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ANALYSIS OF TALL STEPPING VIADUCT PIERS UNDER SEISMIC EXCITATION

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Abstract. *The analysis of peak displacement response in stepping viaduct piers is essential for achieving safe structural design. The conventional iterative analysis methods prescribed in codes of practice are not guaranteed to converge, highlighting the need for an alternative design approach. We present a closed-form solution for the peak quasi-dynamic displacement response of stepping flexible frames subjected to earthquake ground motions.*

Stepping response, first implemented in a tall railway viaduct in New Zealand, offers particular advantages in seismic regions. It significantly reduces design force demands at the base of piers, eliminating the need for heavy footings or deep foundation elements. As a result, substantial reductions in construction costs and seismic damage can be achieved due to period elongation associated with the stepping mechanism. Another benefit is that stepping structures align with the design philosophy of rapid recovery after extreme loading—often referred to as "resiliency"—because they mitigate permanent offsets and cumulative post-elastic cyclic deformations, both of which contribute to increased repair costs and reduced functionality.

We then consider the phenomenon of moving resonance and present an approximate analytical expression to account for its effects on the seismic response of stepping frames. The closed-form solution for displacement demand can be readily adapted to incorporate the influence of moving resonance on the quasi-dynamic response of stepping oscillators.

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1 INTRODUCTION

In modern earthquake engineering, it is prudent and sometimes necessary to permit rocking response (here, we use rocking and stepping response interchangeably), despite the traditional design preference of avoiding such instabilities. Under controlled conditions, temporary loss of stability can be advantageous in reducing construction and repair costs. The first modern structure to intentionally employ this behavior — and still in service today — is the 315-m-long S. Rangitikei Viaduct, featuring 76-meter-tall stepping piers constructed in the 1970s [1]. Observations of the survival of tall and slender structures during the May 1960 Chilean earthquake inspired Housner's seminal work [2] which established the fundamental theory for the dynamic response of an ideal rocking rigid block and enabled subsequent research and development in this area. More recently, the design of structures with minimal damage after natural disasters (a design philosophy termed "resiliency") has led to the successful implementation of controlled rocking of large structural frames, making the study of stepping response more relevant.

The analysis of peak displacement response in stepping viaduct piers is essential to achieve a safe structural design. Despite decades of research, modern seismic design standards and specifications are only beginning to adopt controlled rocking as a viable means of ensuring safe seismic performance [3]. Drawing on observations from earlier rocking structures—and recognizing the substantial economic savings that can be realized by allowing foundations to uplift—Priestley et al. [4] developed an iterative trial-and-error framework to estimate the maximum rocking displacement demand of rocking piers using equivalent linearization and response spectrum techniques. Their work also appears to be the basis of recent developments in the adoption of rocking response into seismic design provisions [5]. However, this iterative procedure occasionally fails to converge to a stable displacement demand, an issue that was also noted by Priestley et al. [4], who observed that "*in some cases no stable response can be achieved.*" This limitation has hindered broader adoption of this promising technology. To address this challenge, we present a closed-form solution for the peak quasi-dynamic displacement response of stepping flexible frames subjected to earthquake ground motions [6].

Next, we discuss Moving Resonance, an inherent feature of the dynamic response of softening nonlinear oscillators under nonstationary stochastic excitation. Moving Resonance occurs when changes in the oscillator's natural frequency track the shift of dominant frequencies in the excitation with increasing amplitudes. As with inverted pendula, where the oscillation period lengthens with increasing sway amplitude, the rocking period of vibration elongates logarithmically with displacement amplitude [1, 2]. Meanwhile, many earthquake ground motions exhibit a temporal shift of frequency content toward lower frequencies, as commonly observed in accelerograms. It has been shown that the effect of temporal nonstationarity in the excitation's frequency content on the response of nonlinear oscillators can be severe, producing response amplifications by factors as high as three [7]. We investigate this phenomenon and propose a simplified approach that may be useful for addressing its effects during design.

2 Analysis of Displacement Demand

Priestley's method [4] is an iterative procedure for determining the peak rocking displacement of an equivalent linear single-degree-of-freedom (SDOF) oscillator. The method incorporates three key components: static equilibrium at the displaced position with the foundation soil at its limiting capacity, the SDOF oscillator period, and the pseudo-acceleration design spectrum. With reference to Fig. 1, the relevant equations to be solved are:

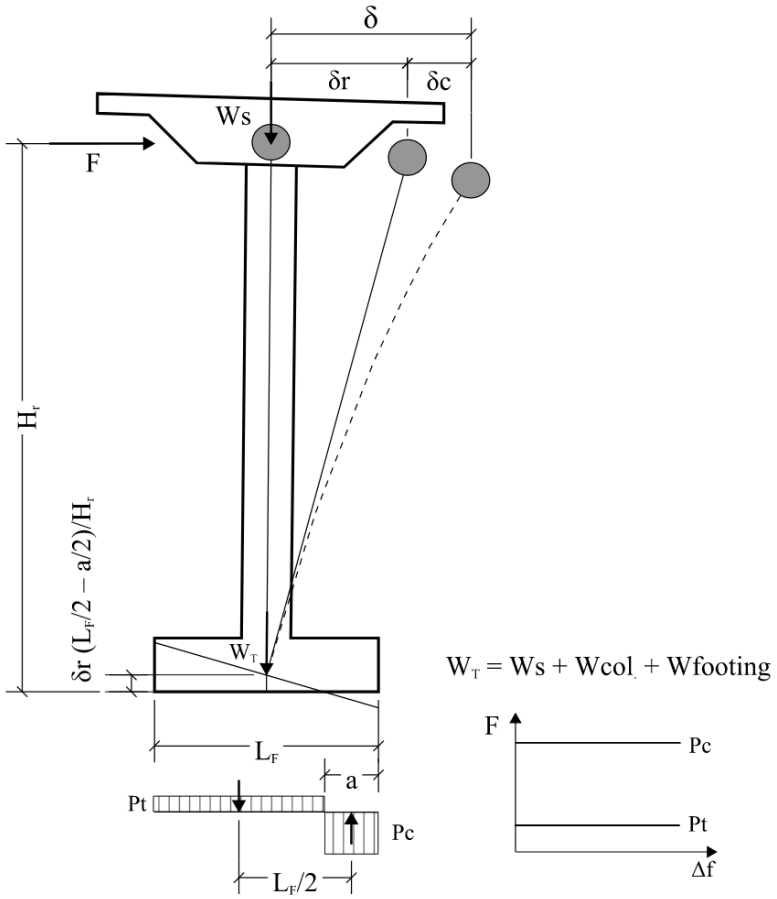


Figure 1: Rocking of a viaduct pier [4].

$$
\begin{aligned}
F &= W_T \frac{(L_F - a)}{2H_r} - W_s \frac{\delta}{H_r} \\
T &= 2\pi \sqrt{\frac{(W_s + 0.5W_{col})}{gK}} \\
\delta &= \left(\frac{T}{2\pi}\right)^2 \cdot \beta \cdot S_a\,(T)
\end{aligned}
\tag{1}
$$

where $F = K\delta$, K, T, and δ are the lateral force, the equivalent lateral stiffness, the equivalent period of vibration, and the total rocking displacement; $W_T = W_s + W_{col}$ is the total weight of the pier, including the superstructure and the column; a is the width of the rectangular compression stress block at the foundation capacity P_c; $S_a\,(\circ)$, β and g are the spectral acceleration of a five-percent damped design spectrum at the site, the spectral acceleration reduction factor due to damping, and the acceleration due to gravity; and L_F and H_r are shown in the figure.

2.1 The Iterated Maps of Design Displacement

For a numerical solution using a simple piece-wise linear design spectrum for $S_a\,(T)$, the set of equations in (1) can be expressed in the form of nonlinear maps that govern the stepping

response of an SDOF oscillator [6]:

$$\begin{cases} \delta_{i+1} = \lambda_1 \left(\frac{\delta_i}{1 - w \cdot \delta_i} \right) \triangleq f_1 \left(\delta_i \right) \\ \delta_{i+1} = \lambda_2 \left(\frac{\delta_i}{1 - w \cdot \delta_i} \right)^{\frac{1}{2}} \triangleq f_2 \left(\delta_i \right) \end{cases} \tag{2}$$

for small-displacements period of the oscillator below, $f_1(\circ)$, and above, $f_2(\circ)$, the characteristic period of ground motion. In (2):

$$w = \frac{2W_s}{W_T \left(L_F - a \right)} \tag{3}$$

and:

$$\begin{cases} \lambda_1 = 2H_r \cdot \frac{\beta S_{DS}}{g} \cdot \frac{W_s + \frac{1}{2} W_{col}}{W_T (L_F - a)} \\ \lambda_2 = \sqrt{H_r} \cdot \frac{\beta S_{D1}}{\pi \sqrt{2g}} \cdot \sqrt{\frac{W_s + \frac{1}{2} W_{col}}{W_T (L_F - a)}} \end{cases} \tag{4}$$

with S_{DS} and S_{D1} as the parameters of the chosen ground motion response spectrum. The stable solution for the displacement demand, δ^*, occurs for the case using f_2 in (2):

$$\delta^* = \frac{1 - \left(1 - 4w\lambda_2^2 \right)^{\frac{1}{2}}}{2w} \tag{5}$$

3 Moving Resonance

Moving resonance [7] is an intrinsic feature of the dynamics of nonlinearly softening oscillators subjected to nonstationary stochastic excitation. It occurs when changes in the natural frequencies of the system track the shift of dominant frequencies in the excitation with increasing amplitudes. While resonance in linear oscillators refers to the exponential growth of response when the natural frequency is at, or close to, the excitation frequency, moving resonance describes a similar phenomenon in the nonlinear oscillator whose natural frequency evolves in step with the changing frequency content of the nonstationary excitation. Because the rocking period of vibration lengthens with displacement amplitude, while earthquake ground motions commonly exhibit a temporal shift of frequency content toward lower frequencies, accounting for moving resonance is important when evaluating the response of a rocking pier.

The mean-square displacement response of a time-varying linear oscillator subjected to a nonstationary ground motion process is governed directly by the amplitude intensity and the time-varying power spectral density of the ground motion, [7–9], the latter of which is modeled as:

$$R_g \left[\omega(t), \xi_0 \omega_0 \middle| \underline{\theta}_g(t) \right] = 2I_c(t) + 2\xi_0 \omega_0 I_s(t) \tag{6}$$

where:

$$I_c(t) = \frac{\left[\xi_0\omega_0 + \alpha_g(t)\right]\left\{\left[\xi_0\omega_0 + \alpha_g(t)\right]^2 + \omega_d^2(t) + \omega_g^2(t) - \alpha_g^2(t)\right\}}{\left[\omega^2(t) - \omega_g^2(t)\right]^2 + 4\omega(t)\omega_g(t)\left[\xi_0\omega_0 + \alpha_g(t)\right]\left[\xi_g(t)\omega(t) + \xi(t)\omega_g(t)\right]} + \frac{\alpha_g(t)\left\{\left[\xi_0\omega_0 + \alpha_g(t)\right]^2 - \omega_d^2(t) + \omega_g^2(t) - \alpha_g^2(t)\right\}}{\left[\omega^2(t) - \omega_g^2(t)\right]^2 + 4\omega(t)\omega_g(t)\left[\xi_0\omega_0 + \alpha_g(t)\right]\left[\xi_g(t)\omega(t) + \xi(t)\omega_g(t)\right]}$$

$$I_s(t) = \frac{\left[\xi_0\omega_0 + \alpha_g(t)\right]^2\left[\omega_d^2(t) - \omega_g^2(t) + \alpha_g^2(t)\right] + 2\left[\xi_0\omega_0 + \alpha_g(t)\right]\alpha_g(t)}{\left[\omega^2(t) - \omega_g^2(t)\right]^2 + 4\omega(t)\omega_g(t)\left[\xi_0\omega_0 + \alpha_g(t)\right]\left[\xi_g(t)\omega(t) + \xi(t)\omega_g(t)\right]} \tag{7}$$

with:

$$\omega_g(t) = \omega_r + \left(\omega_p - \omega_r\right)\left(\frac{\omega_s - \omega_r}{\omega_p - \omega_r}\right)^{\frac{t}{t_{max}}}$$
$$\alpha_g(t) = \omega_p\xi_p + \left(\omega_r\xi_r - \omega_p\xi_p\right)\left(\frac{t}{t_{dur}}\right) \tag{8}$$

where $R_g[\circ]$ is the time-varying power spectral density of the ground motion process for the ground motion acceleration parametrized by $\underline{\theta}_g(t) = \left\{\omega_g(t), \alpha_g(t)\right\}$, which are deterministic and slowly-varying functions describing the dominant ground acceleration frequency and its bandwidth (8); $\omega_d(t) = \left[\omega^2(t) - \xi_0^2\omega_0^2\right]^{0.5}$ is the damped natural frequency while ω_0 and ξ_0 are the small-amplitude frequency and viscous damping ratio of the time-varying oscillator; $\xi_g(t) = \alpha_g(t)\omega_g^{-1}(t)$; ω_p, ω_s, and ω_r are the estimated dominant frequencies of the P, S, and surface waves in the earthquake ground motion; t_{max} and t_{dur} denote the time of the maximum intensity of the ground motion intensity and the time interval over which the amplitude intensity of the ground motion process is greater than a predefined percentage of its maximum, respectively.

The amplification in response due to moving resonance, $\mathcal{A}$, can be approximated by the ratio of the peak time-varying power spectral density of the ground motion acceleration as $t \to t_{max}$, and $\omega(t) \to \omega_g(t)$ to the peak time-varying power spectral density of the ground motion at small-amplitude oscillator frequency and viscous damping ratio:

$$\mathcal{A} \triangleq \frac{R_g\left[\omega(t) \to \omega_g(t), \xi_0\omega_0 \middle| \underline{\theta}_g(t)\right]}{R_g\left[\omega_0, \xi_0\omega_0 \middle| \underline{\theta}_g(t)\right]} \cdot \left[\frac{\omega_0}{\omega(t)}\right]^2 \tag{9}$$

Assuming negligible small-amplitude viscous damping, $\xi_0 = \alpha_0 = 0$, the amplification in response due to moving resonance simplifies to [6]:

$$\mathcal{A} = \frac{\left(\omega_0^2 - \omega_p^2\right)^2 + 4\left(\omega_0\omega_p\xi_p\right)^2}{4\left(\omega_p\xi_p\right)\left(\omega_r\xi_r\right)\omega_p^2} \cdot \left(\frac{\omega_0}{\omega_s}\right)^2 \tag{10}$$

Note that for a small-amplitude oscillator frequency near the P-wave dominant frequency, $\omega_0 \simeq \omega_p$, $\mathcal{A}$ is entirely a geophysical quantity determined by the seismic wave characteristics at the site:

$$\mathcal{A} \simeq \left(\frac{\omega_p}{\omega_s}\right)^2 \tag{11}$$

The moving resonance amplification factor in (10) is shown graphically in Fig. 2 for $0.1 \leq \omega_0 \leq 10$ Hz.

Finally, the resulting demand displacement is expressed in terms of $\mathcal{A}$ as:

$$\delta^* = \frac{1 - \sqrt{1 - 4w\lambda_2^2}}{2w} \cdot \mathcal{A} \tag{12}$$

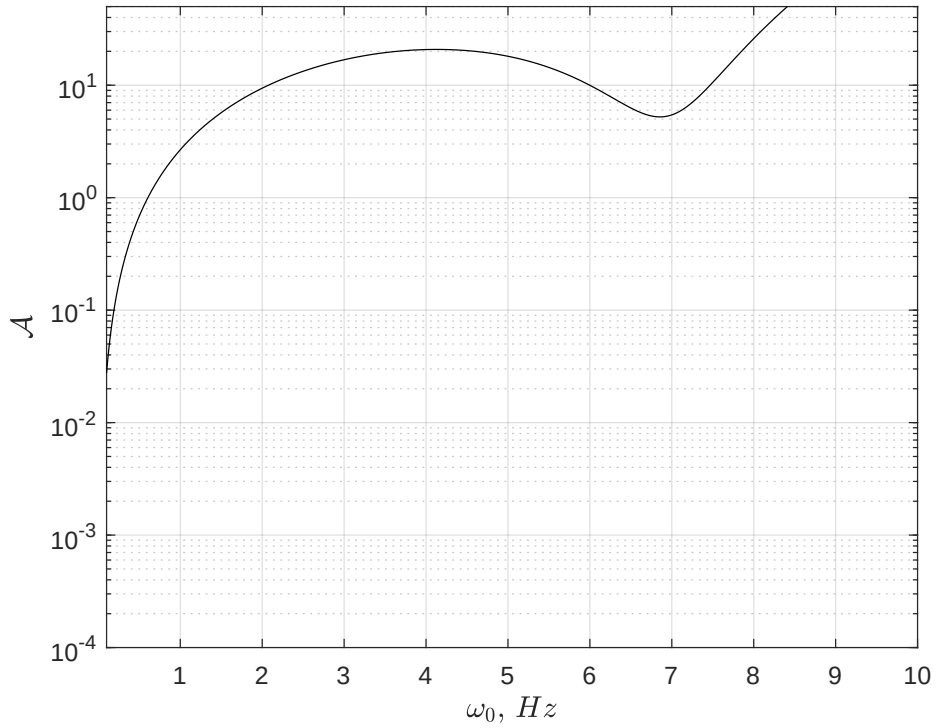


Figure 2: Variation of the moving resonance amplification factor, as a function of small-amplitude oscillator frequency for $\omega_p = 7.0$ Hz, $\omega_s = 3.0$ Hz, $\omega_r = 1.0$ Hz, $\xi_p = 0.1$, and $\xi_r = 0.7$.

4 CONCLUSIONS

- Closed-form solutions and stability criteria for the displacement demand of the stepping (rocking) response of two-dimensional flexible frames under seismic excitations were presented.

- The quasi-dynamic procedures used for design-displacement calculations in current design literature and engineering specifications do not account for nonlinear dynamics, and therefore are suitable only for an initial assessment of approximate displacement demands.
- Site and source characteristics of the seismic excitation that may cause the nonlinear oscillatory system to experience ground motions with shifting frequencies require additional attention. A closed-form estimate of the approximate amplification of dynamic response due to moving resonance was presented, and it was shown to depend on the dominant P- and S-wave frequencies at the site.

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