

## THE MODELLING OF MULTIPLE LEAF MASONRY WALLS OF THE ARQUATA DEL TRONTO FORTRESS AS A NON-SMOOTH DYNAMICAL SYSTEM

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**Abstract.** *In the present paper has been reported the Non-Smooth Contact Dynamics method (NSCD) used for the damage evaluation of the multiple leaf masonry walls of the medieval fortress of Arquata del Tronto, strongly affected by the last Centre Italy earthquakes of August and October 2016. Pursuing this approach, a system of rigid blocks has been used for the assembly of the masonry structure. By means of contacts between blocks, which are governed by the Signorini's impenetrability condition and by dry-friction Coulomb's law, the building exhibits discontinuous dynamics. Finally, the NSCD method has proved to be an effective instrument for investigating the dynamical behaviour of the masonry structures under the ground seismic accelerations. Indeed, several possible failure mechanisms have been confirmed through the numerical results, which have given a deep insight into the seismic vulnerability of this damaged medieval fortress.*

## 1 INTRODUCTION

In the structural mechanics became relevant the attention for the damage assessment of historical masonry buildings, which are commonly complex, with irregular geometries, heterogeneity and absence of a box behaviour due to defective connections between different structural parts, in particular walls and floors, that often play a fundamental role. However, the knowledge of the dynamical behaviour is crucial for a reliable seismic vulnerability assessment, which became more and more important due to recent catastrophic earthquakes that stroked Italy in the last few decades (Umbria-Marche 1997–1998, Abruzzo 2009, Emilia-Romagna 2012, Marche-Lazio-Umbria-Abruzzo 2016) [1]–[3].

In October 2016, two major earthquakes occurred in the Marche region in the Centre of Italy, causing widespread damage especially on the historical structures. The epicentre of the second one stroked Norcia, Visso, Arquata del Tronto, Accumoli and Amatrice, and a lot of damages to cultural heritage were also done in the cities of Tolentino, San Severino, Camerino, and Matelica.

To investigate the mechanical behaviour of masonry structures, commonly Finite Element Methods is utilized, often including very sophisticated constitutive laws taking into account post-elastic behaviours and damage [4]–[8]. This method, while being very appealing, do not focus on the possible non-smooth nature of the dynamic response, which can come sliding and impacting between different blocks, and situation that is common just before and during the collapse [9]–[13].

Moreover, the ancient masonry structures can be considered as discontinuous structural systems, which is composed of units (e.g. bricks, stones, blocks, etc.), bonded together with or without mortar. Thus, defining the constitutive model and the material properties for a numerical model is meaningful to adequately represent the dynamical behaviour of a real structure, to take into account the variation of masonry properties and the range of stress state types that exist in masonry structures [14]–[16].

Hence, the progressive damage of the Arquata del Tronto medieval fortress, in the province of Ascoli Piceno (AP), is examined throughout the paper and it is based on an advanced numerical point of view. The measured damage evaluation is relative to the long sequence of strong earthquakes that struck Central Italy in 2016, plunging the area into chaos for several months (see Figure 1) [17].

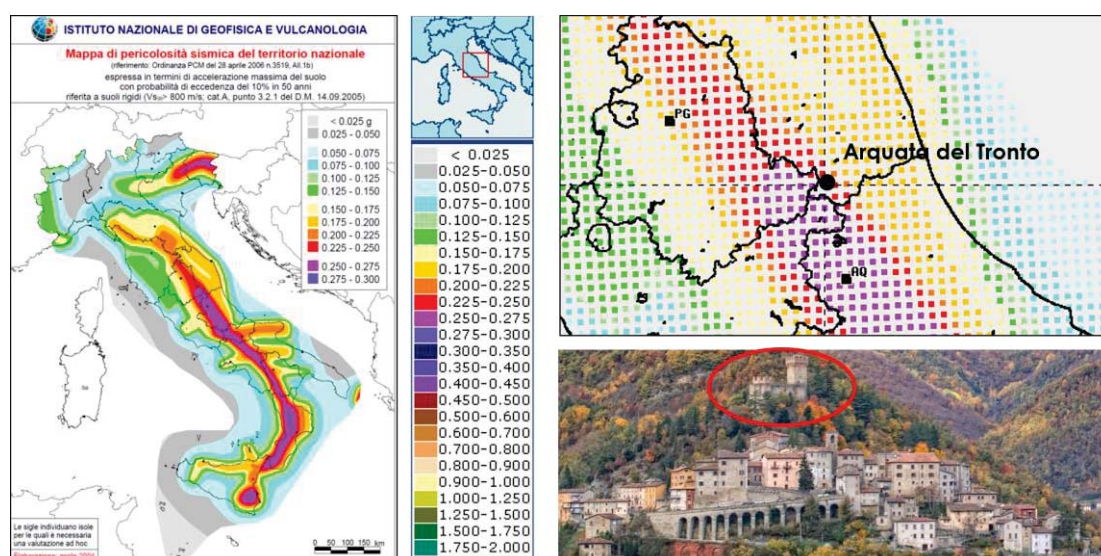


Figure 1: Macroseismic intensity map of the Central Italy area (<https://emidius.mi.ingv.it/>) and localization of the Arquata del Tronto (AP) medieval fortress

An advanced numerical model is here utilized to have an insight into the modalities of progressive damage and the behaviour of the structure under strong nonlinear dynamic excitations, namely a Non-Smooth Contact Dynamic method (NSCD) adopting a full 3D detailed discretization [18], [19].

In accordance with the NSCD method, the fortress is structured as a system of rigid bodies, assuming frictional sliding and perfect plastic impacts between the blocks. The structure exhibited a complex dynamic behaviour, because of the geometrical nonlinearity and the non-smooth nature of the contact laws [13], [20], with a focus on the possible non-smooth nature of the dynamic response, which can come commonly just before and during the collapse.

The main aim of the survey is to confirm the weakness zones, the efficacy of past interventions and to evaluate the acceleration of the activation of different mechanisms, by means of the discontinuous approaches, to address comparison between the numerical and the real damages, and, at the same time, to confirm the powerful of the model.

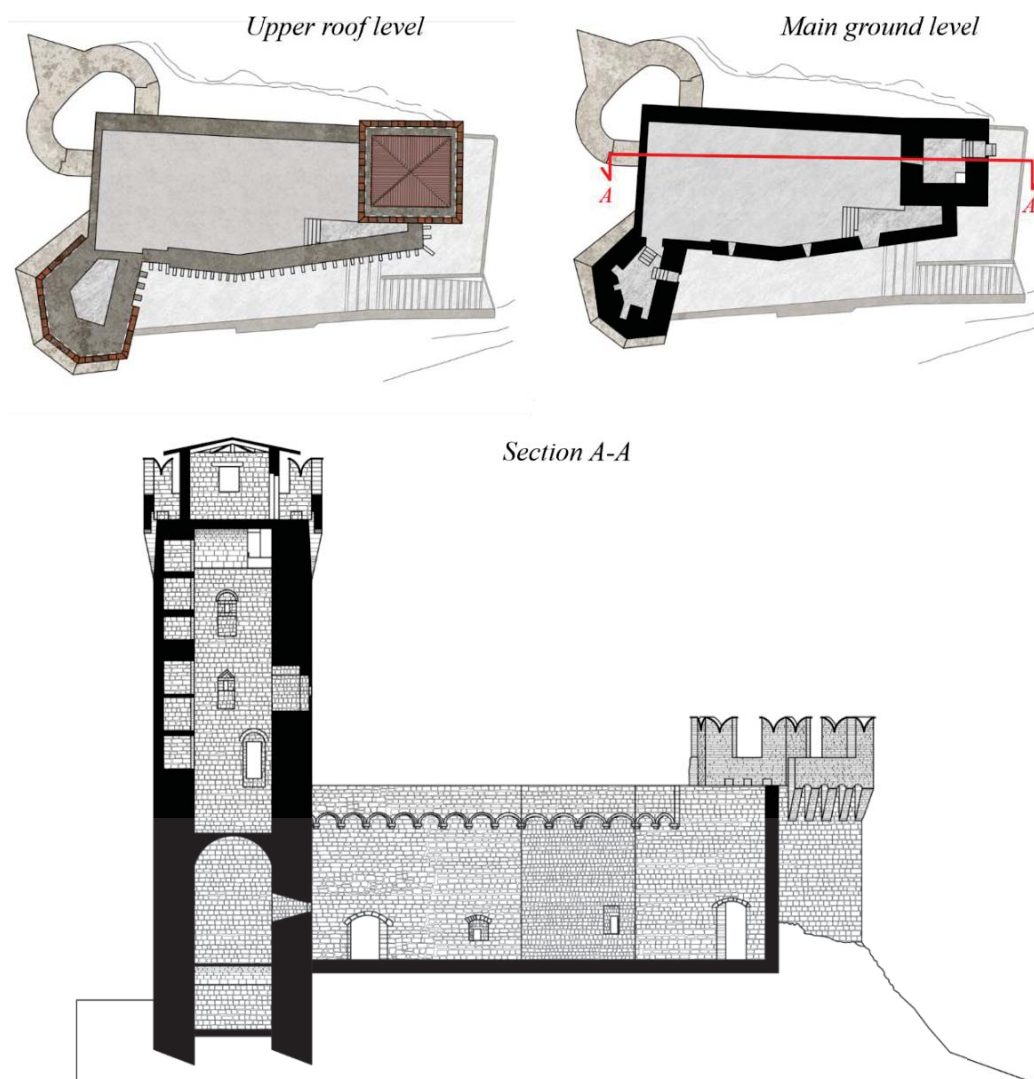


Figure 2: Geometric configuration of the Arquata del Tronto (AP) medieval fortress

## 2 HISTORICAL DEVELOPMENTS OF THE MEDIEVAL FORTRESS

The fortress of Arquata del Tronto, in Figure 2 and Figure 3, which is a typical example of Umbria-Marche Apennines fortified architecture of the XIII century, is mainly made by cal-



careous stone masonry walls made of multiple leaf, which can be prone to disintegration under strong dynamic actions. The fortress is in a strategical position, located on the border between the Ascoli and Norcia's administrations, and was damaged but left standing by the magnitude 6.2 earthquake event occurred in August 2016 that flattened the town centre. The medieval fortress lost *chemin de ronde* and was one of the few buildings in the town to remain still standing after the main shock. It was then severely damaged by further quakes that rattled the region in October 2016, demonstrating a not proper transverse bond for most of the masonry walls.

Moreover, the first built element of the fortress is the tower with the hexagonal plant and height of 12 m, located in the South-East edge, which encloses an irregular pentagonal environment. At the base, it is visible the gate of the ancient walled now escape tunnels. Between the XIV and XV centuries was built the North tower, with a square base and height of 24 m, which was destined sighting and extreme defence. On its top, there is a quadrangular room that was built in the early years of the XX century. The last built part is a circular tower with a diameter of 10 m and a height of 12 m, located at the South-West corner.

Following earthquakes occurred in 1703, the fortress of Arquata del Tronto suffered serious damages. Therefore, various retrofitting interventions were carried out in subsequent years. The last restoration work, before the seismic sequence of 2016, was made in the last decade of XX century.

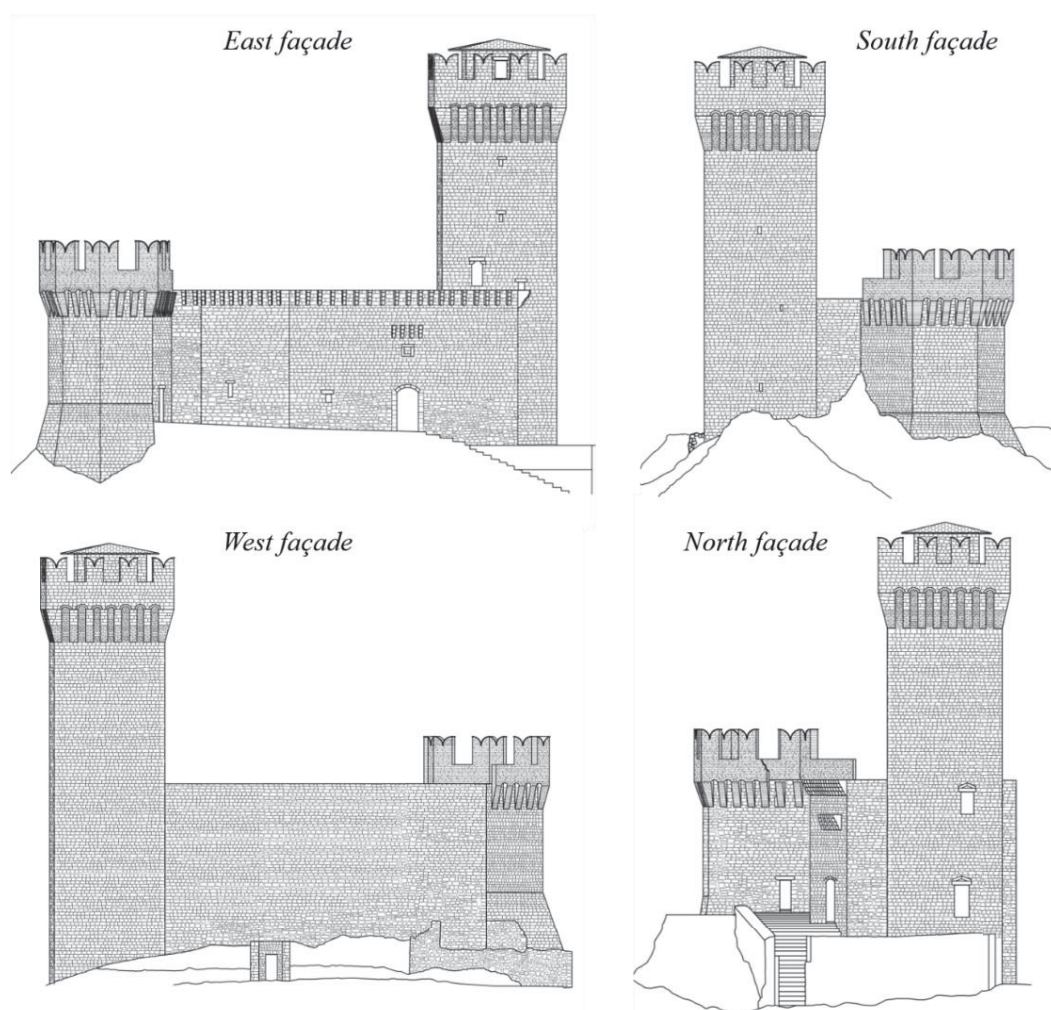


Figure 3: Geometric configuration of the Arquata del Tronto (AP) medieval fortress

## 2.1 Damage of the Arquata del Tronto fortress after the Central Italy earthquakes of 2016

Globally, the Arquata del Tronto medieval fortress has suffered extended damages after the shocks of the Central Italy seismic sequence of 2016.

The major cracks appeared on the upper part of the masonry towers, with the complete overturning of the merlons and the collapse of the protruding turret, visible in Figure 4a-b. Moreover, there are widespread crumbling of the multiple leaf masonry walls, partially showed in Figure 4c.

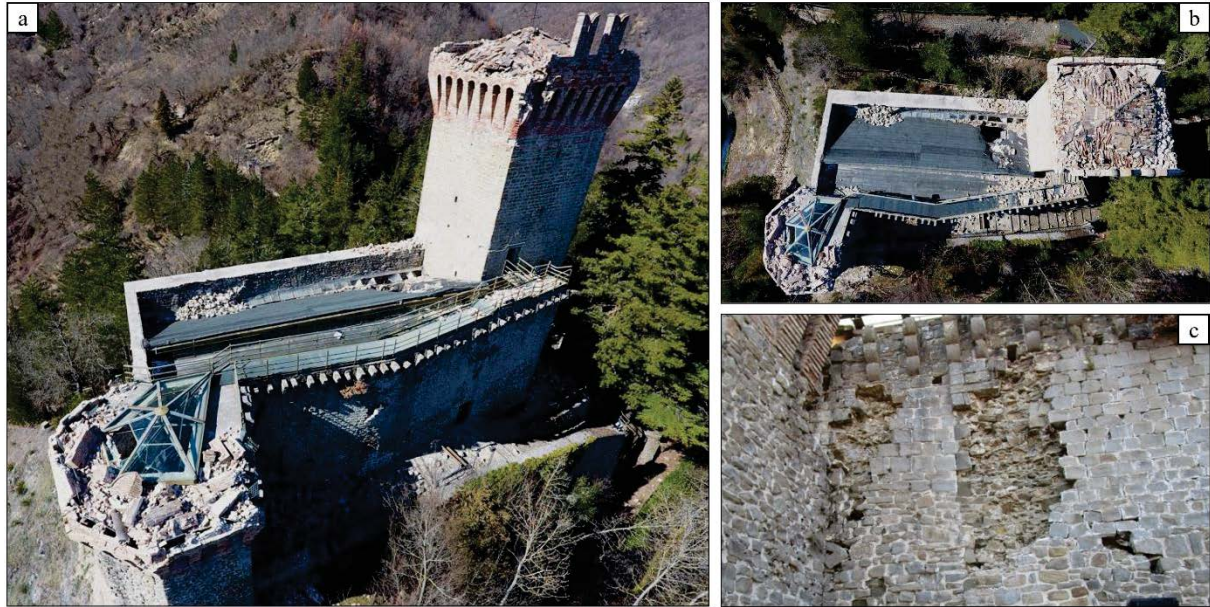


Figure 4: Views of the global (a, b) and of the multiple leaf masonry walls (c) damages of the Arquata del Tronto (AP) medieval fortress after the seismic sequence of 2016

## 3 THE NON-SMOOTH CONTACT DYNAMICS METHOD

The NSCD method belongs to the family of Discrete Element methods, distinguishing from the classical Distinct Element method for three differences: (i) it integrates the non-smooth contact laws directly, (ii) it uses an implicit integration scheme, and (iii) it does not account for any structural damping. It is important to stress the fact that the NSCD method is based on some modelling simplifications. The main assumption is that bodies are rigid, and their dynamics is governed by the equation of motion and by the frictional contact conditions. To describe the frictional contact laws, we must introduce some basic definitions. In the following, the notation adopted in [21] is used (scalars, vectors, and tensors are explicitly declared, and italic letters are used for all of them). Given two arbitrary bodies  $B_i$  and  $B_j$ , let  $P_i$  and  $P_j$  (Figure 5a) be the points of possible contact on the boundaries of  $B_i$  and  $B_j$ , respectively, and let  $n$  be the outer unit vector, orthogonal to the boundary of  $B_i$  in  $P_i$ . We define  $g = (P_j - P_i) \cdot n$  the gap between  $P_i$  and  $P_j$  (a dot means scalar product),  $(\dot{u}_n, \dot{u}_t)$  the normal and tangential velocities of  $P_j$  with respect to  $P_i$ , and  $(r_n, r_t)$  the normal and tangential reactive forces of  $B_i$  on  $B_j$ .

The contact conditions are:

- 1) The Signorini's law of impenetrability (Figure 5b)

$$g \geq 0, r_n \geq 0, gr_n = 0, \quad (1)$$

which, in the case of contact  $g = 0$ , is equivalent to the following Kuhn–Tucker conditions [18]

$$\dot{u}_n \geq 0, r_n \geq 0, \dot{u}_n r_n = 0, \quad (2)$$

written in term of relative normal velocity.

- 2) The dry-friction Coulomb's law (Figure 5c) that governs the behaviour in the tangential direction

$$|r_t| \leq \mu r_n: \begin{cases} r_t < \mu r_n \rightarrow \dot{u}_t = 0 \\ |r_t| = \mu r_n \rightarrow \dot{u}_t = -\lambda \frac{r_t}{|r_t|} \end{cases} \quad (3)$$

with  $\mu$  the friction coefficient and  $\lambda$  an arbitrary positive real number.

If  $q$  is the vector of the system configuration parameters (unknown translations and rotations of each body), and  $p$  is the global vector of reaction forces, the equation of motion can be written as follows

$$M\ddot{q} = f(q, \dot{q}, t) + p, \quad (4)$$

where  $M$  is the mass matrix, and  $f$  is the vector of external forces.

The local pairs  $(\dot{u}_n, \dot{u}_t)$  and  $(r_n, r_t)$ , characteristic of each contact, are related to the global vectors  $\dot{q}$  and  $p$ , respectively, through linear maps which depend on  $q$ . Since the contact laws (1) - (3) are non-smooth, velocities  $\dot{q}$  and reactions  $p$  are discontinuous functions of time. They belong to the set of bounded variation functions, i.e. functions which, at each time, have finite left and right limits. Since the accelerations are not defined when the velocities are discontinuous, Eq. (4) is reformulated in the integral form [18], [21], and it is solved numerically using a time-stepping approach, where time is discretized into time intervals, and, within each time interval  $[t_i, t_{i+1}]$ , the equation of motion is integrated as follows

$$M(\dot{q}_{i+1} - \dot{q}_i) = \int_{t_i}^{t_{i+1}} f(q, \dot{q}, t) dt + \bar{p}_{i+1}, \quad (5a)$$

$$q_{i+1} = q_i + \int_{t_i}^{t_{i+1}} \dot{q}(t) dt. \quad (5b)$$

Where  $\bar{p}_{i+1}$  is the impulse in  $[t_i; t_{i+1}]$ . The primary variables of the problem are the velocity vector  $\dot{q}_{i+1}$  and the impulse vector  $\bar{p}_{i+1}$  at the instant  $t_{i+1}$ . In the NSCD method, the integrals in (5a, 5b) are evaluated by means of an implicit time integrator. The overall set of global Eq. (5a, 5b) and local contact relations (1) and (2), where the reactions are approximated by the average impulses in  $[t_i; t_{i+1}]$ , is condensed at the contact local level, and then they are solved by means of a non-linear Gauss–Seidel by block method.

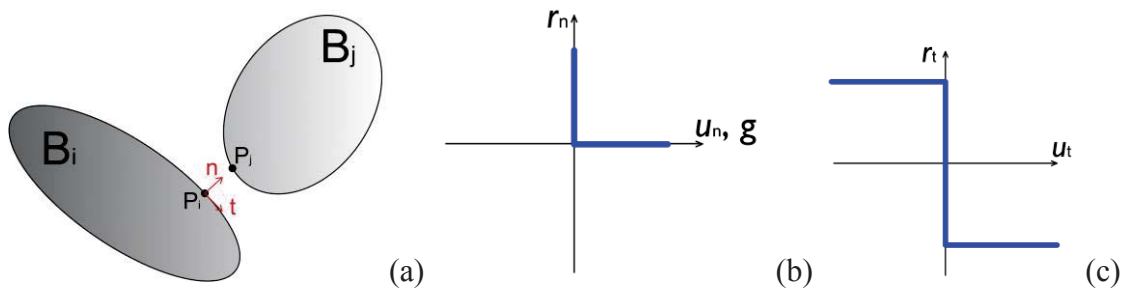


Figure 5: The interaction between two bodies (a) Signorini's law (b) Coulomb's law (c)



The relations (1) imply a perfectly plastic impact, i.e., the Newton law with restitution coefficient equal to zero. A perfect plastic impact law makes impossible to describe bouncing phenomena, and, furthermore, overestimates the energy dissipated during impacts. However, in case of systems of bricks or stones, the restitution coefficient has low values, and bouncing phenomena can be neglected. Since we are interested in the dynamical interactions between different parts of the fortress, we neglect blocks deformability. It follows that the numerical results obtained depict an overall picture of the fortress dynamics and describe the failure mechanisms of the whole structure, due to blocks rocking and sliding, but they do not describe the stresses and strain distributions within each block.

The values of friction coefficient range from 0.3 to 1.2, according to different combinations of units and mortars [22]. As a first attempt, we assume the value  $\mu = 0.5$  for the interface block/block of the external and internal leaf,  $\mu = 0.3$  for the infill and the interface between it and the leaf, and  $\mu = 0.9$  for the interface block/foundation to observe, mainly, the dynamics of the fortress without the structure-foundation interaction. Finally, we note that damping is not considered here and only friction and perfect plastic impacts dissipate energy.

The LMGC90<sup>©</sup> code is used, due to its ability to compute the interaction of a large number of bodies, based on the NSCD method, also assessing the seismic vulnerability of the structures [23]. As visible in Figure 6, very detailed models have been created to understand better the influence of local and global mechanisms within the analysed medieval fortress and, where possible, the presence of some past retrofitting interventions is also considered.

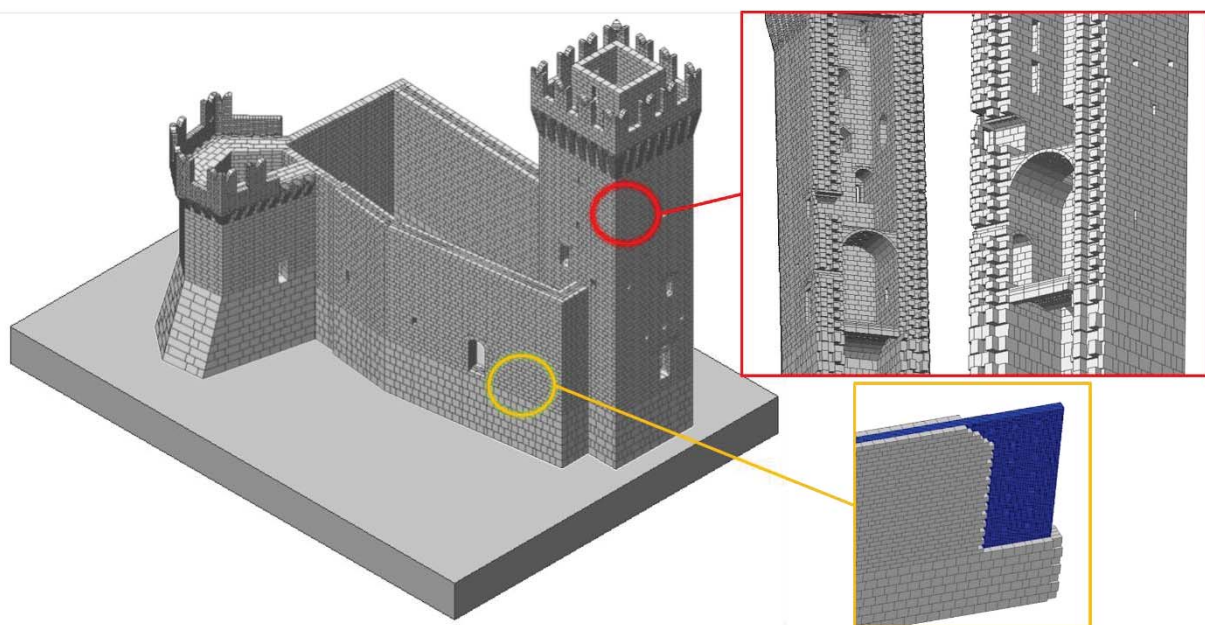


Figure 6: 3D discrete element model of the Arquata del Tronto (AP) medieval fortress

#### 4 NUMERICAL RESULTS

The main results of the nonlinear dynamic analyses are reported in Figure 7. For this purpose, it has been applied to the main shock of the seismic sequence of 2016 in the Centre of Italy. The dynamic action used in the nonlinear analyses is recorded near Arquata del Tronto (AP) in 24<sup>th</sup> August 2016 Amatrice (AMT station in Italian Accelerometric Archive (ITACA)) with  $M_w=6.0$ .

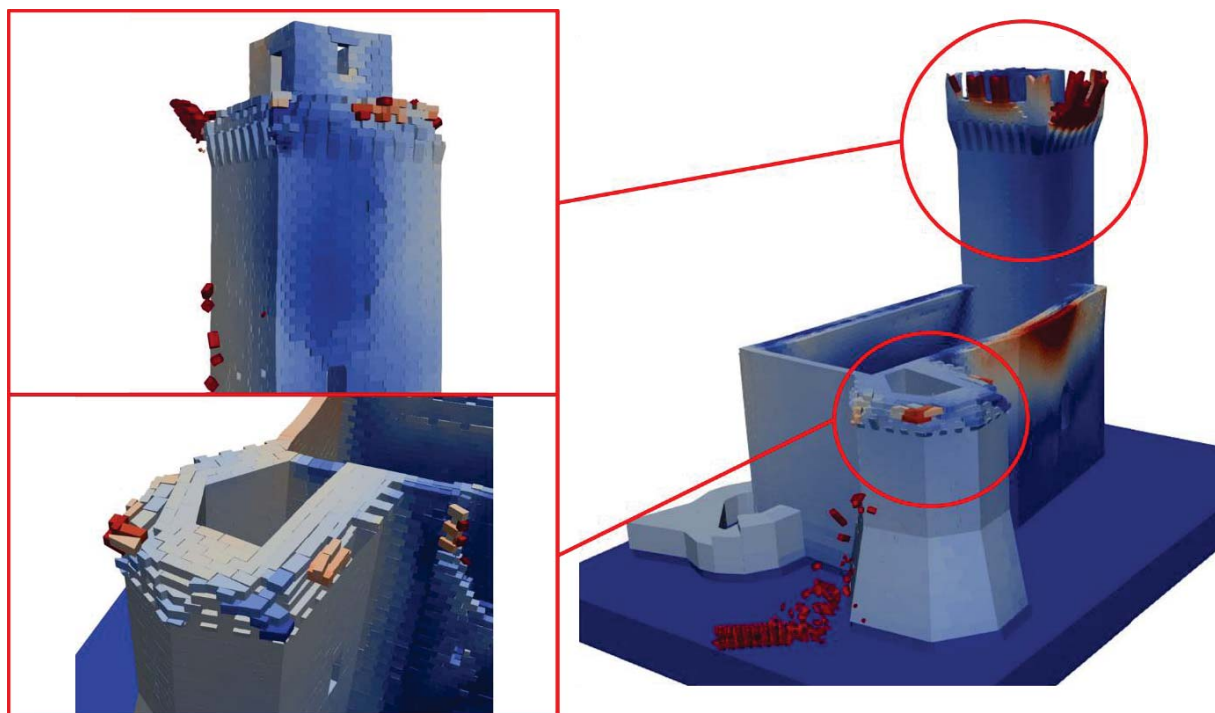


Figure 7: Failure mechanism of the Arquata del Tronto (AP) medieval fortress

Thus, from the numerical results, both the role played by the actual geometry and the insufficient resistance of the constituent materials are envisaged, showing a good match with actual crack patterns and collapses observed after the seismic sequence. Finally, the numerical analyses provide a valuable picture of the actual behaviour of the fortress, thus giving useful hints for the reconstruction and future effective strengthening interventions.

## 5 CONCLUSIONS

The results obtained for the Arquata del Tronto medieval fortress stroked by the Central Italy seismic sequence are here briefly reported and summarized, underline the high vulnerability of this type of multiple leaf masonry walls, and especially of the upper where merlons are placed.

A discontinuous approach and the NSCD method, implemented in the LMGC90<sup>®</sup> is used in order to assess numerically the real damage. The NSCD method combines modelling simplicity and great predictive capabilities.

Its simplicity comes from the following fundamental simplifying assumptions: (i) block rigidity; (ii) simple contact laws between blocks; (iii) absence of any damping. As a result, the mechanical behaviour of the masonry structures is influenced by only the friction coefficient. This is a significant consequence for modelling ancient buildings since the determination of the mechanical properties of these masonries is always uncertain and variable. Despite its simplicity, the model can predict a large variety of dynamical behaviours of the historical structures and their seismic vulnerability.

The numerical investigation pointed out that no collapse mechanism passes the safety check. This outcome suggests that the reconstruction of the collapsed parts on the top of the towers if pursued, should be complemented by specific strengthening devices.



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