWERS AND VALIDATION OF THEIR NUMERICAL MODELS

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**Abstract.** *Masonry towers are quite widespread in Europe and represent an important portion of the built heritage that must be preserved, especially in high-seismicity regions. Very often, such masonry towers exhibit unique peculiar morphologic and typological characteristics, which might affect their different structural behaviors under horizontal loads. For this reason, accurate knowledge of their dynamical parameters is useful for seismic assessment and the design of risk mitigation interventions.*

*In this work, the opportunities provided by dynamic identification techniques for the non-destructive evaluation of heritage structures are discussed with a focus on different bell towers, located in the Marche region (Italy). All of them were stricken by a long seismic sequence (Center Italy earthquakes between August and October 2016), and are investigated in detail in order to have an insight into their dynamic behavior. Furthermore, the experimental investigations and the operational modal analysis results are presented, and they are useful for defining the Finite Element (FE) model of the towers with a continuum approach.*

*The monitoring system consists of several elements adequately connected. Many operative problems have conditioned the positioning of the instrumentation due to the limited accessibility of the structure, not only to the primary access but also to reach the top.*

*However, it has been possible to identify with certain confidence the first three frequencies of the towers and their corresponding mode shapes. The results carried out after the updating procedure may be considered very good. The material data values estimated in this way will constitute an important reference for the evaluation of the state of the building.*

## 1 INTRODUCTION

Europe preserves a large part of the Earth's historical and artistic heritage, Italy is one of the European countries that owns the majority of historical buildings, especially palaces and churches. Because of its particular position, the nation is crossed by African and Euro-Asiatic tectonic plates, the seismic risk is very high and continuously endangers the historical structures [1–12]. One of the last examples is the recently Central Italy Earthquakes that hit the areas between Marche, Abruzzo and Umbria Regions causing widespread damages especially at the construction part of the cultural heritage, highlighting, once again, the necessity to find methodologies to take under control their health, in order to intervene before irreparable damages occur. Currently, the most appropriate technique to achieve this purpose is the Structural Health Monitoring (SHM) using the variation of structures dynamic characteristics as frequencies, damping and mode shapes. To obtain these data, accelerometric sensors are placed on specific points of the structure, in order to record, accelerometric time histories only from ambient noise. The data acquired are filtered and processed by appropriate techniques able to extract the dynamic information. These data can be used for two different purposes, depending on the acquisition duration: if there are acquired for a short period (Short-term monitoring) they can be used to identify the current building state and caliber a numerical model [7,13–16], whereas if there are acquired for a long period (Long-term monitoring), such as more than one year, they can be used to the damage identification [17–21], since, the variation of the dynamic characteristics are linked to stiffness or mass variations [22]. In this last case, particular attention must be paid to the ambient conditions because, as many studies showed, they can provoke variation in frequencies, some authors are looking for algorithms to delete them from the signals [23,24].

In this paper, the monitoring and the Finite Element Model (FEM) calibration are presented taking as a case study a complex composed by civic tower a palace and an arcade in Matelica, a village in the hinterland of Marche region Figure 1. This study wants to be an example of applicability the dynamic monitoring for protection of the heritage masonry constructions.

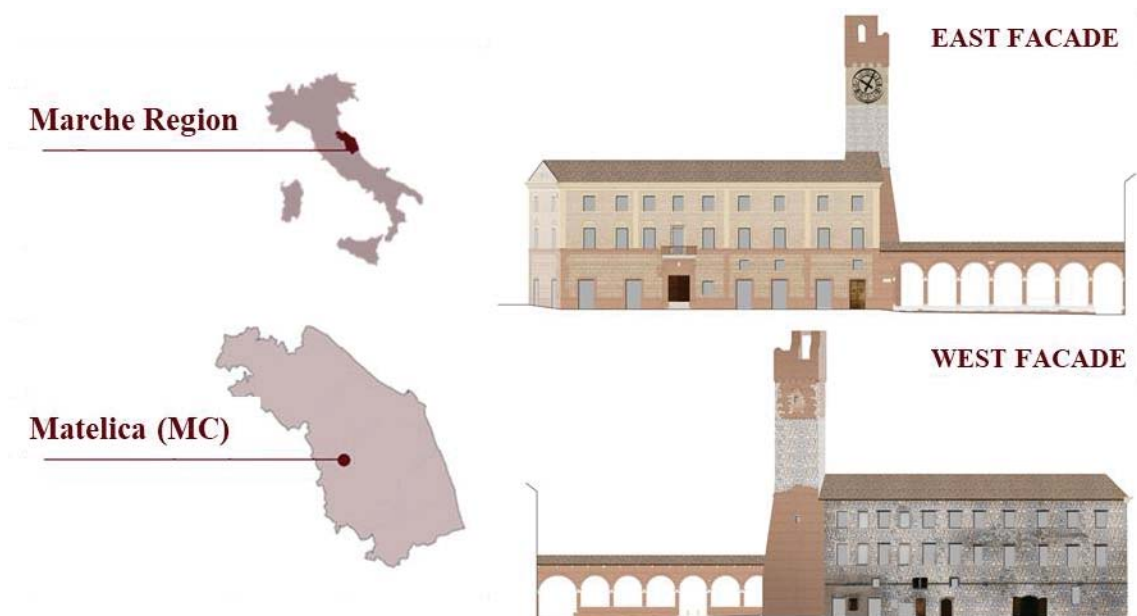


Figure 1 Geographical localization and East and West facades

## 2 CASE STUDY

### 2.1. Historical development

At the start, the Governor's Palace was a porticoed structure used as delimitation between the plaza and the open-air market. This structure was destroyed during the gothic war.

The Civic Tower dates to the XII century, it was built on the ruins of a Romanesque structure, in 1209 it became the imperial lieutenant's residence. The actual configuration of the Governor's Palace goes back to 1284, by the work of the Benincasa da Firenze and Bruno da Fabriano. The gallery construction on the tower and the ogival windows closing occurred in the XIV century. In 1511 started the arcade construction, on the right side of the Palace, designed by Costantino and Giovan Battista da Lugano architects, and was initially used as a textile market (Figure 2). The structures were modified several times, between 1670-1830, when they were subjected to the first restoration interventions; however, at their end, the palace was declared unsafe for static problems. To improve the tower stability in 1893 its base was extended. Similar interventions aimed to improve the stability of all the components of the building continued until 1997 when a big restoration was carried out after the Umbria-Marche earthquake.

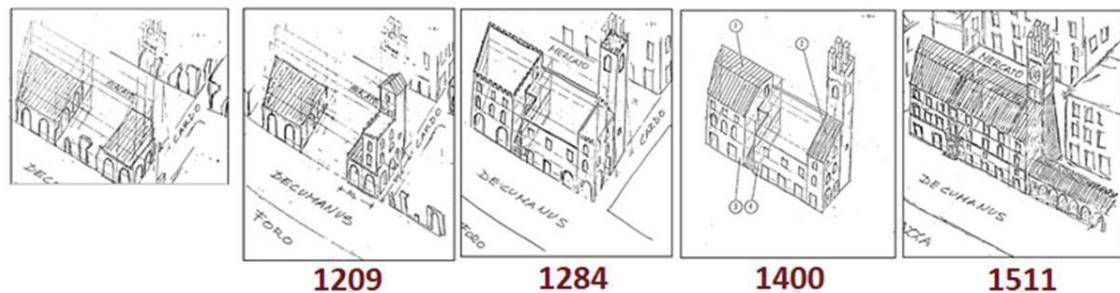


Figure 2 Historical evolution of the complex

### 2.2. Geometric survey

The palace has a rectangular plant of dimensions 13.30mx37.58m, it has three floors and a mezzanine between the ground and first floor. On the North-West side there is the Civic Tower, it has a rectangular plant 7.25mx7.45m and a height of 35m with eight levels divided by masonry and wood floors. The open gallery is on the North side of the palace. It has a rectangular plant characterized by 14 polygonal columns with ionic capitals that support round arches (Figure 3). The materials used are various. The most visible façades in the South and East are covered by regular masonry, whereas on the others are visible the irregular masonry Figure 1. The masonry type present are uncut stone for the palace, uncut stone and solid blocks for the tower and solid bricks for the arcade. The roofs are made of wood trusses.

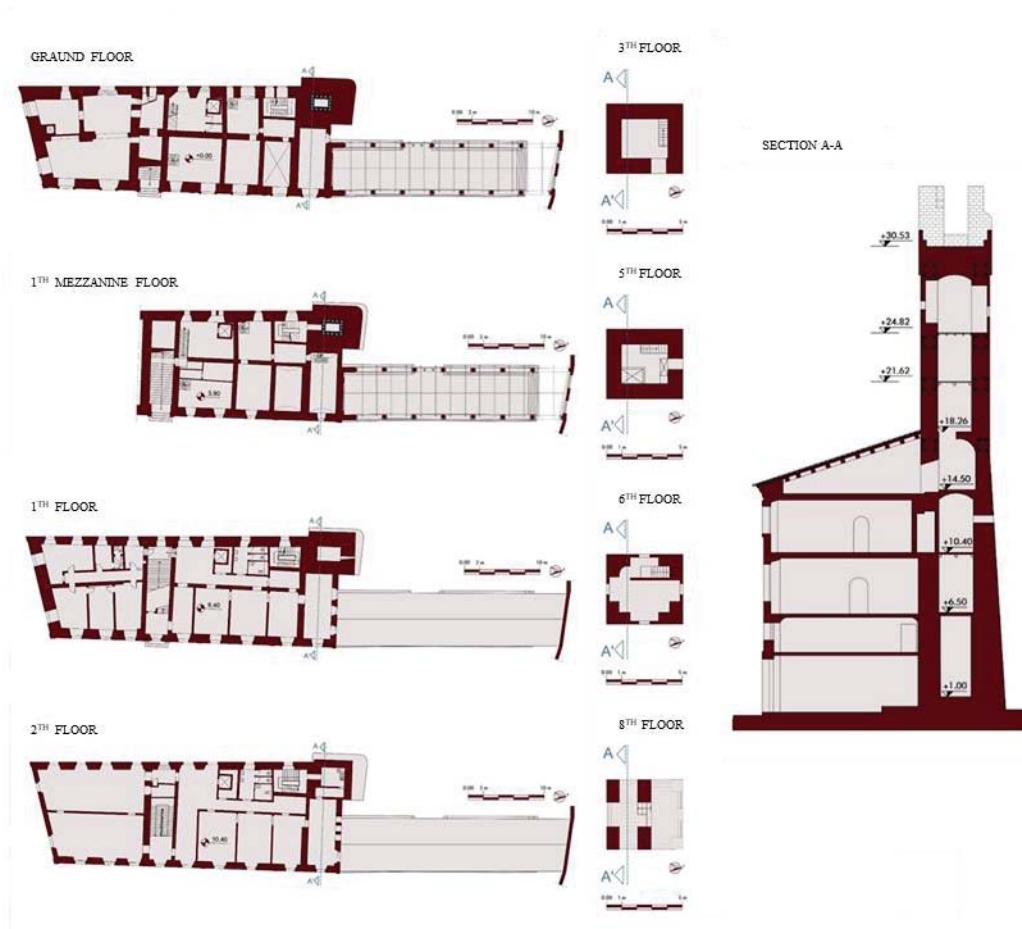


Figure 3 Geometrical configuration of the complex

### 3 DYNAMIC CHARACTERIZATION

#### 3.1 Ambient Vibration Testing

The study of historical buildings structural behavior is usually carried out through linear or non-linear analysis on FEMs. Being the objective to represent the real structure behavior in the most accurate way, the realization of these Numerical Models (NMs) requires a deep knowledge of the structural parameters, such as materials properties, connections among elements, boundary conditions, etc. Among all the possible methodologies which allows to investigate structural properties, these case study has been approached through a method which has become very popular in the last few years, due to its non-invasiveness and the satisfactory results produced: Ambient Vibration Testing (AVT). This method is based on the monitoring of vibrations produced on the building by ambient effects (like wind, traffic...) and on the extraction of modal properties (natural frequencies, modal damping and mode shapes) from the analysis of these data through modal identification techniques. Then these properties are used to build an Experimental Model (EM), whose response is compared with that of the NM. An iterative process, which consists in the variation of the unknown parameters in the model, like Young Elastic Modulus  $E$  and material density  $\rho$ , is applied in order to obtain correspondence between both natural frequencies and associated mode shapes of the two types of models [25].

### 3.2 Monitoring equipment, sensors layout and data

Structural monitoring of the case study has been performed using a wired sensors network, composed of:

- four triaxial accelerometers of GEA System series, with a sensitivity of 1000 mV/g, a measurement range of  $\pm 8g$  and A/D converter with a resolution of 24 bit;
- hub for synchronization;
- coaxial cables.

Sensors layout has been designed applying the concept of Optimal Sensor Placement (OSP) [26], so that, in the different layouts configured for the acquisitions, sensors have been placed in the most sensitive points of the building (in accordance with the possibility of reaching them), in order to better detect the translational and torsional components of the tower displacement [18]. In order to assure a rigid link between the selected point and the sensor epoxy resin has been used.

The monitoring activity of the tower has been conducted through 6 acquisitions, considering as many sensors layouts, which are shown in Figure 4. Data have been collected with a sample rate of 1024 Hz for periods which vary between 40 and 60 minutes, in order to satisfy Rodriguez condition [27].

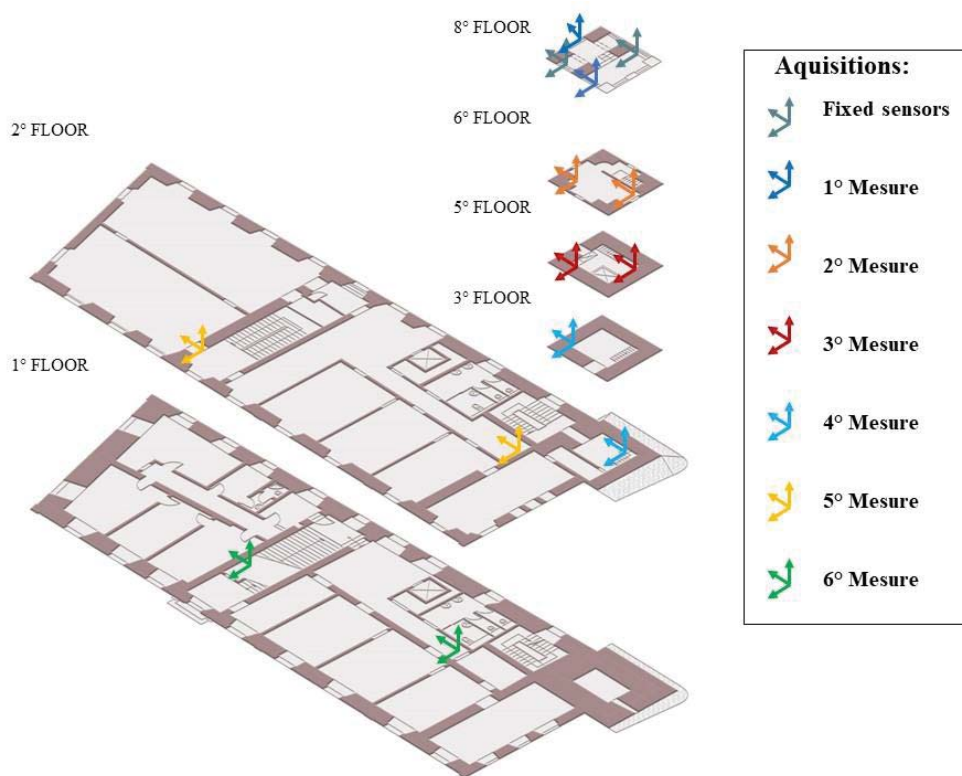


Figure 4 Sensors Layout



### 3.3 Operational Modal Analysis

#### 3.3.1 OMA identification technique

The acquired data, after a filtering process, have been analyzed through modal identification techniques. Among these methodologies some operates in time domain, while others operate in the frequency domain. For the case study under exam, it was decided to apply a time domain technique, called Stochastic Subspace Identification (SSI) method [28]. This methodology consists in a conversion of the second order differential equation of motion into two first order equations, defined “state equation” and “observation equation”, which, for a generic discrete time instant  $t = k\Delta t$  (where  $\Delta t$  is the sampling period and  $k \in \mathbb{N}$ ) can be written as:

$$\begin{aligned}\{\hat{x}_{k+1}\} &= [A]\{\hat{x}_k\} + [B]\{u_k\} \\ \{y_k\} &= [C]\{\hat{x}_k\} + [D]\{u_k\}\end{aligned}\quad (1)$$

where:

- $[A]$  is the matrix of input physical information;
- $[B]$  is the matrix of input statistical parameters;
- $[C]$  is the discrete output matrix;
- $[D]$  is the direct output transmission matrix;
- $x_k = x_k(\Delta t)$  is the discrete-time vector containing the sampled displacement and velocities;
- $\{y_k\}, \{u_k\}$  are the vectors of sampled input and output.

Other parameters describing the dynamic system are considered as deterministic and should not be subdued to the variations linked to excitation changes.

At this point, applying a Singular Value Decomposition (SVD) of the  $[A]$  matrix, the Eigenvalues, corresponding to the natural frequencies, and the correspondent Eigenvectors, which describe the mode shapes, are extracted.

For the examined case study, acquired data have been decimated into an interval of 12.5 Hz, in order to highlight the first five modes, mainly involving the tower, and their modal parameters. The values of natural frequencies and modal damping, for each mode, are reported in Table 1.

Mode	Frequency [Hz]	Damping [%]
1	1.923	0.679
2	2.007	0.873
3	5.058	2.931
4	6.531	1.569
5	7.629	2.705

Table 1 Modal parameters of Experimental Model

#### 3.3.2 Preliminary FE model

The results of the investigation campaign, carried out to assess the geometrical characteristics of the building and to evaluate status of the materials, allowed to realize a preliminary non calibrated NM, created using Midas FEA© software.

This model, even if the analysis is principally focused on the tower behavior, represents the complete structure, composed of the main building, the tower and the open gallery, due to the importance of considering the effects of interaction between the tower and the connected building [29–31]. The internal elements, like floors and stairs, have been modeled in case they

significantly influence the stiffness of the structure, like in case of the concrete floors built after the restoration works, while, in case of secondary elements, they have been considered only as loads distributed on the walls. As concerns the modeling process, the geometries have been discretized through 4-nodes tetrahedral solid elements, while, for the material properties assigned to the meshes, they have been selected from the tables reported in the Italian Code [32,33], scaled with the appropriate factors to take into accounts the knowledge level (KL1). These preliminary parameters are reported in Table 2.

Position	Types	W [kN/m <sup>3</sup> ]	E [N/mm <sup>2</sup> ]
Open Gallery Base	Soft stone masonry	13-16	1440
Open Gallery Walls	Solid bricks masonry	18	1500
Wood	Wood	7.6	1100
Building interior Walls	Solid bricks masonry	18	1500
Interior Arch Walls	Solid bricks masonry	18	1500
Building Perimetric Walls	Rough-hewn masonry	20	1440
Tower Walls (1-2-3-5-6)	Rough-hewn masonry	20	1440
Tower Wall (4)	Rough-hewn masonry	20	1230
Tower Walls (7-8-9)	Solid bricks masonry	18	1500

Table 2 Material parameters of preliminary Numerical Model

The NM, built with these characteristics, has been analyzed using Lanczos method, and has shown the results reported in Table 3, where the values of natural frequencies associated to the first five modes are shown.

Mode	Frequency [Hz]	Damping [%]
1	1.731	5.000
2	1.764	5.000
3	4.953	5.000
4	6.264	5.000
5	7.667	5.000

Table 3 Modal parameters of preliminary Numerical Model

### 3.3.3 NM calibration

The calibration process of the NM model consists, as already said, in an iterative procedure in which some parameters considered unknown, in particular Young Elastic Modulus and density of materials, are modified, at each step of the procedure, in order to obtain a coincidence, or at least a maximum difference ( $\Delta f$ ) of 5%, between the modal frequencies of the experimental and numerical models. Then, also the correspondence between the mode shape, associated to the same mode, is checked and validated, using the CrossMAC criterion [34]. This tool, which is applied in case of complex modes, provides an indication of the consistency of the modal vectors associated to the mode shapes obtained from different models. This indicator is calculated as the normalized scalar product of the modal vector for the  $r$  mode of analytical model  $\{\Phi_A\}$ , and the associated modal vector for the  $q$  mode of the test model  $\{\Phi_X\}$ :

$$MAC(r, q) = \frac{|\{\Phi_A\}_r^T \{\Phi_X\}_q|^2}{(\{\Phi_A\}_r^T \{\Phi_A\}_r)(\{\Phi_X\}_q^T \{\Phi_X\}_q)} \quad (2)$$

These scalar results are assembled in the CrossMAC matrix, whose optimal configuration is that of a principal diagonal with terms which are near or equal to 1 (which indicate good or perfect correspondence) and with terms outside of the diagonal which are near or equal to 0.

The first comparison in terms of frequencies between the EM and NM is shown in Table 4, which shows a too big difference in the values of frequencies associated to the first two modes.

Mode	Frequency EM [Hz]	Frequency NM [Hz]	$\Delta f$ [%]
1	1.923	1.731	9.98
2	2.007	1.764	12.11
3	5.058	4.953	2.08
4	6.531	6.264	4.09
5	7.629	7.667	0.50

Table 4 Comparison of EM and preliminary NM modal frequencies

Better correspondence, instead, is obtained comparing the mode shapes, whose direction of displacement are coincident between EM and NM, being the first two mode translational respectively in Y and X direction, the third and the fifth are flexional in Y and X, respectively, while the fourth shows a distorting behavior (Figure 5).

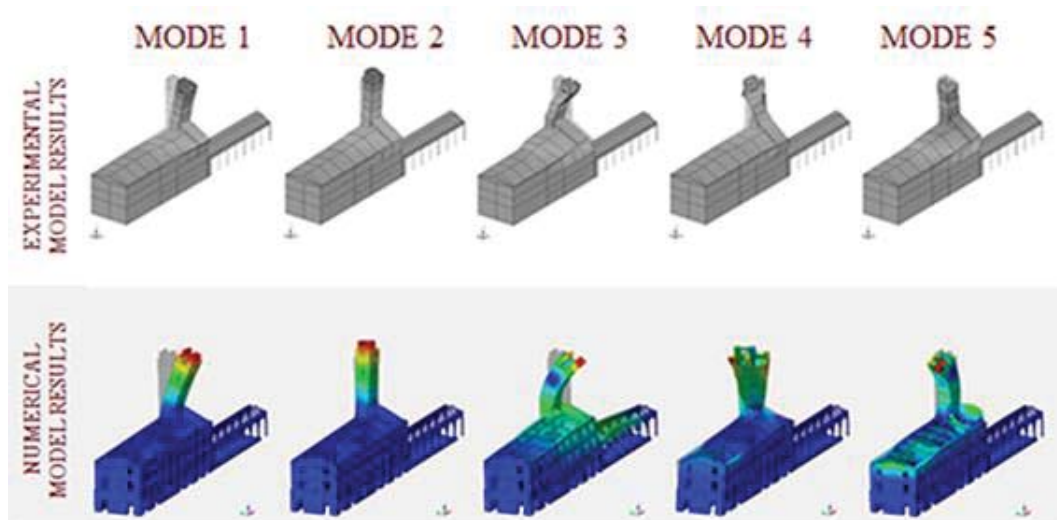


Figure 5 Comparison of mode shapes of preliminary NM and EM

The first step of parameter update has been operated only on the elastic modulus of the materials attributed to the tower elements. The criterion was to increase the stiffness of the element subjected to intervention in 2007 and decrease the density of some other elements, whose characteristics are inferior in respect of the previsions. The result of this operation comported a decrement of the frequencies values difference under the 5%, with the exception of the second mode.

In the second step, also the elastic modulus of elements which were not subdued to interventions has been increased, assuming that their deterioration was not as serious as thought at first. The result comported also a decrement of the difference in the values of frequency for the second mode under 5%.

In the last step it has been decided to intervene on the parameters associated to the building, dividing the perimetric walls in three sections, whose stiffness has been increased for the portion nearer the tower, and decreased for the other two parts. The results in terms of frequencies are reported in Table 5.

Mode	Frequency EM [Hz]	Frequency NM [Hz]	$\Delta f$ [%]
1	1.93	1.88	2.59
2	2.01	1.92	4.48



3	5.06	5.31	4.94
4	6.53	6.77	3.68
5	7.63	7.82	2.49

Table 5 Comparison of modal frequencies of EM and calibrated NM

This modification allowed a better correspondence between mode shapes, as it is possible to see from Figure 6, and as attested in Table 6, where CrossMAC results are shown.

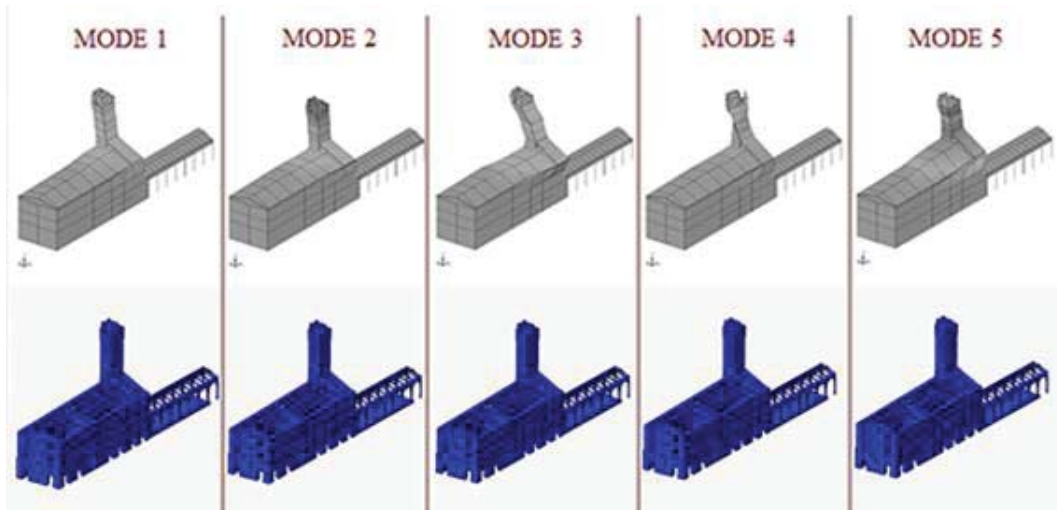


Figure 6 Mode shapes comparison after calibration

CrossMAC		EXPERIMENTAL MODEL				
		MODE 1	MODE 2	MODE 3	MODE 4	MODE 5
		1.93 [Hz]	2.01 [Hz]	5.06 [Hz]	6.53 [Hz]	7.63 [Hz]
NUMERICAL MODEL	MODE 1	0.89	0.03	0.00	0.00	0.01
	1.88 [Hz]					
	MODE 2	0.00	0.98	0.01	0.04	0.02
	1.92 [Hz]					
	MODE 3	0.19	0.02	0.70	0.05	0.30
	5.31 [Hz]					
	MODE 4	0.00	0.02	0.00	0.81	0.16
	6.77 [Hz]					
	MODE 5	0.00	0.09	0.11	0.10	0.78
	7.82 [Hz]					

Table 6 Cross MAC after calibration

These results highlight the low level of interaction between the tower and the palace, whose dynamic behaviors do not influence each other so much, probably due to the different manufacturing and absence of linking elements. In particular, the disorganic movement of the tower, shown for the higher modes, and especially the distorting behavior of the fourth mode, can be linked to the sectional changes and to the variation of material properties. As concerns the damage propagation, the most damages zones are the ones with low values of Young Elastic Modulus.

#### 4 CONCLUSIONS

The model updating process, operated for the tower and the rest of the Governor Palace structure, produced good results in terms of correspondence with the modal parameters and mode shapes extracted from EM. Moreover, it allowed the observation of an interesting

phenomenon in the tower dynamic behavior, such as the sectional distortion caused by the fourth mode.

Once again, dynamic monitoring and modal identification techniques, confirmed their effectiveness in the evaluation of dynamic behavior and consequently on the health status of historical structures, being the method non-invasive and allowing to produce accurate numerical models, which can be used implement rapid, localized and economically sustainable interventions.

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