

This book contains the solutions to a large number of questions that provide a comprehensive workout for students preparing for the Leaving Certificate Maths exam 2019.

It includes the solutions to the SEC Papers from 2018 to 2012 (Sets A to G) in addition to the official sample paper produced in 2014 (Set H). It also includes solutions to our own sample papers (Sample Papers 1 to 4) which follow the style of questioning required to prepare you for the Leaving Certificate exam in 2019. Sample Papers 5, 6 & 7 and accompanying solutions are available online at [www.educateplus.ie](http://www.educateplus.ie).

This book of the solutions to SEC and sample papers provides an excellent guide for students where every aspect of the course is covered. So, enjoy and good luck with your studies.

*Tony Kelly & Kieran Mills*

## **TABLE OF CONTENTS**

**HOW WILL THIS BOOK HELP YOU? ..... 2**

### **SEC PAPERS SOLUTIONS**

#### **SEC 2018 (Set A)**

Paper 1: Exam 2018 ..... 6

Paper 2: Exam 2018..... 19

#### **SEC 2017 (Set B)**

Paper 1: Exam 2017 ..... 32

Paper 2: Exam 2017..... 44

#### **SEC 2016 (Set C)**

Paper 1: Exam 2016 ..... 58

Paper 2: Exam 2016..... 73

#### **SEC 2015 (Set D)**

Paper 1: Exam 2015 ..... 89

Paper 2: Exam 2015..... 106

#### **SEC 2014 (Set E)**

Paper 1: Exam 2014 ..... 124

Paper 2: Exam 2014..... 143

#### **SEC 2013 (Set F)**

Paper 1: Exam 2013..... 164

Paper 2: Exam 2013..... 180

#### **SEC 2012 (Set G)**

Paper 1: Exam 2012..... 198

Paper 2: Exam 2012..... 215

#### **SEC 2014 SAMPLE PAPER (Set H)**

Paper 1: Sample Paper 2014 ..... 234

Paper 2: Sample Paper 2014 ..... 247

## EDUCATE.IE SAMPLE PAPER SOLUTIONS

### SAMPLE PAPER 1

Paper 1 .....	258
Paper 2 .....	272

### SAMPLE PAPER 2

Paper 1 .....	282
Paper 2 .....	290

### SAMPLE PAPER 3

Paper 1 .....	300
Paper 2 .....	312

### SAMPLE PAPER 4

Paper 1 .....	324
Paper 2 .....	333

SAMPLE PAPER 5 [SOLUTIONS AVAILABLE ONLINE]

SAMPLE PAPER 6 [SOLUTIONS AVAILABLE ONLINE]

SAMPLE PAPER 7 [SOLUTIONS AVAILABLE ONLINE]

## HOW WILL THIS BOOK HELP YOU?

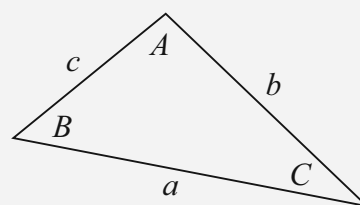
### 1. FORMULAE

The *Formulae and Tables* Book available to you in the exam is an invaluable resource. It is wonderful that students no longer have to learn formulae off by heart.

There is a lot of information in your *Formulae and Tables* Book so you need to be completely familiar with its layout. Our solutions book makes constant reference to where formulae are placed in the book with their page numbers as shown on the right. These references are placed strategically throughout these solutions.

#### FORMULAE AND TABLES BOOK Trigonometry of the triangle:

[page 16]



$$\text{Area: } \frac{1}{2} ab \sin C$$

$$\text{Sine Rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$

## 2. CALCULATOR

Your calculator is your friend. You should be comfortable carrying out all the operations on your calculator from simple arithmetic calculations to being able to change modes to carrying out more complicated statistical operations. Once again, this book contains constant references to how you use your calculator as shown on the right.

The calculator used in this book is a CASIO (*fx-83GT PLUS*). This calculator is by far the most popular calculator used in schools.

### CASIO CALCULATOR (*fx-85GT PLUS*)

Steps to find  $r$ :

Press Mode.

Press 2: Stat

Press 2:  $A + Bx$

Input your  $x$  and  $y$  values

Press AC Button

Press Shift followed by the Number 1

Press 5: Reg

Press 3:  $r$

Press =

## 3. HOW THE MARKING SCHEME WORKS

The marking scheme used to allocate marks to your paper is an extremely precise method to make sure you get exactly the marks you deserve for the type of answer you have written in the exam. Read the official explanation of this scheme presented below before we give you a user-friendly guide as to how it operates.

### Example of the structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No. of categories	2	3	4	5	6
5-mark scale		0, 3, 5	0, 3, 4, 5		
10-mark scale		0, 5, 10	0, 4, 8, 10		
15-mark scale					
20-mark scale			0, 7, 18, 20	0, 7, 10, 18, 20	
25-mark scale				0, 15, 20, 22, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

## Marking scales – level descriptors

### A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

### B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

### C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

### D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

### E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C\** indicates that 9 marks may be awarded.

## USER-FRIENDLY GUIDE TO THE MARKING SCHEME USING AN EXAMPLE:

### MARKING SCHEME NOTES

#### Question 7 (b) (iv) [Scale 10C\* (0, 4, 8, 10)]

**4:** • Calculates or gives one correct distance.

**8:** • Calculates the total distance.

• Sets up correct fraction to calculate time without complete and correct calculation.

• Sets up incorrect fraction to calculate time with correct calculation.

\* Penalise one mark for incorrect or omitted units, provided full marks otherwise.

The first note of importance is that you have no idea of the marking scheme when you are sitting the paper. The original marking scheme may undergo a number of changes over the Summer before the final marking scheme is released. This final marking scheme is constantly referenced throughout the solutions.

Let us examine how it works using the example above for a particular question (7 (b) (iv)):

**[Scale 10C\* (0, 4, 8, 10)] 10 means 10 marks are allocated to this question.**

**[Scale 10C\* (0, 4, 8, 10)] C means there are 4 possible marks you can receive for this question.**

**[Scale 10C\* (0, 4, 8, 10)] \* means 1 mark can be taken off for leaving out units.** So 4, 8 or 10 marks could become 3, 7 or 9 marks if you forgot to put in your correct units.

**[Scale 10C\* (0, 4, 8, 10)] 0 means no credit.** This is what you get when the grid is left blank or you write something totally irrelevant in it. No matter how much pressure you feel, always write something. Even though you may feel overwhelmed by the paper you still have access to formulae, your calculator and your head is full of opinions.

*Do not leave any grids blank. Always write something of relevance to the question.*

**[Scale 10C\* (0, 4, 8, 10)] 4 means low partial credit.** You will get this for a minimal amount of effort - any effort in the right direction. This gives you 40% of the marks. However, other marking schemes may even give you 7 for a low partial credit - 70% of the marks. You just never know. Once again, leave no blanks. Every grid should have something written on it. Correctors want to give you marks. Make sure you give them that opportunity.

**[Scale 10C\* (0, 4, 8, 10)] 8 means high partial credit.**

**[Scale 10C\* (0, 4, 8, 10)] 10 means full credit.**

## 4. OUR SAMPLE PAPERS GET YOU READY FOR THE REAL THING

Our sample papers were written and researched over a number of years to try and capture the essence and style of the new method of questioning brought in by the new Project Maths course.

Initially when students start using our sample papers, they think they are difficult. But when they persevere they actually start enjoying them because they feel they are cracking the course. This is what learning is all about: being challenged, struggling and fighting with a problem and then it all becomes clear - that lovely *ah ha* moment. Students who complete our sample papers feel totally prepared for the real thing.

One thing you will notice is the SEC Leaving Certificate questions bear a remarkable similarity to a number of our sample questions. What a compliment!

# LC 2018 (SET A): PAPER 1

## QUESTION 1 (25 MARKS)

### Question 1 (a)

$$2x + 3y - z = -4 \dots (1)$$

$$3x + 2y + 2z = 14 \dots (2)$$

$$x - 3z = -13 \dots (3)$$

$$2x + 3y - z = -4 \dots (1) \times (-3)$$

$$x - 3z = -13 \dots (3)$$

$$-6x - 9y + 3z = 12$$

$$x - 3z = -13$$

$$\underline{-5x - 9y = -1 \dots (4)}$$

$$2x + 3y - z = -4 \dots (1) \times (2)$$

$$3x + 2y + 2z = 14 \dots (2)$$

$$4x + 6y - 2z = -8$$

$$\underline{3x + 2y + 2z = 14}$$

$$7x + 8y = 6 \dots (5)$$

$$-5x - 9y = -1 \dots (4) \times (7)$$

$$7x + 8y = 6 \dots (5) \times (5)$$

$$-35x - 63y = -7$$

$$\underline{35x + 40y = 30}$$

$$-23y = 23 \Rightarrow y = -1$$

Into (5):  $7x + 8(-1) = 6 \Rightarrow x = 2$

Into (3):  $(2) - 3z = -13 \Rightarrow z = 5$

ANSWER:  $(x, y, z) = (2, -1, 5)$

#### MARKING SCHEME NOTES

#### Question 1 (a) [Scale 15D (0, 5, 7, 11, 15)]

5: • Matches coefficient of 1 variable in 2 equations

• Writes  $x$  in terms of  $z$

7: • 1 unknown found with errors

• Eliminates one unknown

• 1 unknown found and stops

11: • 2 unknowns found

### Question 1 (b)

$$\frac{2x-3}{x+2} \geq 3, x \in \mathbb{R}, x \neq -2$$

Multiply both sides by  $(x+2)^2$ :  $(2x-3)(x+2) \geq 3(x+2)^2$

$$(2x-3)(x+2) - 3(x+2)^2 \geq 0$$

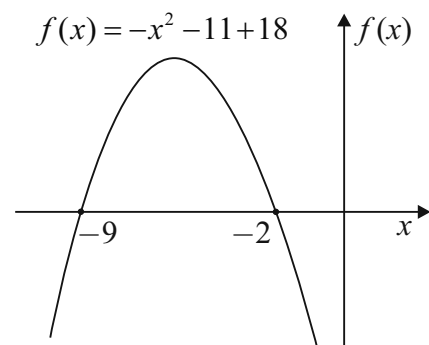
$$(x+2)[(2x-3) - 3(x+2)] \geq 0$$

$$\boxed{(x+2)(-x-9) \geq 0} \dots \text{Test box}$$

Solve  $(x+2)(-x-9) = 0 \Rightarrow x = -9, -2$

Region test:

←	$\boxed{-9}$	↔	$\boxed{-2}$	→
$(x+2)(-x-9) \geq 0$	$(x+2)(-x-9) \geq 0$	$(x+2)(-x-9) \geq 0$		
$-10: (-8)(1) \geq 0$	$-3: (-1)(-6) \geq 0$	$-1: (1)(-8) \geq 0$		
$-8 \geq 0$ [False]	$6 \geq 0$ [True]	$-8 \geq 0$ [False]		
	↓			
	$-9 \leq x < -2$			



ANSWER:  $-9 \leq x < -2, x \in \mathbb{R}, x \neq -2$

**MARKING SCHEME NOTES**

**Question 1 (b) [Scale 10D (0, 3, 5, 8, 10)]**

- 3:** • Use of  $(x + 2)^2$   
 • Relevant work but with linear inequality  
 • Squares both sides with some subsequent work (low partial credit at most)
- 5:** • Quadratic inequality involving 0
- 8:** • Roots of quadratic found
- Note:** Accept  $-9 \leq x \leq -2$

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**

Geometric series:  $x^2, 5x - 8, x + 8$

$$r = \frac{x+8}{5x-8} = \frac{5x-8}{x^2}$$

$$x^2(x+8) = (5x-8)^2$$

$$x^3 + 8x^2 = 25x^2 - 80x + 64$$

$$x^3 - 17x^2 + 80x - 64 = 0$$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 10C (0, 4, 8, 10)]**

- 0:** • Treats as an arithmetic sequence
- 4:** •  $\frac{5x-8}{x^2}$  or  $\frac{x+8}{5x-8}$   
 • Some effort at finding  $x$  in a geometric sequence (must use at least one of the terms)  
 •  $r = \frac{T_n}{T_{n-1}}$  or similar
- 8:** •  $\frac{5x-8}{x^2} = \frac{x+8}{5x-8}$   
 •  $(5x-8)^2$  and  $x^2(x+8)$

**Question 2 (b)**

$$f(x) = x^3 - 17x^2 + 80x - 64$$

$$f(1) = (1)^3 - 17(1)^2 + 80(1) - 64 = 1 - 17 + 80 - 64 = 0$$

1 is a root  $\Rightarrow (x-1)$  is a factor

$$\begin{aligned} x^3 - 17x^2 + 80x - 64 &= (x-1)(x^2 + kx + 64) \\ &= x^3 + (k-1)x^2 + (64-k)x - 64 \end{aligned}$$

$$k-1 = -17 \Rightarrow k = -16$$

$$x^3 - 17x^2 + 80x - 64 = (x-1)(x^2 - 16x + 64) = 0$$

$$(x-1)(x-8)(x-8) = 0 \Rightarrow x = 1, 8, 8$$

**Answer:**  $x = 8$

*or*

$$\begin{array}{r} x^2 - 16x + 64 \\ x-1 \overline{) x^3 - 17x^2 + 80x - 64} \\ \underline{\mp x^3 \pm 1x^2} \\ -16x^2 + 80x - 64 \\ \underline{\pm 16x^2 \mp 16x} \\ 64x - 64 \\ \underline{\mp 64x \pm 64} \\ 0 \end{array}$$

**MARKING SCHEME NOTES**

**Question 2 (b) [Scale 10C (0, 4, 8, 10)]**

- 4:** • Shows  $f(1) = 0$   
 • Any correct substitution
- 8:** • Quotient in quadratic form found  
 • Accept  $x = 8$  without work if  $f(1) = 0$  has been shown

**Question 2 (c)**

$$x = 1: r = \frac{x+8}{5x-8} = \frac{9}{-3} = -3$$

$$x = 8: r = \frac{16}{32} = \frac{1}{2}$$

$S_\infty$  exists for  $|r| < 1$ .

$$a = x^2 = 64, r = \frac{1}{2}$$

$$S_\infty = \frac{a}{1-r} = \frac{64}{1-\frac{1}{2}} = 128$$

**MARKING SCHEME NOTES****Question 2 (c) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- Substitution used to identify  $x = 8$  as the required value
  - Substitution used to exclude  $x = 1$  as the required value
  - Finds  $\frac{a}{1-r}$  for  $x = 1$
  - $S_\infty = \frac{x^2}{1-\frac{5x-8}{x^2}}$
  - Relevant substitution into correct formula
- 4:**
- GP identified ( $a$  and  $r$ )
  - If the candidate works with both  $x = 1$  and  $x = 8$  but fails to eliminate  $x = 1$  or chooses the incorrect answer
- Note:** If  $|r| > 1$  then Low Partial Credit at most

**QUESTION 3 (25 MARKS)****Question 3 (a)**

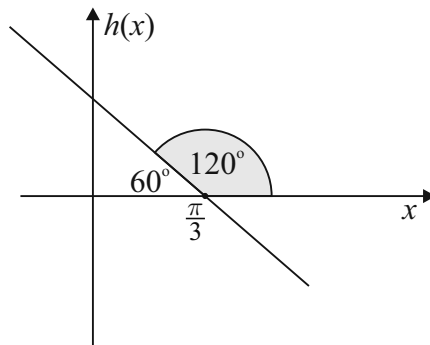
$$y = h(x) = \cos(2x)$$

$$\frac{dy}{dx} = -2 \sin 2x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} = -2 \sin 2\left(\frac{\pi}{3}\right) = -\sqrt{3}$$

$$\text{Slope } m = \tan \theta = -\sqrt{3}$$

$$\therefore \theta = 120^\circ$$

**MARKING SCHEME NOTES****Question 3 (a) [Scale 10D (0, 3, 5, 8, 10)]**

- 3:**
- Differentiation indicated
  - Use of 2
- 5:**
- Derivative found
- 8:**
- $\tan \theta =$  evaluated derivative
  - $\theta = -60^\circ$

**Note:** Must use differentiation to gain any credit

**Note:** If integration symbol appears then 0 credit

**Question 3 (b)**

$$y = h(x) = \cos(2x), 0 \leq x \leq \frac{\pi}{4}$$

$$\bar{y} = \frac{\int_0^{\frac{\pi}{4}} \cos(2x) dx}{\frac{\pi}{4} - 0} = \frac{\frac{1}{2}[\sin 2x]_0^{\frac{\pi}{4}}}{\frac{\pi}{4}} = \frac{2}{\pi}[\sin \frac{\pi}{2} - \sin 0] = \frac{2}{\pi}$$

The average value  $\bar{y}$  of a continuous function  $y = g(x)$  on an interval  $[a, b]$  is given by:

$$\bar{y} = \frac{\int_a^b g(x) dx}{b-a} = \frac{\int_a^b y dx}{b-a}$$

**MARKING SCHEME NOTES****Question 3 (b) [Scale 15D (0, 5, 7, 11, 15)]****5:** • Integration indicated**7:** •  $\cos 2x$  integrated correctly  $\left(\frac{\sin(2x)}{2}\right)$ •  $-2\sin 2x$  and finishes correctly**11:** • Substitutes limits into integral and stops• Integral evaluated at  $x = \frac{\pi}{4}$  (i.e. omits  $\frac{1}{\frac{\pi}{4} - 0}$ ) and finishes**Note:** Errors in integration could include:

An error in the trig function (including sign)

An error in the angle

An error in the application of the chain rule

**Note:** Must have integration to gain any credit**QUESTION 4 (25 MARKS)****Question 4 (a)****Step 1:**  $n = 1$ : Prove  $[r(\cos \theta + i \sin \theta)]^1 = r^1(\cos 1\theta + i \sin 1\theta)$ **Proof:**  $r(\cos \theta + i \sin \theta) = r(\cos \theta + i \sin \theta)$ . Therefore, it is true for  $n = 1$ .**Step 2:**  $n = k$ : Assume  $[r(\cos \theta + i \sin \theta)]^k = r^k(\cos k\theta + i \sin k\theta)$ .**Step 3:**  $n = k + 1$ : Prove  $[r(\cos \theta + i \sin \theta)]^{k+1} = r^{k+1}[\cos(k+1)\theta + i \sin(k+1)\theta]$ .

**Proof :**  $[r(\cos \theta + i \sin \theta)]^{k+1} = [r(\cos \theta + i \sin \theta)]^k [r(\cos \theta + i \sin \theta)]$   
 $= r^k(\cos k\theta + i \sin k\theta)r(\cos \theta + i \sin \theta)$  [From **Step 2.**]  
 $= r^{k+1}[(\cos k\theta \cos \theta - \sin k\theta \sin \theta) + i(\sin k\theta \cos \theta + \cos k\theta \sin \theta)]$   
 $= r^{k+1}[\cos(k+1)\theta + i \sin(k+1)\theta]$

Therefore, assuming it is true for  $n = k$  means it is true for  $n = k + 1$ .So, true for  $n = 1$  and true for  $n = k$  implies it is true for  $n = k + 1$  means it is true for all  $n \in \mathbb{N}$ .**MARKING SCHEME NOTES****Question 4 (a) [Scale 15D (0, 5, 7, 11, 15)]****5:** • Step 1**7:** • Step 2 or Step 3**11:** • Uses Step 2 to prove Step 3**Note:** Accept Step 1, Step 2, Step 3 in any order

Full credit -1: • Omits conclusion but otherwise correct

**15:** •  $[r(\cos \theta + i \sin \theta)]^n = r^n(\cos(n\theta) + i \sin(n\theta))$  proved correctly

**Question 4 (b)**

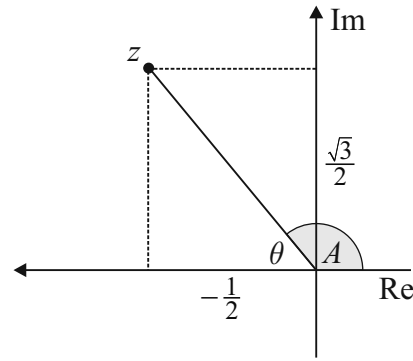
$$z = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \Rightarrow r = |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$\theta = \tan^{-1} \left| \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} \right| = \tan^{-1} \sqrt{3} = \frac{\pi}{3}$$

$$A = \arg z = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\therefore z = \cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right)$$

$$z^3 = \left[ \cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right) \right]^3 = \cos 2\pi + i \sin 2\pi = 1 + 0i$$



**MARKING SCHEME NOTES**

**Question 4 (b) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- Modulus or argument correct
  - Some correct multiplication
  - Apply De Moivre correctly with incorrect modulus and argument

- 8:**
- $\left( \cos(3) \frac{2\pi}{3} + i \sin(3) \frac{2\pi}{3} \right)$
  - Multiplication correct but un-simplified

Full credit -1: •  $\cos 2\pi + i \sin 2\pi$  or  $\cos 360^\circ + i \sin 360^\circ$

**Accept:** Answer with reference to cube root of unity

**QUESTION 5 (25 MARKS)**

**Question 5 (a) (i)**

First row:  $a = 4, d = 3$

$$S_{45} = \frac{45}{2} [2(4) + 44(3)] = 3150$$

Second row:  $a = 7, d = 5$

$$S_{45} = \frac{45}{2} [2(7) + 44(5)] = 5265$$

$$\text{Difference} = 5265 - 3150 = 2115$$

4	7	10	13		
7	12	17	22		
10	17	24	31		
13	22	31	40		

**MARKING SCHEME NOTES**

**Question 5 (a) (i) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- Formulates  $S_{45}$  for row 1 or row 2
  - $3 + 5 + 7 \dots$
  - Identifies  $a$  or  $r$  for either row 1 or row 2

- 8:**
- $S_{45}$  found for row 1 or row 2

Full credit -1: • Fails to subtract

**Question 5 (a) (ii)**

Find the value of the sixtieth cell in first column: 4, 7, 10, 13,...

$$a = 4, d = 3, n = 60$$

$$T_{60} = 4 + (59)(3) = 181$$

Find the common difference of the numbers in the sixtieth column: 3, 5, 7, 9,...

$$a = 3, d = 2, n = 60$$

$$T_{60} = 3 + (59)(2) = 121$$

Find the value of the seventieth cell in sixtieth column: 181, 302,...

$$a = 181, d = 121, n = 70$$

$$T_{70} = 181 + (69)(121) = 8530$$

**MARKING SCHEME NOTES****Question 5 (a) (ii) [Scale 10D (0, 3, 5, 8, 10)]**

**3:** • Identifies  $T_{60}$  in column 1 or  $T_{70}$  in row 1 or equivalent

• Some relevant substitution into correct formula

• Identifies  $a$  or  $d$  for either row 1 or row 2

**5:** • Finds  $a$  in row 60 or row 70

• Finds  $d$  in row 60 or row 70

**8:** • Formulates substituted expression for  $T_{70}$  in row 60 or  $T_{60}$  in column 70

• Finds both  $a$  and  $d$  in row 60 or row 70

**Question 5 (b)**

$$a_1 = 4, a_2 = 2$$

$$a_n = a_{n-1} - a_{n-2}$$

$$a_3 = a_2 - a_1 = 2 - 4 = -2$$

$$a_4 = a_3 - a_2 = -2 - 2 = -4$$

$$a_5 = a_4 - a_3 = -4 - (-2) = -2$$

$$a_6 = a_5 - a_4 = -2 - (-4) = 2$$

$$a_7 = a_6 - a_5 = 2 - (-2) = 4$$

$$a_8 = a_7 - a_6 = 4 - 2 = 2$$

Sequence: 4, 2, -2, -4, -2, 2, 4, 2

The values repeat every 6 numbers.

$$\frac{2019}{6} = 336 + \text{Remainder} = 3 \Rightarrow a_{2019} = -2$$

**MARKING SCHEME NOTES****Question 5 (b) [Scale 5C (0, 3, 4, 5)]**

**3:** •  $a_3 = -2$

•  $a_3 = a_2 - a_1$  or similar

**5:** • Any 4 from  $a_3, a_4, a_5, a_6, a_7$  and  $a_8$  found

Full credit -1: •  $a_3, a_4, a_5, a_6,$  and  $a_{2019}$  found

**QUESTION 6 (25 MARKS)****Question 6 (a)**

$$k(x) = h(x)$$

$$x^3 = x$$

$$x^3 - x = 0$$

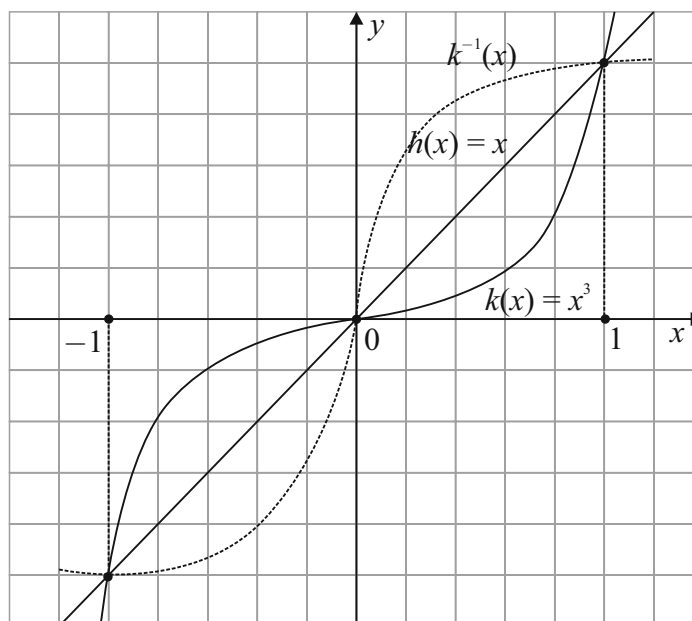
$$x(x^2 - 1) = 0$$

$$x(x-1)(x+1) = 0$$

$$x = -1, 0, 1$$

$$h(-1) = -1, h(0) = 0, h(1) = 1$$

$$\text{Co-ordinates: } (-1, -1), (0, 0), (1, 1)$$

**Question 6 (b) (i)**

$$\begin{aligned} \text{Area} &= \int_{-1}^0 (x^3 - x) dx + \int_0^1 (x - x^3) dx \\ &= \left[ \frac{1}{4}x^4 - \frac{1}{2}x^2 \right]_{-1}^0 + \left[ \frac{1}{4}x^4 - \frac{1}{2}x^2 \right]_0^1 \\ &= \left[ 0 - \left( \frac{1}{4} - \frac{1}{2} \right) \right] + \left[ \left( \frac{1}{4} - \frac{1}{2} \right) - 0 \right] = \frac{1}{2} \end{aligned}$$

**Question 6 (b) (ii)**

$k^{-1}(x)$  is a reflection of  $k(x)$  through the line  $h(x) = x$ .

**MARKING SCHEME NOTES****Question 6 (a) [Scale 10C (0, 4, 8, 10)]**

- 4: • Equation written  
 • One correct solution from the graph  
 • Solution of the form  $(a, a)$  where  $a \neq 0, 1$
- 8: • Equation factorised (3 factors)  
 • 2 correct points  
 •  $x$  values only

**Question 6 (b) (i) [Scale 10C (0, 4, 8, 10)]**

- 4: • Integral indicated  
 • One relevant area found
- 8: • Integral evaluated at  $x = 1$  (upper limit)  
 •  $\int_{-1}^1 (x - x^3) dx = 0$

**Question 6 (b) (ii) [Scale 5B (0, 2, 5)]**

- 2: • Incomplete image  
 • 2 correct image points  
 •  $k^{-1}(x) = x^{\frac{1}{3}}$

**QUESTION 7 (55 MARKS)**

**Question 7 (a)**

$$t(x) = k \left[ \ln \left( 1 - \frac{x}{80} \right) \right]$$

$$x = 35 \text{ wpm, } t(35) = 35 \cdot 96 \text{ days}$$

$$k = \frac{t(x)}{\left[ \ln \left( 1 - \frac{x}{80} \right) \right]} = \frac{35 \cdot 96}{\left[ \ln \left( 1 - \frac{35}{80} \right) \right]} = -62 \cdot 5$$

**Question 7 (b)**

$$t(x) = 100 \text{ days: } 100 = -62 \cdot 5 \left[ \ln \left( 1 - \frac{x}{80} \right) \right]$$

$$-\frac{100}{62 \cdot 5} = \ln \left( 1 - \frac{x}{80} \right)$$

$$e^{-\frac{100}{62 \cdot 5}} = 1 - \frac{x}{80}$$

$$\frac{x}{80} = 1 - e^{-\frac{100}{62 \cdot 5}}$$

$$x = 80(1 - e^{-\frac{100}{62 \cdot 5}}) \approx 64 \text{ wpm}$$

**MARKING SCHEME NOTES**

**Question 7 (a) [Scale 15C (0, 5, 10, 15)]**

- 5:**
- Effort at transposing
  - Some substitution into function
  - Full substitution and stops
- 10:**
- Function written in terms of  $k$  and fully substituted
  - One incorrect substitution worked correctly and with some reference to  $k \neq -62 \cdot 5$

**Question 7 (b) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- Some substitution into function
  - Trial and improvement ( more than 1 iteration)
  - Correct answer without work
- 4:**
- $e^{-\frac{100}{62 \cdot 5}} = 1 - \frac{x}{80}$
  - Equation rewritten in terms of  $x$  or  $\frac{x}{80} =$

**Question 7 (c)**

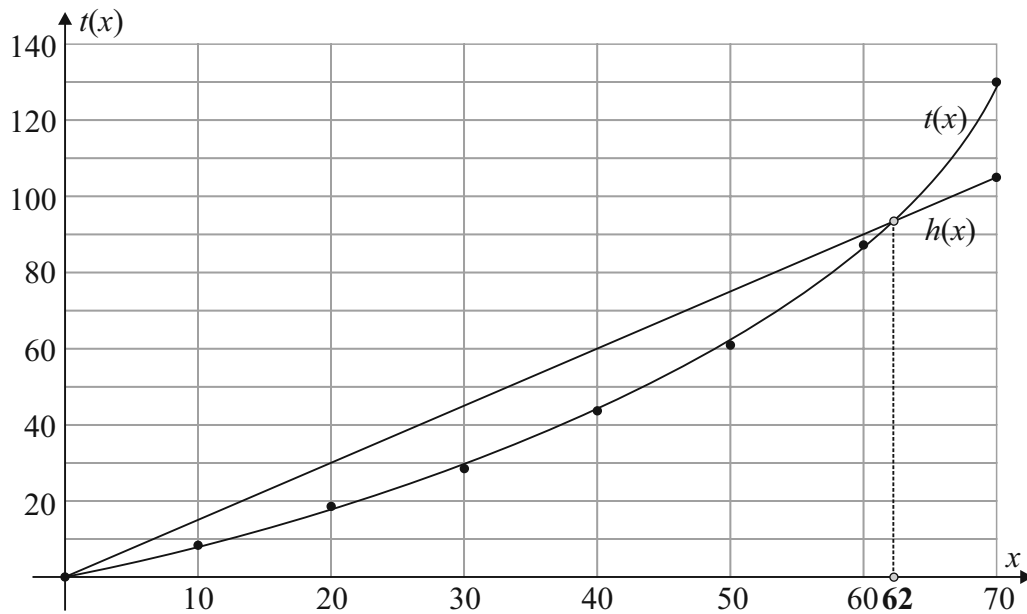
$x$ (wpm)	0	19	20	30	40	50	60	70
$t(x)$ (days)	0	8	18	29	43	61	87	130

**MARKING SCHEME NOTES**

**Question 7 (c) [Scale 20D (0, 5, 10, 15, 20)]**

- 5:**
- One entry correct
  - One plot (from candidates table) correct
- 10:**
- 4 entries correct and 4 plots of table values
- 15:**
- All plots consistent with candidates table values (with at least 1 correct value)
  - Table correct but incorrect plots

**Question 7 (d)**



$$p(x) = 1.5x$$

$$p(0) = 1.5(0) = 0$$

$$p(70) = 1.5(70) = 105$$

**MARKING SCHEME NOTES**

**Question 7 (d) [Scale 5C (0, 3, 4, 5)]**

- 3: • One point on line identified  
 • One point (not origin) plotted
- 4: • 2 points on line identified and plotted

**Question 7 (e)**

$$h(x) = p(x) - t(x)$$

(i)  $h(x) = 0 \Rightarrow p(x) = t(x)$   
 $x \approx 62$

(ii)  $h(x) = 1.5x + 62.5 \left[ \ln \left( 1 - \frac{x}{80} \right) \right]$

$$\frac{dh}{dx} = 1.5 + 62.5 \times \frac{1}{\left( 1 - \frac{x}{80} \right)} \times \left( -\frac{1}{80} \right)$$

$$= 1.5 - \frac{62.5}{(80-x)}$$

$$\frac{dh}{dx} = 0: 1.5 = \frac{62.5}{(80-x)} \Rightarrow 1.5(80-x) = 62.5$$

$$120 - 1.5x = 62.5 \Rightarrow 57.5 = 1.5x$$

$$\therefore x = \frac{115}{3}$$

$$h\left(\frac{115}{3}\right) = 1.5\left(\frac{115}{3}\right) + 62.5 \left[ \ln \left( 1 - \frac{\left(\frac{115}{3}\right)}{80} \right) \right]$$

$$\approx 17 \text{ days}$$

**MARKING SCHEME NOTES**

**Question 7 (e) (i) [Scale 5B (0, 2, 5)]**

- 2: • Point of intersection indicated on graph  
 •  $h(x)$  written in terms of  $x$

**Tolerance:**  $\pm 2$  wpm

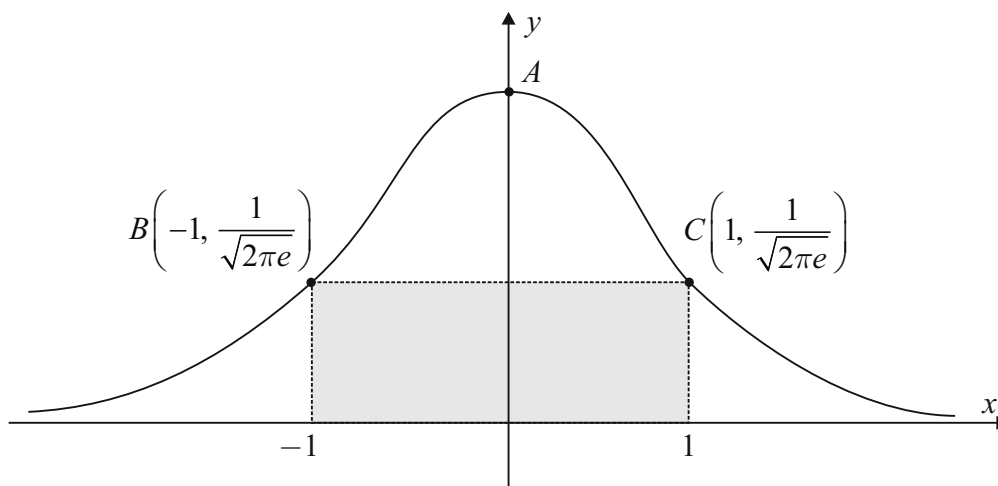
**Question 7 (e) (ii) [Scale 5B (0, 3, 4, 5)]**

- 3: • Any correct differentiation  
 •  $h(x) = 1.5x + 62.5 \ln \left( 1 - \frac{x}{80} \right)$   
 •  $h'(x) = 0$
- 4: • Differentiation correct but un-simplified  
 • Value for  $x$  and stops

$$y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = \ln f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{f(x)} \times f'(x)$$

**QUESTION 8 (40 MARKS)**



**Question 8 (a)**

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$x = 0: f(0) = \frac{1}{\sqrt{2\pi}} e^0 = \frac{1}{\sqrt{2\pi}}$$

$$\therefore A\left(0, \frac{1}{\sqrt{2\pi}}\right)$$

**Question 8 (b)**

$$\text{Area} = 2 \times \frac{1}{\sqrt{2\pi e}} = 0.484$$

**MARKING SCHEME NOTES**

**Question 8 (a) [Scale 10C (0, 4, 8, 10)]**

- 4: •  $x = 0$   
• Value for  $x$  substituted into  $f(x)$

8: •  $\frac{1}{\sqrt{2\pi}}$

Full credit – 1: • (0, 0.3989)

**Question 8 (b) [Scale 10C (0, 4, 8, 10)]**

- 0: • Integrating original function

- 4: • length = 2  
• Width = [y co-ordinate]

8: •  $\left[1\left(\frac{1}{\sqrt{2\pi e}}\right)\right]$

Full credit – 1: • Area = 0.484

**Question 8 (c)**

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \times (-x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$f'(1) = -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}} = -\frac{1}{\sqrt{2\pi e}} < 0$$

$\therefore f(x)$  is decreasing at  $C$

**MARKING SCHEME NOTES**

**Question 8 (c) [Scale 10C (0, 4, 8, 10)]**

- 4: •  $x = 1$  identified  
• Some correct differentiation  
• Indicates significance of  $\frac{dy}{dx} < 0$

8: • Derivative found

**Question 8 (d)**

$$f'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \times (-x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$f''(x) = -\frac{1}{\sqrt{2\pi}} [xe^{-\frac{1}{2}x^2} \times (-x) + e^{-\frac{1}{2}x^2}]$$

$$= -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} [1 - x^2]$$

Point of inflection:  $f''(x) = 0 \Rightarrow -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} [1 - x^2] = 0$

$$1 - x^2 = 0 \Rightarrow x^2 = 1$$

$$\therefore x = \pm 1$$

$\therefore$  Point of inflection at  $C$  where  $x = -1$ .

**MARKING SCHEME NOTES**

**Question 8 (d) [Scale 10D (0, 3, 5, 8, 10)]**

**3:** •  $f'(x)$  transferred or found

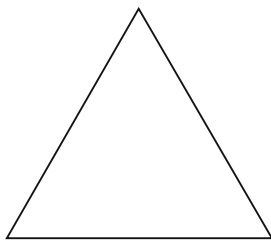
- Mention of  $f''(x)$
- Identifies  $x = -1$

**5:** •  $f''(x)$  identified and some correct differentiation

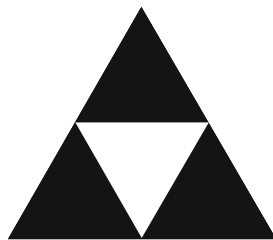
**8:** •  $f''(x)$  found

**Note:** If the product rule and chain rule are not applied in finding  $f''(x)$  then the candidate can be awarded mid-partial credit at most

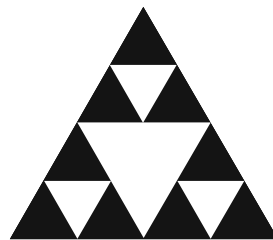
**QUESTION 9 (55 MARKS)**



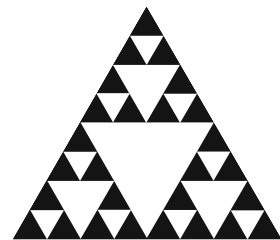
Step 0



Step 1



Step 2



Step 3

**Question 9 (a)**

Step	0	1	2	3
Number of black triangles	1	3	9	27
Fraction of the original triangle remaining	1	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{27}{64}$

**MARKING SCHEME NOTES**

**Question 9 (a) [Scale 10C (0, 4, 8, 10)]**

**4:** • One correct entry

**8:** • Three correct entries

Full credit –1: • Answers as decimals

**Question 9 (b)**

- (i) Number of black triangles: 1, 3, 9, 27, ...  
 $n = 0$ : Number of black triangles =  $1 = 3^0$   
 $n = 1$ : Number of black triangles =  $3 = 3^1$   
 $n = 2$ : Number of black triangles =  $9 = 3^2$   
 $n = 3$ : Number of black triangles =  $27 = 3^3$   
 $n = n$ : Number of black triangles =  $3^n$   
*Or*  
 Formula:  $T_n = ar^{n-1}$   
 [ $n = 1$  is first term in formula]  
 Sequence: 3, 9, 27, ...  
 $a = 3, r = 3$   
 $T_n = 3 \times 3^{n-1} = 3^n$

- (ii)  $3^k > 1 \times 10^9$   
 $k > \log_3(1 \times 10^9) = 18.9$   
 $\therefore k$  is the 19<sup>th</sup> step.

**Question 9 (c)**

- (i) Fraction remaining:  $1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \dots$   
 $n = 0$ : Fraction remaining =  $1 = (\frac{3}{4})^0$   
 $n = 1$ : Fraction remaining =  $\frac{3}{4} = (\frac{3}{4})^1$   
 $n = 2$ : Fraction remaining =  $\frac{9}{16} = (\frac{3}{4})^2$   
 $n = 3$ : Fraction remaining =  $\frac{27}{64} = (\frac{3}{4})^3$   
 $n = n$ : Fraction remaining =  $(\frac{3}{4})^n$   
*Or*  
 Formula:  $T_n = ar^{n-1}$   
 [ $n = 1$  is first term in formula]  
 Sequence:  $\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \dots$  [ $a = \frac{3}{4}, r = \frac{3}{4}$ ]  
 $T_n = \frac{3}{4} \times (\frac{3}{4})^{n-1} = (\frac{3}{4})^n$   
 $(\frac{3}{4})^h < \frac{1}{100} \Rightarrow h < \log_{\frac{3}{4}}(\frac{1}{100}) = 16.01$   
 $\therefore h = 17$

- (ii)  $\lim_{n \rightarrow \infty} (\frac{3}{4})^n = 0$

**MARKING SCHEME NOTES**

**Question 9 (b) (i) [Scale 5B (0, 2, 5)]**

- 2: •  $3n$  written  
 •  $n^3$  written  
 Full credit – 1: •  $3^{n-1}$  written

**Question 9 (b) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: •  $3^n > 1\ 000\ 000\ 000$   
 4: • Inequality with  $k$  not written as an index  
**Note:** if  $3k$  or  $k^3$  from above used fully here then award low partial credit at most

**MARKING SCHEME NOTES**

**Question 9 (c) (i) [Scale 10C (0, 4, 8, 10)]**

- 4: • Correct answer without work  
 •  $(\frac{3}{4})^h$  or candidates ratio to the power of  $h$   
 •  $r = \frac{3}{4}$   
 • Lists two or more terms  
 8: • Inequality with  $h$  not written as an index  
 Full credit – 1: •  $(\frac{3}{4})^{h-1} < \frac{1}{100}$  and finishes correctly

**Question 9 (c) (ii) [Scale 5B (0, 2, 5)]**

- 2: •  $\lim_{n \rightarrow \infty}$   
 • Some use of  $\frac{3}{4}$   
 5: • Correct answer without work  
 •  $\frac{1}{\infty}$  or equivalent

**Question 9 (d)**

(i) Table:

Step	0	1	2	3	4
Perimeter	3	$\frac{9}{2}$	$\frac{27}{4}$	$\frac{81}{8}$	$\frac{243}{16}$

(ii) Perimeter:  $3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \frac{243}{16}, \dots$

$$n = 0: \text{Perimeter} = 3 = \frac{3^1}{2^0}$$

$$n = 1: \text{Perimeter} = \frac{9}{2} = \frac{3^2}{2^1}$$

$$n = 2: \text{Perimeter} = \frac{27}{4} = \frac{3^3}{2^2}$$

$$n = 3: \text{Perimeter} = \frac{81}{8} = \frac{3^4}{2^3}$$

$$n = 4: \text{Perimeter} = \frac{243}{16} = \frac{3^5}{2^4}$$

$$n = n: \text{Perimeter} = \frac{3^{n+1}}{2^n}$$

Or Formula:  $T_n = ar^{n-1}$

Sequence:  $\frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \frac{243}{16}, \dots$

$$a = \frac{9}{2}, r = \frac{3}{2}$$

$$T_n = \frac{9}{2} \times \left(\frac{3}{2}\right)^{n-1} \Rightarrow T_{35} = \frac{9}{2} \times \left(\frac{3}{2}\right)^{34} = 4\,368\,329$$

(iii) As  $n \rightarrow \infty$ , Area  $\rightarrow 0$

As  $n \rightarrow \infty$ , Perimeter  $\rightarrow \infty$

**MARKING SCHEME NOTES**

**Question 9 (d) (i) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- One correct entry
  - All numerators correct with all incorrect denominators
  - All denominators correct with all incorrect numerators
- 8:**
- Two correct entries

**Question 9 (d) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- Pattern identified
  - Recognises  $r = \frac{3}{2}$
  - Some relevant substitution into
  - $T_n = ar^{n-1}$
  - $a = 3$  or  $a = 4.5$
- 4:**
- Step 35 =  $\frac{3^{36}}{2^{35}}$  or equivalent

Full credit – 1: •  $T_{35} = (3) \left(\frac{3}{2}\right)^{34}$

**Question 9 (d) (iii) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- $\lim_{n \rightarrow \infty} \left(\frac{3^{n+1}}{2^n}\right)$  or equivalent
  - Area is getting smaller
  - Perimeter is increasing
- 4:**
- Area approaches 0
  - **Perimeter**  $\rightarrow \infty$  identified
  - Area is getting smaller and Perimeter is increasing

## LC 2018 (SET A): PAPER 2

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

Let  $x$  = Mary's net winnings

	First	Second	Third	No place
$x$ (€)	7000	5000	1000	-2000
$P(x)$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{3}{5}$

$$E(x) = \sum xP(x) = 7000 \times \frac{1}{20} + 5000 \times \frac{1}{10} + 1000 \times \frac{1}{4} - 2000 \times \frac{3}{5} = -100$$

ANSWER: Mary's expected loss is €100.

#### MARKING SCHEME NOTES

**Question 1 (a) [Scale 15C (0, 4, 11, 15)]**

**4:** •  $E(x)$  partially formulated (1 or 2 terms)

**11:** •  $E(x)$  fully formulated (sum of all three/all four terms)

#### Question 1 (b)

	First	Second	Third	No place
$x$ (€)	$7000 + x$	$5000 + x$	$1000 + x$	-2000
$P(x)$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{3}{5}$

$$E(x) = \sum xP(x) = (7000 + x) \times \frac{1}{20} + (5000 + x) \times \frac{1}{10} + (1000 + x) \times \frac{1}{4} + (-2000) \times \frac{3}{5} = 0$$

$$7000 + x + 2(5000 + x) + 5(1000 + x) + 12(-2000) = 0$$

$$7000 + x + 10\ 000 + 2x + 5000 + 5x - 24\ 000 = 0$$

$$8x = 2000$$

$$\therefore x = \text{€}250$$

#### MARKING SCHEME NOTES

**Question 1 (b) [Scale 10D (0, 3, 5, 8, 10)]**

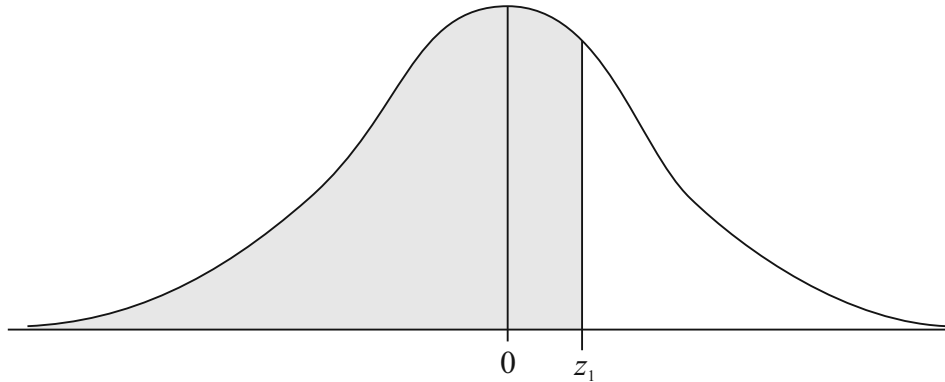
**3:** • Any relevant use of  $x$

**5:** •  $E(x)$  fully formulated (LHS)

**8:** • Relevant equation in  $x$

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**



$$P(z \leq z_1) = 0.67 \Rightarrow z_1 = 0.44$$

**Question 2 (b)**

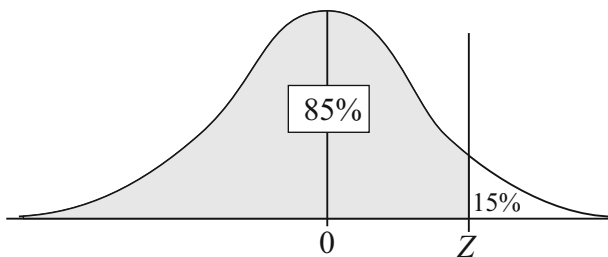
- (i) Maths exam:  $\mu = 70, \sigma = 15, x = 65$   
 English exam:  $\mu = 72, \sigma = 10, x = 68$

$$\text{Maths: } z_1 = \frac{x - \mu}{\sigma} = \frac{65 - 70}{15} = -\frac{1}{3} = -0.33$$

$$\text{English: } z_2 = \frac{x - \mu}{\sigma} = \frac{68 - 72}{10} = -\frac{2}{5} = -0.4$$

Mary did better relatively in Maths.  
 In Maths she is 0.33 standard deviations from the mean whereas in English she is 0.4 standard deviations from the mean.

(ii)

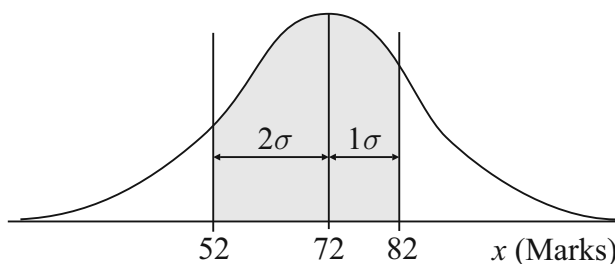


$$P(Z \leq z) = 0.85 \Rightarrow Z = 1.04$$

$$1.04 = \frac{X - 72}{10} \Rightarrow X = 82.4$$

Answer: 83 marks

(iii)



$$P(52 \leq x \leq 82) = \frac{68}{2} + \frac{95}{2} = 81.5\%$$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 10D (0, 2, 5)]**

- 2: •  $P(z < z_1) = 0.67$

**Question 2 (b) (i) [Scale 10C (0, 3, 7, 10)]**

- 3: • Relevant formula with some correct substitution

•  $\frac{65 - 70}{15}$  or  $\frac{68 - 72}{10}$

- 7: •  $\frac{65 - 70}{15}$  and  $\frac{68 - 72}{10}$

**Question 2 (b) (ii) [Scale 5C (0, 2, 4, 5)]**

- 2: • 0.15

• 1.04

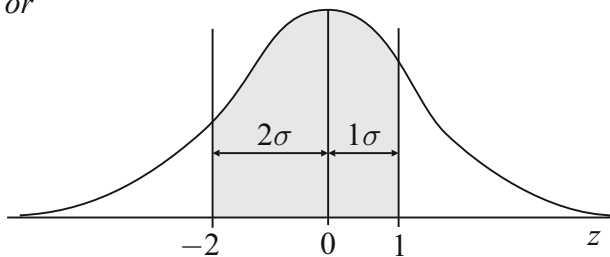
- Relevant formula with some correct substitution

- 4: • Relevant equation in  $x$

The **empirical rule** states that in a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ :

1. 68% of the data falls within  $1\sigma$  of the mean  $\mu$ .
2. 95% of the data falls within  $2\sigma$  of the mean  $\mu$ .
3. 99.7% of the data falls within  $3\sigma$  of the mean  $\mu$ .

or



$$\begin{aligned}
 P(-2 < z < 1) &= P(z < 1) - P(z < -2) \\
 &= P(z < 1) - P(z > 2) \\
 &= P(z < 1) - [1 - P(z < 2)] \\
 &= P(z < 1) + P(z < 2) - 1 \\
 &= 0.8413 + 0.9772 - 1 = 0.8185 = 81.85\%
 \end{aligned}$$

**MARKING SCHEME NOTES**

**Question 2 (b) (iii) [Scale 5C (0, 2, 4, 5)]**

2: • Evidence of relevant linking of deviation and mean

- $\frac{68}{2}$  or  $\frac{95}{2}$
- $\frac{52-72}{10}$  or  $\frac{82-72}{10}$
- $\frac{0.6826}{2}$  or  $\frac{0.9544}{2}$

4: •  $\frac{68}{2}$  and  $\frac{95}{2}$

- $\frac{52-72}{10}$  and  $\frac{82-72}{10}$
- $\frac{0.6826}{2}$  and  $\frac{0.9544}{2}$

**QUESTION 3 (25 MARKS)**

**Question 3 (a)**

0	7	1	7	3	7
---	---	---	---	---	---

(i) Codes ending in zero:

1st digit	2nd digit	3rd digit	4th digit	5th digit	6th digit				
<input type="text"/>	and then <input type="text"/>	and then <input type="text"/>	and then <input type="text"/>	and then <input type="text"/>	and then <input type="text"/>	and then <input type="text" value="0"/>			
$10$	$\times$	$10$	$\times$	$10$	$\times$	$10$	$\times$	$1$	$= 100\,000$

(ii) Codes containing digits 2018 together in this order:

1st digit	2nd digit	3rd digit	
<input type="text"/>	and then <input type="text"/>	and then <input type="text" value="2018"/>	
$10$	$\times$	$10$	$\times$
		$1$	$\times$
			$3! = 300$

**MARKING SCHEME NOTES**

**Question 3 (a) (i) [Scale 15C (0, 4, 11, 15)]**

- 4: • Some use of 10  
 • Identifies that 5 other digits are required to complete code
- 11: •  $9^5$  or equivalent  
 •  $10^6$

**Question 3 (a) (ii) [Scale 5B (0, 2, 5)]**

- 2: •  $10 \times 10$

**Question 3 (b)**

$$\frac{(n+3)!(n+2)!}{(n+1)!(n+1)!} = \frac{(n+3)(n+2)(n+1)!(n+2)(n+1)!}{(n+1)!(n+1)!} = (n+3)(n+2)^2$$

$$= (n+3)(n^2 + 4n + 4)$$

$$= n^3 + 7n^2 + 16n + 12$$

$$a=1, b=7, c=16, d=12$$

**MARKING SCHEME NOTES**

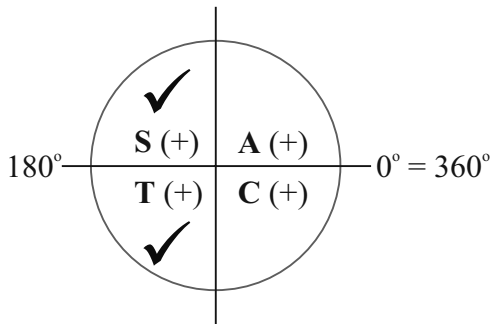
**Question 3 (b) [Scale 5C (0, 2, 4, 5)]**

- 2: • Factorial expansion (e.g.  $(n+3)! = (n+3)(n+2)(n+1)\dots\dots\dots 1$ )  
 • Effort at a numerical value for n on both LHS and RHS (method 2)
- 4: •  $(n+3)(n+2)(n+2)$   
 • Four simultaneous equations

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**

$$\cos 2x = -\frac{\sqrt{3}}{2}, 0^\circ \leq x \leq 360^\circ$$



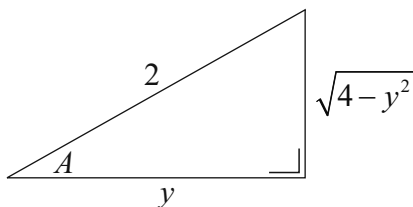
Reference angle:  $2x = \cos^{-1} \left| -\frac{\sqrt{3}}{2} \right| = 30^\circ$

$$2x = \begin{cases} 180^\circ - 30^\circ = 150^\circ, 510^\circ \text{ [2nd quadrant]} \\ 180^\circ + 30^\circ = 210^\circ, 570^\circ \text{ [3rd quadrant]} \end{cases}$$

$$x = \begin{cases} 75^\circ, 255^\circ \\ 105^\circ, 285^\circ \end{cases}$$

$$x = 75^\circ, 105^\circ, 255^\circ, 285^\circ$$

**Question 4 (b)**



$$\cos A = \frac{y}{2} \Rightarrow \sin A = \frac{\sqrt{4-y^2}}{2}$$

$$\begin{aligned} \sin 2A &= 2 \sin A \cos A = 2 \times \frac{y}{2} \times \frac{\sqrt{4-y^2}}{2} \\ &= \frac{y\sqrt{4-y^2}}{2} \end{aligned}$$

**MARKING SCHEME NOTES**

**Question 4 (a) [Scale 20C (0, 7, 13, 20)]**

- 7: •  $0^\circ$  or  $150^\circ$  or  $210^\circ$   
 13: • 2 relevant values of x

**Question 4 (b) [Scale 5C (0, 2, 4, 5)]**

- 2: •  $\sqrt{4-y^2}$   
 •  $2\sin A \cos A$  without substitution  
 •  $\sin 2A$  expressed in  $\tan A$  format  
 • Relevant labelled diagram (2, y, A)

- 4: • Substitution for  $\sin A$  or  $\cos A$  in formula

$$\bullet \sin A = \left( \frac{\sqrt{4-y^2}}{2} \right)$$

$$\bullet \tan A = \left( \frac{\sqrt{4-y^2}}{y} \right)$$

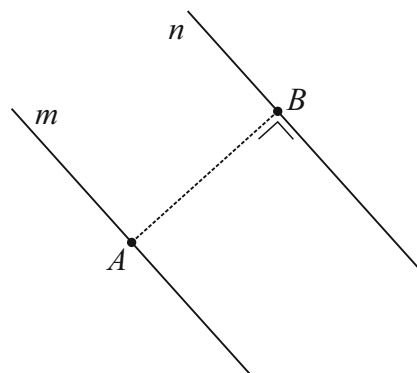
**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

$$m : 2x + 3y + 1 = 0$$

$$A(-2, 1) : 2(-2) + 3(1) + 1 = 0$$

$$\therefore A(-2, 1) \in m$$



**MARKING SCHEME NOTES**

**Question 5 (a) [Scale 10C (0, 3, 7, 10)]**

**3:** • Substitution for  $x$  or  $y$  in equation of line

**7:** • Substitution for  $x$  and  $y$  in eq. of line (LHS when no indication of 0)

**Question 5 (b)**

$$\text{Slope of } m = -\frac{2}{3}$$

$$\text{Slope of } AB : m_{\perp} = \frac{3}{2}$$

$$\text{Equation of } AB : (x_1, y_1) = A(-2, 1), m = \frac{3}{2}$$

$$y - 1 = \frac{3}{2}(x + 2)$$

$$2y - 2 = 3x + 6$$

$$AB : 3x - 2y + 8 = 0$$

$$AB \cap n = \{B\}$$

$$AB : 3x - 2y + 8 = 0 \dots (1)(\times 3)$$

$$n : 2x + 3y - 51 = 0 \dots (2)(\times 2)$$

$$9x - 6y + 24 = 0$$

$$4x + 6y - 102 = 0$$

$$\hline 13x - 78 = 0 \Rightarrow x = 6$$

$$\text{Into (1): } 3(6) - 2y + 8 = 0 \Rightarrow y = 13$$

$$\therefore B(6, 13)$$

**MARKING SCHEME NOTES**

**Question 5 (b) [Scale 10D (0, 3, 5, 8, 10)]**

**3:** • Slope of  $AB$   
• Equation of line formula with some substitution

**5:** • Equation of  $AB$

**8:** • Effort at finding intersection of lines

**Note:** Point of intersection, found correctly, of  $n$  and a relevant  $AB$  (with errors) merits Mid-Partial Credit at least

**Question 5 (c)**

$a = 1, b = 3; A(-2, 1), B(6, 13)$

$$D\left(\frac{1(6)+3(-2)}{1+3}, \frac{1(13)+3(1)}{1+3}\right) = D(0, 4)$$

Equation of  $s$ :

End points of diameter:  $A(-2, 1), D(0, 4)$

$$\text{Centre} = \left(\frac{-2+0}{2}, \frac{1+4}{2}\right) = \left(-1, \frac{5}{2}\right)$$

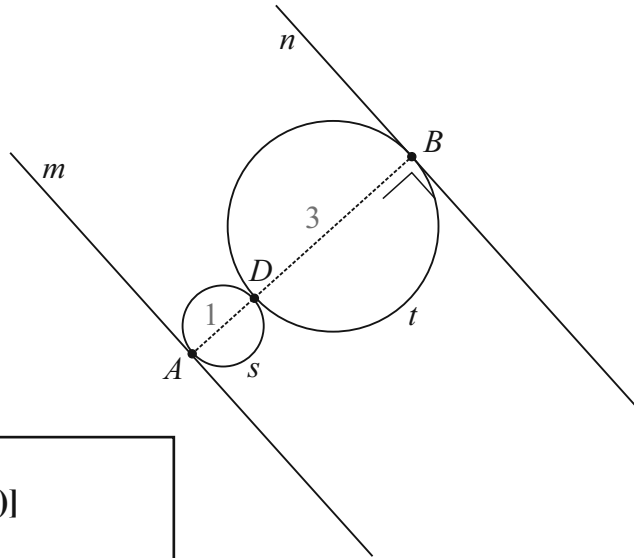
$$r = \frac{1}{2}\sqrt{(-2-0)^2 + (1-4)^2} = \frac{1}{2}\sqrt{13}$$

$$s: (x+1)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{1}{2}\sqrt{13}\right)^2$$

$$(x+1)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$4(x+1)^2 + 4\left(y - \frac{5}{2}\right)^2 = 13$$

Point dividing  $[AB]$  in the ratio  $a:b$

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$


**MARKING SCHEME NOTES**

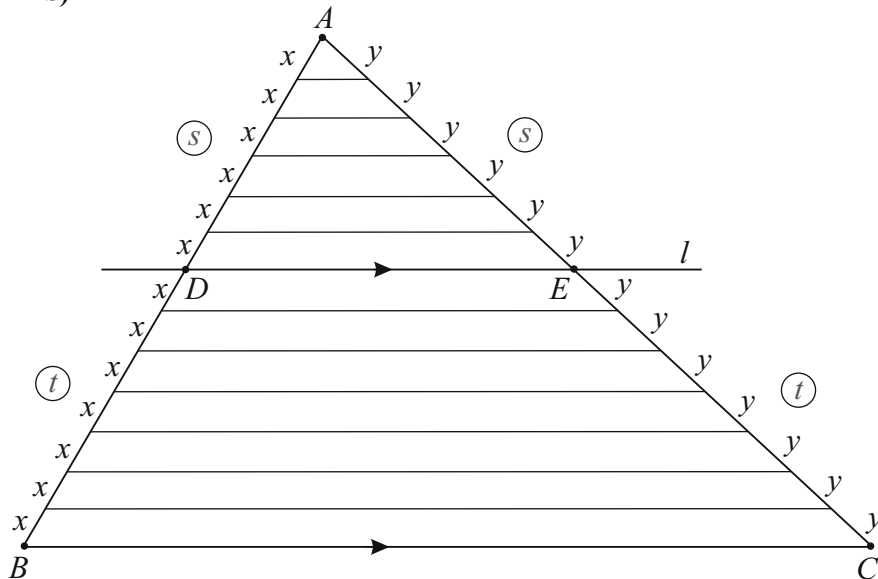
**Question 5 (c) [Scale 5C (0, 2, 4, 5)]**

2: • Some relevant use of 1:3  
 • Midpoint of  $AB$  found once but no further work of relevance  
 • Formula with some relevant substitution

4: • Centre and radius of circle

**QUESTION 6 (25 MARKS)**

**Question 6 (a)**



GIVEN:  $\triangle ABC$  with a line  $DE$  parallel to  $[BC]$  dividing  $[AB]$  in the ratio  $s:t$ .

TO PROVE: In  $\triangle ABC$ , if a line  $DE$  is parallel to  $BC$  and cuts  $[AB]$  in the ratio  $s:t$ , then it also cuts  $[AC]$  in the same ratio.

CONSTRUCTION: Divide  $[AB]$  into  $(s + t)$  segments of equal length  $x$  by drawing equally spaced parallel lines starting at  $A$  and ending at  $B$ .

PROOF:

By the transversal theorem, the segments on  $[AC]$  have equal length  $y$ .

$$\therefore \frac{|AD|}{|DB|} = \frac{sx}{tx} = \frac{s}{t} \Rightarrow \frac{|AE|}{|EC|} = \frac{sy}{ty} = \frac{s}{t}$$

Therefore,  $DE$  divides  $[AC]$  in the same ratio.

**MARKING SCHEME NOTES**

**Question 6 (a) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Relevant diagram drawn
- 7: • Construction clearly indicated
- 11: • Proof missing 1 relevant step

**Question 6 (b)**

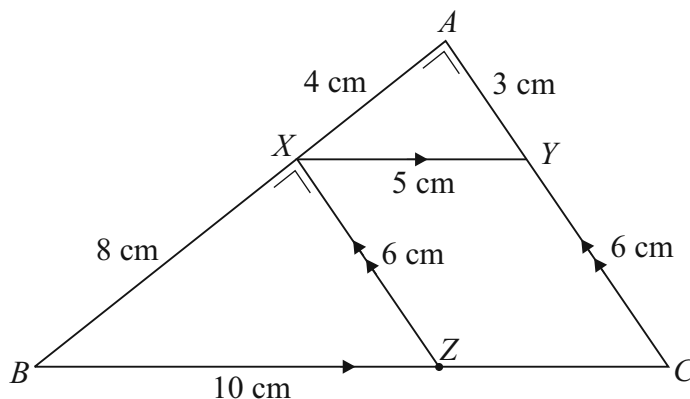
$$|XY| = \sqrt{4^2 + 3^2} = 5 \text{ cm [Pythagoras]}$$

$$|BX| = 8 \text{ cm [}|AX| : |XB| = 1 : 2]$$

$$|YC| = 6 \text{ cm [Ratio theorem]}$$

$$|XZ| = |YC| = 6 \text{ cm [ZCYX is a parallelogram]}$$

$$|BZ| = \sqrt{8^2 + 6^2} = 10 \text{ cm [Pythagoras]}$$



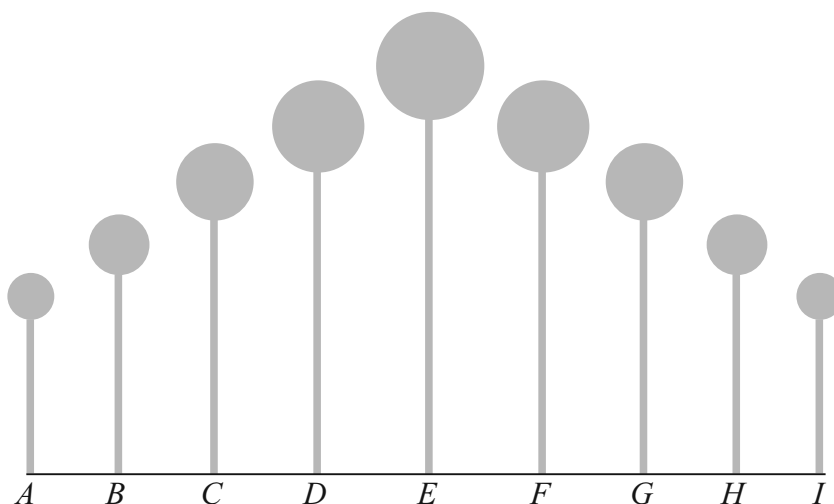
**MARKING SCHEME NOTES**

**Question 6 (b) [Scale 10C (0, 3, 7, 10)]**

- 3: •  $|XY|$  or  $|BX|$  or  $|CY|$  found
  - Pythagoras with some substitution
- 7: •  $|ZC|$  or  $|BC|$  found
  - Ratios formulated with  $|BZ|$  the sole unknown

**QUESTION 7 (50 MARKS)**

**Question 7 (a)**



Volume of sphere A:  $V = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

Geometric sequence:  $36\pi, 36\pi \times 1.75, 36\pi \times 1.75^2, \dots$

$a = 36\pi, r = 1.75, n = 5$

$$S_5 = \frac{36\pi(1-1.75^5)}{1-1.75} = 2324 \text{ cm}^3$$

**MARKING SCHEME NOTES**

**Question 7 (a) [Scale 10D (0, 3, 5, 8, 10)]**

3: • Volume formula with some substitution

5: • Volume of 2 spheres  
• GP formula with some substitution

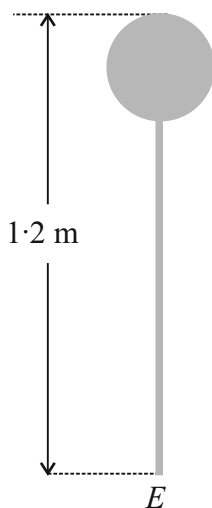
8: • Volume of 5 spheres  
• GP formula fully substituted

**Question 7 (b)**

(i) Surface area  $A = 4\pi r^2 = 503 \text{ cm}^2$

$$r = \sqrt{\frac{503}{4\pi}} = 6.33 \text{ cm}$$

Height =  $120 - 2(6.33) = 107.3 \text{ cm}$



(ii) Bar A:  $r = 1 \text{ cm}, V = 71.3\pi \text{ cm}^3$

$$\pi(1)^2 h = 71.3\pi \Rightarrow h = 71.3 \text{ cm}$$

Arithmetic sequence:  $71.3, 71.3 + d, 71.3 + 2d, 71.3 + 3d, 71.3 + 4d$

$$\therefore 71.3 + 4d = 107.3 \Rightarrow d = 9$$

A:  $h = 71.3 \text{ cm}, B: h = 80.3 \text{ cm}, C: h = 89.3 \text{ cm}, D: h = 98.3 \text{ cm}, E: h = 107.3 \text{ cm}$

**MARKING SCHEME NOTES**

**Question 7 (b) (i) [Scale 10C (0, 3, 7, 10)]**

3: •  $4\pi r^2 = 503$

•  $\frac{4}{3}\pi r^3 = \text{volume from (a)}$

7: •  $r$  found

**Question 7 (b) (ii) [Scale 10D (0, 3, 5, 8, 10)]**

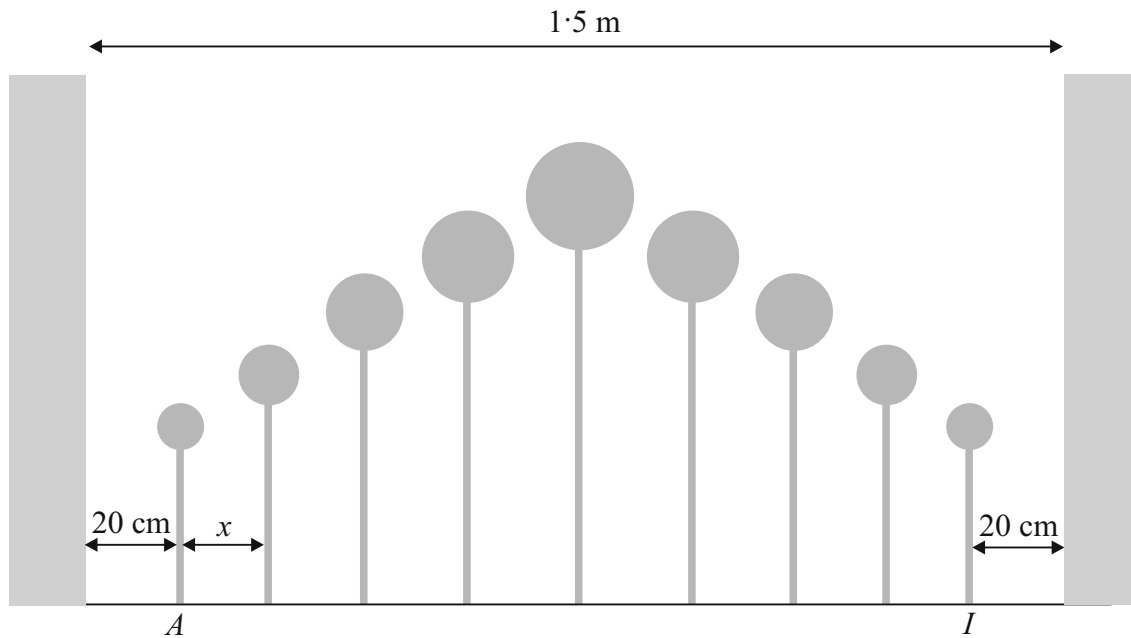
3: • Vol formula with some substitution

•  $\pi r^2 h = 71.3\pi$

5: • Height of bar A

8: • Difference in height between bar A and bar E

**Question 7 (c)**



$$150 = 40 + 9 \times 2 + 8x$$

$$92 = 8x$$

$$x = 11.5 \text{ cm}$$

**MARKING SCHEME NOTES**

**Question 7 (c) [Scale 15C (0, 4, 11, 15)]**

- 4:**
- Recognises 8 equal divisions
  - Indicates subtraction of one relevant length
  - $9 \times 2$
- 11:**
- $150 - 40 - 18$  or equivalent

**Question 7 (d)**

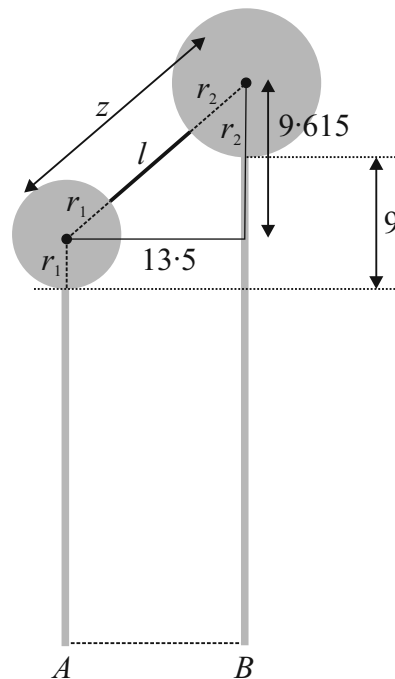
$$r_1 = 3 \text{ cm}$$

$$\frac{4}{3}\pi r_2^3 = 36\pi \times 1.75 \Rightarrow r_2^3 = 47.25$$

$$\therefore r_2 = 3.615 \text{ cm}$$

$$z = \sqrt{13.5^2 + 9.615^2} = 16.57 \text{ cm}$$

$$l = 16.57 - 3 - 3.615 = 9.955 \approx 10 \text{ cm}$$



**MARKING SCHEME NOTES**

**Question 7 (d) [Scale 5B (0, 2, 5)]**

- 2:**
- $V_B$  formulated with some substitution
  - $|XY|$  formulated
  - $|TW|$  evaluated
  - $\text{rod} = |TW| - r_b - r_a$  formulated with 2 relevant values

**QUESTION 8 (60 MARKS)**

**Question 8 (a)**

(i) Population mean:

Population:  $\mu = 4.64, \sigma = 0.12$

Sample:  $n = 10, \bar{\mu} = 4.64, \bar{\sigma} = \frac{0.12}{\sqrt{10}} = 0.038$

$$\bar{x} = 4.6: \bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}} = \frac{4.6 - 4.64}{0.038} = -1.05$$

$$\bar{x} = 4.7: \bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}} = \frac{4.8 - 4.64}{0.038} = 1.58$$

$$P(4.6 < \bar{x} < 4.7) = P(-1.05 < \bar{z} < 1.58)$$

$$= P(\bar{z} < 1.58) - P(\bar{z} < -1.05)$$

$$= P(\bar{z} < 1.58) - P(\bar{z} > 1.05)$$

$$= P(\bar{z} < 1.58) - [1 - P(\bar{z} < 1.05)]$$

$$= P(\bar{z} < 1.58) - 1 + P(\bar{z} < 1.05)$$

$$= 0.9429 - 1 + 0.8531 = 0.796$$

(ii) Population sample:

$$n = 400, \hat{p} = \frac{324}{400} = 0.81$$

$$\hat{\sigma} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.81(1-0.81)}{400}} = 0.0196$$

$$\text{Confidence interval: } \hat{p} - 1.96\hat{\sigma} \leftrightarrow \hat{p} + 1.96\hat{\sigma}$$

$$0.81 - 1.96(0.0196) \leftrightarrow 0.81 + 1.96(0.0196)$$

$$0.77 \leftrightarrow 0.85$$

The company is 95% confident that between 77% and 85% of the population like the new bar.

**MARKING SCHEME NOTES**

**Question 8 (a) (i) [Scale 20D (0, 5, 10, 15, 20)]**

- 5:**
- $z_1$  formulated with some correct substitution
  - $z_2$  formulated with some correct substitution

**10:** •  $z_1$  and  $z_2$  fully substituted

**15:** •  $-1.05$  and  $1.58$  or equivalents

**Question 8 (a) (ii) [Scale 15C (0, 4, 11, 15)]**

**4:** • CI formulated with some correct substitution

• 1.96

•  $\hat{p} \pm \frac{1}{\sqrt{n}}$  incomplete or completed

**11:** • CI fully substituted

**Question 8 (b)**

(i) Table of statements:

Statement	Always True	Sometimes True	Never True
1. When forming confidence intervals (for fixed $n$ and $\hat{p}$ ), an increased confidence level implies a wider interval.	✓		
2. As the value of $\hat{p}$ increases (for fixed $n$ ), the estimated standard error of the population proportion increases.		✓	
3. As the value of $\hat{p}(1-\hat{p})$ increases (for fixed $n$ ), the estimated standard error of the population proportion increases.	✓		
4. As $n$ , the number of people sampled, increases (for fixed $\hat{p}$ ), the estimated standard error of the population proportion increases.			✓

(ii) Let  $p = \hat{p}(1 - \hat{p}) = \hat{p} - \hat{p}^2$

$$\frac{dp}{d\hat{p}} = 1 - 2\hat{p}$$

$$\frac{dp}{d\hat{p}} = 0 \Rightarrow 1 - 2\hat{p} = 0$$

$$\therefore \hat{p} = 0.5$$

Maximum value:  $p = \hat{p}(1 - \hat{p}) = 0.5(1 - 0.5) = 0.25$

(iii)  $1.96 \times \hat{\sigma} = 1.96 \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 1.96 \times \sqrt{\frac{0.25}{800}} = 0.0346$

**NOTE:** The radius of the confidence interval is the margin of error =  $1.96 \times \hat{\sigma}$ .

**MARKING SCHEME NOTES**

**Question 8 (b) (i) [Scale 5C (0, 2, 4, 5)]**

2: • Any 1 correct

4: • Any 2 correct

**Question 8 (b) (ii) + (iii) [Scale 10D (0, 3, 5, 8, 10)]**

3: • Any relevant calculus

- $\frac{1}{\sqrt{n}} = \frac{1}{\sqrt{800}}$

- Effort at completing the square

5: •  $\hat{p} = \frac{1}{2}$  or equivalent

8: • Maximum value

**Question 8 (c)**

$$\begin{aligned} \text{Fund} &= 20\,000 + \frac{20\,000 \times 1.01}{1.024} + \frac{20\,000 \times 1.01^2}{1.024^2} + \dots + \frac{20\,000 \times 1.01^{25}}{1.024^{25}} \\ &= 20\,000 \left[ 1 + \frac{1.01}{1.024} + \left(\frac{1.01}{1.024}\right)^2 + \dots + \left(\frac{1.01}{1.024}\right)^{25} \right] \end{aligned}$$

**Note:**  $a = 1, r = \frac{1.01}{1.024}, n = 26$

$$\text{Fund} = 20\,000 \times \frac{1 \left( 1 - \left(\frac{1.01}{1.024}\right)^{26} \right)}{1 - \frac{1.01}{1.024}} = \text{€}440\,132.40$$

**MARKING SCHEME NOTES**

**Question 8 (c) [Scale 10D (0, 3, 5, 8, 10)]**

3: •  $20000(1.01)$  or  $\frac{20\,000}{1.024}$

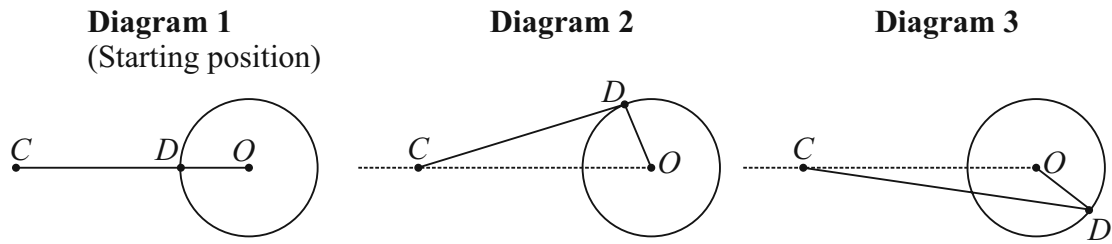
5: •  $\frac{20\,000(1.01)}{1.024}$  or similar term

- Correctly handles inflation element or completes correctly present values element and finishes

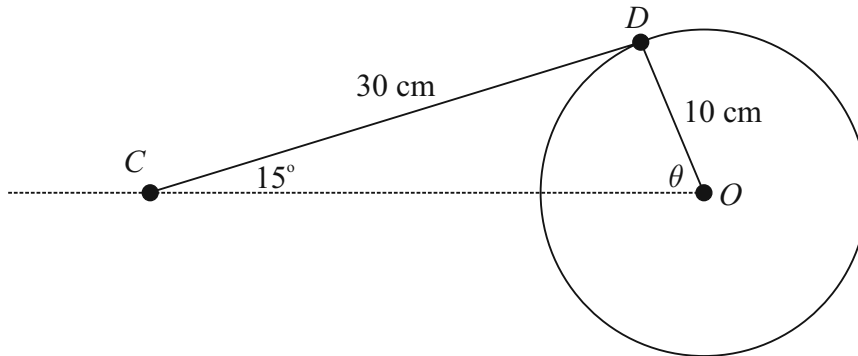
8: • GP with  $a, r$  and  $n$  identified

**Note:** Treat  $n = 25$  as a misreading

**QUESTION 9 (40 MARKS)**



**Question 9 (a)**



$$\frac{\sin \theta}{30} = \frac{\sin 15^\circ}{10} \Rightarrow \theta = \sin^{-1} \left( \frac{30 \sin 15^\circ}{10} \right) = 50.9^\circ$$

$$|\angle COD| = 51^\circ$$

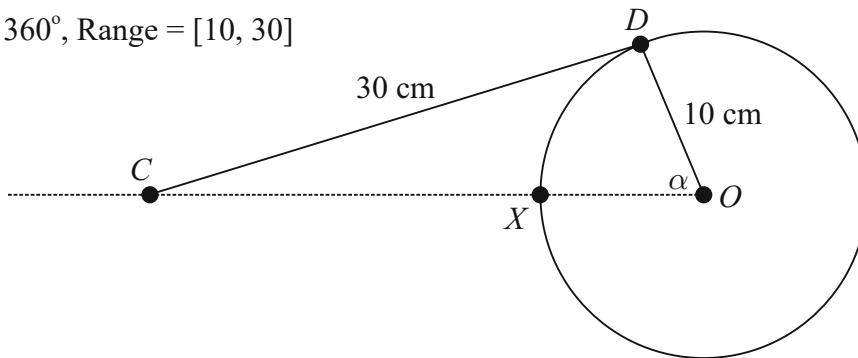
**MARKING SCHEME NOTES**  
**Question 9 (a) [Scale 10C (0, 3, 7, 10)]**

3: • Sine rule formulated with some substitution

7: •  $\sin x$

**Question 9 (b)**

(i) Period =  $360^\circ$ , Range = [10, 30]



(ii) Table:

$\alpha$	$0^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
$f(\alpha)$ (cm)	30	18.28	10	18.28	30

**MARKING SCHEME NOTES**

**Question 9 (b) (i) [Scale 5C (0, 2, 4, 5)]**

2: • Period or range correct

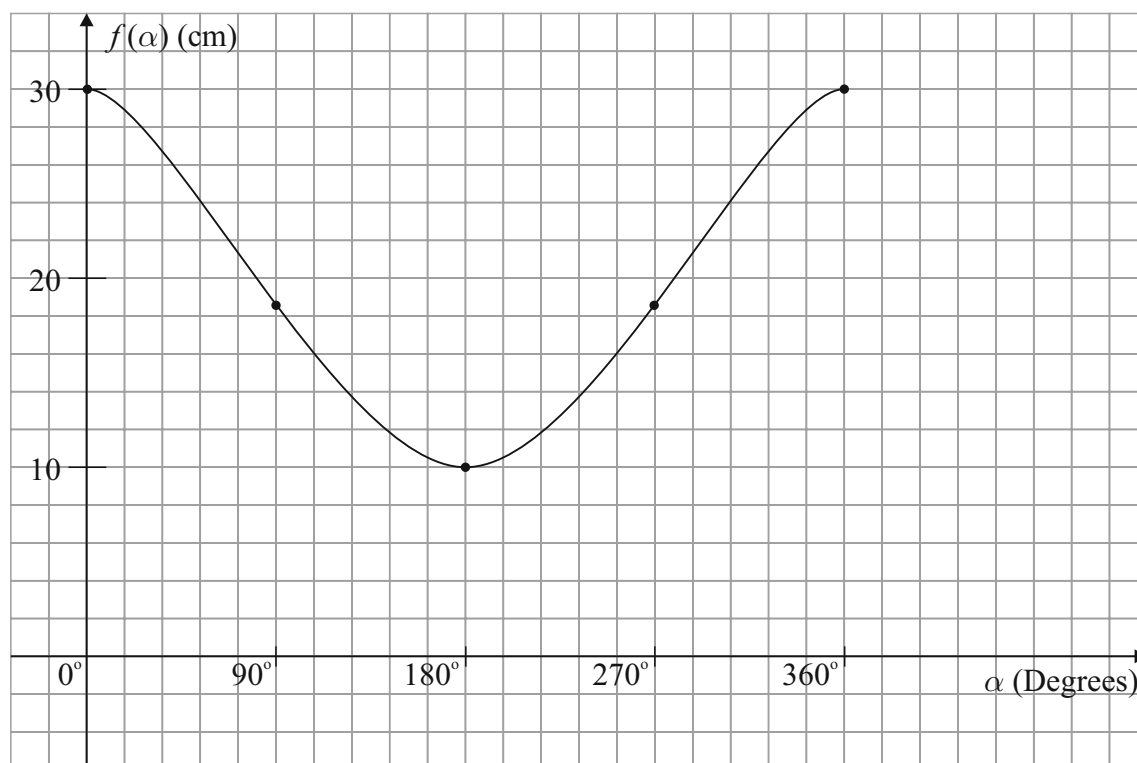
4: • Period correct and range partly correct  
 • Period **and** range in incorrect order

**Question 9 (b) (ii) [Scale 5C (0, 2, 4, 5)]**

2: • 1 correct new value

4: • 2 correct new values

(iii) Graph:



(iv) Diagram 2 - graph has a greater rate of change in this position.

**MARKING SCHEME NOTES**

**Question 9 (b) (iii) [Scale 10C (0, 3, 7, 10)]**

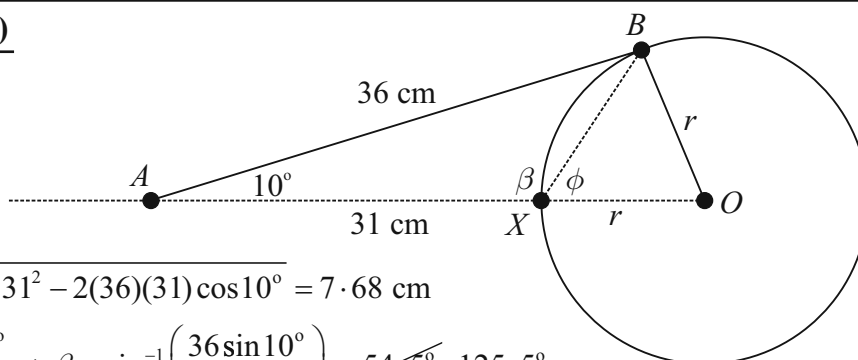
3: • 1 point from table plotted

7: • 3 points from table plotted

**Question 9 (b) (iv) [Scale 5B (0, 2, 5)]**

2: • Diagram 2 identified but without reason or with invalid reason

**Question 9 (c)**



$$|XB| = \sqrt{36^2 + 31^2 - 2(36)(31)\cos 10^\circ} = 7.68 \text{ cm}$$

$$\frac{\sin \beta}{36} = \frac{\sin 10^\circ}{31} \Rightarrow \beta = \sin^{-1}\left(\frac{36 \sin 10^\circ}{7.68}\right) = 54.5^\circ, 125.5^\circ$$

$$\phi = 180^\circ - 125.5^\circ = 54.5^\circ$$

$$r^2 = r^2 + 7.68^2 - 2r(7.68)\cos 54.5^\circ$$

$$2r(7.68)\cos 54.5^\circ = 7.68^2$$

$$2r \cos 54.5^\circ = 7.68$$

$$r = \frac{7.68}{2 \cos 54.5^\circ} \approx 7 \text{ cm}$$

**MARKING SCHEME NOTES**

**Question 9 (c) [Scale 5C (0, 2, 4, 5)]**

2: • Cosine rule formulated with some substitution

•  $(31 + r)$

4: • Relevant equation in  $r$

## LC 2017 (SET B): PAPER 1

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

$$\begin{aligned}
 f(x) &= 2x^2 - 7x - 10 = 2\left(x^2 - \frac{7}{2}x - 5\right) \left[k = -\frac{7}{2}\right] \\
 &= 2\left(\left(x - \frac{7}{4}\right)^2 - \left(-\frac{7}{4}\right)^2 - 5\right) \\
 &= 2\left(\left(x - \frac{7}{4}\right)^2 - \frac{49}{16} - 5\right) \\
 &= 2\left(\left(x - \frac{7}{4}\right)^2 - \frac{129}{16}\right) \\
 &= 2\left(x - \frac{7}{4}\right)^2 - \frac{129}{8}
 \end{aligned}$$

$$\begin{aligned}
 \left(x + \frac{k}{2}\right)^2 &= x^2 + kx + \left(\frac{k}{2}\right)^2 \\
 \therefore x^2 + kx &= \left(x + \frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2
 \end{aligned}$$

#### Question 1 (b)

Minimum value of  $f(x)$  occurs when

$$\left(x - \frac{7}{4}\right)^2 = 0 \Rightarrow x = \frac{7}{4}$$

Minimum value occurs at  $f(x) = -\frac{129}{8}$

Minimum point =  $\left(\frac{7}{4}, -\frac{129}{8}\right)$

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 5D (0, 2, 3, 4, 5)]

- 2: •  $a = 2$  identified explicitly or as factor  
 3: • Completed square  
 4: •  $h$  or  $k$  identified from work

##### Question 1 (b) [Scale 10B (0, 4, 10)]

- 4: • One relevant co-ordinate identified

#### Question 1 (c) (i)

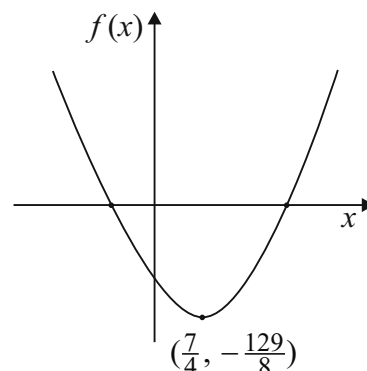
Two real roots:  $b^2 - 4ac > 0$

$$a = 2, b = -7, c = -10$$

$$(-7)^2 - 4(2)(-10) = 49 + 80 = 129 > 0 \Rightarrow f \text{ has two real roots}$$

Or

As the coefficient of  $x^2$  is positive the graph is shaped as shown to the right. The minimum point is below the  $x$ -axis giving two real roots.



#### Question 1 (c) (ii)

$$f(x) = 2\left(x - \frac{7}{4}\right)^2 - \frac{129}{8} = 0$$

$$2\left(x - \frac{7}{4}\right)^2 = \frac{129}{8}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{129}{16}$$

$$\left(x - \frac{7}{4}\right) = \pm \sqrt{\frac{129}{16}}$$

$$x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$$

#### MARKING SCHEME NOTES

##### Question 1 (c) (i) [Scale 5B (0, 3, 5)]

- 3: • Mention of  $a > 0$   
 •  $b^2 - 4ac$   
 • Identifies location of one or two roots, e.g. between 4 and 5.

##### Question 1 (c) (ii) [Scale 5C (0, 3, 4, 5)]

- 3: • Formula with some substitution  
 • Equation rewritten with some transpose  
 4: •  $x = \frac{7}{4} \pm \sqrt{\frac{129}{16}}$  or equivalent

**QUESTION 2 (25 MARKS)**

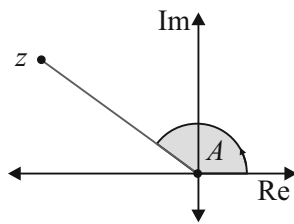
**Question 2 (a)**

$$z = -\sqrt{3} + i$$

$$|z| = \sqrt{(-\sqrt{3})^2 + 1^2} \\ = \sqrt{3+1} = \sqrt{4} = 2$$

$$\theta = \tan^{-1} \left| \frac{1}{\sqrt{3}} \right| = \frac{\pi}{6}$$

$$\arg z = A = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ (Second quadrant)}$$



$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

$$z = 2 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$z^4 = \left[ 2 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \right]^4$$

$$= 2^4 \left( \cos \frac{20\pi}{6} + i \sin \frac{20\pi}{6} \right)$$

$$= 16 \left( \cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3} \right)$$

$$= 16 \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = -8 - 8\sqrt{3}i$$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 15D (0, 5, 8, 12, 15)]**

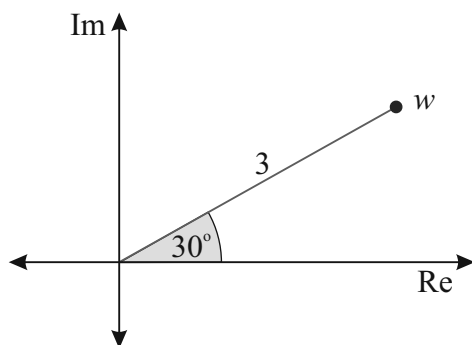
**5:** •  $\theta$  or  $|z|$  found

**8:** •  $z$  written in polar form

**12:** • De Moivre's Theorem applied correctly

**Note:** Not using De Moivre: Low partial credit for fully correct work

**Question 2 (b)**



$$|w| = 3, \arg w = 30^\circ = \frac{\pi}{6}$$

$$w = 3 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$t = zw = 2 \left( \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right) \times 3 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$= 6 \left( \cos \left( \frac{5\pi}{6} + \frac{\pi}{6} \right) + i \sin \left( \frac{5\pi}{6} + \frac{\pi}{6} \right) \right)$$

$$= 6(\cos \pi + i \sin \pi) = 6(-1) = -6$$

**MARKING SCHEME NOTES**

**Question 2 (b) [Scale 10D (0, 4, 7, 8, 10)]**

**4:** • Work towards  $w$  in Cartesian or polar form

**7:** •  $zw$  expressed as a product

**8:** •  $zw$  in Cartesian or polar form

**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$f(x) = \frac{1}{3}x^2 - x + 3$$

$$f(x+h) = \frac{1}{3}(x+h)^2 - (x+h) + 3 = \frac{1}{3}x^2 + \frac{2}{3}hx + \frac{1}{3}h^2 - x - h + 3$$

$$f(x+h) - f(x) = \frac{1}{3}x^2 + \frac{2}{3}hx + \frac{1}{3}h^2 - x - h + 3 - \frac{1}{3}x^2 + x - 3 = \frac{2}{3}hx + \frac{1}{3}h^2 - h$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right) = \lim_{h \rightarrow 0} \left( \frac{\frac{2}{3}hx + \frac{1}{3}h^2 - h}{h} \right) = \lim_{h \rightarrow 0} \left( \frac{2}{3}x + \frac{1}{3}h - 1 \right) = \frac{2}{3}x - 1$$

**MARKING SCHEME NOTES****Question 3 (a) [Scale 20D (0, 5, 14, 17, 20)]**

5: • any  $f(x+h)$

14: •  $f(x+h) - f(x)$  with some correct work

17: •  $\frac{\frac{1}{3}(x+h)^2 - (x+h) + 3 - (\frac{1}{3}x^2 - x + 3)}{h}$  simplified

**Notes:**

- omission of limit sign penalised once only
- answer not from 1st Principles merits 0 marks

**Question 3 (b)**

$$y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$y = \ln f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{f(x)} \times f'(x)$$

**THE RULE IN WORDS:**

$$y = \ln f(x) \Rightarrow \frac{dy}{dx} = \text{one over the function}$$

inside the ln multiplied by the derivative of the function inside the ln.

$$f(x) = \ln(3x^2 + 2)$$

$$g(x) = x + 5$$

$$f(g(x)) = \ln(3(x+5)^2 + 2) = \ln(3(x^2 + 10x + 25) + 2)$$

$$= \ln(3x^2 + 10x + 25) + 2)$$

$$= \ln(3x^2 + 30x + 75 + 2)$$

$$= \ln(3x^2 + 30x + 77)$$

$$[f(g(x))]' = \frac{1}{3x^2 + 30x + 77} \times 6x + 30 = \frac{6x + 30}{3x^2 + 30x + 77}$$

$$[f(g(\frac{1}{4}))]' = \frac{6(\frac{1}{4}) + 30}{3(\frac{1}{4})^2 + 30(\frac{1}{4}) + 77} = \frac{504}{1355} = 0.372$$

**MARKING SCHEME NOTES****Question 3 (b) [Scale 5C (0, 3, 4, 5)]**

3: • Any correct differentiation

•  $fg(x)$  formulated

4: •  $\frac{d(fg(x))}{dx}$  found

**Note:** Work with  $f(x) \times g(x)$  merits low partial credit at most

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**

Day	1	2	3	4
Percentage of substance (%)	95	42.75	19.2375	8.6569

**METHOD 1:**

$$a = 95, r = \frac{42.75}{95} = \frac{9}{20} = 0.45$$

$$T_n = ar^{n-1} = 95 \times 0.45^{n-1}$$

$$T_n = 0.01 \Rightarrow 95 \times 0.45^{n-1} = 0.01$$

$$0.45^{n-1} = \frac{0.01}{95}$$

$$\log_{10} 0.45^{n-1} = \log_{10} \left( \frac{0.01}{95} \right)$$

$$(n-1) \log_{10} 0.45 = \log_{10} \left( \frac{0.01}{95} \right)$$

$$n = \frac{\log_{10} \left( \frac{0.01}{95} \right)}{\log_{10} 0.45} + 1 = 12.47$$

Answer: 13th day

**METHOD 2:**

$$P = ka^t$$

$$t = 1: 95 = ka^1 = ka \dots (1)$$

$$t = 2: 42.75 = ka^2 \dots (2)$$

$$(2) \div (1): \frac{ka^2}{ka} = \frac{42.75}{95} \Rightarrow a = 0.45$$

$$\text{Into (1): } k \times 0.45 = 95 \Rightarrow k = \frac{95}{0.45} = \frac{1900}{9}$$

$$P = ka^t = \frac{1900}{9} \times 0.45^t = 0.01$$

$$0.45^t = \frac{0.09}{1900}$$

$$\log_{10} 0.45^t = \log_{10} \left( \frac{0.09}{1900} \right)$$

$$t \log_{10} 0.45 = \log_{10} \left( \frac{0.09}{1900} \right)$$

$$t = \frac{\log_{10} \left( \frac{0.09}{1900} \right)}{\log_{10} 0.45} = 12.47$$

Answer: 13th day

**MARKING SCHEME NOTES**

**Question 4 (a) [Scale 15D (0, 5, 8, 12, 15)]**

**5:** •  $r$  found

•  $T_n$  of a GP with some substitution

**8:** • Inequality in  $n$  written

**12:** • Inequality in  $n$  simplified (log handled)

**15:** • Accept  $n = 12.47$

**AUTHOR'S NOTE:** The marking scheme gives the answer as the 12th day. Based on the way the question is asked it is unclear whether the correct answer occurs on the 12th or the 13th day. The question tells us that the percentage is measured at the same time each day. We do not know how early or late in the day the percentage was measured. In any case, full marks are obtained when you produce an answer of  $n = 12.47$ .

**Question 4 (b)**

Perimeter of first square =  $4 \times 2 = 8$  cm

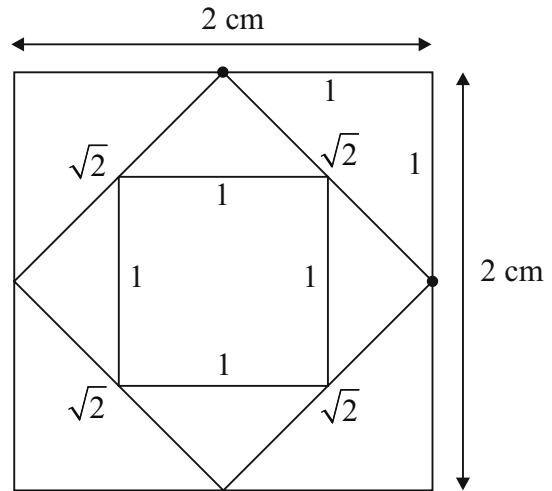
Perimeter of second square =  $4 \times \sqrt{2} = 4\sqrt{2}$  cm

Perimeter of third square =  $4 \times 1 = 4$  cm

Sum  $S = 8 + 4\sqrt{2} + 4 + \dots$

$$a = 8, r = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{8}{1-\frac{\sqrt{2}}{2}} = (16 + 8\sqrt{2}) \text{ cm}$$



**MARKING SCHEME NOTES**

**Question 4 (b) [Scale 10C (0, 5, 8, 10)]**

**5:** • length of one side of new square

**8:** •  $S_{\infty}$  fully substituted

• Correct work with one side only

**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

$$f(x) = 2x^3 + 5x^2 - 4x - 3$$

$$f(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3 = -54 + 45 + 12 - 3 = 0$$

$\therefore (x + 3)$  is a factor

**FINDING THE QUADRATIC FACTOR**

**METHOD 1: LINING UP COEFFICIENTS**

$$2x^3 + 5x^2 - 4x - 3 = (x + 3)(2x^2 + kx - 1)$$

$$= 2x^3 + (k + 6)x^2 + \dots$$

$$5 = k + 6 \Rightarrow k = -1$$

**METHOD 2: DIVISION**

$$\begin{array}{r}
 2x^2 - x - 1 \\
 x + 3 \overline{) 2x^3 + 5x^2 - 4x - 3} \\
 \underline{\mp 2x^3 \mp 6x^2} \phantom{-3} \\
 -x^2 - 4x - 3 \\
 \underline{\pm x^2 \pm 3x} \phantom{-3} \\
 -x - 3 \\
 \underline{\pm x \pm 3} \\
 0
 \end{array}$$

$$\therefore 2x^3 + 5x^2 - 4x - 3 = (x + 3)(2x^2 - x - 1) = 0$$

$$2x^2 - x - 1 = (2x + 1)(x - 1) = 0 \Rightarrow x = -\frac{1}{2}, 1$$

$$\therefore f(x) = (x + 3)(2x + 1)(x - 1)$$

$$f(x) = 0 \Rightarrow x = -3, -\frac{1}{2}, 1$$

**MARKING SCHEME NOTES**

**Question 5 (a) [Scale 15C (0, 5, 10, 15)]**

**5:** • Shows  $f(-3) = 0$

**10:** • quadratic factor of  $f(x)$  found

**Note:** No remainder in division may be stated as reason for  $x = -3$  as root

**Question 5 (b)**

$$f(x) = 2x^3 + 5x^2 - 4x - 3$$

$$f'(x) = 6x^2 + 10x - 4$$

$$f''(x) = 12x + 10$$

$$\text{Turning points: } f'(x) = 0 \Rightarrow 6x^2 + 10x - 4 = 0$$

$$3x^2 + 5x - 2 = 0$$

$$(3x - 1)(x + 2) = 0 \Rightarrow x = \frac{1}{3}, -2$$

$$f\left(\frac{1}{3}\right) = 2\left(\frac{1}{3}\right)^3 + 5\left(\frac{1}{3}\right)^2 - 4\left(\frac{1}{3}\right) - 3 = -\frac{100}{27}$$

$$f(-2) = 2(-2)^3 + 5(-2)^2 - 4(-2) - 3 = 9$$

$$f''\left(\frac{1}{3}\right) = 12\left(\frac{1}{3}\right) + 10 = 14 > 0 \Rightarrow \left(\frac{1}{3}, -\frac{100}{27}\right) \text{ is a local minimum}$$

$$f''(-2) = 12(-2) + 10 = -14 < 0 \Rightarrow (-2, 9) \text{ is a local maximum}$$

**MARKING SCHEME NOTES**

**Question 5 (b) [Scale 5C (0, 3, 4, 5)]**

**3:** •  $\frac{dy}{dx}$  found (Some correct differentiation)

**4:** • roots and one  $y$  value found

**Note:** One of Max/Min must be identified for full credit

**Question 5 (c)**

$f(x) + a$ : Roots are on opposite sides of  $x$ -axis

Translate  $f(x)$  **up** by  $\frac{100}{27} \Rightarrow a > \frac{100}{27}$

Translate  $f(x)$  **down** by 9  $\Rightarrow a < -9$

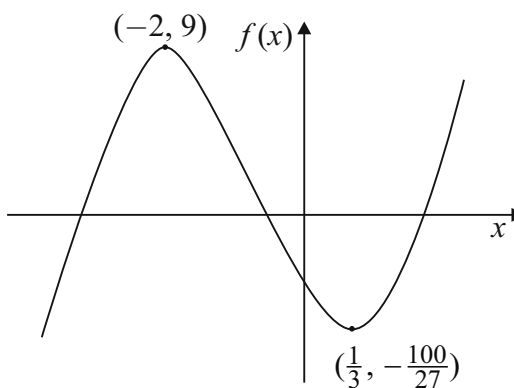
$$\therefore a < -9, a > \frac{100}{27}, a \in \mathbb{R}$$

**MARKING SCHEME NOTES**

**Question 5 (c) [Scale 5B (0, 3, 5)]**

**3:** • one value identified

• no range identified (from 2 values)



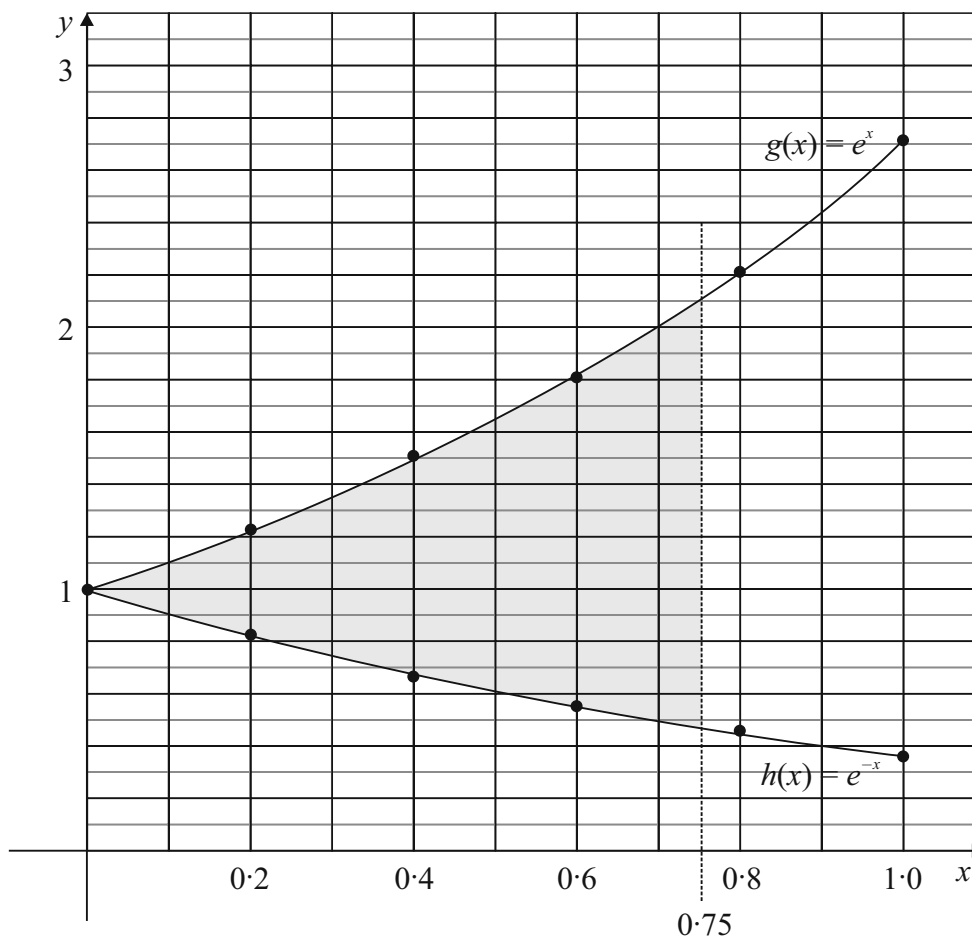
**QUESTION 6 (25 MARKS)**

**Question 6 (a)**

$$g(x) = e^x, x \in \mathbb{R}, 0 \leq x \leq 1$$

$$h(x) = e^{-x}, x \in \mathbb{R}, 0 \leq x \leq 1$$

$x$	0	0.2	0.4	0.6	0.8	1.0
$h(x)$	1	0.82	0.67	0.55	0.45	0.37



**Question 6 (b)**

$$\begin{aligned} \text{Shaded area} &= \int_0^{0.75} (e^x - e^{-x}) dx = [e^x + e^{-x}]_0^{0.75} \\ &= [(e^{0.75} + e^{-0.75}) - (e^0 + e^0)] \\ &= e^{0.75} + e^{-0.75} - 2 \\ &= 0.5894 \end{aligned}$$

**MARKING SCHEME NOTES**

**Question 6 (a) [Scale 15C (0, 5, 10, 15)]**

**5:** • one point correct

**10:** • Graph not in required domain

**Question 6 (b) [Scale 10C (0, 5, 8, 10)]**

**5:** • Formulates integration for area under one curve with limits

**8:** • integrates twice for correct area under both curves

**Note:** Trapezoidal rule must have at least 5 divisions AND fully correct work gets Low Partial Credit

**QUESTION 7 (55 MARKS)**

**Question 7 (a)**

$$p(t) = Se^{0.1t} \times 10^6$$

$$t = 0 \text{ (2010): } p(0) = Se^{0.1(0)} \times 10^6 = 1\,100\,000$$

$$S = \frac{1\,100\,000}{10^6} = 1.1$$

**Question 7 (b)**

$$p(t) = 1.1e^{0.1t} \times 10^6$$

$$t = 5 \text{ (2015): } p(5) = 1.1e^{0.1(5)} \times 10^6 = 1\,813\,593$$

**Question 7 (c)**

$$\begin{aligned} \text{Change in population during 2015} &= p(6) - p(5) \\ &= 1.1e^{0.1(6)} \times 10^6 - 1.1e^{0.1(5)} \times 10^6 = 190\,737 \end{aligned}$$

**Question 7 (d)**

$$q(t) = 3.9e^{kt} \times 10^6$$

$$t = 1 \text{ (2011): } q(1) = 3.9e^{k(1)} \times 10^6 = 3\,709\,795$$

$$e^k = \frac{3\,709\,795}{3.9 \times 10^6}$$

$$\ln(e^k) = \ln\left(\frac{3\,709\,795}{3.9 \times 10^6}\right)$$

$$k = -0.05$$

**MARKING SCHEME NOTES**

**Question 7 (a) [Scale 10B (0, 4, 10)]**

- 4: • equation in  $S$  with substitution

**Question 7 (b) [Scale 10B (0, 4, 10)]**

- 4: • substitution into formula for  $p(5)$

**Question 7 (c) [Scale 5C (0, 3, 4, 5)]**

- 3: • substitution into formula for  $p(6)$   
 • use of  $p(5)$  from previous part  
 •  $p(6) - p(5)$  written or implied  
 4: • Formulates  $p(6) - p(5)$  with some substitution

**Question 7 (d) [Scale 15C (0, 5, 10, 15)]**

- 5: • Either substitution into formula for  $k$   
 • Verifies  $k$  value only  
 10: • relevant equation in  $k$

**Question 7 (e)**

$$p(t) = q(t) \Rightarrow 1.1e^{0.1t} \times 10^6 = 3.9e^{-0.05t} \times 10^6$$

$$1.1e^{0.1t} = 3.9e^{-0.05t} \Rightarrow \frac{e^{0.1t}}{e^{-0.05t}} = \frac{3.9}{1.1}$$

$$e^{0.15t} = \frac{3.9}{1.1} \Rightarrow \ln(e^{0.15t}) = \ln\left(\frac{3.9}{1.1}\right)$$

$$0.15t = \ln\left(\frac{3.9}{1.1}\right)$$

$$t = \frac{1}{0.15} \ln\left(\frac{3.9}{1.1}\right) = 8.44$$

Populations are the same during 2018.

**Question 7 (f)**

The average value  $\bar{y}$  of a continuous function  $y = g(x)$  on an interval  $[a, b]$  is given by:

$$\bar{y} = \frac{\int_a^b g(x) dx}{b-a} = \frac{\int_a^b y dx}{b-a}$$

$$q(t) = 3.9e^{-0.05t} \times 10^6$$

$$\bar{q} = \frac{1}{15-0} \int_0^{15} (3.9e^{-0.05t} \times 10^6) dt$$

$$= \frac{3.9 \times 10^6}{15} \int_0^{15} e^{-0.05t} dt$$

$$= -\frac{3.9 \times 10^6}{15(0.05)} [e^{-0.05t}]_0^{15}$$

$$= -\frac{3.9 \times 10^6}{15(0.05)} [e^{-0.05(15)} - e^{-0.05(0)}]$$

$$= -\frac{3.9 \times 10^6}{15(0.05)} [e^{-0.05(15)} - 1] = 2\,743\,694$$

**MARKING SCHEME NOTES**

**Question 7 (e) [Scale 5C (0, 3, 4, 5)]**

- 3: •  $p(t) = q(t)$  written or implied  
 4: • relevant equation in  $t$

**Question 7 (f) [Scale 5C (0, 3, 4, 5)]**

- 3: • integral formulated (with limits)  
 4: • integration with full substitution

**Question 7 (g)**

$$q(t) = 3 \cdot 9e^{-0.05t} \times 10^6$$

$$\frac{dq}{dt} = 3 \cdot 9e^{-0.05t} \times 10^6 \times (-0.05) = -195\,000e^{-0.05t}$$

$$\left(\frac{dq}{dt}\right)_{t=8} = -195\,000e^{-0.05(8)}$$

$$= -130\,712 \text{ people/year}$$

**MARKING SCHEME NOTES**

**Question 7 (g) [Scale 5C (0, 3, 4, 5)]**

3: •  $q'(t)$

4: •  $q'(t)$  fully substituted

**QUESTION 8 (55 MARKS)**

**Question 8 (a)**

$$P = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \dots + \frac{A}{(1+i)^t}$$

$$a = \frac{A}{(1+i)}, r = \frac{1}{(1+i)}$$

$$S_t = P = \frac{a(1-r^t)}{1-r} = \frac{\frac{A}{(1+i)}\left(1 - \frac{1}{(1+i)^t}\right)}{1 - \frac{1}{(1+i)}}$$

$$= \frac{A\left(\frac{(1+i)^t - 1}{(1+i)^t}\right)}{(1+i)\left(\frac{1+i-1}{(1+i)}\right)} = \frac{A\left(\frac{(1+i)^t - 1}{(1+i)^t}\right)}{i} = \frac{A((1+i)^t - 1)}{i(1+i)^t}$$

$$\therefore A = P \frac{i(1+i)^t}{((1+i)^t - 1)}$$

**MARKING SCHEME NOTES**

**Question 8 (a) [Scale 5C (0, 3, 4, 5)]**

3: •  $P = \frac{A}{1+i}$

•  $A = P(1+i)$

•  $S_n$  formula with some substitution

4: • full substitution for  $P$  (or  $A$ ) into  $S_n$  formula

**Question 8 (b) (i)**

Monthly repayment  $A = 5000 \times 0.025 = \text{€}125$

**Question 8 (b) (ii)**

Annual rate = 21.75%  $\Rightarrow i = 0.2175$

$$i_M = (1+i)^{\frac{1}{12}} - 1 = 1.2175^{\frac{1}{12}} - 1 = 0.0165$$

Monthly rate = 1.65%

**MARKING SCHEME NOTES**

**Question 8 (b) (i) [Scale 10B (0, 4, 10)]**

4: • Any one unknown

**Question 8 (b) (ii) [Scale 10B (0, 4, 10)]**

4: • Formula with some substitution

**Question 8 (b) (iii)**

You pay €125 every month. Part of this payment pays for the interest accrued that month and part of it pays off the overall debt.

**End of month 1:** Interest = €5000 × 0.0165 = €82.50

Balance reduced by: €125 – €82.50 = €42.50

New balance = €5000 – €42.50 = €4957.50

**End of month 2:** Interest = €4957.50 × 0.0165 = €81.80

Balance reduced by: €125 – €81.80 = €43.20

New balance = €4957.50 – €43.20 = €4914.30

**End of month 3:** Interest = €4914.30 × 0.0165 = €81.09

Balance reduced by: €125 – €81.09 = €43.91

New balance = €4914.30 – €43.91 = €4870.39

Payment Number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1	125	82.50	42.50	4957.50
2	125	81.80	43.20	4914.30
3	125	81.09	43.91	4870.39

**MARKING SCHEME NOTES**

**Question 8 (b) (iii) [Scale 10C (0, 5, 8, 10)]**

**5:** • One correct additional entry

**8:** • 6 correct additional entries

**Note:** Where interest rate in b(ii) is not 1.65%, then check the validity of all values given.

**Question 8 (b) (iv)**

$P = 5000, A = 125, i_M = 0.0165, t = ?$

$$A = P \times \frac{i_M(1+i_M)^t}{(1+i_M)^t - 1}$$

$$125 = 5000 \times \frac{0.0165(1.0165)^t}{(1.0165)^t - 1}$$

$$\frac{125}{5000 \times 0.0165} = \frac{(1.0165)^t}{(1.0165)^t - 1}$$

$$\frac{50}{33} = \frac{1.0165^t}{1.0165^t - 1}$$

$$50(1.0165^t - 1) = 33 \times 1.0165^t$$

$$50 \times 1.0165^t - 50 = 33 \times 1.0165^t$$

$$17 \times 1.0165^t = 50$$

$$1.0165^t = \frac{50}{17}$$

$$\log_{10} 1.0165^t = \log_{10} \frac{50}{17}$$

$$t \log_{10} 1.0165 = \log_{10} \frac{50}{17}$$

$$\therefore t = \frac{\log_{10} \frac{50}{17}}{\log_{10} 1.0165} = 65.92$$

Answer: 66 months

**Question 8 (b) (v)**

$i = 0.085$

$i_w = (1+i)^{\frac{1}{52}} - 1 = (1.085)^{\frac{1}{52}} - 1 = 0.00157$

$P = 5000, i_w = 0.00157, t = 156 \text{ weeks}, A = ?$

$$A = P \times \frac{i_w(1+i_w)^t}{(1+i_w)^t - 1}$$

$$A = 5000 \times \frac{0.00157(1.00157)^{156}}{(1.00157)^{156} - 1}$$

= €36.16

**Question 8 (b) (vi)**

$$\text{Saving} = 125 \times 66 - 36 \cdot 16 \times 156 = \text{€}2609 \cdot 04$$

**MARKING SCHEME NOTES**

**Question 8 (b) (iv) [Scale 5C (0, 3, 4, 5)]**

- 3: • Formula with some substitution  
• Some relevant manipulation of formula
- 4: • Equation in  $t$  ( $t$  no longer an index)

**Question 8 (b) (v) [Scale 10C (0, 5, 8, 10)]**

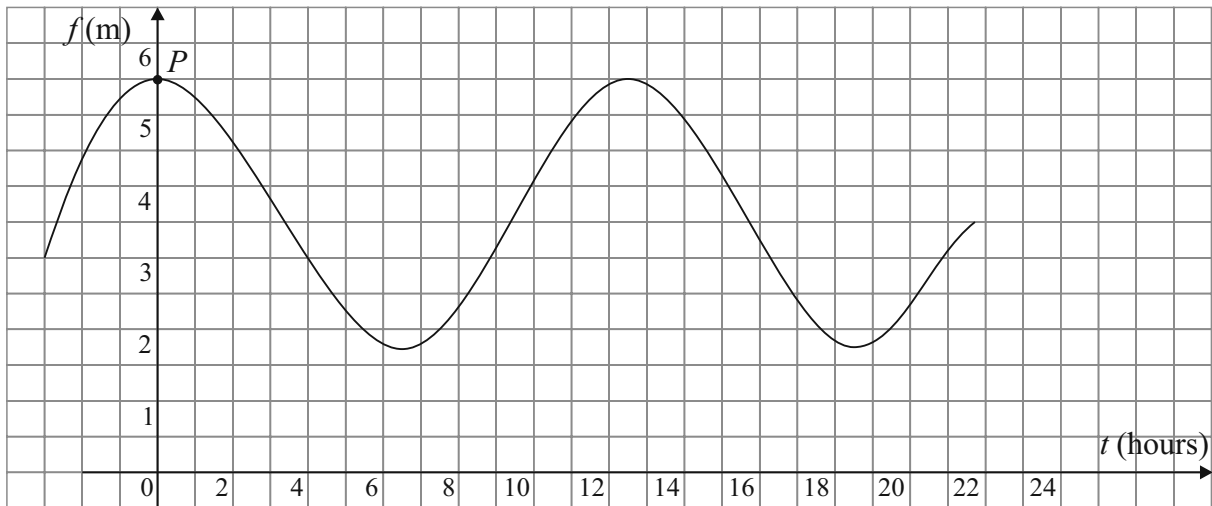
- 5: •  $r$  (weekly) found
- 8: • Fully substituted equation

**Question 8 (b) (vi) [Scale 5B (0, 3, 5)]**

- 3: • Total repayment by either method found

**QUESTION 9 (40 MARKS)**

**Question 9 (a)**



**MARKING SCHEME NOTES**

**Question 9 (a) [Scale 20C (0, 10, 18, 20)]**

- 10: • Vertical axis drawn  
• Horizontal axis drawn
- 18: • Horizontal axis fully scaled and positioned OR  
• Vertical axis fully scaled  
Use relevant portions of axes

**Note:**  $P$  can be on vertical axis

**Question 9 (b) (i)**

$$y = a + b \cos kx$$

$$\text{Period } P = \frac{2\pi}{k}$$

$$\text{Range} = [a - b, a + b], b > 0$$

$$f(t) = a + b \cos ct$$

$$\text{Range} = [a - b, a + b] = [1.7, 5.5]$$

$$a - b = 1.7 \dots (1)$$

$$a + b = 5.5 \dots (2)$$

$$2a = 7.2 \Rightarrow a = 3.6$$

$$\text{Into (2): } 3.6 - b = 1.7 \Rightarrow b = 1.9$$

**Question 9 (b) (ii)**

High tide:  $t = 2:00, 14:34$

$$\text{Period } P = 12:34 \text{ hours} = 12 \frac{17}{30} \text{ hours}$$

$$P = \frac{2\pi}{c} = 12 \frac{17}{30} \Rightarrow c = \frac{2\pi}{12 \frac{17}{30}} = 0.5$$

**Question 9 (c)**

$$f(t) = 3.6 + 1.9 \cos 0.5t = 5.2$$

$$1.9 \cos 0.5t = 1.6$$

$$\cos 0.5t = \frac{16}{19} \Rightarrow 0.5t = \cos^{-1}\left(\frac{16}{19}\right) = 0.57 \text{ (Radians)}$$

$$0.5t = \begin{cases} 0.57, 6.853 \text{ [First quadrant]} \\ 5.713 \text{ [Fourth quadrant]} \end{cases}$$

$$t = \begin{cases} 1.14, 13.706 \\ 11.426 \end{cases}$$

1.14 hours = 1 hour 8 mins  
 11.426 hours = 11 hours 26 mins  
 13.706 hours = 13 hours 42 mins  
 Afternoon times: 2:00 + 11:26 = 13:26; 2:00 + 13:42 = 15:42

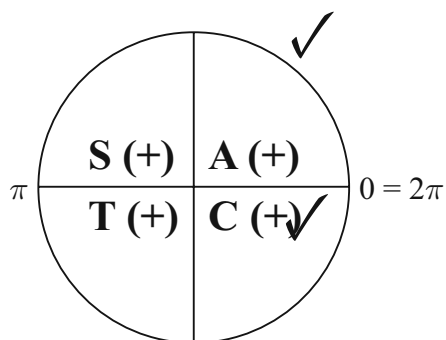
**MARKING SCHEME NOTES**

**Question 9 (b) (i) [Scale 10C (0, 5, 8, 10)]**

- 5:** • one equation in  $a$  and  $b$   
 • Range in terms of  $a$  and  $b$
- 8:** •  $a$  or  $b$  found
- Note:** Accept correct answer without work

**Question 9 (b) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3:** • Period identified or  $\frac{2\pi}{c}$  or 12.34
- 4:** • equation in  $c$  with some substitution



**MARKING SCHEME NOTES**

**Question 9 (c) [Scale 5C (0, 3, 4, 5)]**

**3:** • equation with some substitution

**4:** • solution for  $t$

**Note:** Low partial at most if formula not used

## LC 2017 (SET B): PAPER 2

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

$$P(r) = {}^n C_r (p)^r (q)^{n-r}$$

$$\text{Success } S: p = \frac{1}{5} = 0.2$$

$$\text{Failure } F: q = \frac{4}{5} = 0.8$$

$$P(F, S, F, S, F, S, F)$$

$$= 0.8 \times 0.2 \times 0.8 \times 0.2 \times 0.8 \times 0.2 \times 0.8$$

$$= \frac{256}{78\,125}$$

#### Question 1 (b)

$P(3S \text{ in first 6 trials and then } S \text{ in last trial})$

$$= {}^6 C_3 \times (0.2)^3 \times (0.8)^3 \times (0.2)$$

$$= \frac{256}{15\,625}$$

#### Question 1 (c)

$P(\text{At least one } S) = 1 - P(\text{No successes})$

$$= 1 - {}^n C_0 \times (0.2)^0 \times (0.8)^n = 1 - 0.8^n$$

#### Question 1 (d)

$$P(\text{At least one } S) = 1 - 0.8^n = 0.99$$

$$1 - 0.99 = 0.8^n$$

$$0.01 = 0.8^n$$

$$\log_{10} 0.01 = \log_{10} 0.8^n$$

$$\log_{10} 0.01 = n \log_{10} 0.8$$

$$n = \frac{\log_{10} 0.01}{\log_{10} 0.8} = 20.63$$

Minimum value of  $n = 21$

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 10C (0, 4, 5, 10)]

4: •  $\frac{4}{5}$

•  $(\frac{1}{5})^3$

5: •  $\frac{4}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{1}{5} \times \frac{4}{5}$  in any order

##### Question 1 (b) [Scale 5D (0, 2, 3, 4, 5)]

2: •  $\binom{6}{3}$  or  $(\frac{1}{5})^3$  or  $(\frac{4}{5})^3$

•  $\frac{1}{5}$  for last day

3: •  $\binom{6}{3} (\frac{1}{5})^3 (\frac{4}{5})^3$  stops or continues

•  $\binom{7}{4} (\frac{1}{5})^4 (\frac{4}{5})^3$  continues

4: •  $\binom{6}{3} (\frac{1}{5})^3 (\frac{4}{5})^3 (\frac{1}{5})$

##### Question 1 (c) [Scale 5B (0, 3, 5)]

3: • 1 or  $(\frac{4}{5})^n$

• any correct term from the expansion

##### Question 1 (d) [Scale 5C (0, 2, 4, 5)]

2: • Ans (c)  $> 0.99$

4: • viable solution to inequality

•  $n = 20.6377$  and stops

**QUESTION 2 (25 MARKS)**

Table 1							
Speed (km/hour)	40	48	56	64	88	96	112
Fuel consumption (km/litre)	21	16	18	16	13	11	9

**Question 2 (a)**

Correlation coefficient  $r = -0.957$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 5A (0, 5)]**

**Question 2 (b) [Scale 5C (0, 2, 4, 5)]**

2: • 3 correct plots

4: • All plots correct with an incorrect line of best fit  
• All plots correct and no line of best fit

**CALCULATOR: FINDING CORRELATION**

**COEFFICIENT  $r$**

- Press MODE
- Press 2: STAT
- Press 2: A + BX

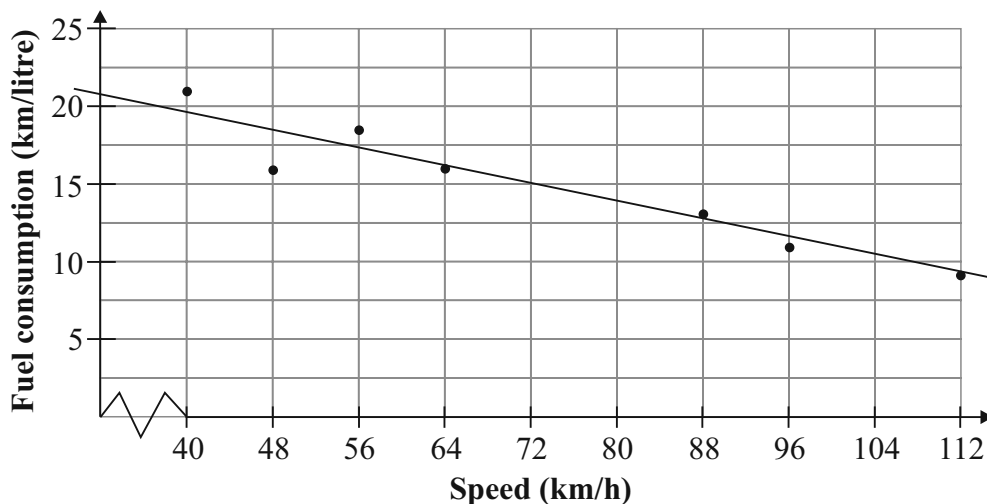
Now input your data.

When you are finished:

- Press AC.
- Press SHIFT followed by the number 1 button
- Press 5: Reg
- Press 3: r
- Press equals



**Question 2 (b)**



**Question 2 (c)**

A slope of the line of best fit of  $-0.15$  means that for every  $0.15$  km/h reduction in speed there is a saving of fuel consumption of  $1$  km/litre.

**Question 2 (d) (i)**

Mary:  $v = 96$  km/h,  $s = 260$  km

$$t = \frac{s}{v} = \frac{260}{96} = \frac{65}{24} \text{ h} = 162.5 \text{ mins}$$

Jane:  $v = 112$  km/h,  $s = 260$  km

$$t = \frac{s}{v} = \frac{260}{112} = \frac{65}{28} \text{ h} = 139.3 \text{ mins}$$

Difference in journey times =  $162.5 - 139.3 \approx 23$  mins

**MARKING SCHEME NOTES**

**Question 2 (c) [Scale 5B (0, 3, 5)]**

- 3: • Some reference to speed and fuel consumption  
• Reference to rate of change

**Question 2 (d) (i) [Scale 5C (0, 2, 4, 5)]**

- 2: •  $\frac{260}{96}$  or  $\frac{260}{112}$   
4: •  $\frac{260}{96} - \frac{260}{112}$  or equivalent  
• Answer in hours

**Question 2 (d) (ii)**

Mary:  $v = 96$  km/h,  $s = 260$  km, Fuel consumption = 11 km/l

$$\text{Number of litres} = \frac{260}{11} = 23.636$$

$$\text{Cost} = 23.636 \times 132.9 \text{ c} = \text{€}31.41$$

Jane:  $v = 112$  km/h,  $s = 260$  km, Fuel consumption = 9 km/l

$$\text{Number of litres} = \frac{260}{9} = 28.889$$

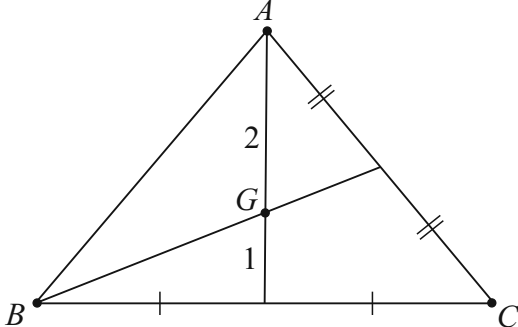
$$\text{Cost} = 28.889 \times 132.9 \text{ c} = \text{€}38.39$$

$$\text{Difference in cost} = \text{€}38.39 - \text{€}31.41 = \text{€}6.98$$

**MARKING SCHEME NOTES**  
**Question 2 (d) (ii) [Scale 5C (0, 2, 4, 5)]**  
 2: • Amount of Mary's fuel or amount of Jane's fuel  
 4: •  $(\frac{260}{96} - \frac{260}{112}) \times 1.329$  or equivalent

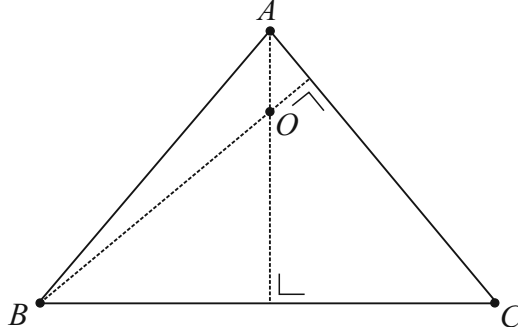
**QUESTION 3 (25 MARKS)**

**Centroid of a triangle**

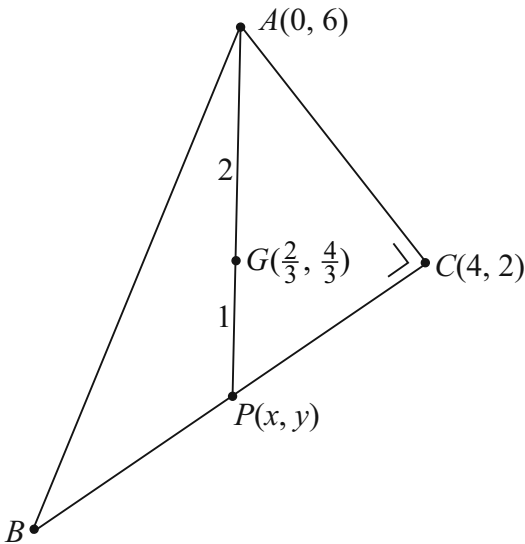


A median is a line from a vertex to the midpoint of the opposite side. The **centroid**  $G$  is the intersection of the medians.  $G$  divides the median in the ratio 2:1.

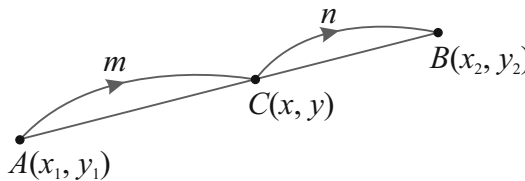
**Orthocentre of a triangle**



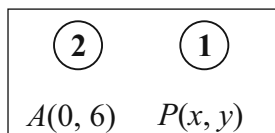
An altitude is a line from a vertex perpendicular to the opposite side. The **orthocentre**  $O$  is the intersection of the altitudes.



**Internal division of a line**  
 The point between  $A(x_1, y_1)$  and  $B(x_2, y_2)$  that divides  $[AB]$  in the ratio  $m:n$  is given by  $C(x, y)$ , where:

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$


**Question 3 (a)**



Centroid  $G(\frac{2}{3}, \frac{4}{3})$

$$\frac{2}{3} = \frac{2x+0}{2+1} = \frac{2x}{3} \Rightarrow x = 1$$

$$\frac{4}{3} = \frac{2y+6}{2+1} = \frac{2y+6}{3} \Rightarrow 4 = 2y+6 \Rightarrow y = -1$$

$\therefore P(1, -1)$

**Question 3 (b)**

$P$  is the midpoint of  $BC$ .

$$C(4, 2) \rightarrow P(1, -1) \rightarrow B(-2, -4)$$

**Question 3 (c)**

In order for  $C$  to be the orthocentre of the triangle  $ABC$ : The altitudes from vertices  $A$  and  $B$  are the sides  $AC$  and  $BC$  which will be perpendicular to each other at  $C$ .

$$A(0, 6), B(-2, -4), C(4, 2)$$

$$\text{Slope of } AC: m_1 = \frac{2-6}{4-0} = \frac{-4}{4} = -1$$

$$\text{Slope of } BC: m_2 = \frac{2+4}{4+2} = \frac{6}{6} = 1$$

$$m_1 \times m_2 = -1 \times 1 = -1 \Rightarrow AC \perp BC$$

$\therefore C$  is the orthocentre of  $\triangle ABC$ .

**MARKING SCHEME NOTES**

**Question 3 (a) [Scale 10C (0, 4, 5, 10)]**

- 4:**
- $P(\frac{4}{3}, -\frac{10}{3})$  or equivalent, i.e ratio 1:1
  - $\frac{2}{3}$  or  $\frac{1}{3}$  identified as part of change in  $x$  ordinate
  - $-\frac{14}{3}$  or  $-\frac{7}{3}$  identified as part of change in  $y$  ordinate
  - Ratio formula with some substitution
- 5:**
- one relevant co-ordinate of  $P$  found

**Question 3 (b) [Scale 5C (0, 2, 4, 5)]**

- 2:**
- $P$  as mid-point of  $BC$
- 4:**
- one relevant co-ordinate of  $B$  found
- Note:** Accept  $(-2, -4)$  without work  
Accept correct graphical solution

**MARKING SCHEME NOTES**

**Question 3 (c) [Scale 10C (0, 4, 5, 10)]**

- 4:**
- Identifies significance of right-angled triangle
  - one equation of perpendicular from vertex to opposite side found
- 5:**
- slope of  $AC$  and slope of  $BC$  found but no conclusion
  - two equations of perpendiculars from vertex to opposite side found

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**

**METHOD 1 [SUBSTITUTION]**

$$A(0, 0), B(6.5, 0), C(10, 7)$$

$$s: x^2 + y^2 + 2gx + 2fy + c = 0$$

$$A(0, 0) \in s: (0)^2 + (0)^2 + 2g(0) + 2f(0) + c = 0 \Rightarrow c = 0$$

$$B(6.5, 0) \in s: (6.5)^2 + (0)^2 + 2g(6.5) + 2f(0) + (0) = 0$$

$$(6.5)^2 + 2g(6.5) = 0$$

$$(6.5) + 2g = 0$$

$$g = -\frac{13}{4}$$

$$C(10, 7) \in s: (10)^2 + (7)^2 + 2(-\frac{13}{4})(10) + 2f(7) + (0) = 0$$

$$100 + 49 - 65 + 14f = 0$$

$$14f = -84 \Rightarrow f = -6$$

$$s: x^2 + y^2 + 2(-\frac{13}{4})x + 2(-6)y + (0) = 0$$

$$x^2 + y^2 - \frac{13}{2}x - 12y = 0$$

$$2x^2 + 2y^2 - 13x - 24y = 0$$

**METHOD 2 [PERPENDICULAR BISECTORS]**

$$l_1: x = \frac{13}{4}$$

$$\text{Midpoint of } AC = E(5, \frac{7}{2}), \text{ Slope of } AC = \frac{7}{10}$$

$$l_2: (x_1, y_1) = E(5, \frac{7}{2}), m = -\frac{10}{7}$$

$$y - \frac{7}{2} = -\frac{10}{7}(x - 5)$$

$$14y - 49 = -20(x - 5) \Rightarrow 14y - 49 = -20x + 100$$

$$20x + 14y - 149 = 0$$

$$\text{Centre } O = l_1 \cap l_2: 20(\frac{13}{4}) + 14y - 149 = 0 \Rightarrow y = 6$$

$$\therefore O(\frac{13}{4}, 6)$$

$$r = |OA| = \sqrt{(\frac{13}{4} - 0)^2 + (6 - 0)^2} = \frac{\sqrt{745}}{4}$$

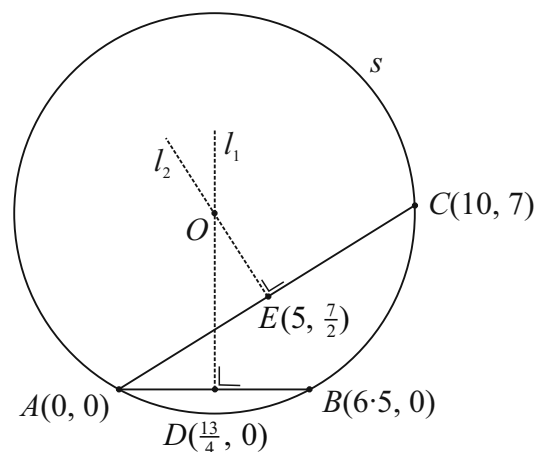
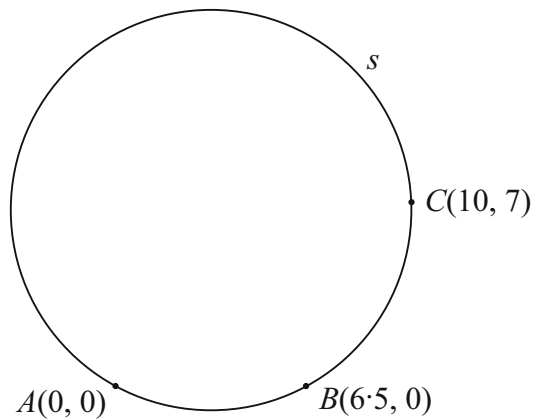
$$\text{Circle } s: \left(x - \frac{13}{4}\right)^2 + (y - 6)^2 = \frac{745}{16}$$

Equation of a circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre =  $(-g, -f)$

$$r = \sqrt{g^2 + f^2 - c}$$



Equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Centre =  $(h, k)$ , radius  $r$

**METHOD 3 [CIRCUMCIRCLE]**

$(-g, -f) \in$  mediator  $(0, 0)$  and  $(6.5, 0)$

$\therefore -g = 3.25$

Centre  $(3.25, -f)$

Since  $(0, 0) \in$  of circle:  $c = 0$

Equation of circle  $s: x^2 + y^2 - 6.5x + 2fy + 0 = 0$

$(10, 7)$  on circle:  $100 + 49 - 65 + 14f = 0$

$84 + 14f = 0$

$\therefore f = -6$

$s: x^2 + y^2 - 6.5x - 12y + 0 = 0$

$s: 2x^2 + 2y^2 - 13x - 24y = 0$

**MARKING SCHEME NOTES**

**Question 4 (a) [Scale 10D (0, 3, 5, 8, 10)]**

**Method 1**

3: •  $c = 0$

• One relevant equation in  $g$  and/or  $f$

5: • 2 of  $g, f, c$  found

8: •  $g, f,$  and  $c$  found or equivalent

**Method 2**

3: • Effort at formulating equation of 1 perpendicular bisector

5: • Point of intersection of 2 perpendicular bisectors found

8: • Point of intersection of 2 perpendicular bisectors and radius

**Method 3**

3: •  $c = 0$

• One point substituted into equation of circle

• Midpoint  $(0,0)$  and  $(6.5, 0)$  formulated

5: • 2 of  $g, f, c$  found

8: •  $g, f,$  and  $c$  found or equivalent

**Question 4 (b)**

**METHOD 1 [tan  $\theta$  formula]**

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$A(0, 0), B(6.5, 0), C(10, 7)$

Slope of  $BC: m_1 = \frac{7-0}{10-6.5} = \frac{7}{3.5} = 2$

Slope of  $AC: m_2 = \frac{7-0}{10-0} = \frac{7}{10}$

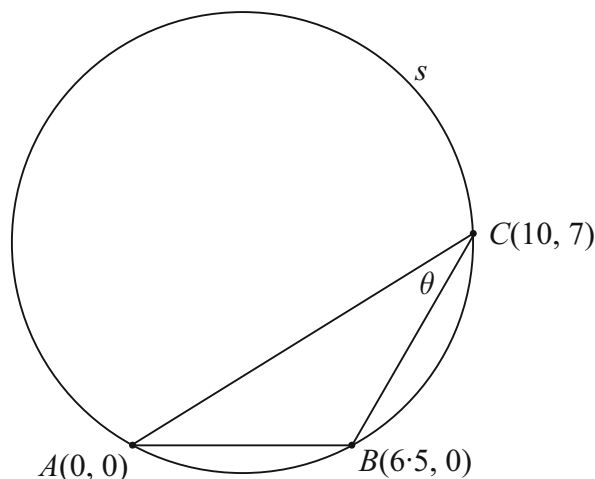
$\tan \theta = \left| \frac{2 - \frac{7}{10}}{1 + 2(\frac{7}{10})} \right| \Rightarrow \theta = 28.44^\circ$

**METHOD 2 [Cosine rule]**

$|AB|^2 = 42.25, |AC|^2 = 149, |BC|^2 = 61.25$

$\cos \theta = \frac{149 + 61.25 - 42.25}{2 \times \sqrt{149} \times \sqrt{61.25}}$

$\therefore \theta = \cos^{-1} \left( \frac{149 + 61.25 - 42.25}{2 \times \sqrt{149} \times \sqrt{61.25}} \right) = 28.44^\circ$



**MARKING SCHEME NOTES**

**Question 3 (b) [Scale 15C (0, 6, 9, 15)]**

**Method 1**

6: • one relevant slope

9: •  $\tan \theta$  fully substituted

**Method 2**

6: • one relevant length

9: •  $\cos \theta$  fully substituted

**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

Consider  $\triangle AFE$ :

$$|\angle AEF| = 90^\circ$$

$$\text{Let } |\angle FAE| = \theta$$

$$\therefore |\angle EFA| = 90^\circ - \theta$$

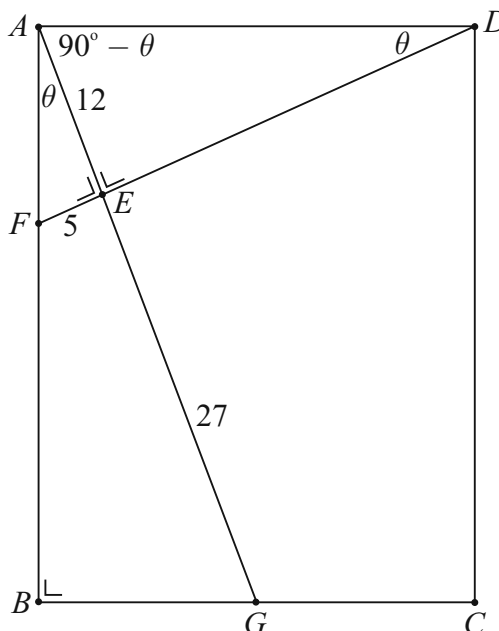
Consider  $\triangle DAE$ :

$$|\angle AED| = 90^\circ$$

$$|\angle EAD| = 90^\circ - \theta$$

$$\therefore |\angle ADE| = \theta$$

$\therefore \triangle AFE$  and  $\triangle DAE$  are similar.



**MARKING SCHEME NOTES**

**Question 5 (a) [Scale 10C (0, 4, 5, 10)]**

4: • Identifies one angle of same size in each triangle

5: • Identifies second angle of same size in each triangle

• Implies triangles are similar without justifying  $|\angle FAE| = |\angle ADE|$

**Question 5 (b)**

$$\triangle AFE \text{ and } \triangle DAE \text{ are similar: } \frac{|AD|}{|AF|} = \frac{|AE|}{|FE|}$$

$$|AF|^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$|AF| = \sqrt{169} = 13 \text{ cm}$$

$$\therefore \frac{|AD|}{13} = \frac{12}{5} \Rightarrow |AD| = \frac{12 \times 13}{5} = 31.2 \text{ cm}$$

**Question 5 (c)**

Consider  $\triangle AFE$ :

$$|\angle AEF| = 90^\circ$$

$$\text{Let } |\angle FAE| = \theta$$

$$\therefore |\angle EFA| = 90^\circ - \theta$$

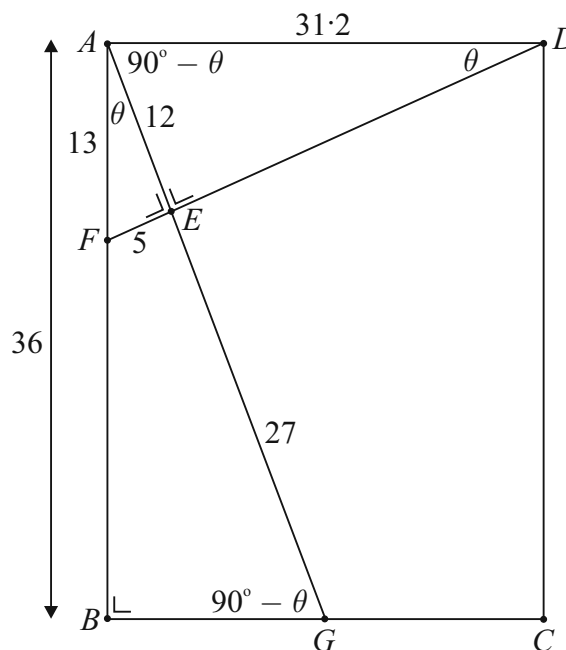
Consider  $\triangle AGB$ :

$$|\angle ABG| = 90^\circ$$

$$|\angle BAG| = \theta \text{ [Common angle]}$$

$$\therefore |\angle AGB| = 90^\circ - \theta$$

$\therefore \triangle AFE$  and  $\triangle AGB$  are similar.



$$\frac{|AB|}{|AE|} = \frac{|AG|}{|AF|} \Rightarrow \frac{|AB|}{12} = \frac{12 + 27}{13}$$

$$\therefore |AB| = \frac{12 \times 39}{13} = 36 \text{ cm}$$

**MARKING SCHEME NOTES**

**Question 5 (b) [Scale 5C (0, 2, 4, 5)]**

2: •  $|AF| = 13$

• One set of corresponding sides identified, e.g.  $\frac{|AD|}{13}$  or  $\frac{12}{5}$

4: •  $\frac{|AD|}{13} = \frac{12}{5}$  or equivalent

**Question 5 (c) [Scale 5C (0, 2, 4, 5)]**

2: •  $|AG| = 39$

• One set of corresponding sides identified

4: •  $\frac{39}{13} = \frac{|AD|}{12}$  or equivalent

**Question 5 (d)**

Area of  $GCDE = \text{Area } ABCD - \text{Area } \Delta AGB - \text{Area } \Delta DAE$

$$= |AD| \times |AB| - \frac{1}{2} \times |BG| \times |AB| - \frac{1}{2} \times |AE| \times |DE|$$

$$= 31 \cdot 2 \times 36 - \frac{1}{2} \times \sqrt{39^2 - 36^2} \times 36 - \frac{1}{2} \times 12 \times \sqrt{31 \cdot 2^2 - 12^2}$$

$$= 680 \cdot 4 \text{ cm}^2$$

**MARKING SCHEME NOTES**

**Question 5 (d) [Scale 5C (0, 2, 4, 5)]**

2: • One relevant area formulated

• Relevant equation for area  $GCDE$

4: • Relevant individual areas found but fails to finish

• Area calculated but with one relevant area omitted

**QUESTION 6 (25 MARKS)**

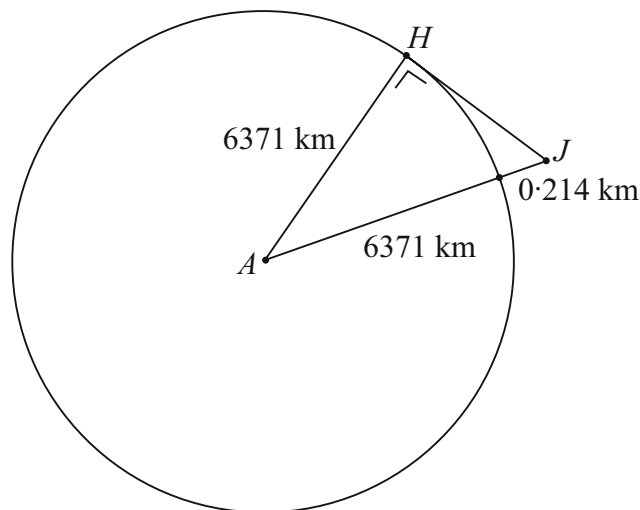
**Question 6 (a)**

$JH$  is a tangent to the circle. Therefore,  $HJ$  is perpendicular to  $AH$ .

$$|AJ|^2 = |AH|^2 + |JH|^2$$

$$(6371 + 0 \cdot 214)^2 = 6371^2 + |JH|^2$$

$$|JH| = \sqrt{6317 \cdot 214^2 - 6317^2} \approx 52 \text{ km}$$



**MARKING SCHEME NOTES**

**Question 6 (a) [Scale 15C (0, 6, 9, 15)]**

6: •  $|AJ|$  formulated

• indication of Pythagoras

9: • Pythagoras fully substituted

**Question 6 (b)**

Consider  $\triangle ABC$ :  $\sin 37^\circ = \frac{|BC|}{6371}$

$|BC| = 6371 \sin 37^\circ = 3834.2 \text{ km}$

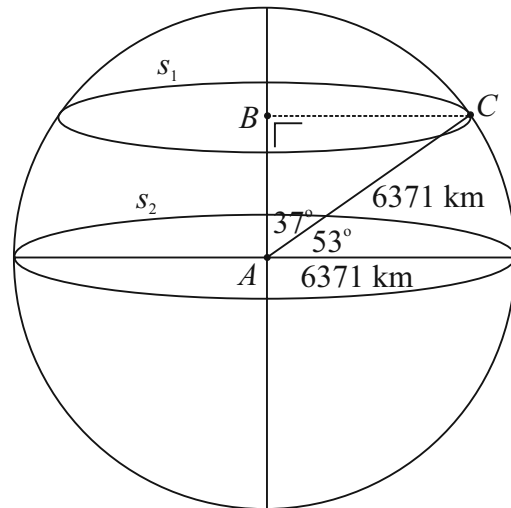
Length of  $s_1 = 2\pi(3834.2) \approx 24\,091 \text{ km}$

**MARKING SCHEME NOTES**

**Question 6 (b) [Scale 10C (0, 4, 5, 10)]**

4: •  $\cos 53^\circ$  or  $\sin 37^\circ$

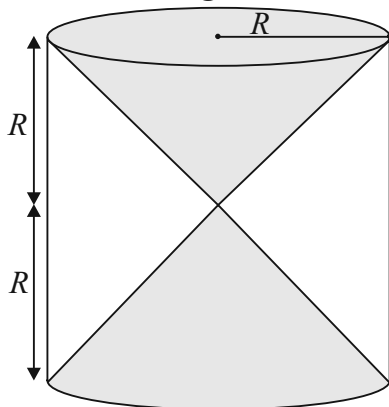
5: • radius of  $s_1$  calculated and stops  
• length of circle formula fully substituted



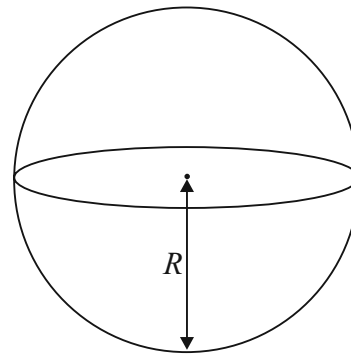
**QUESTION 7 (40 MARKS)**

**Question 7 (a)**

**Figure 1**



**Figure 2**



Volume of empty space in cylinder  
 = Volume of cylinder  $- 2(\text{Volume of cone})$   
 $= \pi R^2(2R) - 2\left(\frac{1}{3}\pi R^2 \times R\right)$   
 $= 2\pi R^3 - \frac{2}{3}\pi R^3 = \frac{4}{3}\pi R^3$   
 = Volume of sphere

Volume of cylinder  $= \pi r^2 h$   
 Volume of cone  $= \frac{1}{3}\pi r^2 h$   
 Volume of sphere  $= \frac{4}{3}\pi r^3$

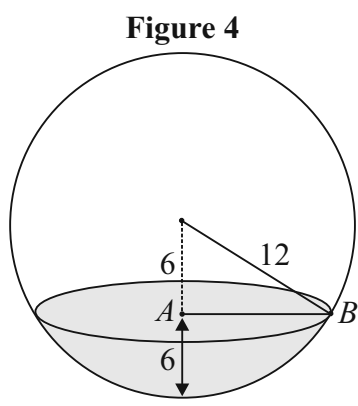
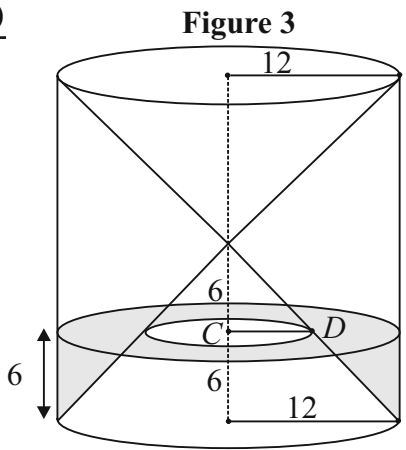
**MARKING SCHEME NOTES**

**Question 7 (a) [Scale 10C (0, 4, 5, 10)]**

4: • A relevant volume formulated

5: • Vol of space formulated in terms of  $\pi$  and  $R$

**Question 7 (b)**



(i)  $12^2 = 6^2 + |AB|^2$   
 $|AB| = \sqrt{12^2 - 6^2} = 6\sqrt{3}$  cm

(ii) There are two similar triangles in the cone.  
 $\therefore \frac{|CD|}{12} = \frac{6}{12} \Rightarrow |CD| = 6$  cm

(iii) Sphere: Area of surface of water  
 $= \pi(6\sqrt{3})^2 = 108\pi$  cm<sup>2</sup>  
 Cylinder: Area of surface of water  
 $= \pi(12)^2 - \pi(6)^2 = 108\pi$  cm<sup>2</sup>

**MARKING SCHEME NOTES**

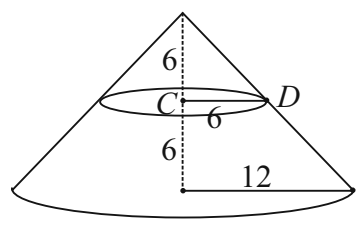
**Question 7 (b) (i) [Scale 10B (0, 5, 10)]**  
 5: • indication of Pythagoras

**Question 7 (b) (ii) [Scale 10C (0, 4, 5, 10)]**  
 4: • indication of similar triangles  
 • indication of a relevant ratio  
 5: • corresponding ratios identified but fails to finish  
**Note:** Accept correct answer without work

**Question 7 (b) (iii) [Scale 5C (0, 2, 4, 5)]**  
 2: • Surface Area in Fig. 3 substituted  
 • Surface Area in Fig. 4 substituted  
 4: • One Surface Area found

**Question 7 (c)**

Volume of cylinder under water  $= \frac{1}{3}\pi(12)^2(12) - \frac{1}{3}\pi(6)^2(6) = 504\pi$  cm<sup>3</sup>  
 Volume of water in cylinder  $= \pi(12)^2(6) - 504\pi = 360\pi$  cm<sup>3</sup>  
 $=$  Volume of water in sphere



**MARKING SCHEME NOTES**

**Question 7 (c) [Scale 5C (0, 2, 4, 5)]**  
 2: • Vol of cylinder found  
 • Vol of truncated cone substituted  
 • Vol of one cone found (12 or 6)  
 4: • Volume fully substituted but fails to finish  
 • Volume of truncated cone found

**QUESTION 8 (60 MARKS)**

**Question 8 (a)**

**FORMULAE AND TABLES BOOK**  
**Statistics and Probability: Probability distribution** (standardising formula) [page 34]

$$z = \frac{x - \mu}{\sigma}$$

$n$  = Number in the sample

$\sigma$  = standard deviation of the sample

Population:  $\mu = 63.5$  kg,  $\sigma = 10$  kg

(i) Mariska:  $X = 50$  kg

$$Z = \frac{x - \mu}{\sigma} = \frac{50 - 63.5}{10} = -1.35$$

$$P(x > 50) = P(z > -1.35) = P(z \leq 1.35) \\ = 0.9115 = 91.15\%$$

(ii) Kamal:  $X = ?$

$$P(Z > z) = 0.015$$

$$1 - P(Z \leq z) = 0.015$$

$$P(Z \leq z) = 1 - 0.015 = 0.985 \Rightarrow Z = 2.17$$

$$Z = \frac{X - \mu}{\sigma} \Rightarrow 2.17 = \frac{X - 63.5}{10}$$

$$21.7 + 63.5 = X$$

$$X = 85.2 \text{ kg}$$

(iii) Null hypothesis  $H_0: \mu = 63.5$  kg

[Weight has **not** changed from 2015 to 2016]

Alternative hypothesis  $H_1: \mu \neq 63.5$  kg

[Weight has changed from 2015 to 2016]

Sample:  $n = 150$ ,  $\bar{x} = 62$  kg,

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{150}}$$

$$z \text{ statistic: } \bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}} = \frac{62 - 63.5}{\frac{10}{\sqrt{150}}} = -1.84$$

$$\bar{z} = -1.84 > -1.96 \text{ [Do not reject } H_0]$$

**or**

$$p = 2\{1 - P(z < |-1.84|)\}$$

$$= 2\{1 - P(z < 1.84)\}$$

$$= 2\{1 - (0.9671)\} = 0.0658$$

$$p = 0.0658 > 0.05 \text{ [Do not reject } H_0]$$

**or**

Confidence interval:  $\bar{x} - 1.96\bar{\sigma} \leftrightarrow \bar{x} + 1.96\bar{\sigma}$

$$62 - 1.96 \times \frac{10}{\sqrt{150}} \leftrightarrow 62 + 1.96 \times \frac{10}{\sqrt{150}}$$

Confidence interval:  $60.3997 \leftrightarrow 63.6003$

$63.5$  falls within this interval

[Do not reject  $H_0$ ]

**MARKING SCHEME NOTES**

**Question 8 (a) (i) [Scale 10D (0, 3, 5, 8, 10)]**

3: •  $\mu$  or  $\sigma$  identified

5: •  $z$  found

8: •  $P(z < 1.35)$  and stops

**Question 8 (a) (ii) [Scale 5D (0, 2, 3, 4, 5)]**

2: • identifies 0.985

3: • identifies 2.17

4: • formula for  $x$  fully substituted

**Question 8 (a) (iii) [Scale 15D (0, 5, 7, 9, 15)]**

5: •  $z$  formulated with some substitution

• states null/alternative hypothesis only

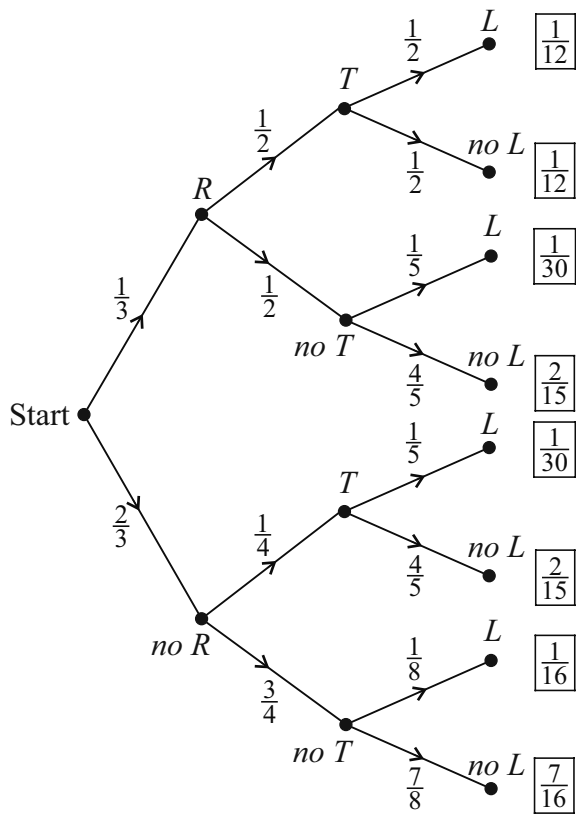
• reference to  $\pm 1.96$

7: •  $z$  fully substituted

9: •  $z = -1.8371 > -1.96$

• fails to contextualise the answer

**Question 8 (b) (i)**



**Question 8 (b) (ii)**

$$P(L) = \frac{1}{12} + \frac{1}{30} + \frac{1}{30} + \frac{1}{16} = \frac{17}{80}$$

**Question 8 (b) (iii)**

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(R|L) = \frac{P(R \cap L)}{P(L)} = \frac{P(R \text{ and then } L)}{P(L)}$$

$$= \frac{\frac{1}{12} + \frac{1}{30}}{\frac{17}{80}} = \frac{28}{51}$$

**MARKING SCHEME NOTES**

**Question 8 (b) (i) [Scale 15C (0, 6, 9, 15)]**

- 6: • 3 boxes filled correctly
- 9: • 10 boxes filled correctly

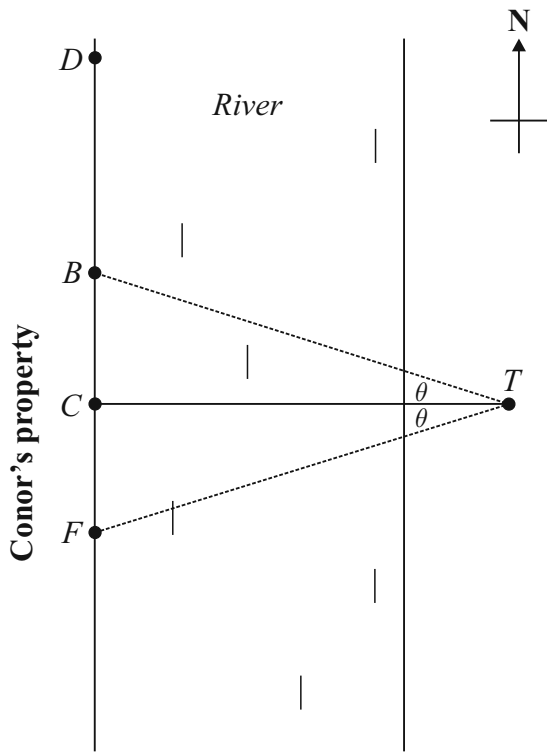
**Question 8 (b) (ii) [Scale 10C (0, 4, 5, 10)]**

- 4: • 2 relevant fractions transferred
- 5: • 4 relevant fractions identified but fails to complete

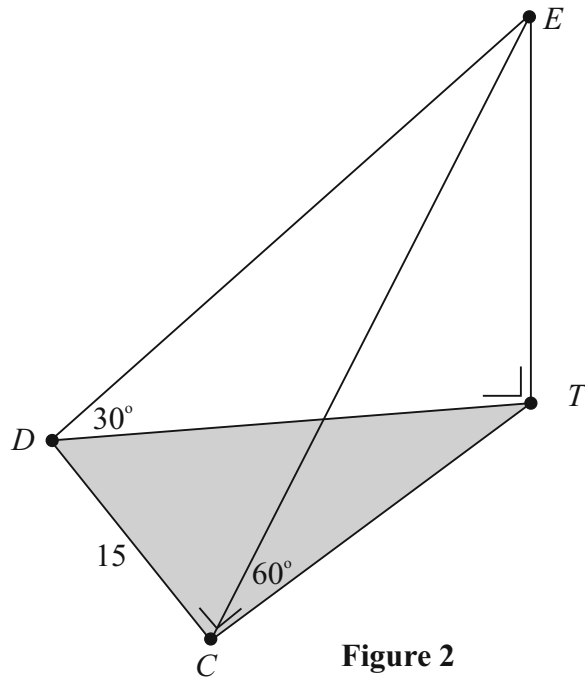
**Question 8 (b) (iii) [Scale 5C (0, 2, 4, 5)]**

- 2: •  $P(L)$
- $P(R \cap L)$
- 4: • Formula fully substituted

**QUESTION 9 (50 MARKS)**



**Figure 1**



**Figure 2**

**Question 9 (a)**

Consider  $\triangle CTE$ :

$$\tan 60^\circ = \frac{|ET|}{|CT|} \Rightarrow |ET| = |CT| \tan 60^\circ = \sqrt{3}|CT|$$

**Question 9 (b)**

Consider  $\triangle DTE$ :

$$\tan 30^\circ = \frac{|TE|}{|DT|} \Rightarrow \frac{1}{\sqrt{3}} = \frac{|TE|}{|DT|} \Rightarrow |DT| = \sqrt{3}|TE|$$

Consider the right-angled  $\triangle DCT$ :

$$15^2 + |CT|^2 = |DT|^2$$

$$225 + |CT|^2 = (\sqrt{3}|TE|)^2$$

$$225 + |CT|^2 = 3|TE|^2$$

$$|TE|^2 = \frac{225 + |CT|^2}{3} \Rightarrow |TE| = \sqrt{\frac{225 + |CT|^2}{3}} \text{ m}$$

**MARKING SCHEME NOTES**

**Question 9 (a) [Scale 10B (0, 5, 10)]**

- 5: •  $\tan 60^\circ$   
 • effort to express  $|TE|$  in terms of another side of the triangle

**Question 9 (b) [Scale 5C (0, 2, 4, 5)]**

- 2: •  $\tan 30^\circ$   
 • Use of Pythagoras for  $|DT|$   
 • Effort at expressing  $|DT|$  in terms of another side of  $\triangle DET$
- 4: •  $|TE| = |DT| \frac{1}{\sqrt{3}}$

**Question 9 (c)**

$$|TE| = \frac{\sqrt{225 + |CT|^2}}{3} = \sqrt{3}|CT|$$

$$\frac{225 + |CT|^2}{3} = 3|CT|^2$$

$$225 + |CT|^2 = 9|CT|^2 \Rightarrow 225 = 8|CT|^2$$

$$|CT| = \sqrt{\frac{225}{8}} = 5.3 \text{ m}$$

**Question 9 (d)**

$$|TE| = \sqrt{3}|CT| = \sqrt{3} \times 5.3 = 9.2 \text{ m}$$

**Question 9 (e)**

$$\cos|\angle FTC| = \cos\theta = \frac{|CT|}{|TF|} = \frac{5.3}{9.2}$$

$$\theta = \cos^{-1}\left(\frac{5.3}{9.2}\right) = 54.8^\circ$$

**Question 9 (f)**

$$P(\text{Tree lands on bank}) = \frac{2 \times 54.8^\circ}{360^\circ} \times 100\% = 30.4\%$$

**MARKING SCHEME NOTES**

**Question 9 (c) [Scale 10C (0, 4, 5, 10)]**

- 4: • equates both expressions  
5: • Isolate  $|CT|$  in equation

**Question 9 (d) [Scale 10B (0, 5, 10)]**

- 5: • Substitution into formula for  $|TE|$

**Question 9 (e) [Scale 5C (0, 2, 4, 5)]**

- 2: • Some relevant substitution for  $\cos\theta$   
4: • Formula for  $\cos\theta$  substituted in terms of  $|CT|$

**Question 9 (f) [Scale 10C (0, 4, 5, 10)]**

- 4: • (Answer to **part (e)**) $\times 2$   
•  $360^\circ$   
5: •  $P$  fully formulated

# LC 2016 (SET C): PAPER 1

## QUESTION 1 (25 MARKS)

### Question 1 (a)

Use the conjugate root theorem to find the other root. Therefore, if  $z_1$  is one of the roots, the conjugate of  $z_1$  is also a root. Call this root  $z_2$ .

$$z_1 = -4 + 3i \Rightarrow z_2 = -4 - 3i$$

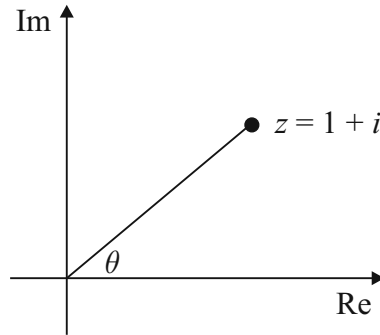
### Question 1 (b)

$$z = 1 + i$$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|\tan \theta| = \left| \frac{1}{1} \right| = 1 \Rightarrow \theta = \tan^{-1} 1 = \frac{\pi}{4}$$

$$\therefore \theta = \frac{\pi}{4} \text{ [1st quadrant]}$$



$$z = 1 + i = \sqrt{2} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$\begin{aligned} z^8 &= (\sqrt{2})^8 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^8 \\ &= 2^4 (\cos 2\pi + i \sin 2\pi) \\ &= 16(1 + 0i) = 16 \end{aligned}$$

#### FORMULAE: Complex Numbers

Modulus  $|z|$ :  $|z| = |a + bi| = \sqrt{a^2 + b^2}$

$z = a + bi$  (Cartesian form)

$z = r(\cos \theta + i \sin \theta)$  (Polar form)

#### DE MOIVRE'S THEOREM (page 20)

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 5B (0, 2, 5)]

2: • real or imaginary part correct

##### Question 1 (b) [Scale 10C (0, 3, 7, 10)]

3: • correct answer without use of De Moivre's

• modulus or argument correct

• formula

• statement of de Moivre's

7: •  $16(\cos 2\pi + i \sin 2\pi)$

NOTE: not De Moivre and incorrect answer merits 0 marks

**Question 1 (c)****METHOD 1**

Forming a quadratic equation from its roots.

$$z^2 - Sz + P = 0$$

**S:** Sum of the roots**P:** Product of the roots

$$z^2 + (-2+i)z + 3-i = 0$$

Roots:  $1+i$ ,  $\alpha$ 

$$\text{Sum} = \alpha + 1 + i = -(-2+i)$$

$$\alpha + 1 + i = 2 - i$$

$$\therefore \alpha = 1 - 2i$$

**METHOD 2**

$$z^2 + (-2+i)z + 3-i = 0$$

$$a = 1, b = (-2+i), c = (3-i)$$

$$z = \frac{-(-2+i) \pm \sqrt{(-2+i)^2 - 4(1)(3-i)}}{2}$$

$$= \frac{2-i \pm \sqrt{4-4i+i^2-12+4i}}{2}$$

$$= \frac{2-i \pm \sqrt{4-4i-1-12+4i}}{2} = \frac{2-i \pm \sqrt{-9}}{2}$$

$$= \frac{2-i \pm 3i}{2} = \frac{2+2i}{2}, \frac{2-4i}{2}$$

$$= 1+i, 1-2i$$

Other root =  $(1-2i)$ **MARKING SCHEME NOTES****Question 1 (c) [Scale 10C (0, 3, 7, 10)]****3:** • root formula with some substitution**7:** • formula fully substituted*Or***3:** • equation rearranged

•  $-\frac{b}{a}$

**7:** • correct substitution

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**

**Step 1:**  $|x - 4| \geq 2$  ...Test box

**Step 2:**  $|x - 4| = 2$   
 $x - 4 = \pm 2 \Rightarrow x = 2, 6$

**Step 3:** Region test

←	2	↔	6	→
$ x - 4  \geq 2$	$ x - 4  \geq 2$	$ x - 4  \geq 2$	$ x - 4  \geq 2$	$ x - 4  \geq 2$
$0:  0 - 4  \geq 2$	$4:  4 - 4  \geq 2$	$4:  4 - 4  \geq 2$	$8:  8 - 4  \geq 4$	$8:  8 - 4  \geq 4$
$4 \geq 2$ [True]	$0 \geq 2$ [False]	$0 \geq 2$ [False]	$4 \geq 2$ [True]	$4 \geq 2$ [True]
↓		↓		↓
$x \leq 2$		$x \geq 6$		$x \geq 6$

**Step 4:**  $x \leq 2, x \geq 6, x \in \mathbb{R}$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 10C (0, 3, 7, 10)]**

- 3:**
- either side squared
  - one correct linear inequality written
  - stating range of natural numbers only

**7:** • correct solutions to quadratic

**10:** • correct answer without work

**NOTE:** use of natural numbers in range merits High Partial Credit at most

**Question 2 (b)**

$$2x + 3y = -1 \Rightarrow x = \frac{-1 - 3y}{2} \dots(1)$$

$$x^2 + xy + 2y^2 = 4 \dots(2)$$

$$\left(\frac{-1 - 3y}{2}\right)^2 + \left(\frac{-1 - 3y}{2}\right)y + 2y^2 = 4$$

$$\frac{1 + 6y + 9y^2}{4} + \frac{-y - 3y^2}{2} + 2y^2 = 4 \text{ [}\times 4\text{]}$$

$$1 + 6y + 9y^2 - 2y - 6y^2 + 8y^2 = 16$$

$$11y^2 + 4y - 15 = 0$$

$$(11y + 15)(y - 1) = 0$$

$$\therefore y = -\frac{15}{11}, 1$$

$$y = 1: x = \frac{-1 - 3(1)}{2} = \frac{-4}{2} = -2$$

$$y = -\frac{15}{11}: x = \frac{-1 - 3(-\frac{15}{11})}{2} = \frac{17}{11}$$

**Answer:**  $(-2, 1), (\frac{17}{11}, -\frac{15}{11})$

**MARKING SCHEME NOTES**

**Question 2 (b) [Scale 15C (0, 5, 10, 15)]**

**5:** • effort to isolate  $x$  (or  $y$ )

**10:** • fully correct substitution into quadratic

**QUESTION 3 (25 MARKS)**

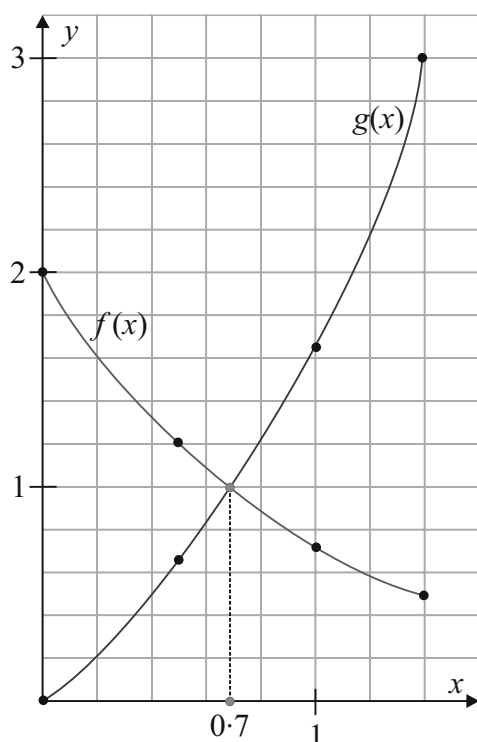
**Question 3 (a) (i)**

$$f(0) = \frac{2}{e^0} = \frac{2}{1} = 2, f(0.5) = \frac{2}{e^{0.5}} = 1.21, f(1) = \frac{2}{e^1} = 0.74, f(\ln 4) = \frac{2}{e^{\ln 4}} = \frac{2}{4} = 0.5$$

$$g(0) = e^0 - 1 = 1 - 1 = 0, g(0.5) = e^{0.5} - 1 = 0.65, g(1) = e^1 - 1 = 1.72, g(\ln 4) = e^{\ln 4} - 1 = 4 - 1 = 3$$

$x$	0	0.5	1	$\ln(4) \approx 1.4$
$f(x) = \frac{2}{e^x}$	2	1.21	0.74	0.5
$g(x) = e^x - 1$	0	0.65	1.72	3

**Question 3 (a) (ii)**



**Question 3 (b)**

$$f(x) = g(x) \Rightarrow \frac{2}{e^x} = e^x - 1 \text{ [} \times e^x \text{]}$$

$$2 = (e^x)^2 - e^x$$

$$(e^x)^2 - e^x - 2 = 0$$

$$(e^x + 1)(e^x - 2) = 0$$

$$e^x = -1 \text{ [No solutions]}$$

$$e^x = 2 \Rightarrow \ln(e^x) = \ln 2$$

$$x \ln e = \ln 2$$

$$\therefore x = \ln 2 = 0.693$$

**Question 3 (a) (iii)**

$$f(x) = g(x) \Rightarrow x \approx 0.7$$

**MARKING SCHEME NOTES**

**Question 3 (a) (i) [Scale 5C (0, 2, 4, 5)]**

2: • one entry correct

4: • 5 entries correct

**Question 3 (a) (ii) [Scale 5C (0, 2, 4, 5)]**

2: • one plot correct

4: • 5 plots correct

• one correct graph

• no labelling

**NOTES:**

- straight lines NOT acceptable

- one clear label merits full credit

- one ambiguous label merits High Partial Credit at most

**Question 3 (a) (iii) [Scale 5B (0, 2, 5)]**

2: • point of intersection clearly indicated on graph, but value of  $x$  not stated

**MARKING SCHEME NOTES**

**Question 3 (b) [Scale 10C (0, 3, 7, 10)]**

3: • substitution correct

7: • correct factors of quadratic

• root formula correctly substituted

$$e^x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$$

**NOTE:** oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most

**QUESTION 4 (25 MARKS)****Question 4 (a)****Step 1:**  $n = 1$ : Prove  $8^1 - 1$  is divisible by 7.**Proof:**  $8^1 - 1 = 7$ . Therefore, it is true for  $n = 1$ .**Step 2:**  $n = k$ : Assume  $8^k - 1$  is divisible by 7.In other words, assume  $8^k - 1 = 7m$ ,  $m \in \mathbb{N}$ .

$$\therefore 8^k = (7m + 1)$$

**Step 3:**  $n = (k + 1)$ : Prove  $8^{k+1} - 1$  is divisible by 7.

$$\begin{aligned} \text{Proof: } 8^{k+1} - 1 &= 8 \times 8^k - 1 \\ &= 8(7m + 1) - 1 \text{ [By Step 2.]} \\ &= 56m + 8 - 1 \\ &= 56m + 7 \\ &= 7(8m + 1) \\ &= 7 \times (\text{whole number}) \\ \therefore 8^{k+1} - 1 &\text{ is divisible by 7.} \end{aligned}$$

Therefore, assuming it is true for  $n = k$  means it is true for  $n = k + 1$ . So true for  $n = 1$  and true for  $n = k$  means it is true for  $n = k + 1$  implies it is true for all  $n \in \mathbb{N}$ .

**MARKING SCHEME NOTES****Question 4 (a) [Scale 15D (0, 4, 7, 11, 15)]****4:** •  $P_1$  step**7:** •  $P_k$  step•  $P_{k+1}$  step**11:** • use of  $P_k$  step to prove  $P_{k+1}$  step**NOTE:** accept  $P_1$  step,  $P_k$  step and  $P_{k+1}$  step in any order**Question 4 (b)**

$$\log_a 2 = p, \log_a 3 = q, a > 0$$

**(i)**  $\log_a \frac{8}{3} = \log_a 8 - \log_a 3$

$$= \log_a 2^3 - \log_a 3$$

$$= 3 \log_a 2 - \log_a 3$$

$$= 3p - q$$

**(ii)**  $\log_a \frac{9a^2}{16} = \log_a 9 + \log_a a^2 - \log_a 16$

$$= \log_a 3^2 + \log_a a^2 - \log_a 2^4$$

$$= 2 \log_a 3 + 2 \log_a a - 4 \log_a 2$$

$$= 2q - 4p + 2$$

**FORMULAE AND TABLES BOOK****Indices and logs [page 21]**

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a \left( \frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a (x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a \left( \frac{1}{x} \right) = -\log_a x$$

**MARKING SCHEME NOTES****Question 4 (b) (i) [Scale 5C (0, 2, 4, 5)]****2:** •  $\log_a 8 - \log_a 3$ **4:** •  $\log_a 8 = 3 \log_a 2$  (and/or  $= 3p$ )**Question 4 (b) (ii) [Scale 5D (0, 2, 3, 4, 5)]****2:** •  $\log_a 9a^2 - \log_a 16$ **3:** •  $2 \log_a 3$ •  $2 \log_a a$ •  $4 \log_a 2$ •  $4p$  or  $2q$  or  $2$ **4:** •  $2(\log_a 3 + \log_a a) - 4 \log_a 2$ 

or equivalent

**QUESTION 5 (25 MARKS)**

**Question 5 (a) (i)**

$$(5x - 9)^2 = (x - 1)^2 + (4x)^2$$

$$25x^2 - 90x + 81 = x^2 - 2x + 1 + 16x^2$$

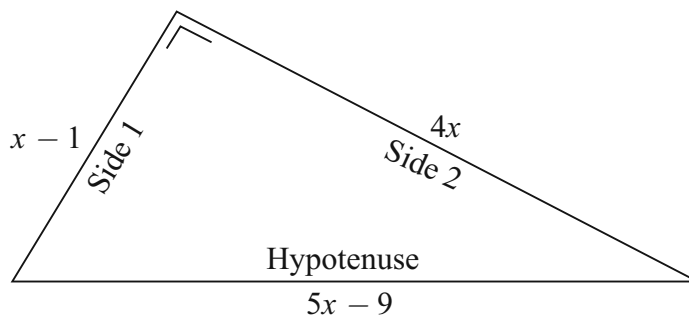
$$8x^2 - 88x + 80 = 0$$

$$x^2 - 11x + 10 = 0$$

$$(x - 1)(x - 10) = 0$$

$$x = 1, 10$$

$$\therefore x = 10$$



**Question 5 (a) (ii)**

$$x = 10$$

Hypotenuse:  $5x - 9 = 5(10) - 9 = 41$

Side 1:  $x - 1 = (10) - 1 = 9$

Side 2:  $4x = 4(10) = 40$

$$(\text{Side 1})^2 + (\text{Side 2})^2 = 9^2 + 40^2 = 1681$$

$$(\text{Hypotenuse})^2 = 41^2 = 1681$$

$$\therefore (\text{Side 1})^2 + (\text{Side 2})^2 = (\text{Hypotenuse})^2$$

**MARKING SCHEME NOTES**

**Question 5 (a) (i) [Scale 10D (0, 2, 5, 8, 10)]**

- 2:** • any use of Pythagoras
- 5:** • fully correct substitution
- 8:** • both roots correct

**Question 5 (a) (ii) [Scale 5B (0, 2, 5)]**

- 2:** • 9 or 40 or 41
- using 1 or -10 from candidates work

Reject  $x = 1$  as it will give one of the sides a length of zero.

**Question 5 (b) (i)**

An **injective** function from a set  $A$  to a set  $B$  is a function in which **every** value of  $x$  in  $A$  has a **different** value of  $y$  in  $B$ .

**METHOD 1**

If  $f(x)$  is an injective function from  $A$  to  $B$ , it follows that  $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ .

$$f(x_1) = 3x_1 - 2$$

$$f(x_2) = 3x_2 - 2$$

$$f(x_1) = f(x_2) \Rightarrow 3x_1 - 2 = 3x_2 - 2$$

$$3x_1 = 3x_2$$

$$\therefore x_1 = x_2 \Rightarrow f(x) \text{ is an injective function, } x \in \mathbb{R}.$$

**Question 5 (b) (ii)**

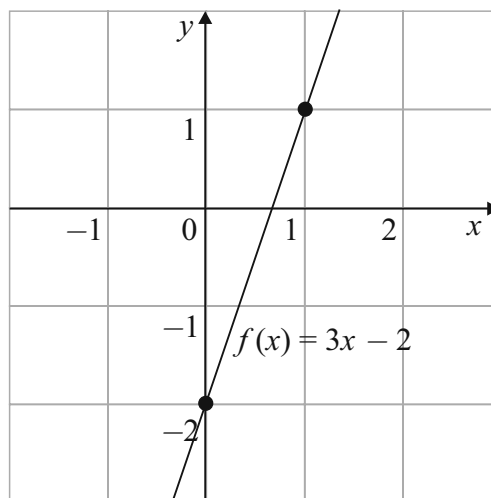
$$y = 3x - 2$$

$$3x = y + 2$$

$$x = \frac{y + 2}{3}$$

$$\therefore f^{-1}(x) = \frac{x + 2}{3}$$

**METHOD 2**



A graph of the function is drawn. Horizontal lines in the codomain cut the graph **at most once**. Therefore, the function is injective.

**MARKING SCHEME NOTES****Question 5 (b) (i) [Scale 5B (0, 2, 5)]**

- 2: •  $f^{-1}(x)$  written  
 •  $f(x)$  drawn  
 •  $f(a) = f(b)$

**Question 5 (b) (ii) [Scale 5B (0, 2, 5)]**

- 2: • any relevant transpose

**QUESTION 6 (25 MARKS)****Question 6 (a)**

$$f(x) = (2x + 4)^2 = 4x^2 + 16x + 16$$

$$\begin{aligned} f(x+h) &= 4(x+h)^2 + 16(x+h) + 16 \\ &= 4(x^2 + 2hx + h^2) + 16x + 16h + 16 \\ &= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16 \end{aligned}$$

$$\begin{aligned} f(x+h) - f(x) &= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16 - (4x^2 + 16x + 16) \\ &= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16 - 4x^2 - 16x - 16 \\ &= 8hx + 4h^2 + 16h \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{8hx + 4h^2 + 16h}{h} = 8x + 4h + 16$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right) = \lim_{h \rightarrow 0} (8x + 4h + 16) = 8x + 16$$

**MARKING SCHEME NOTES****Question 6 (a) [Scale 10D (0, 2, 5, 8, 10)]**

- 2: • any  $f(x+h)$

- 5: • limit of  $\frac{f(x+h) - f(x)}{h}$

- 8: • limit of  $\frac{(2x+2h+4)^2 - (2x+4)^2}{h}$

**NOTES:**

- omission of limit sign penalised once only
- answer not from 1st Principles merits 0 marks

**Question 6 (b) (i)**

$$y = x \sin\left(\frac{1}{x}\right) = x \sin(x^{-1})$$

$$\frac{dy}{dx} = x(\cos(x^{-1}) \times -1x^{-2}) + \sin(x^{-1}) \times 1$$

$$= -\frac{x}{x^2}(\cos(x^{-1})) + \sin(x^{-1})$$

$$= -\frac{1}{x} \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Derivatives [page 25]**

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$$

**Question 6 (b) (ii)**

$$x = \frac{4}{\pi} : \frac{dy}{dx} = -\frac{\pi}{4} \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)$$

$$= -\frac{\pi}{4} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left(1 - \frac{\pi}{4}\right) = 0.15$$

**MARKING SCHEME NOTES**

**Question 6 (b) (i) & (ii) [Scale 15D (0, 4, 7, 11, 15)]**

4: • any correct differentiation

7: • product rule applied

11: • correct differentiation

**NOTE:** one penalty for calculator in wrong mode

**QUESTION 7 (40 MARKS)**

**Question 7 (a) (i)**

$$\frac{dV}{dt} = 250 \text{ cm}^3/\text{s}$$

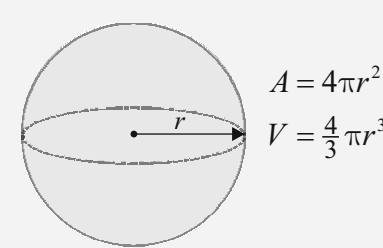
$$V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2 = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 250 = 4\pi r^2 \times \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{250}{4\pi r^2}$$

$$r = 20 : \frac{dr}{dt} = \frac{250}{4\pi(20)^2} = \frac{5}{32\pi} \text{ cm/s}$$

**FORMULAE AND TABLES BOOK**  
**Surface area and volume:**  
**Sphere [page 8]**



**MARKING SCHEME NOTES**

**Question 7 (a) (i) [Scale 10C (0, 3, 7, 10)]**

3: • work towards  $\frac{dv}{dr}$  or  $\frac{dv}{dt}$  or  $\frac{dr}{dt}$

7: • correct expression for  $\frac{dr}{dt}$

**Question 7 (a) (ii) [Scale 10C (0, 3, 7, 10)]**

3: • work towards  $\frac{dA}{dr}$  or  $\frac{dA}{dt}$

7: • correct expression for  $\frac{dA}{dt}$

**Question 7 (a) (ii)**

$$A = 4\pi r^2 \Rightarrow \frac{dA}{dr} = 8\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 8\pi r \times \frac{dr}{dt}$$

$$r = 20 : \frac{dA}{dt} = 8\pi(20) \times \frac{5}{32\pi} = 25 \text{ cm}^2/\text{s}$$

**Question 7 (b) (i)**

$$f(x) = -x^2 + 10x$$

$$f(x) = 0 \Rightarrow -x^2 + 10x = 0$$

$$x^2 - 10x = 0$$

$$x(x - 10) = 0$$

$$\therefore x = 0 \text{ m, } 10 \text{ m}$$

**MARKING SCHEME NOTES****Question 7 (b) (i) [Scale 10C (0, 3, 7, 10)]**

3: • quadratic equation formed

• gets  $x = 0$  only

7: • quadratic factorised

NOTE:  $f'(x) = 0 \Rightarrow 2x - 10 = 0 \Rightarrow x = 5$

merits 0 marks

**Question 7 (b) (ii) [Scale 10C (0, 3, 7, 10)]**

3: • integration set up

7: • correct integration with some substitution

**Question 7 (b) (ii)**

The average value  $\bar{y}$  of a continuous function  $y = f(x)$  on an interval  $[a, b]$  is given by:

$$\bar{y} = \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{b-a} \int_a^b y dx$$

Average  $f(x)$

$$= \frac{1}{10-0} \int_0^{10} (-x^2 + 10x) dx$$

$$= \frac{1}{10} \left[ -\frac{x^3}{3} + \frac{10x^2}{2} \right]_0^{10}$$

$$= \frac{1}{10} \left[ -\frac{x^3}{3} + 5x^2 \right]_0^{10}$$

$$= \frac{1}{10} \left[ \left( -\frac{(10)^3}{3} + 5(10)^2 \right) - \left( -\frac{(0)^3}{3} + 5(0)^2 \right) \right]$$

$$= \frac{1}{10} \left[ -\frac{1000}{3} + 500 \right] = \frac{50}{3} \text{ m}$$

**QUESTION 8 (55 MARKS)**

**Question 8 (a) (i)**

$$f(x) = -0.274x^2 + 1.193x + 3.23$$

$$A(-0.5, 2.565), B(4.5, 3.05)$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Derivatives [page 25]**

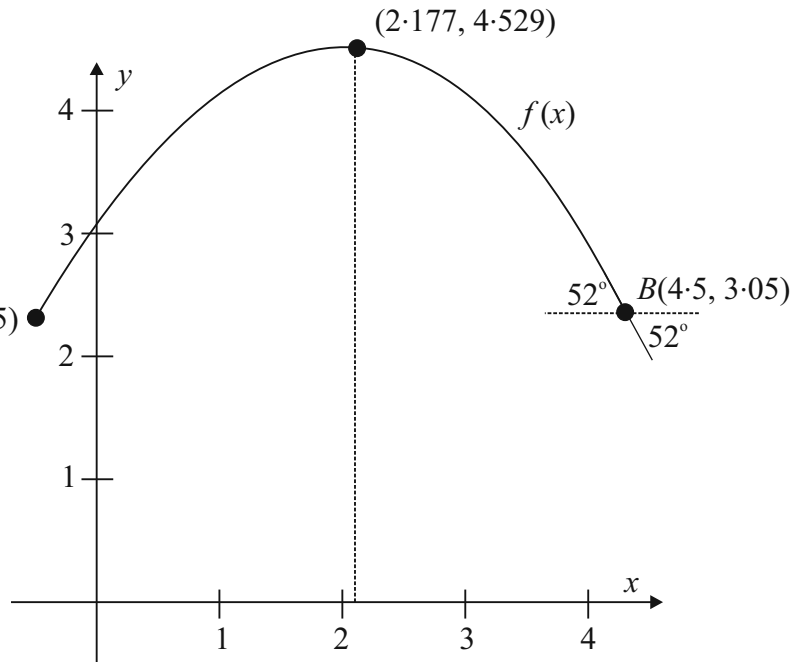
$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

$$f(x) = -0.274x^2 + 1.193x + 3.23$$

$$f'(x) = -0.548x + 1.193$$

FIND TURNING POINTS  
(LOCAL MAXIMUM/MINIMUM)

Put  $\frac{dy}{dx} = 0$  and solve for  $x$



Maximum:

$$f'(x) = 0 \Rightarrow -0.548x + 1.193 = 0$$

$$0.548x = 1.193$$

$$\therefore x = \frac{1.193}{0.548} = 2.177 \text{ m}$$

$$f(2.177) = -0.274(2.177)^2 + 1.193(2.177) + 3.23 = 4.529 \text{ m}$$

The maximum height reached by the ball is 4.529 m.

**MARKING SCHEME NOTES**

**Question 8 (a) (i) [Scale 10C (0, 3, 7, 10)]**

3: • any correct differentiation  
• effort made at completing square  
• trial and error with more than one value of  $x$  tested

7: •  $x$  value correct

**NOTE:** if correct answer by trial and error, must show points on each side of max point to be lower to earn full credit

**Question 8 (a) (ii)**

Basket  $B(4.5, 3.05)$

$$f'(x) = -0.548x + 1.193$$

$$f'(4.5) = -0.548(4.5) + 1.193 = -1.273 = \text{Slope } m = \tan \theta$$

$$\text{Acute angle} = \tan^{-1}|-1.273| \approx 52^\circ$$

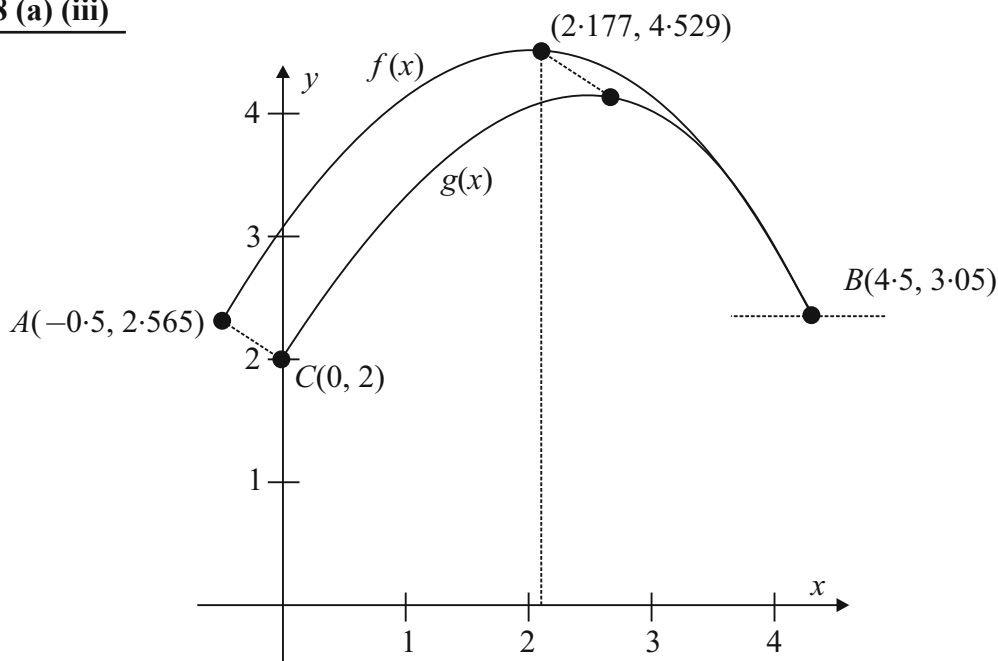
**MARKING SCHEME NOTES**

**Question 8 (a) (ii) [Scale 5B (0, 2, 5)]**

2: •  $\tan$

**NOTE:** right angled triangles may appear in diagram given in equation

**Question 8 (a) (iii)**



$$A(-0.5, 2.565) \rightarrow C(0, 2) [+0.5, -0.565]$$

$$(2.177, 4.529) \rightarrow (2.177 + 0.5, 4.529 - 0.565) = (2.677, 3.964)$$

**Question 8 (a) (iv)**

**VERTICAL AND HORIZONTAL SHIFTS**

If  $y = f(x)$ , then:

1.  $f(x) + v$  shifts  $f(x)$  vertically up by  $v$  units, if  $v > 0$ .
2.  $f(x) - v$  shifts  $f(x)$  vertically down by  $v$  units, if  $v > 0$ .
3.  $f(x + h)$  shifts  $f(x)$  horizontally left by  $h$  units, if  $h > 0$ .
4.  $f(x - h)$  shifts  $f(x)$  horizontally right by  $h$  units, if  $h > 0$ .

$f(x)$  is shifted right and down:  $h = 0.5, v = 0.565$

$$f(x) = -0.274x^2 + 1.193x + 3.23$$

$$g(x) = f(x - h) - v = f(x - 0.5) - 0.565$$

$$= -0.274(x - 0.5)^2 + 1.193(x - 0.5) + 3.23 - 0.565$$

$$= -0.274(x^2 - x + 0.25) + 1.193(x - 0.5) + 3.23 - 0.565$$

$$= -0.274x^2 + 0.274x - 0.0685 + 1.193x - 0.5965 + 3.23 - 0.565$$

$$= -0.274x^2 + 1.467x + 2$$

**MARKING SCHEME NOTES**

**Question 8 (a) (iii) [Scale 5B (0, 2, 5)]**

2: •  $(-0.5, 2.565) \rightarrow (0, 2)$

**Question 8 (a) (iv) [Scale 10D (0, 2, 5, 8, 10)]**

2: • equation of curve

• use of  $C$

5: • using peak value

8: • value of  $a$  found

**Question 8 (b)**

Event	$x$	Formula	$a$	$b$	$c$
200 m race	Time (s)	$y = a(b - x)^c$	4.99087	42.5	1.81
Javelin	Distance (m)	$y = a(x - b)^c$	15.9803	3.8	1.04

$x$  = Time taken (s) or distance achieved (m)

$y$  = Points scored

**Question 8 (b) (i)**

200 m

$$x = 23.8 \text{ s}$$

$$y = a(b - x)^c = 4.99087(42.5 - 23.8)^{1.81}$$

$$\approx 1000 \text{ points}$$

Javelin

$$x = 58.2 \text{ m}$$

$$y = a(x - b)^c = 15.9803(58.2 - 3.8)^{1.04}$$

$$\approx 1020 \text{ points}$$

**Question 8 (b) (ii)**

Javelin

$$y = 1295 \text{ points}$$

$$y = a(x - b)^c$$

$$1295 = 15.9803(x - 3.8)^{1.04}$$

$$\frac{1295}{15.9803} = (x - 3.8)^{1.04}$$

$$\left(\frac{1295}{15.9803}\right)^{\frac{1}{1.04}} = x - 3.8$$

$$\therefore x = \left(\frac{1295}{15.9803}\right)^{\frac{1}{1.04}} + 3.8 = 72.23 \text{ m}$$

**Question 8 (b) (iii)**

Event	$x$	Formula	$a$	$b$	$c$
200 m race	Time (s)	$y = a(b - x)^c$	4.99087	42.5	1.81
800 m race	Distance (m)	$y = a(b - x)^c$	0.11193	254	?

800 m:  $x = 2$  minutes and  $1.84 \text{ s} = 121.84 \text{ s}$ ,  $y = 1087$  points

$$y = a(b - x)^c$$

$$1087 = 0.11193(254 - 121.84)^c$$

$$\frac{1087}{0.11193} = 132.16^c$$

$$9711.43 = 132.16^c$$

$$\therefore c = \frac{\log_{10} 9711.43}{\log_{10} 132.16} \approx 1.88$$

**MARKING SCHEME NOTES****Question 8 (b) (i) [Scale 10D (0, 2, 5, 8, 10)]**

**2:** • some relevant substitution into one formula

**5:** • one value of  $y$  found

• some relevant substitution into both formulas

**8:** • one value correct and some relevant substitution into second formula

• uses incorrect formula (once only)

**Question 8 (b) (ii) [Scale 5B (0, 2, 5)]**

**2:** • some relevant substitution into formula

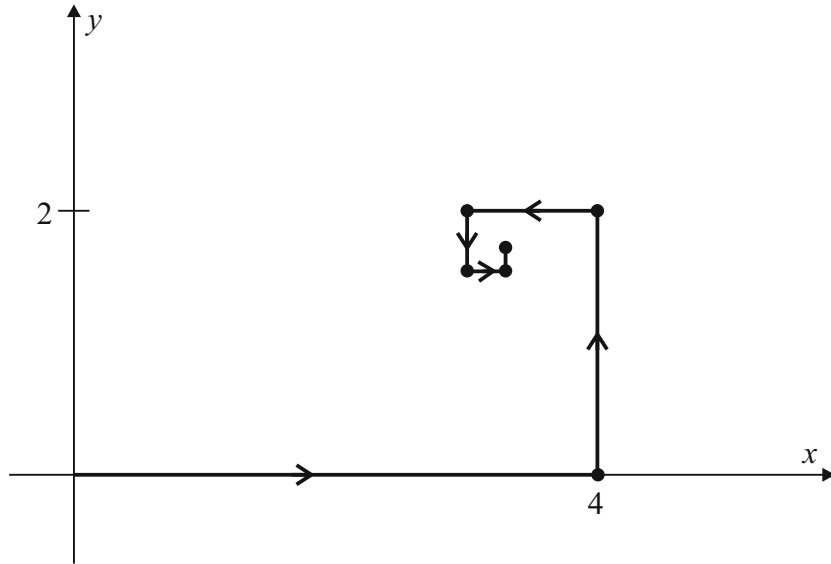
**Question 8 (b) (iii) [Scale 10C (0, 3, 7, 10)]**

**3:** • some relevant substitution into formula

**7:** • fully correct substitution into formula

**QUESTION 9 (55 MARKS)**

**Question 9 (a)**



**MARKING SCHEME NOTES**

**Question 9 (a) (i) [Scale 10C (0, 3, 7, 10)]**

- 3: • some listing of terms  
 •  $S_n$  formula  
 7: • listing of exactly 7 correct terms  
 • formula fully substituted

**Question 9 (a) (ii) [Scale 10C (0, 3, 7, 10)]**

- 3: •  $S_\infty$  formula  
 7: • formula fully substituted

**Question 9 (a) (iii) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • 2 extra entries correct in either row  
 7: • either row fully correct  
 11: • one co-ordinate correct

**NOTES:**

- need to see  $S_\infty$  correctly used to move beyond Mid Partial Credit  
 - no  $S_\infty$  merits Mid Partial Credit at most

**Question 9 (b) (i)**

$G_1$	$G_2$	$G_3$	$G_4$	$G_5$
			Female	
		Female		
Male	→ Female		Male	
		Male	→ Female	

**Question 9 (b) (ii)**

$$G_{n+2} = G_{n+1} + G_n$$

$$G_1 = 1$$

$$G_2 = 1$$

$$G_3 = G_2 + G_1 = 1 + 1 = 2$$

$$G_4 = G_3 + G_2 = 2 + 1 = 3$$

$$G_5 = G_4 + G_3 = 3 + 2 = 5$$

$$G_6 = G_5 + G_4 = 5 + 3 = 8$$

$$G_7 = G_6 + G_5 = 8 + 5 = 13$$

**Question 9 (b) (iii)**

**METHOD 1**

$$G_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \sqrt{5}}$$

$$G_3 = \frac{(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3}{2^3 \sqrt{5}} = 2 \text{ [Using calculator]}$$

**METHOD 2**

$$G_3 = \frac{(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3}{2^3 \sqrt{5}} \text{ [Difference of two cubes]}$$

$$= \frac{(1 + \sqrt{5} - 1 + \sqrt{5})(1 + \sqrt{5})^2 + (1 + \sqrt{5})(1 - \sqrt{5}) + (1 - \sqrt{5})^2}{8\sqrt{5}}$$

$$= \frac{(2\sqrt{5})(1 + 2\sqrt{5} + 5 + 1 - 5 + 1 - 2\sqrt{5} + 5)}{8\sqrt{5}}$$

$$= \frac{8}{4} = 2$$

**MARKING SCHEME NOTES**

**Question 9 (b) (i) [Scale 5B (0, 2, 5)]**

2: • one correct entry

**Question 9 (b) (ii) [Scale 10C (0, 3, 7, 10)]**

3: •  $G_6 = G_5 + G_4$   
•  $G_7 = G_6 + G_5$   
•  $G_7$  or  $G_6$  correct  
• 8 and/or 13 without work

7: • correct substitution in both

**Question 9 (b) (iii) [Scale 5B (0, 2, 5)]**

2: • some correct substitution  
• using approximate value for  $\sqrt{5}$   
•  $G_3 = 2$   
• some effort at cubing

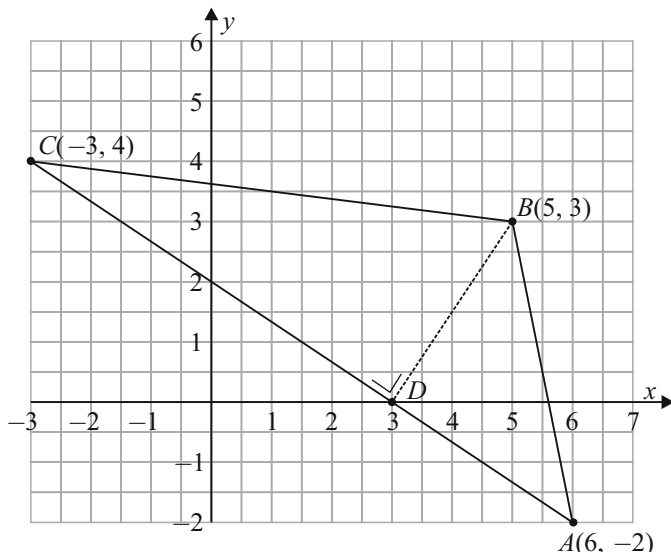
**NOTE:** use of  $\sqrt{5}$  as approximation, even if rounded off to 2 at end of work merits at most Partial Credit

## LC 2016 (SET C): PAPER 2

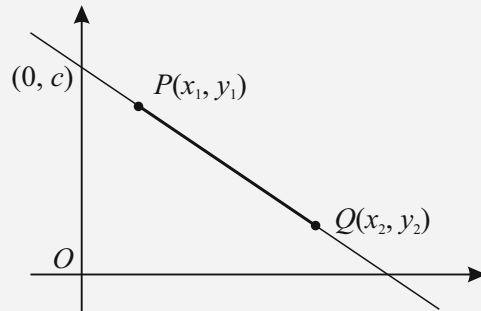
### QUESTION 1 (25 MARKS)

#### Question 1 (a)

Points:  $A(6, -2)$ ,  $B(5, 3)$ ,  $C(-3, 4)$



#### FORMULAE AND TABLES BOOK Co-ordinate geometry: Line



Slope of  $PQ$  [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of  $PQ$  [page 18]

$$y - y_1 = m(x - x_1)$$

$$\text{Slope of } AC = \frac{4 - (-2)}{-3 - 6} = \frac{6}{-9} = -\frac{2}{3}$$

$$\text{Slope of } BD = \frac{3}{2}$$

$$\text{Equation of } BD: (x_1, y_1) = B(5, 3), m = \frac{3}{2}$$

$$(y - 3) = \frac{3}{2}(x - 5)$$

$$2y - 6 = 3x - 15$$

$$3x - 2y - 9 = 0$$

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 10C (0, 3, 7, 10)]

- 3: • slope formula with some relevant substitution  
 •  $3 = 5m + c$   
 •  $y - y_1 = m(x - x_1)$  with  $x_1$  or  $y_2$  or both substituted
- 7: • perpendicular slope  
 • equation of line through  $B$  parallel to  $AC$

#### Question 1 (b)

$BD$  is an altitude (height). The orthocentre is the intersection of the altitudes. Find the equation of another altitude, say the line from  $C$  perpendicular to  $AB$ .

$$\text{Slope of } AB = \frac{3 - (-2)}{5 - 6} = \frac{5}{-1} = -5$$

$$\text{Perpendicular slope of } AB = \frac{1}{5}$$

$$\text{Equation of altitude: } (x_1, y_1) = C(-3, 4), m = \frac{1}{5}$$

$$(y - 4) = \frac{1}{5}(x - (-3))$$

$$5y - 20 = x + 3$$

$$x - 5y + 23 = 0$$

Find the point of intersection of the equations of the two altitudes:

$$3x - 2y = 9 \dots (1)$$

$$x - 5y = -23 \dots (2) (\times -3)$$

$$3x - 2y = 9$$

$$\underline{-3x + 15y = 69}$$

$$13y = 78 \Rightarrow y = 6$$

Into (2):  $x - 5(6) = -23 \Rightarrow x = 30 - 23 = 7$

Orthocentre = (7, 6)

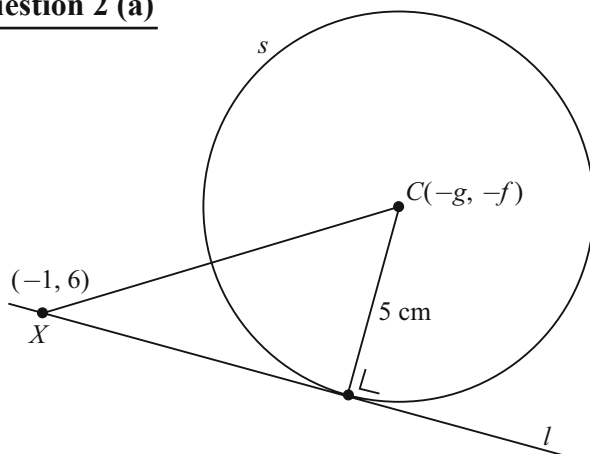
**MARKING SCHEME NOTES**

**Question 1 (b) [Scale 15D (0, 4, 7, 11, 15)]**

- 4:**
- demonstration of understanding of orthocentre (e.g. mentions altitude)
  - slope formula with some relevant substitution
  - altitude from part (a)
- 7:**
- equation of an altitude other than (a)
  - some relevant substitution towards finding a second altitude and altitude from (a)
  - correct construction
- 11:**
- two correct altitudes
  - correct construction with orthocentre (7, 6)

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**



Point  $(x_1, y_1) = X(-1, 6)$ ,  $m = \frac{1}{7}$

Equation of  $XC$ :  $(y - 6) = \frac{1}{7}(x - (-1))$

$$7y - 42 = x + 1$$

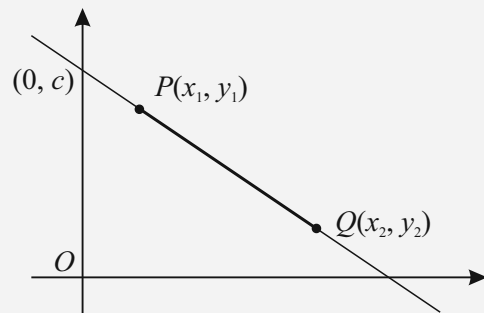
$$x - 7y + 43 = 0$$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 10C (0, 3, 7, 10)]**

- 3:**
- equation of line formula with some relevant substitution
- 7:**
- equation of line not in required form

**FORMULAE AND TABLES BOOK**  
**Co-ordinate geometry: Line**



Equation of  $PQ$  [page 18]

$$y - y_1 = m(x - x_1)$$

Slope of  $PQ$  [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Distance from  $(x_1, y_1)$  to the line

$$ax + by + c = 0 \text{ [page 19]}$$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

**Question 2 (b)**

$$C(-g, -f), d = 5$$

$$5 = \frac{|3(-g) + 4(-f) - 21|}{\sqrt{3^2 + 4^2}} \Rightarrow 5 = \frac{|-3g - 4f - 21|}{5}$$

$$|-3g - 4f - 21| = 25$$

$$-3g - 4f - 21 = \pm 25$$

$$\therefore 3g + 4f = -46 \dots (1)$$

$$\text{or } 3g + 4f = 4 \dots (1)$$

$$C(-g, -f), X(-1, 6), m = \frac{1}{7}$$

$$\frac{6 + f}{-1 + g} = \frac{1}{7}$$

$$42 + 7f = -1 + g$$

$$g - 7f = 43 \dots (2)$$

**Circle 1:**

$$3g + 4f = -46 \dots (1)$$

$$g - 7f = 43 \dots (2) (\times -3)$$

$$3g + 4f = -46$$

$$-3g + 21f = -129$$

$$\hline 25f = -175 \Rightarrow f = -7$$

$$\text{Into (2): } g - 7(-7) = 43 \Rightarrow g + 49 = 43$$

$$\therefore g = -6$$

$$\text{Centre} = (-g, -f) = (6, 7), r = 5$$

$$(x - 6)^2 + (y - 7)^2 = 25$$

**Circle 2:**

$$3g + 4f = 4 \dots (1)$$

$$g - 7f = 43 \dots (2) (\times -3)$$

$$3g + 4f = 4$$

$$-3g + 21f = -129$$

$$\hline 25f = -125 \Rightarrow f = -5$$

$$\text{Into (2): } g - 7(-5) = 43 \Rightarrow g + 35 = 43$$

$$\therefore g = 8$$

$$\text{Centre} = (-g, -f) = (-8, 5), r = 5$$

$$(x + 8)^2 + (y - 5)^2 = 25$$

**FORMULAE AND TABLES BOOK****Co-ordinate geometry: Circle [page 19]**Given centre  $(h, k)$  and radius  $r$ 

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre  $(-g, -f)$ 

$$\text{Radius } \sqrt{g^2 + f^2 - c}$$

**MARKING SCHEME NOTES****Question 2 (b) [Scale 15D (0, 4, 7, 11, 15)]**

4: • some correct substitution into relevant formula (line, circle, perpendicular distance).

7: • one relevant equation in  $g$  and  $f$ 

• ( either(i) or (ii) or (iii))

11: • two relevant equations ( either (i) and (iii) or (ii) and (iii))

**QUESTION 3 (25 MARKS)****Question 3 (a)****LHS**

$$\begin{aligned} & \frac{\cos 7A + \cos A}{\sin 7A - \sin A} \\ &= \frac{2 \cos \left( \frac{7A + A}{2} \right) \cos \left( \frac{7A - A}{2} \right)}{2 \cos \left( \frac{7A + A}{2} \right) \sin \left( \frac{7A - A}{2} \right)} \\ &= \frac{2 \cos 4A \cos 3A}{2 \cos 4A \sin 3A} = \frac{\cos 3A}{\sin 3A} \\ &= \frac{1}{\tan 3A} \end{aligned}$$

**RHS**

$$\cot 3A = \frac{1}{\tan 3A}$$

**FORMULAE AND TABLES BOOK  
Trigonometry (page 13/15)**

$$\tan A = \frac{\sin A}{\cos A}, \cot A = \frac{\cos A}{\sin A}$$

$$\cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$$

$$\sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)$$

**MARKING SCHEME NOTES****Question 3 (a) [Scale 15C (0, 5, 10, 15)]**

**5:** • sum to product formula with some substitution

**10:** • sum to product formula fully substituted

**Question 3 (b)****METHOD 1**

$$\begin{aligned} \cos 2\theta &= \frac{1}{9} \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \cos^2 \theta - (1 - \cos^2 \theta) \\ &= \cos^2 \theta - 1 + \cos^2 \theta \\ &= 2 \cos^2 \theta - 1 \end{aligned}$$

$$2 \cos^2 \theta - 1 = \frac{1}{9}$$

$$2 \cos^2 \theta = \frac{10}{9}$$

$$\cos^2 \theta = \frac{5}{9}$$

$$\therefore \cos \theta = \pm \frac{\sqrt{5}}{3}$$

**FORMULAE AND TABLES BOOK  
Trigonometry (page 13/14)**

$$\cos^2 A + \sin^2 A = 1$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

**METHOD 2**

$$\begin{aligned} \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta) \\ &= \frac{1}{2}\left(1 + \frac{1}{9}\right) \\ &= \frac{1}{2}\left(\frac{10}{9}\right) = \frac{5}{9} \\ \therefore \cos \theta &= \pm \frac{\sqrt{5}}{3} \end{aligned}$$

**MARKING SCHEME NOTES****Question 3 (b) [Scale 10D (0, 3, 5, 8, 10)]**

**3:** • Use of a relevant formula in  $\cos 2\theta$

•  $\cos^{-1}\left(\frac{1}{9}\right) = 83.62^\circ$

•  $\theta = 41.8^\circ$

**5:** • correct substitution

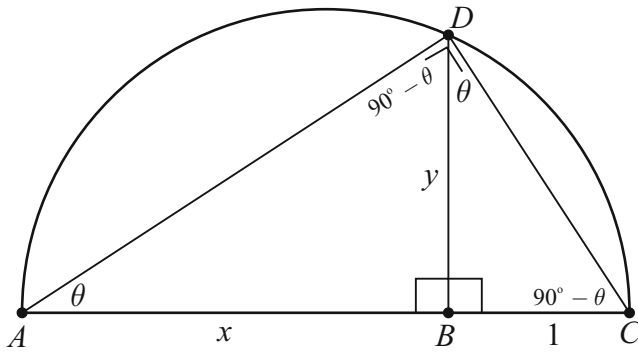
•  $\theta = 41.8^\circ$  and  $\theta = 132.2^\circ$  or  $\theta = 221.8^\circ$

**8:** • one value only (e.g.  $+\frac{\sqrt{5}}{3}$ )

• values found for  $\cos 41.8^\circ$  and  $\cos 138.2^\circ$  or  $\cos 221.8^\circ$

**QUESTION 4 (25 MARKS)**

**Question 4 (a) (i)**



Consider triangle  $ABD$ .

Let  $\theta = |\angle BAD|$ .  $|\angle ABD| = 90^\circ$ .

$\therefore |\angle ADB| = 90^\circ - \theta$ .

$|\angle ADC| = 90^\circ$  [Angle standing on a diameter is a right angle]

Consider triangle  $DBC$ .

$|\angle BDC| = 90^\circ - (90^\circ - \theta) = \theta$ .  $|\angle BDC| = 90^\circ$ .

$\therefore |\angle DCB| = 90^\circ - \theta$ .

$\therefore$  Triangles  $ABD$  and  $DBC$  are similar.

**Question 4 (a) (ii)**

$$\frac{|AB|}{|DB|} = \frac{|DB|}{|BC|} \Rightarrow \frac{x}{y} = \frac{y}{1}$$

$$x = y^2$$

$$\therefore y = \sqrt{x}$$

**MARKING SCHEME NOTES**

**Question 4 (a) (i) [Scale 15C (0, 5, 10, 15)]**

**5:** • identifies one angle of same size in each triangle

**10:** • identifies second angle of same size in each triangle

• implies triangles are similar without justifying (ii) in model solution or equivalent

**Question 4 (a) (ii) [Scale 5C (0, 2, 4, 5)]**

**2:** • one set of corresponding sides identified

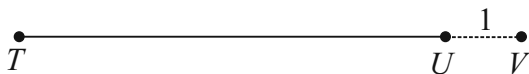
• indicates relevant use of Pythagoras

**4:** • corresponding sides fully substituted

• expression in  $y^2$  or  $y^4$ , i.e. fails to finish

**Question 4 (b)**

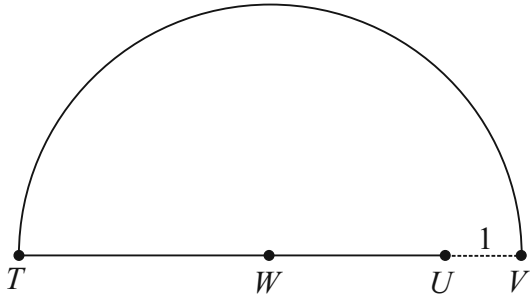
**STEP 1:** Draw a line  $[UV]$  of length 1 cm such that  $[TV]$  is a straight line.



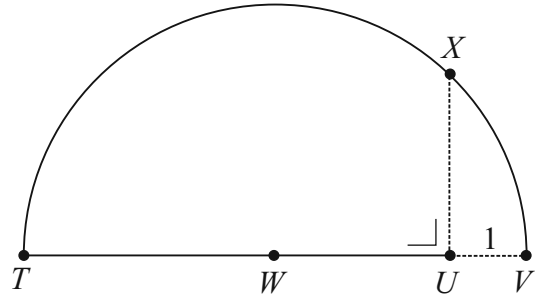
**STEP 2:** Find the centre  $W$  of line segment  $[TV]$  using a ruler or using a compass to bisect line segment  $[TV]$ .



**STEP 3:** With  $W$  as centre and  $[TW]$  as radius, draw a semi-circle.



**STEP 4:** Using a set-square, draw the line  $UX$  perpendicular to  $[TV]$ .



**CONCLUSION:** The length in centimetres of the line segment  $[UX]$  is the square root of the length of line segment  $[TV]$ .

**MARKING SCHEME NOTES**

**Question 4 (b) [Scale 5C (0, 2, 4, 5)]**

- 2: • perpendicular line drawn at  $U$  or  $T$   
 • relevant use of 1 cm length  
 • mid point of incorrect extended segment constructed
- 4: • correct mid-point constructed

**QUESTION 5 (25 MARKS)**

**Question 5 (a) (i)**

	Way 1	Way 2	Way 3	Way 4
John	✓	✓	×	✓
David	✓	×	✓	✓
Mike	×	✓	✓	✓

✓ = Hit  
 × = Miss

**Question 5 (a) (ii)**

John:  $P(\text{Hit}) = \frac{1}{5}$ ,  $P(\text{Miss}) = \frac{4}{5}$

David:  $P(\text{Hit}) = \frac{1}{6}$ ,  $P(\text{Miss}) = \frac{5}{6}$

Mike:  $P(\text{Hit}) = \frac{1}{4}$ ,  $P(\text{Miss}) = \frac{3}{4}$

$P(\text{Win}) = P(\text{Way 1}) \text{ or } P(\text{Way 2}) \text{ or } P(\text{Way 3}) \text{ or } P(\text{Way 4})$

$$= \frac{1}{5} \times \frac{1}{6} \times \frac{3}{4} + \frac{1}{5} \times \frac{5}{6} \times \frac{1}{4} + \frac{4}{5} \times \frac{1}{6} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{6} \times \frac{1}{4}$$

$$= \frac{13}{120}$$

**MARKING SCHEME NOTES**

**Question 5 (a) (i) [Scale 5B (0, 2, 5)]**

- 2: • 1 correct column

**Question 5 (a) (ii) [Scale 10C (0, 3, 7, 10)]**

- 3: • one correct triple (numerical or descriptive)  
 • probability of any one *Miss*
- 7: • 4 correct triples (numerical)

**Question 5 (b)**

$$P(A \cap B) = 0.1$$

$$P(B \setminus A) = 0.3$$

$$P(A \setminus B) = x$$

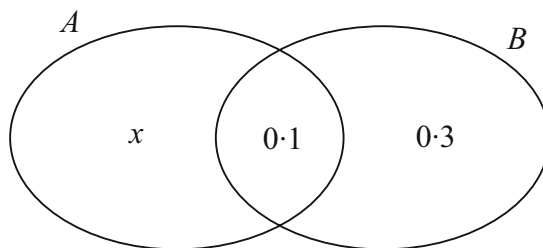
$$P(A) = x + 0.1$$

$$P(B) = 0.4$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.4} = 0.25$$

$$P(A|B) = P(A) \text{ [} A \text{ and } B \text{ independent]}$$

$$\therefore 0.25 = x + 0.1 \Rightarrow x = 0.15$$



**MARKING SCHEME NOTES**

**Question 5 (b) [Scale 10C (0, 3, 7, 10)]**

3: • formula written or implied

• writes  $P(A) = x + 0.1$

7: • formula fully substituted

**QUESTION 6 (25 MARKS)**

**Question 6 (a)**

$$P(\text{Letter and then first number and then second number}) = \frac{1}{26} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{2600}$$

**MARKING SCHEME NOTES**

**Question 6 (a) [Scale 10C (0, 3, 7, 10)]**

3: • any correct relevant probability

7: • correct probabilities but not expressed as single fraction or equivalent

**NOTE:** Accept correct answer without supporting work

**Question 6 (b)**

$$P(\text{Letter and then first number and then not second number}) = \frac{1}{26} \times \frac{1}{10} \times \frac{9}{10} = \frac{9}{2600}$$

$$P(\text{Letter and then not first number and then second number}) = \frac{1}{26} \times \frac{9}{10} \times \frac{1}{10} = \frac{9}{2600}$$

$$P(\text{Letter and then not first number and then not second number}) = \frac{1}{26} \times \frac{9}{10} \times \frac{9}{10} = \frac{81}{2600}$$

$$P(\text{Fail to win}) = 1 - \frac{1}{2600} - \frac{9}{2600} - \frac{9}{2600} - \frac{81}{2600} = \frac{25}{26}$$

Fill in the table from the club's point of view. If someone wins the jackpot, the club ends up paying the winner €998 (€1000 – €2).

Event	Payout ( $x$ ) in euro	Probability $P(x)$	$xP(x)$
Win Jackpot	-998	$\frac{1}{2600}$	$-\frac{998}{2600}$
Match letter and first number only	-48	$\frac{9}{2600}$	$-\frac{432}{2600}$
Match letter and second number only	-48	$\frac{9}{2600}$	$-\frac{432}{2600}$
Match letter and neither number	-48	$\frac{81}{2600}$	$-\frac{3888}{2600}$
Fail to win	2	$\frac{25}{26}$	$\frac{25}{13}$

$$E(x) = \sum xP(x) = -\frac{998}{2600} - 2 \times \frac{432}{2600} - \frac{3888}{2600} + \frac{25}{13} = -\frac{15}{52} = -\text{€}0.29 = -29 \text{ c}$$

The club expects to lose 29 c per €2 play.

**MARKING SCHEME NOTES**

**Question 6 (b) [Scale 10C (0, 3, 7, 10)]**

- 3: • 1 correct entry to table  
 7: • all entries correct but fails to finish or finishes incorrectly  
 • no conclusion

**Question 6 (c)**

If you increase or decrease each value of the discrete random variable by the same number, the expected value also increases or decreases by the same number.

$$\text{Profit needed per game} = \frac{\text{€}600}{845} = \text{€}0.71$$

$$\text{Change in ticket price} = \text{€}0.71 - (-\text{€}0.29) = \text{€}1.00$$

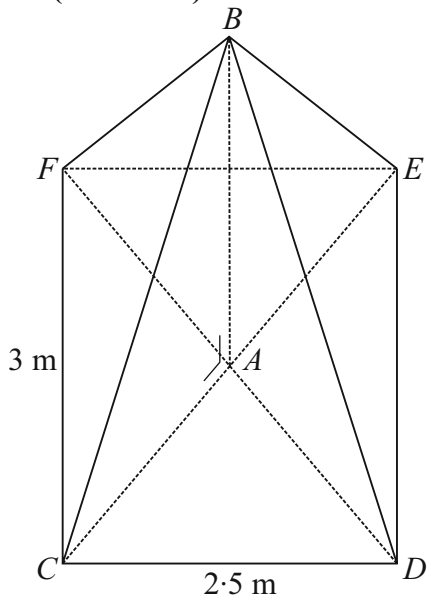
$$\text{New ticket price} = \text{€}3$$

**MARKING SCHEME NOTES**

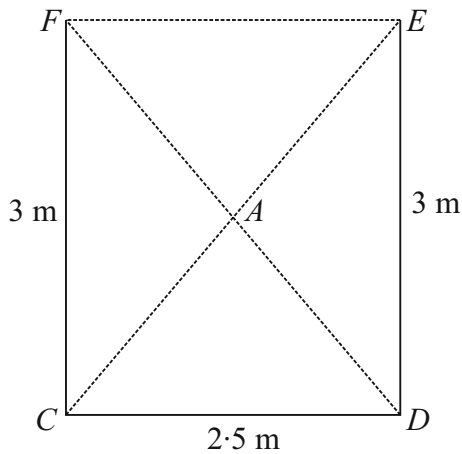
**Question 6 (c) [Scale 5C (0, 2, 4, 5)]**

- 2: • links profit, revenue and payout  
 4: • formula fully substituted

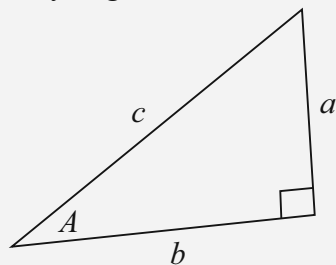
**QUESTION 7 (55 MARKS)**



**Question 7 (a) (i)**



**FORMULAE AND TABLES BOOK**  
**Trigonometry** (page 16)  
 Pythagoras' theorem



$$c^2 = a^2 + b^2$$

$$|EC|^2 = |CD|^2 + |ED|^2$$

$$|EC| = \sqrt{2 \cdot 5^2 + 3^2}$$

$$|AC| = \frac{1}{2}|EC| = \frac{1}{2}\sqrt{2 \cdot 5^2 + 3^2} = 1.95 \text{ m}$$

**Question 7 (a) (ii)**

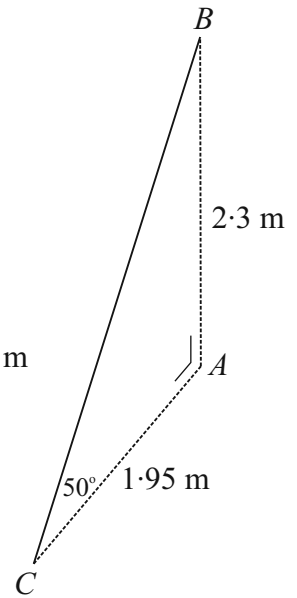
$$\tan 50^\circ = \frac{|AB|}{1.95}$$

$$|AB| = 1.95 \times \tan 50^\circ$$

$$= 2.3 \text{ m}$$

**Question 7 (a) (iii)**

$$|BC| = \sqrt{2 \cdot 3^2 + 1.95^2} \approx 3 \text{ m}$$



**MARKING SCHEME NOTES**

**Question 7 (a) (i) [Scale 10C (0, 3, 7, 10)]**

- 3: • Pythagoras with relevant substitution  
 7: •  $|EC|$  correct  
 •  $|AC| = \frac{1}{2}\sqrt{15 \cdot 25}$

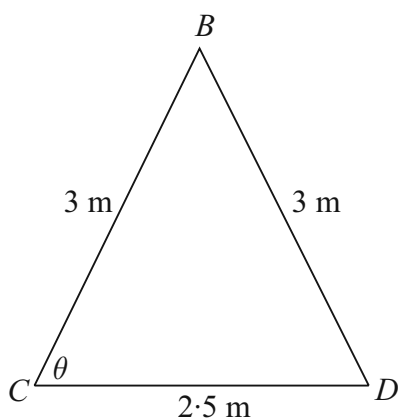
**Question 7 (a) (ii) [Scale 10B (0, 5, 10)]**

- 5: • tan formulated correctly

**Question 7 (a) (iii) [Scale 10C (0, 3, 7, 10)]**

- 3: • Pythagoras with relevant substitution  
 7: • Pythagoras fully substituted  
 •  $|BC| = \frac{1.95}{\sin 40^\circ}$  (i.e.  $|BC|$  isolated)

**Question 7 (a) (iv)**



**FORMULAE AND TABLES BOOK**  
**Trigonometry of the triangle:**

**Cosine rule [page 16]**

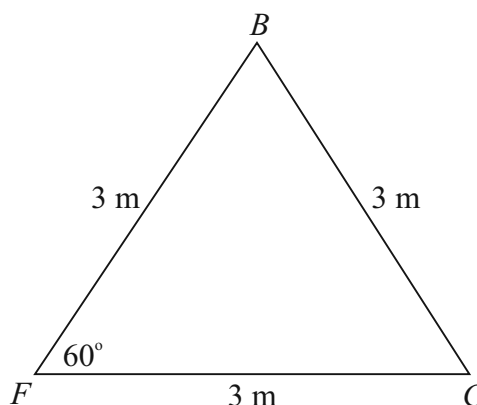
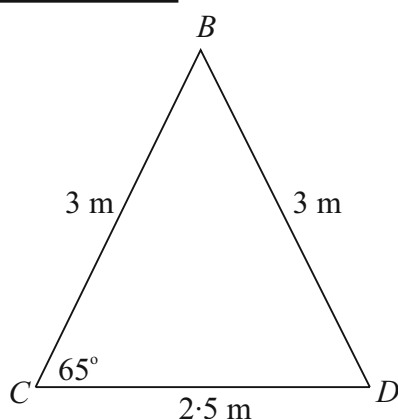
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$3^2 = 3^2 + 2 \cdot 5^2 - 2(3)(2 \cdot 5) \cos \theta$$

$$\cos \theta = \frac{3^2 + 2 \cdot 5^2 - 3^2}{2(3)(2 \cdot 5)} = \frac{5}{12}$$

$$\theta = \cos^{-1}\left(\frac{5}{12}\right) \approx 65^\circ$$

**Question 7 (a) (v)**



Total area of 4 triangular faces

= 2 faces of area of triangle  $BCD$  + 2 faces of area of triangle  $BFC$

$$= 2 \times \frac{1}{2} \times 2 \cdot 5 \times 3 \times \sin 65^\circ + 2 \times \frac{1}{2} \times 3 \times 3 \times \sin 60^\circ$$

$$\approx 15 \text{ m}^2$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

**MARKING SCHEME NOTES**

**Question 7 (a) (iv) [Scale 10C (0, 3, 7, 10)]**

- 3:**
- cosine rule with some relevant substitution
  - cosine ratio with some relevant substitutions
  - identifies three sides of triangle  $BCD$
- 7:**
- cosine rule with full relevant substitutions
  - cosine ratio with full relevant substitutions

**Question 7 (a) (v) [Scale 10D (0, 3, 5, 8, 10)]**

- 3:**
- recognises area of 4 triangles
- 5:**
- Area of 1 triangle correct
- 8:**
- area of isosceles triangle and equilateral triangle

**NOTE:** Area = 4 isosceles or 4 equilateral triangles merit *HPC* at most

**Question 7 (b)**

$$\tan 60^\circ = \frac{3}{|AC|} \Rightarrow |AC| = \frac{3}{\tan 60^\circ} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

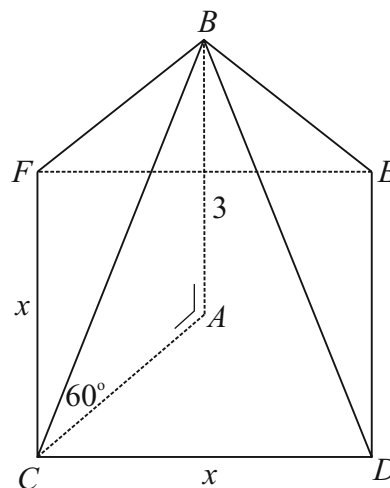
$$\tan 60^\circ = \frac{3}{|AC|} \Rightarrow |AC| = \frac{3}{\tan 60^\circ} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\therefore |EC| = 2\sqrt{3}$$

$$x^2 + x^2 = (2\sqrt{3})^2$$

$$2x^2 = 12$$

$$x^2 = 6 \Rightarrow x = \sqrt{6} \text{ m}$$



**MARKING SCHEME NOTES**

**Question 7 (b) [Scale 5C (0, 2, 4, 5)]**

2: • effort at Pythagoras but without  $|CA|$  (or  $|CE|$ )

•  $|CA|$  found

4: •  $|CE| = 2\sqrt{3}$

**QUESTION 8 (45 MARKS)**

**Question 8 (a)**

$$h(t) = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$$

$$a = 1.6, b = 1.5, k = \frac{\pi}{6}$$

$$P = \frac{2\pi}{\frac{\pi}{6}} = 12 \text{ hours}$$

$$\text{Range} = [1.6 - 1.5, 1.6 + 1.5] = [0.1 \text{ m}, 3.1 \text{ m}]$$

$$y = a + b \cos(kx + \alpha)$$

$$\text{Period } P = \frac{2\pi}{k}$$

$$\text{Range } [a - b, a + b], b > 0 \text{ and } [a + b, a - b], b < 0$$

**Question 8 (b)**

The maximum height is 3.1 m.

**Question 8 (c)**

$$h = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$$

$$\frac{dh}{dt} = -1.5 \sin\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6} = -\frac{\pi}{4} \sin\left(\frac{\pi}{6}t\right)$$

$$\left(\frac{dh}{dt}\right)_{t=2} = -\frac{\pi}{4} \sin\left(\frac{\pi}{6} \times 2\right) = -\frac{\pi}{4} \sin\left(\frac{\pi}{3}\right)$$

$$= -\frac{\pi}{4} \times \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}\pi}{8}$$

$$= -0.68 \text{ m/h}$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Derivatives** [page 25]

$$y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x$$

$$y = \cos f(x) \Rightarrow \frac{dy}{dx} = -\sin f(x) \times f'(x)$$

The height of water is decreasing as the tide goes out.

**MARKING SCHEME NOTES****Question 8 (a) [Scale 5C (0, 2, 4, 5)]**

- 2: • some use of  $2\pi$  or  $\frac{\pi}{6}$   
 • range of cos function

- 4: • period or range correct

**NOTE:** Accept correct period and/or range without work

**Question 8 (b) [Scale 5B (0, 2, 5)]**

- 2: • max occurs when  $\cos A = 1$  or  $t = 0$   
 • effort at  $h'(t)$

**NOTE:** Accept correct answer without work

**Question 8 (c) [Scale 5C (0, 2, 4, 5)]**

- 2: • effort at differentiation

- 4: • correct numerical answer but not in context

**Question 8 (d) (i)**

$$h(t) = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$$

$$h(0) = 1.6 + 1.5 \cos\left(\frac{\pi}{6} \times 0\right) = 1.6 + 1.5 = 3.1 \text{ m}$$

$$h(3) = 1.6 + 1.5 \cos\left(\frac{\pi}{6} \times 3\right) = 1.6 + 0 = 1.6 \text{ m}$$

$$h(6) = 1.6 + 1.5 \cos\left(\frac{\pi}{6} \times 6\right) = 1.6 - 1.5 = 0.1 \text{ m}$$

**MARKING SCHEME NOTES****Question 8 (d) (i) [Scale 10C (0, 3, 7, 10)]**

- 3: • one correct height

- 7: • five correct heights

**Question 8 (d) (ii) [Scale 10C (0, 3, 7, 10)]**

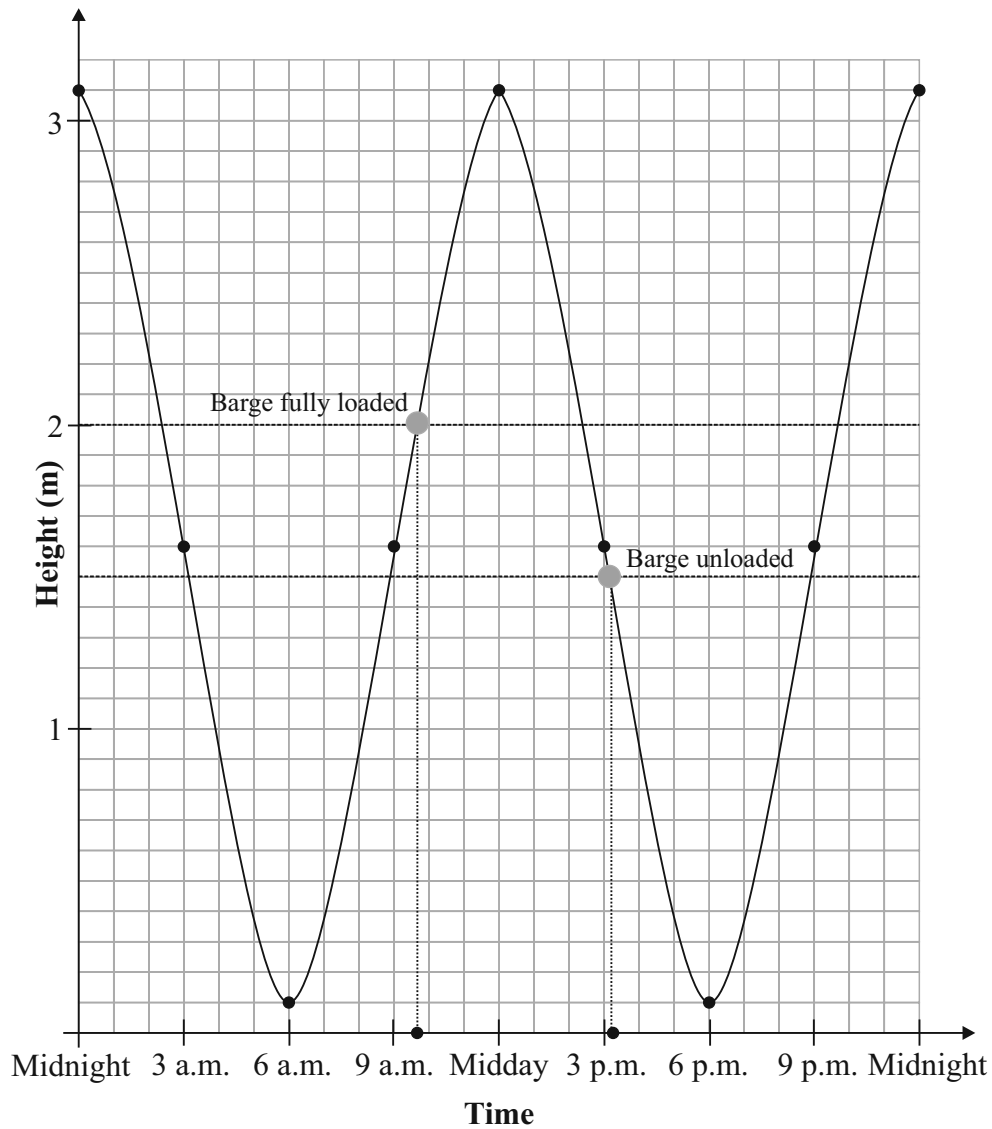
- 3: • one correct plot

- 7: • at least 7 correct plots

- plots correct but graph not sketched or sketched incorrectly

$h(t) = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
$t$ (hours)	0	3	6	9	12	15	18	21	24
$h(t)$ (m)	3.1	1.6	0.1	1.6	3.1	1.6	0.1	1.6	3.1

**Question 8 (d) (ii)**



**Question 8 (e)**

High tide = 3.1 m

Low tide = 0.1 m

Difference in water height =  $3.1 - 0.1 = 3$  m

**Question 8 (f)**

Barge enters port when height is 2 m and leaves when height is 1.5 m. This can happen a number of times during the day. One such time is shown on the graph.

Time  $\approx$  5.5 hours

**MARKING SCHEME NOTES****Question 8 (e) [Scale 5B (0, 2, 5)]**

2: • height of Low tide or High tide correctly identified

**NOTES:**

- (i) candidates may show work for this section on graph
- (ii) accept values from candidate's graph
- (iii) accept correct answer from graph without work

**Question 8 (f) [Scale 5B (0, 2, 5)]**

2: • time of entry to port or leave port correctly identified  
 • value(s) for  $h = 2$  and/or  $h = 1.5$  on sketch  
 • time estimated using relevant values other than those required for the maximum time.

**NOTES:**

- (i) candidates may show relevant work for this section on graph
- (ii) accept values from candidate's graph

**QUESTION 9 (50 MARKS)****Question 9 (a) (i)**

$$\mu = \text{€}39\,400$$

$$\sigma = \text{€}12\,920$$

$$x = \text{€}60\,000$$

$$z = \frac{x - \mu}{\sigma} = \frac{60\,000 - 39\,400}{12\,920} = 1.59$$

$$\begin{aligned} P(x \geq 60\,000) &= P(z \geq 1.59) \\ &= 1 - P(z \leq 1.59) \\ &= 1 - 0.9441 \\ &= 0.0559 \approx 5.6\% \end{aligned}$$

**MARKING SCHEME NOTES****Question 9 (a) (i) [Scale 10D (0, 3, 5, 8, 10)]**

3: •  $\mu$  and  $\sigma$  identified

5: •  $z = 1.59$

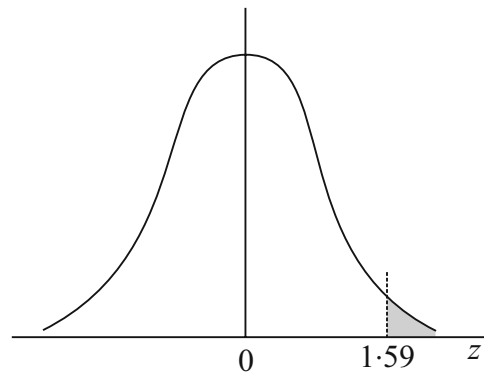
8: • identifies 0.9441

**FORMULAE AND TABLES BOOK**  
**Statistics and Probability: Probability**  
**distribution (standardising formula) [page 34]**

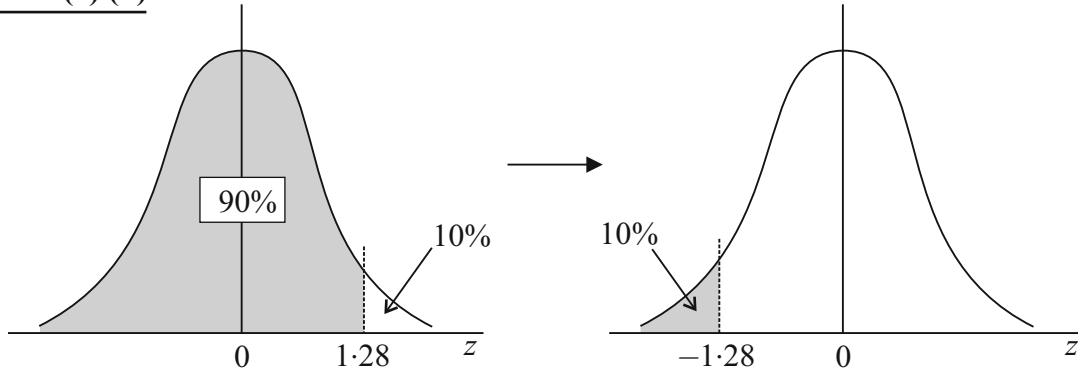
$$z = \frac{x - \mu}{\sigma}$$

$n$  = Number in the sample

$\sigma$  = standard deviation of the sample



**Question 9 (a) (ii)**



$$P(z \leq Z) = 0.9 \Rightarrow Z = 1.28$$

$$\therefore P(z \leq Z) = 0.1 \Rightarrow Z = -1.28$$

$$z = \frac{x - \mu}{\sigma} \Rightarrow -1.28 = \frac{x - 39\,400}{12\,920}$$

$$\therefore x = -1.28 \times 12\,920 + 39\,400 \approx \text{€}22\,862$$

**MARKING SCHEME NOTES**

**Question 9 (a) (ii) [Scale 5C (0, 2, 4, 5)]**

**2:** • identifies 1.28 but fails to progress

**4:** • formula for  $x$  fully substituted

**Question 9 (a) (iii)**

$$n = 1000, \bar{x} = 38\,280$$

$$\bar{Z} = \frac{38\,280 - 39\,400}{\frac{12\,290}{\sqrt{1000}}} = -2.74$$

$$\bar{z} = \frac{\bar{x} - \bar{\mu}}{\bar{\sigma}}, \text{ where } \bar{\mu} = \mu \text{ and } \bar{\sigma} = \frac{\sigma}{\sqrt{n}}.$$

Null hypothesis  $H_0$ : There is no change in the mean annual income.

Alternative hypothesis  $H_1$ : There is a change in the mean annual income.

$\bar{Z} = -2.74 < -1.96$ : Reject the null hypothesis

The result is significant. The mean annual income has changed.

**MARKING SCHEME NOTES**

**Question 9 (a) (iii) [Scale 15D (0, 4, 7, 11, 15)]**

**4:** •  $z$  formulated with some substitution  
• states null and/or alternative hypothesis only  
• reference to 1.96

**7:** •  $z$  fully substituted

**11:** •  $z = -2.74$  and stops  
• fails to state the null and alternative hypothesis correctly  
• fails to contextualise the answer

### Question 9 (b)

At a 95% confidence level,  $\bar{x} - 1.96 \times \frac{\sigma}{\sqrt{n}} \leftarrow \mu \rightarrow \bar{x} + 1.96 \times \frac{\sigma}{\sqrt{n}}$  is the best estimate for  $\mu$ , where  $\bar{x}$  is the mean of the sample and  $\sigma$  is the standard deviation of the population or the sample and is the 95% confidence interval for  $\mu$ .

$$n = 400, \bar{x} = 26\,974, \sigma = 5120$$

$$\bar{x} - 1.96 \times \frac{\sigma}{\sqrt{n}} \leftarrow \mu \rightarrow \bar{x} + 1.96 \times \frac{\sigma}{\sqrt{n}}$$

$$26\,974 - 1.96 \times \frac{5120}{\sqrt{400}} \leftarrow \mu \rightarrow 26\,974 + 1.96 \times \frac{5120}{\sqrt{400}}$$

$$26\,472.24 \leftarrow \mu \rightarrow 27\,475.76$$

$$\text{Confidence interval} = [\text{€}26\,472.24, \text{€}27\,475.76]$$

#### MARKING SCHEME NOTES

##### Question 9 (b) [Scale 10C (0, 3, 7, 10)]

3: • interval formulated with some correct substitution

7: • interval formulated with fully correct substitution

### Question 9 (c)

The central limit theorem states that the sampling distribution of any statistic will be normal or nearly normal, if the sample size is large enough, allowing you to find important information like the mean farm size.

#### MARKING SCHEME NOTES

##### Question 9 (c) [Scale 5B (0, 2, 5)]

2: • mentions 30 (or more) but not contextualised

### Question 9 (d)

$$\text{Margin of error} = 0.045 = \frac{0.98}{\sqrt{n}}$$

$$\sqrt{n} = \frac{0.98}{0.045} \Rightarrow n = \left( \frac{0.98}{0.045} \right)^2 \approx 474$$

#### MARKING SCHEME NOTES

##### Question 9 (d) [Scale 5C (0, 2, 4, 5)]

2: •  $\frac{1}{\sqrt{n}}$

4: •  $n$  formulated with fully correct substitution

NOTE: Accept 493 farmers or 494 farmers

#### AUTHOR'S NOTE:

Our solution is different to the solution given in the marking scheme.

**Question 9 (d)** is clearly a problem on population proportions. We have no estimate for  $\hat{p}$  to work

out  $\text{ME} = 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ . In this case we assume  $\hat{p} = 0.5$  which is the value which maximises

$$\hat{p}(1-\hat{p}). \text{ Therefore, the margin of error is maximised at } \text{ME} = 1.96 \sqrt{\frac{0.5(1-0.5)}{n}} = \frac{0.98}{\sqrt{n}}.$$

## LC 2015 (SET D): PAPER 1

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

Bounce 1 is obtained by multiplying 2 m by  $\frac{3}{4}$  to give a height of  $\frac{3}{2}$  m.

This process is repeated up until the fourth bounce.

Bounce	0	1	2	3	4
Height (m)	$\frac{2}{1}$	$\frac{2}{1} \times \frac{3}{4} = \frac{3}{2}$	$\frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$	$\frac{9}{8} \times \frac{3}{4} = \frac{27}{32}$	$\frac{27}{32} \times \frac{3}{4} = \frac{81}{128}$

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 5C (0, 2, 4, 5)]

2: • Any term correct

4: • Any two terms correct

Note: Dividing by  $\frac{3}{4}$  gets high partial credit at most. Correct decimal values high partial at most.

#### Question 1 (b)

Add up the distances travelled by the ball going up and down. Make sure you multiply the distances moved by the bounced balls by 2 (up and down).

$$S = 2 + 2\left(\frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128}\right) = \frac{653}{64} \text{ m [Using calculator]}$$

or

$$S = 2 + 2\left(\frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128}\right) = 2 + \left(3 + \frac{9}{4} + \frac{27}{16} + \frac{81}{64}\right)$$

$$[a = 3, r = \frac{3}{4}, n = 4]$$

$$S = 2 + S_4 = 2 + \frac{3(1 - (\frac{3}{4})^4)}{1 - \frac{3}{4}} = \frac{653}{64} \text{ m}$$

#### FORMULAE AND TABLES BOOK

##### Sequences and series:

##### Geometric series [page 22]

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$a$  is the first term

$r$  is the common ratio

#### MARKING SCHEME NOTES

##### Question 1 (b) [Scale 10C (0, 4, 8, 10)] NOTE: two solutions

##### 1st solution

4: • Indicates addition of terms

8: • Recognises double distance after first hop  
• Sum of all rises or drops

or

##### 2nd solution

4: • Indicates addition of terms

• Indicates Geometric Progression

8: • Correct Geometric Progression formula with correct substitution

### Question 1 (c)

$$S_{\infty} = 2 + \left(3 + \frac{9}{4} + \frac{27}{16} + \frac{81}{64} + \dots\right) \leftarrow \text{This bracket is an infinite geometric series with } |r| < 1.$$

$$[a = 3, r = \frac{3}{4}]$$

$$S_{\infty} = 2 + \frac{3}{1 - \frac{3}{4}} = 14 \text{ m}$$

**FORMULAE AND TABLES BOOK**  
**Sequences and series:**  
**Geometric series [page 22]**

$$S_{\infty} = \frac{a}{1-r}, |r| < 1$$

$a$  is the first term  
 $r$  is the common ratio

#### MARKING SCHEME NOTES

#### Question 1 (c) [Scale 10C (0, 4, 8, 10)]

- 4:**
- Recognition of sum to infinity
  - $S_{\infty}$  formula
- 8:**
- Correct formula with correct substitution
  - Sum of all rises or drops

### QUESTION 2 (25 MARKS)

#### FACTOR THEOREM

If  $k$  is a root of a polynomial equation  $P(x) = 0$ , then  $(x - k)$  is a factor of  $P(x)$  and vice versa.  
*or*

For a polynomial  $P(x)$ ,  $P(k) = 0 \Rightarrow P(x) = (x - k)Q(x)$ , where  $Q(x)$  is a polynomial of degree one less than  $P(x)$ .

Call the polynomial function  $P(x)$ . Substitute different integer values of  $x$  into this polynomial until you get an answer of 0.

**HINT:** The only integer values that work are divisors of the constant term in  $P(x)$ . So try 1, -1, 11 and -11 in order.

$$P(x) = x^3 - 3x^2 - 9x + 11$$

$$P(1) = (1)^3 - 3(1)^2 - 9(1) + 11 = 1 - 3 - 9 + 11 = 0 \leftarrow \text{This is successful.}$$

$$\therefore (x - 1) \text{ is a factor of } P(x)$$

$(x - 1)$  is a linear factor of the cubic polynomial  $P(x)$ . The other factor will be a quadratic. You can find this quadratic factor by lining up or by division.

#### LINING UP:

Cubic = Linear  $\times$  Quadratic

$$x^3 - 3x^2 - 9x + 11 = (x - 1)(x^2 + px - 11)$$

$$x^3 - 3x^2 - 9x + 11 = x^3 + (p - 1)x^2 + (-p - 11)x + 11$$

$$\text{Line up } x^2: -3 = p - 1 \Rightarrow p = -2$$

$$\therefore P(x) = 0 \Rightarrow x^3 - 3x^2 - 9x + 11 = (x - 1)(x^2 - 2x - 11) = 0$$

#### DIVISION:

$$\begin{array}{r}
 x^2 - 2x - 11 \\
 x - 1 \overline{) x^3 - 3x^2 - 9x + 11} \\
 \underline{\mp x^3 \pm x^2} \phantom{+ 11} \\
 -2x^2 - 9x + 11 \\
 \underline{\pm 2x^2 \mp 2x} \\
 -11x + 11 \\
 \underline{\pm 11x \mp 11} \\
 0
 \end{array}$$

Finally solve the quadratic equation.

$$x^2 - 2x - 11 = 0$$

$$a = 1, b = -2, c = -11$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 + 44}}{2} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2}$$

$$= 1 \pm 2\sqrt{3}$$

Answers :  $x = 1, 1 \pm 2\sqrt{3}$

**FORMULAE AND TABLES BOOK**  
**Algebra: Roots of the quadratic equation**  
 $ax^2 + bx + c = 0$  [page 20]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**MARKING SCHEME NOTES**

**Question 2 [Scale 25E (0, 5, 10, 15, 20, 25)]**

**5:** • Effort at finding root, i.e.  $f(1), f(-1)$ , etc.

**10:** • Finds one root correctly  
 •  $x^2$  after division by incorrect factor  
 • Correct answers in decimal form from calculator with or without work

**15:** • Tries division and gets  $x^2$  at very minimum

**20:** • Having got a quadratic equation with no remainder, fills in quadratic formula  
 •  $1 \pm \sqrt{12}$

**Note:** If there is a remainder after division can only get maximum of 15 marks.

**QUESTION 3 (25 MARKS)**

**Question 3 (a) (i)**

$x$	3	4	5	6	7	8	9
$f(x)$	0	5	8	9	8	5	0

$$f(x) = -x^2 + 12x - 27$$

$$f(5) = -(5)^2 + 12(5) - 27 = -25 + 60 - 27 = 8$$

$$f(6) = -(6)^2 + 12(6) - 27 = -36 + 72 - 27 = 9$$

$$f(8) = -(8)^2 + 12(8) - 27 = -64 + 96 - 27 = 5$$

$$f(9) = -(9)^2 + 12(9) - 27 = -81 + 108 - 27 = 0$$

**Question 3 (a) (ii)**

$$h = 1$$

$$A = \frac{1}{2} \{0 + 0 + 2(5 + 8 + 9 + 8 + 5)\} = 35$$

**FORMULAE AND TABLES BOOK**  
**Area approximations** [page 12]

**Trapezoidal Rule:**

$$A \approx \frac{h}{2} [y_1 + y_n + 2(y_2 + y_3 + y_4 + \dots + y_{n-1})]$$

**MARKING SCHEME NOTES****Question 3 (a) (i) (ii) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Any one correct value  
• Writes formula
- 7: • Correct table
- 11: • Correct formula for trapezoidal rule, and some correct substitution with  $h = 1$   
• Completely incorrect table but applied correctly in a (ii)  
• Correct table and 35 without work

**Note 1:** Answers in terms of  $h$  merit Mid Partial at most

**Note 2:** Correct formula and some substitution gets High Partial

**Note 3:** No formula and  $\frac{1}{2}[5 + 5 + 2(8 + 9 + 8)] = 30$  gets High Partial

**Question 3 (b) (i)**

$$\begin{aligned} & \int_3^9 (-x^2 + 12x - 27) dx \\ &= \left[ -\frac{x^3}{3} + 6x^2 - 27x \right]_3^9 \\ &= \left( -\frac{(9)^3}{3} + 6(9)^2 - 27(9) \right) - \left( -\frac{(3)^3}{3} + 6(3)^2 - 27(3) \right) \\ &= 36 \end{aligned}$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Integrals [page 26]**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

**Question 3 (b) (ii)****FORMULA: % ERRORS**

Absolute error in a quantity = |Measured value of quantity – Accepted value of quantity|

Percentage error in a quantity =  $\frac{\text{Absolute error in quantity}}{\text{Accepted value of quantity}} \times 100\%$

$$\% \text{ error} = \frac{(36 - 35)}{36} \times 100\% = 2.8\%$$

**MARKING SCHEME NOTES****Question 3 (b) (i) (ii) [Scale 10C (0, 4, 8, 10)]**

- 4: • Any correct integration  
• Correct substitution of  $f(x)$   
• Correct % error formula  
• Correct substitution of  $f(x)$  i.e.  $-x^2 + 12x - 27$
- 8: • Correct integration with some correct substitution  
• 97.2%
- 10: • Accept 2.8% without work for full credit

**QUESTION 4 (25 MARKS)****Question 4 (a)**

$$z_2 = 2 + 3i, z_3 = 3 - 2i$$

$$\begin{aligned} \frac{2}{z_1} &= \frac{1}{z_2} + \frac{1}{z_3} = \frac{1}{2+3i} + \frac{1}{3-2i} \\ &= \frac{3-2i+2+3i}{(2+3i)(3-2i)} = \frac{5+i}{12+5i} \end{aligned}$$

$$\begin{aligned} \frac{z_1}{2} &= \frac{12+5i}{5+i} \\ &= \frac{12+5i}{5+i} \times \frac{5-i}{5-i} = \frac{65+13i}{26} = \frac{13(5+i)}{26} = \frac{5+i}{2} \end{aligned}$$

$$\therefore z_1 = 5+i$$

or

$$\frac{1}{z_2} = \frac{1}{2+3i} = \frac{1}{2+3i} \times \frac{2-3i}{2-3i} = \frac{2-3i}{13}$$

$$\frac{1}{z_3} = \frac{1}{3-2i} = \frac{1}{3-2i} \times \frac{3+2i}{3+2i} = \frac{3+2i}{13}$$

$$\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$$

$$\frac{2}{z_1} = \frac{2-3i}{13} + \frac{3+2i}{13} = \frac{2-3i+3+2i}{13} = \frac{5-i}{13}$$

$$\therefore z_1 = \frac{26}{5-i} = \frac{26}{5-i} \times \frac{5+i}{5+i} = \frac{26(5+i)}{26} = 5+i$$

**FORMULA: Complex Numbers**

Conjugates  $\bar{z}$

$$z = a + bi \Rightarrow \bar{z} = a - bi$$

Multiplying a complex number by its conjugate:

$$z\bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

**DIVISION OF COMPLEX NUMBERS**

$$\frac{1}{a+bi} = \frac{1}{a+bi} \times \frac{a-bi}{a-bi} = \frac{a-bi}{a^2+b^2}$$

**MARKING SCHEME NOTES****Question 4 (a) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Some rationalisation  
• Some relevant rearrangement

- 7: • Gets  $z_1$  or  $\frac{2}{z_1}$  in the form of  $\frac{a+bi}{c+di}$

- 11: • Correct use of conjugate in  $\frac{12+5i}{5+i}$

or

- 4: • One complex number correct  
7: • Two complex numbers correct

- 11: • Correct use of conjugate in  $\frac{2}{a+bi} = \frac{5-i}{13}$

**Question 4 (b)**

$$S_n = 1 + \omega + \omega^2 + \dots + \omega^{n-1}$$

$$a = 1, r = \omega$$

$$\therefore S_n = \frac{1(1-\omega^n)}{1-\omega}$$

$$\omega^n = 1$$

$$\therefore S_n = \frac{1(1-1)}{1-\omega} = 0$$

**FORMULAE AND TABLES BOOK****Sequences and series:  
Geometric series [page 22]**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$a$  is the first term

$r$  is the common ratio

**MARKING SCHEME NOTES****Question 4 (b) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- Correct Geometric Progression formula
  - Correct first term
  - Correct ratio
- 8:**
- Values substituted in formula

**QUESTION 5 (25 MARKS)****Question 5 (a)**

$$x = \sqrt{x+6} \quad \leftarrow \text{Square both sides.}$$

$$x^2 = x + 6$$

$$x^2 - x - 6 = 0 \quad \leftarrow \text{Factorise the quadratic.}$$

$$(x+2)(x-3) = 0$$

$$x = \cancel{-2}, 3$$

$$\therefore x = 3$$

Check each solution:

$$x = -2: \text{LHS: } x = -2$$

$$\text{RHS: } \sqrt{x+6} = \sqrt{-2+6} = \sqrt{4} = 2$$

Therefore,  $x = -2$  is not a solution.

$$x = 3: \text{LHS: } x = 3$$

$$\text{RHS: } \sqrt{x+6} = \sqrt{3+6} = \sqrt{9} = 3$$

Therefore,  $x = 3$  is a solution.

**MARKING SCHEME NOTES****Question 5 (a) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- Indication of squaring
- 8:**
- Correct roots
- Note:** must indicate required root

**Question 5 (b)**

$$y = x - \sqrt{x+6} = x - (x+6)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 1 - \frac{1}{2}(x+6)^{-\frac{1}{2}} = 1 - \frac{1}{2\sqrt{x+6}}$$

**FORMULAE AND TABLES BOOK****Calculus: Derivatives [page 25]**

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

$$y = [f(x)]^n \Rightarrow \frac{dy}{dx} = n[f(x)]^{n-1} \times f'(x)$$

**MARKING SCHEME NOTES****Question 5 (b) [Scale 5B (0, 2, 5)]**

- 2:**
- Any correct differentiation
  - Indication of  $(x+6)^{\frac{3}{2}}$

**Question 5 (c)**

$$\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{1}{2\sqrt{x+6}} = 0$$

$$1 = \frac{1}{2\sqrt{x+6}}$$

$$2\sqrt{x+6} = 1 \leftarrow \text{Square both sides.}$$

$$4(x+6) = 1$$

$$4x + 24 = 1$$

$$4x = -23$$

$$x = -\frac{23}{4}$$

$$x = -\frac{23}{4} : y = x - \sqrt{x+6} = -\frac{23}{4} - \sqrt{-\frac{23}{4} + 6} = -\frac{23}{4} - \sqrt{\frac{1}{4}} = -\frac{23}{4} - \frac{1}{2} = -\frac{25}{4}$$

$$\text{Turning point } \left(-\frac{23}{4}, -\frac{25}{4}\right)$$

FIND TURNING POINTS (LOCAL MAXIMUM/MINIMUM)

Put  $\frac{dy}{dx} = 0$  and solve for  $x$

**MARKING SCHEME NOTES****Question 5 (c) [Scale 10C (0, 4, 8, 10)]****4:** • Differentiation equals 0**8:** • Finds  $x$  value**Note 1:** A linear equation from  $f'(x)$  gets low partial at most**Note 2:** Must put  $f'(x) = 0$  in (c) to get any marks**Note 3:**  $f'(x)$  only and  $f''(x)$  only: no credit**QUESTION 6 (25 MARKS)****Question 6 (a)**(i) **Bank A:** Monthly interest rate  $r_M = 0.35\% \Rightarrow i_M = 0.0035$  $(1 + i_M)^{12} = (1 + i_A)$ , where  $i_A$  is the annual interest rate

$$(1 + 0.0035)^{12} = (1 + i_A)$$

$$(1.0035)^{12} = (1 + i_A)$$

$$1.042818 = 1 + i_A$$

$$i_A = 0.042818$$

 $\therefore$  Annual percentage rate (APR)  $r_A = 4.28\%$  [Given to 3 significant figures](ii) **Bank B:** Annual interest rate  $r_A = 4.5\% \Rightarrow i_A = 0.045$ Monthly interest rate:  $i_M = ?$ ,  $r_M = ?$ 

$$(1 + i_M)^{12} = (1 + 0.045)$$

$$(1 + i_M)^{12} = (1.045)$$

$$i_M = 1.045^{\frac{1}{12}} - 1 = 0.0036748$$

 $\therefore r_M = 0.367\%$  [Given to 3 significant figures]

**MARKING SCHEME NOTES****Question 6 (a) (i) (ii) [Scale 10C (0, 4, 8, 10)]**

- 4:** • Correct formula in either part  
• Correct substitution in incorrect formula

- 8:** • Any one section correct

**Note:** Rate as 0.367% or 0.00367 gets High Partial.

**Question 6 (b)****FORMULAE AND TABLES BOOK**

**Financial mathematics: Amortisation - mortgages and loans**  
(equal repayments at equal intervals) [page 31]

$$A = P \frac{i(1+i)^t}{(1+i)^t - 1}$$

$t$  = Time period (in years)

$i$  = (Annual) rate of interest expressed as a decimal

$A$  = (Annual) repayment amount

$P$  = Principal

**NOTE:** The time period can be months or weeks instead of years provided the interest rate is given for that time period.

**METHOD 1:** Use the amortisation formula

Time period: Months

$$r_M = 0.35\% \Rightarrow i_M = 0.0035$$

$$t = 10 \times 12 = 120 \text{ months}$$

$$P = \text{€}80\,000$$

$$\begin{aligned} A &= P \frac{i(1+i)^t}{(1+i)^t - 1} \\ &= 80\,000 \frac{0.0035(1.0035)^{120}}{(1.0035)^{120} - 1} \\ &= \text{€}817.59 \approx \text{€}818 \end{aligned}$$

**METHOD 2:** Use a geometric series

$$\begin{aligned} \frac{A}{1.0035} + \frac{A}{1.0035^2} + \dots + \frac{A}{1.0035^{120}} &= 80\,000 \\ A \left[ \frac{1}{1.0035} + \frac{1}{1.0035^2} + \dots + \frac{1}{1.0035^{120}} \right] &= 80\,000 \\ A \left[ \frac{\frac{1}{1.0035} \left( 1 - \left( \frac{1}{1.0035} \right)^{120} \right)}{1 - \frac{1}{1.0035}} \right] &= 80\,000 \\ \therefore A = \frac{80\,000 \left( 1 - \frac{1}{1.0035} \right)}{\frac{1}{1.0035} \left( 1 - \left( \frac{1}{1.0035} \right)^{120} \right)} &= \text{€}817.59 \approx \text{€}818 \end{aligned}$$

**FORMULAE AND TABLES BOOK**  
**Sequences and series:**  
**Geometric series** [page 22]

$$S_n = \frac{a(1-r^n)}{1-r}$$

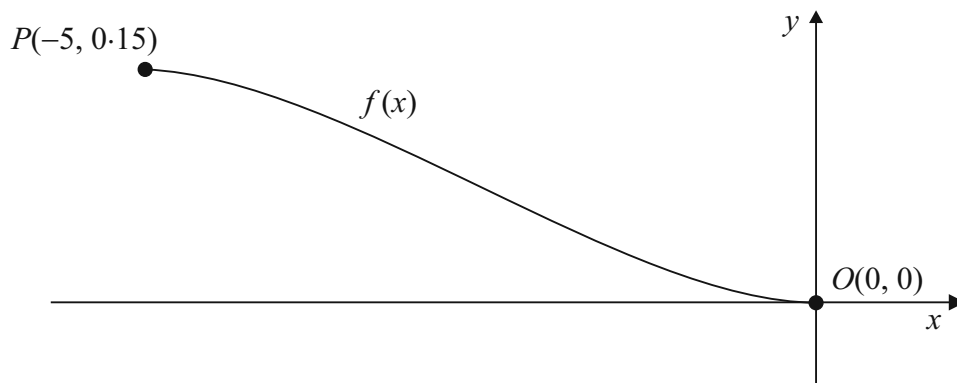
**MARKING SCHEME NOTES****Question 6 (b) [Scale 15C (0, 5, 10, 15)]** Note: two solutions1st solution**5:** • Any correct step, i.e. correct formula**10:** • Substitution in correct formula

or

2nd solution**5:** • Correct equation.

• Listing some terms

• Some substitution

**10:** • Complete substitution and effort at evaluation.**Note:** If  $A$  and 80 000 interchanged and remainder of work correct, may get High Partial credit.**QUESTION 7 (50 MARKS)****Question 7 (a) (i)**

$$f(x) = 0.0024x^3 + 0.018x^2 + cx + d \leftarrow (0, 0) \in f(x) \Rightarrow f(0) = 0.$$

$$f(0) = 0 \Rightarrow 0.0024(0)^3 + 0.018(0)^2 + c(0) + d = 0$$

$$\therefore d = 0$$

**MARKING SCHEME NOTES****Question 7 (a) (i) [Scale 5B (0, 2, 5)]****2:** • Recognises  $x = 0$ **Question 7 (a) (ii)**

$$f(x) = 0.0024x^3 + 0.018x^2 + cx \leftarrow (-5, 0.15) \in f(x) \Rightarrow f(-5) = 0.15.$$

$$f(-5) = 0.15 \Rightarrow 0.0024(-5)^3 + 0.018(-5)^2 + c(-5) = 0.15$$

$$0.15 - 5c = 0.15$$

$$5c = 0$$

$$\therefore c = 0$$

or

The plane land horizontally at  $O$ . Therefore  $f'(x) = 0$  when  $x = 0$ .

$$f(x) = 0.0024x^3 + 0.018x^2 + cx$$

$$f'(x) = 0.0072x^2 + 0.036x + c$$

$$f'(0) = 0 \Rightarrow f'(0) = 0.0072(0)^2 + 0.036(0) + c = 0$$

$$\therefore c = 0$$

**MARKING SCHEME NOTES**

**Question 7 (a) (ii) [Scale 5B (0, 2, 5)]** Note: two solutions

1st solution

2: • Uses  $x = -5$  or  $f(x) = 0.15$

5: • Begins with  $c = 0$  and shows  $f(-5) = 0.15$  or similar

or

2nd solution

2: • Uses  $x = -5$

• Gets  $f'(x)$

• Uses  $f'(x) = 0$  when  $x = 0$

**Question 7 (b) (i)**

$$f(x) = 0.0024x^3 + 0.018x^2$$

$$f'(x) = 0.0072x^2 + 0.036x$$

$$f'(-4) = 0.0072(-4)^2 + 0.036(-4)$$

$$= -\frac{18}{625}$$

$$= -0.0288$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Derivatives [page 25]**

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

$$y = [f(x)]^n \Rightarrow \frac{dy}{dx} = n[f(x)]^{n-1} \times f'(x)$$

**MARKING SCHEME NOTES**

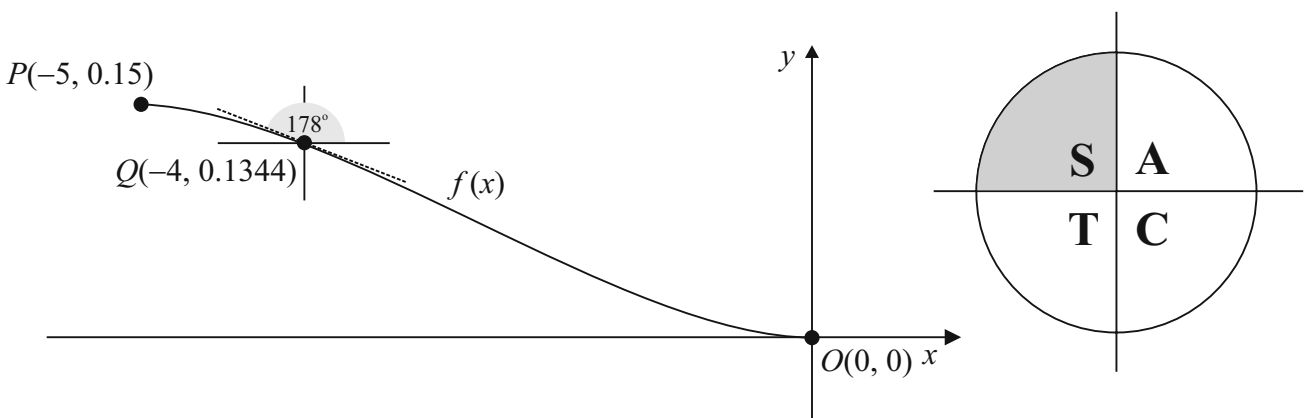
**Question 7 (b) (i) [Scale 10C (0, 3, 7, 10)]**

3: • Any term correctly differentiated

7: • Correct differentiation

10: •  $-\frac{18}{625}$  is a correct answer

**Question 7 (b) (ii)**



The slope of the tangent to  $f(x)$  at  $x = -4$  is given by  $f'(-4)$ . The slope is also the tan the angle makes with the  $+x$ -axis.

$$\tan \theta = -0.0288 \text{ [Second quadrant]}$$

$$\alpha = \tan^{-1}(0.0288) = 1.65^\circ \text{ [Reference angle in first quadrant]}$$

$$\theta = 180^\circ - 1.65^\circ \approx 178^\circ \text{ [Second quadrant]}$$

Angle of descent =  $2^\circ$

**MARKING SCHEME NOTES**

**Question 7 (b) (ii) [Scale 5B (0, 2, 5)]**

- 2: • Recognition of connection between slope and  $\tan \theta$   
 • Any right angled triangle

**Question 7 (c)**

$$f'(x) = 0.0072x^2 + 0.036x$$

$$f''(x) = 0.0144x + 0.036$$

$$f''(x) = 0 \Rightarrow 0.0144x + 0.036 = 0$$

$$\therefore x = -2.5$$

$$f(x) = 0.0024x^3 + 0.018x^2$$

$$f(-2.5) = 0.0024(-2.5)^3 + 0.018(-2.5)^2 = 0.075$$

Point of inflection  $(-2.5, 0.075)$

POINT OF INFLECTION: Put  $\frac{d^2y}{dx^2} = 0$   
 or  $f''(x) = 0$  and solve for  $x$ .

**MARKING SCHEME NOTES**

**Question 7 (c) [Scale 10D (0, 2, 5, 8, 10)]**

- 2: • Some correct differentiation of  $f'(x)$   
 • Mention of  $f'(x)$   
 5: • Correct  $f''(x)$   
 8: • Value of  $x$  substituted

**Question 7 (d) (i)**

$$y = 0.0024x^3 + 0.018x^2$$

$$\begin{aligned} f(-x-5) &= 0.0024(-x-5)^3 + 0.018(-x-5)^2 \\ &= (-x-5)^2 [0.0024(-x-5) + 0.018] \\ &= (x^2 + 10x + 25)[-0.0024x - 0.012 + 0.018] \\ &= (x^2 + 10x + 25)[-0.0024x + 0.006] \\ &= -0.0024x^3 + 0.006x^2 - 0.024x^2 + 0.06x - 0.06x + 0.15 \\ &= -0.0024x^3 - 0.018x^2 + 0.15 \\ &= -(0.0024x^3 + 0.018x^2) + 0.15 \\ &= -y + 0.15 \end{aligned}$$

**MARKING SCHEME NOTES**

**Question 7 (d) (i) [Scale 5C (0, 2, 4, 5)]**

- 2: • Some correct substitution  
 4: • Correct expansions

**Question 7 (d) (ii)**

$$-x - 5 \rightarrow -2 \cdot 5 [x + 2.5]$$

$$-y + 0 \cdot 15 \rightarrow 0 \cdot 075 [y - 0 \cdot 075]$$

$$-x - 5 \rightarrow -2 \cdot 5 \rightarrow x$$

$$-y + 0 \cdot 15 \rightarrow 0 \cdot 075 \rightarrow y$$

$$\therefore (-x - 5, -y + 0 \cdot 15) \rightarrow (-2 \cdot 5, 0 \cdot 075) \rightarrow (x, y)$$

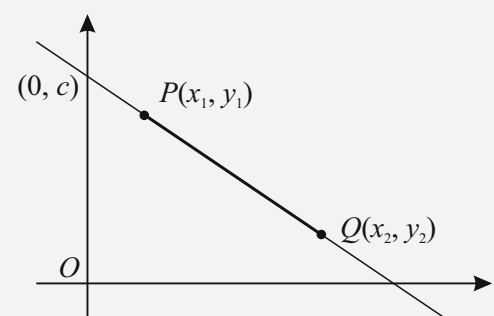
**or**

Let  $(x, y)$  be the image.

The point of inflection will be the midpoint of  $(-x - 5, -y + 0 \cdot 15)$  and  $(x, y)$ .

$$\begin{aligned} \text{Midpoint} &= \left( \frac{-x - 5 + x}{2}, \frac{-y + 0 \cdot 15 + y}{2} \right) \\ &= (-2 \cdot 5, 0 \cdot 075) \end{aligned}$$

**FORMULAE AND TABLES BOOK**  
**Co-ordinate geometry: Line**



Midpoint of  $PQ$  [page 18]  
 $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

**MARKING SCHEME NOTES**

**Question 7 (d) (ii) [Scale 10C (0, 4, 8, 10)] Note:** two solutions

1st solution

**4:** • Work leading to change in  $x$ -value or  $y$ -value

**8:** • Correct change in  $x$  and  $y$  values

**or**

2nd solution

**4:** • Uses  $(x, y)$  as image, and no more

**8:** • Effort at calculating mid-point

**QUESTION 8 (50 MARKS)**

**Question 8 (a) (i)**

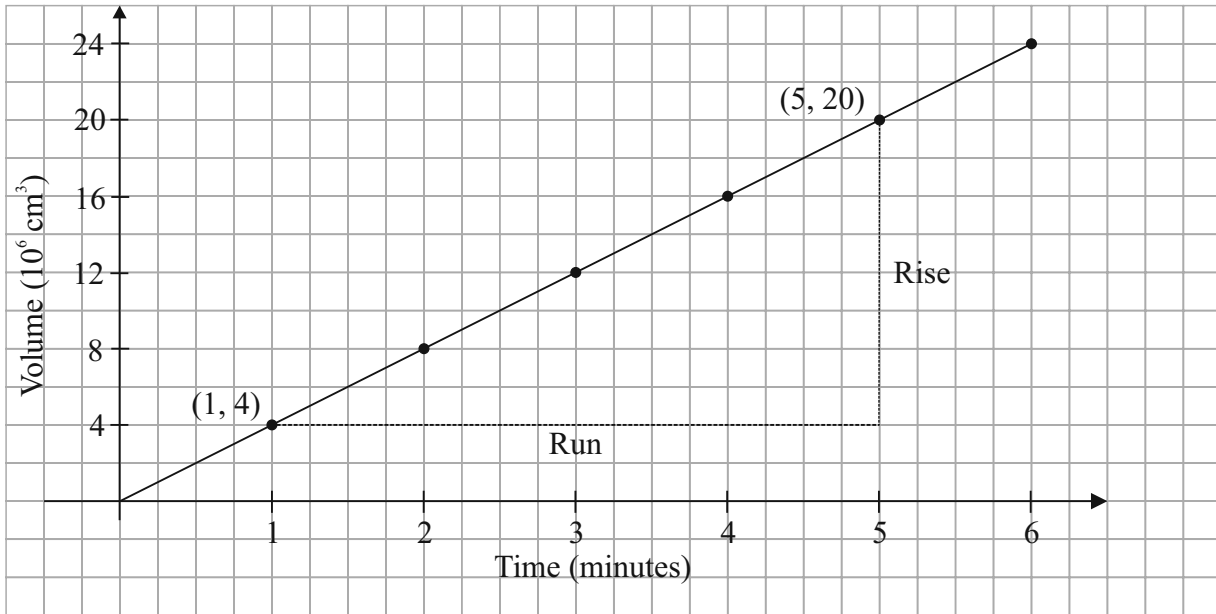
Volume  $V = 4 \times 10^6 \text{ cm}^3$  per minute

Time  $t = 1$  minute  $\Rightarrow V = 4 \times 10^6 \text{ cm}^3$

Time  $t = 2$  minutes  $\Rightarrow V = 4 \times 10^6 \times 2 = 8 \times 10^6 \text{ cm}^3$

Time (minutes)	1	2	3	4	5	6
Volume ( $10^6 \text{ cm}^3$ )	4	8	12	16	20	24

**Question 8 (a) (ii)**



**Question 8 (a) (iii)**

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{20 \times 10^6 - 4 \times 10^6}{5 - 1} = \frac{16 \times 10^6}{4} = 4 \times 10^6$$

$$V = 4 \times 10^6 t \text{ [} y = mx \text{]}$$

**MARKING SCHEME NOTES**

**Question 8 (a) (i) [Scale 5B (0, 2, 5)]**

2: • One correct box

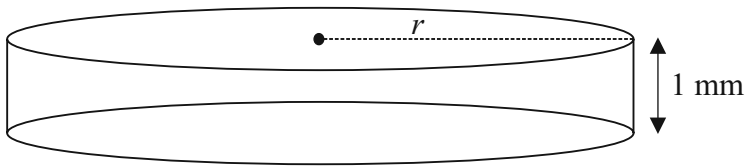
**Question 8 (a) (ii) [Scale 5B (0, 2, 5)]**

0: • Bar chart  
2: • At least two points plotted

**Question 8 (a) (iii) [Scale 5B (0, 2, 5)]**

2: • Incomplete equation for volume  
•  $V =$  any function of  $t$   
• Attempt at finding slope

**Question 8 (b) (i)**



$$V = \pi r^2 h = \pi r^2 (0.1) = \frac{1}{10} \pi r^2 \text{ cm}^3$$

**Question 8 (b) (ii)**

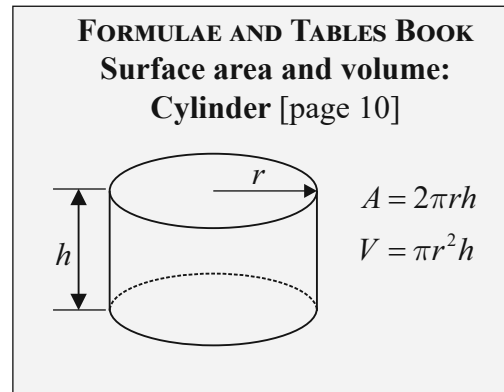
$$V = 4 \times 10^6 t \Rightarrow \frac{dV}{dt} = 4 \times 10^6$$

$$V = \frac{1}{10} \pi r^2 \Rightarrow \frac{dV}{dt} = \frac{1}{5} \pi r \times \frac{dr}{dt} = 4 \times 10^6$$

$$r = 50 \text{ m} = 5000 \text{ cm} : 4 \times 10^6 = \frac{1}{5} \pi (5000) \times \frac{dr}{dt}$$

$$4000 = \pi \times \frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = \frac{4000}{\pi} \text{ cm/minute} = 1273.3 \text{ cm/minute}$$



**MARKING SCHEME NOTES**

**Question 8 (b) (i) [Scale 5B (0, 2, 5)]**

- 2: • Correct volume formula  
 • Converting mm to cm

**Question 8 (b) (ii) [Scale 10D (0, 2, 5, 8, 10)]**

- 2: • Mentions a relevant rate of change  
 5: • Gets  $\frac{dr}{dt}$  from  $\frac{dV}{dr}$  and  $\frac{dV}{dt}$   
 • Writing down chain rule  
 8: • Substitution of values

**Question 8 (c)**

$$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \times \frac{dr}{dt}$$

$$\frac{1}{5} \pi r \times \frac{dr}{dt} = 4 \times 10^6 \Rightarrow \frac{dr}{dt} = \frac{5 \times 4 \times 10^6}{\pi r}$$

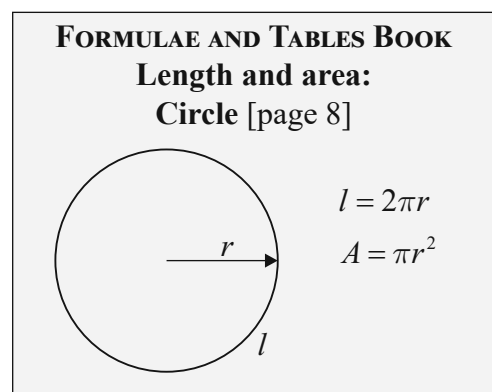
$$\therefore \frac{dA}{dt} = 2\pi r \times \frac{5 \times 4 \times 10^6}{\pi r} = 4 \times 10^7 \text{ cm}^2/\text{minute}$$

or

$$\frac{1}{10} \pi r^2 = (4 \times 10^6) t$$

$$A = \pi r^2 = (4 \times 10^7) t$$

$$\therefore \frac{dA}{dt} = 4 \times 10^7 \text{ cm}^2/\text{minute}$$



**MARKING SCHEME NOTES****Question 8 (c) [Scale 10C (0, 4, 8, 10)]** Note: two solutions1st solution**4:** • Mentions relevant rate of change**8:** • States chain rule i.e.  $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$   
or2nd solution**4:** • Effort to establish value of  $A$ **8:** •  $A$  in terms of  $t$ **NOTE:** Must use calculus to get any credit**Question 8 (d)**

$$r = 1 \text{ km} = 10^5 \text{ cm}$$

$$A = \pi r^2 = \pi(10^5)^2 = 10^{10} \pi \text{ cm}^2$$

$$\frac{dA}{dt} = 4 \times 10^7 \text{ cm}^2/\text{minute}$$

$$\text{Time } t = \frac{10^{10} \pi \text{ cm}^2}{4 \times 10^7 \text{ cm}^2/\text{minute}} = 785.4 \text{ minutes} \approx 13 \text{ hours}$$

**or**

$$r = 1 \text{ km} = 10^5 \text{ m}$$

$$\frac{dr}{dt} = \frac{20 \times 10^6}{\pi r}$$

$$\pi r dr = 20 \times 10^6 dt$$

$$\int_0^{10^5} \pi r dr = \int_0^t 20 \times 10^6 dt$$

$$\left[ \frac{\pi r^2}{2} \right]_0^{10^5} = [20 \times 10^6 t]_0^t$$

$$\frac{\pi(10^5)^2}{2} - \frac{\pi(0)^2}{2} = 20 \times 10^6 t$$

$$10^{10} \pi = 40 \times 10^6 t$$

$$10^4 \pi = 40t$$

$$\therefore t = \frac{10^4 \pi}{40} = 250\pi \text{ minutes} = \frac{250\pi}{60} \text{ hours} \approx 13 \text{ hours}$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Integrals [page 26]**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

**MARKING SCHEME NOTES****Question 8 (d) [Scale 10C (0, 4, 8, 10)]****4:** •  $r$  in centimetres  
• Effort at expression of area**8:** • Correct expression for time

**QUESTION 9 (50 MARKS)****Question 9 (a)**

$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) \leftarrow$  Replace  $t$  by 76 in this function and use your calculator in radian mode.

$$f(76) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times 76\right) = 16 \cdot 835 \text{ hours} = 16 \text{ h } 50 \text{ mins}$$

**MARKING SCHEME NOTES****Question 9 (a) [Scale 10C (0, 4, 8, 10)]**

4: • Uses  $t = 76$

8: • Correct substitution

**Note:** Using  $\pi = 90^\circ \Rightarrow$  one error, but do not penalise again in (b)

**Question 9 (b)**

$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) = 15 \leftarrow$  Put  $f(t)$  equal to 15 and use your calculator to find  $t$ .

$$4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) = 2 \cdot 75$$

$$\sin\left(\frac{2\pi}{365}t\right) = \frac{2 \cdot 75}{4 \cdot 75} = \frac{11}{19}$$

$$t = \frac{365}{2\pi} \sin^{-1}\left(\frac{11}{19}\right) \approx 35 \cdot 87 \text{ days}$$

36 days after 21st March is 26th April

**MARKING SCHEME NOTES****Question 9 (b) [Scale 10C (0, 4, 8, 10)]**

4: • Correct  $f(t)$

•  $f(15)$  substituted

8: • Correct equation with  $t$  only

**Note:** Accept 35 or 36 substituted correctly and tested

**Question 9 (c)**

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$f'(t) = 4 \cdot 75 \cos\left(\frac{2\pi}{365}t\right) \times \frac{2\pi}{365} = \frac{19\pi}{730} \cos\left(\frac{2\pi}{365}t\right)$$

**FORMULAE AND TABLES BOOK****Calculus: Integrals [page 25]**

$$y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$$

$$y = \sin f(x) \Rightarrow \frac{dy}{dx} = \cos f(x) \times f'(x)$$

**MARKING SCHEME NOTES****Question 9 (c) [Scale 10B (0, 5, 10)]**

5: • Any correct differentiation (note: '0' could be correct differentiation here)

**Note:** Substituting  $180^\circ$  for  $\pi \Rightarrow$  one error

**Question 9 (d)**

$$f'(t) = 0 \Rightarrow \frac{19\pi}{730} \cos\left(\frac{2\pi}{365}t\right) = 0$$

$$\cos\left(\frac{2\pi}{365}t\right) = 0$$

$$\frac{2\pi}{365}t = \cos^{-1} 0 = \frac{\pi}{2}$$

$$t = \frac{365}{4}$$

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$\therefore f\left(\frac{365}{4}\right) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times \frac{365}{4}\right) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{\pi}{2}\right) = 12 \cdot 25 + 4 \cdot 75 = 17 \text{ hours}$$

FIND TURNING POINTS (LOCAL MAXIMUM/MINIMUM)

Put  $\frac{dy}{dx} = 0$  and solve for  $x$

**MARKING SCHEME NOTES****Question 9 (d) [Scale 10D (0, 2, 5, 8, 10)] – both solutions**2: •  $f'(t) = 0$ 5: • Value of  $t$ 8: • Value of  $t$  substituted into  $f(t)$ •  $f(t)$  maximum when  $\sin \theta = 1$ **Note:** Accept 91 or 92 substituted and evaluated correctly for full marks**Question 9 (e)**

$$\bar{L} = \frac{1}{184-0} \int_0^{184} \left[ 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) \right] dt$$

$$= \frac{1}{184} \left[ 12 \cdot 25t - 4 \cdot 75 \cos\left(\frac{2\pi}{365}t\right) \times \frac{365}{2\pi} \right]_0^{184}$$

$$= \frac{1}{184} \left[ 12 \cdot 25t - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365}t\right) \right]_0^{184}$$

$$= \frac{1}{184} \left[ \left( 12 \cdot 25(184) - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365} \times 184\right) \right) - \left( 12 \cdot 25(0) - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365}(0)\right) \right) \right]$$

$$= \frac{1}{184} \left[ \left( 12 \cdot 25(184) - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365} \times 184\right) + \frac{4 \cdot 75 \times 365}{2\pi} \right) \right]$$

$$= 15 \cdot 2488 \text{ hours} = 15 \text{ hours } 15 \text{ minutes}$$

**FORMULAE AND TABLES BOOK****Calculus: Integrals [page 26]**

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + c$$

**MARKING SCHEME NOTES****Question 9 (e) [Scale 10D (0, 2, 5, 8, 10)]**2: • Correct expression in  $x$  or  $t$ 

• Correct formula

• Correct limits

5: • Any correct integration

8: • Correct integration and effort at substitution

**Note:** Integration with one error but finished correctly gets High Partial Credit

## LC 2015 (SET D): PAPER 2

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

W = Win, L = Loss

		Die 2					
		1	2	3	4	5	6
Die 1	1	[2] = L	[3] = L	[4] = L	[5] = L	[6] = L	[7] = L
	2	[3] = L	[4] = L	[5] = L	[6] = L	[7] = L	[8] = L
	3	[4] = L	[5] = L	[6] = L	[7] = L	[8] = L	[9] = W
	4	[5] = L	[6] = L	[7] = L	[8] = L	[9] = W	[10] = W
	5	[6] = L	[7] = L	[8] = L	[9] = W	[10] = W	[11] = W
	6	[7] = L	[8] = L	[9] = W	[10] = W	[11] = W	[12] = W

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 10C (0, 4, 8, 10)]

- 4:**
- At least one other correct entry
  - Partially correct table with at least 5 correct totals or couples
- 8:**
- Five or more correct entries including at least one other loss and one other win
  - Table correctly completed with totals or couples but no indication of W or L

#### Question 1 (b)

$$P(\text{Event}) = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

$$(i) P(W) = \frac{\text{Number of wins}}{\text{Number of possible outcomes}} = \frac{10}{36} = \frac{5}{18}$$

$$(ii) P(L) = \frac{\text{Number of losses}}{\text{Number of possible outcomes}} = \frac{26}{36} = \frac{13}{18}$$

$$P(\text{L and then L and then L}) = \frac{13}{18} \times \frac{13}{18} \times \frac{13}{18} = 0.3767$$

#### MARKING SCHEME NOTES

##### Question 1 (b) [Scale 10C (0, 4, 8, 10)]

- 4:**
- Favourable outcomes identified
  - (i) correct only ( $\frac{10}{36}$ ,  $\frac{5}{18}$ , 0.27, 0.28, 0.3)
- 8:**
- (i) omitted or of no merit but (ii)  $\left(\frac{13}{18}\right)^3$

### Question 1 (c)

This is a possible outcome: L W L L L W L L L W

The last W *has* to happen. The other 9 letters can be in any order.

$$P(3W\text{'s with a W on tenth throw}) = {}^9C_2 \left(\frac{5}{18}\right)^2 \left(\frac{13}{18}\right)^7 \times \left(\frac{5}{18}\right) = 0.0791$$

#### MARKING SCHEME NOTES

##### Question 1 (c) [Scale 5C (0, 2, 4, 5)]

- 2: • Relevant binomial formula with some substitution  
• Identifies  $p^7$  or  $(1-p)^3$  or  $(1-p)^2$  or  $1-p$   
• Listing at least any two of the ten throws
- 4: • Probability of two wins in nine throws

### QUESTION 2 (25 MARKS)

#### Question 2 (a)

Number in the sample  $n = 100$

Mean shopping spend  $\bar{x} = 90.45$

Standard deviation  $\sigma = 20.73$

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{20.73}{\sqrt{100}} = 2.073$$

Confidence interval:

$$\bar{x} - 1.96\bar{\sigma} \leftrightarrow \bar{x} + 1.96\bar{\sigma}$$

$$90.45 - 1.96(2.073) \leftrightarrow 90.45 + 1.96(2.073)$$

$$€86.39 \leftrightarrow €94.51$$

You can be 95% confident that the mean amount spent was in the range  $€86.39 < \mu < €94.51$ .

**FORMULAE AND TABLES BOOK**  
**Statistics and Probability: Sampling**  
(standard error of the mean) [page 34]

$$\bar{\sigma} = \frac{\sigma}{\sqrt{n}}$$

$n$  = Number in the sample

$\sigma$  = standard deviation of the sample

$$\text{Confidence interval: } \bar{x} - 1.96\bar{\sigma} \leftrightarrow \bar{x} + 1.96\bar{\sigma}$$

#### MARKING SCHEME NOTES

##### Question 2 (a) [Scale 10C (0, 4, 8, 10)]

- 4: • Relevant formula with or without substitution  
•  $\frac{1}{\sqrt{n}}$  with further work
- 8: •  $1.96 \times \frac{\sigma}{\sqrt{n}}$  evaluated

#### Question 2 (b)

$H_0$ : Mean  $\mu = €94$  ← Null hypothesis: Mean spend is €94

$H_1$ : Mean  $\mu \neq €94$  ← Alternative hypothesis: Mean spend is not €94

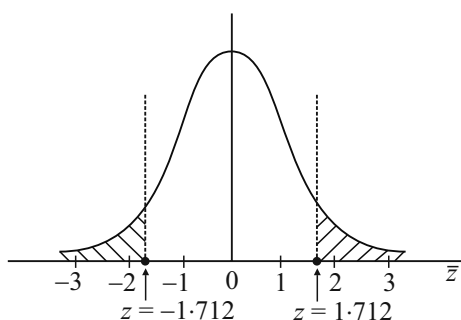
Since the mean  $\mu$  is in the confidence interval, you cannot reject the null hypothesis.

**MARKING SCHEME NOTES****Question 2 (b) [Scale 10D (0, 2, 5, 8, 10)]**

- 2: • One relevant step e.g. null hypothesis or alternative hypothesis stated  
 • Some work towards finding  $z$   
 • Mention of  $\pm 1.96$
- 5: •  $z$  calculated  
 • Either null or alternative hypothesis stated and relevant work towards finding  $z$   
 • Confidence interval from (a) and either null or alternative hypothesis stated  
 • Confidence interval based on 100 (i.e. 89.94, 98.06) and either null or alternative hypothesis stated
- 8: •  $z$  calculated and compared to  $\pm 1.96$  but:  
 o Not stating null hypothesis and/or alternative hypothesis correctly  
 o Not accepting or rejecting hypothesis  
 o Incorrect conclusion for hypothesis  
 • Incorrect use of 94 and confidence interval  
 • Incorrect use of 90.45 and confidence interval

**Question 2 (c)**Mean shopping spend  $\bar{x} = 90.45$ Standard error of the mean  $\bar{\sigma} = 2.073$ Mean amount spend  $\mu = 94$ 

$$\bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}} = \frac{90.45 - 94}{2.073} = -1.712$$

 $p$ -value = 0.0872

**Explanation:** Because  $p = 8.72\%$  is greater than 5% there is not a significant difference between the sample mean and the population mean. Any difference may be due to chance.

**FORMULAE AND TABLES BOOK**  
**Statistics and Probability: Probability distribution** (standardising formula) [page 34]

$$\bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}}$$

 $n$  = Number in the sample $\sigma$  = standard deviation of the sample

$$\begin{aligned} p\text{-value} &= 1 - P(\bar{z} < 1.712) + P(\bar{z} < -1.712) \\ &= 1 - P(\bar{z} < 1.712) + 1 - P(\bar{z} < 1.712) \\ &= 2(1 - P(\bar{z} < 1.712)) \\ &= 2(1 - 0.9564) \\ &= 0.0872 > 0.05 \end{aligned}$$

**MARKING SCHEME NOTES****Question 2 (c) [Scale 5C (0, 2, 4, 5)]**

- 2: • Effort at finding  $P(z < -1.71)$
- 4: •  $p$  value correct  
 • Not contextualising answer correctly

**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$A(4, -1), B(7, t)$$

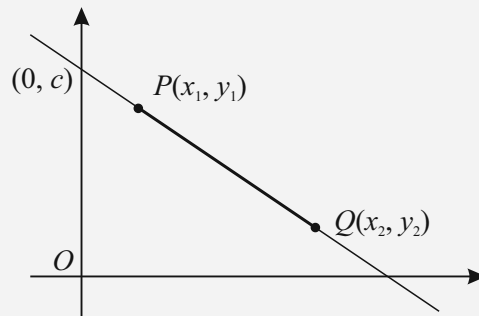
$$m = \frac{t - (-1)}{7 - 4} = \frac{t + 1}{3}$$

$$l_1: 3x - 4y - 12 = 0 \Rightarrow m = \frac{3}{4}$$

$$\therefore m_{\perp} = -\frac{4}{3}$$

$$\therefore \frac{t + 1}{3} = -\frac{4}{3}$$

$$t + 1 = -4 \Rightarrow t = -5$$

**FORMULAE AND TABLES BOOK****Co-ordinate geometry: Line**Slope of  $PQ$  [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**MARKING SCHEME NOTES****Question 3 (a) [Scale 10D (0, 2, 5, 8, 10)]**

- 2: • Slope  $AB$  or  $l_1$
- 5: • Both slopes found
- 8: • Slopes linked to perpendicularity

**Question 3 (b)**Call this distance  $d_1$ .

$$l_1: 3x - 4y - 12 = 0$$

$$P(10, k)$$

$$d_1 = \frac{|3(10) - 4k - 12|}{\sqrt{3^2 + (-4)^2}} = \frac{|18 - 4k|}{5}$$

Finding the slope from the equation of a line:

$$l: ax + by + c = 0$$

$$m = -\frac{a}{b} \Rightarrow m_{\perp} = \frac{b}{a}$$

**MARKING SCHEME NOTES****Question 3 (b) [Scale 10C (0, 4, 8, 10)]**

- 4: • Relevant formula with some correct substitution
- 8: • Substitution into formula fully correct

**Question 3 (c) (i)**

If  $P(10, k)$  lies on the bisector of the angles between  $l_1$  and  $l_2$ , it is the same perpendicular distance from each line.

$$l_2 : 5x + 12y - 20 = 0$$

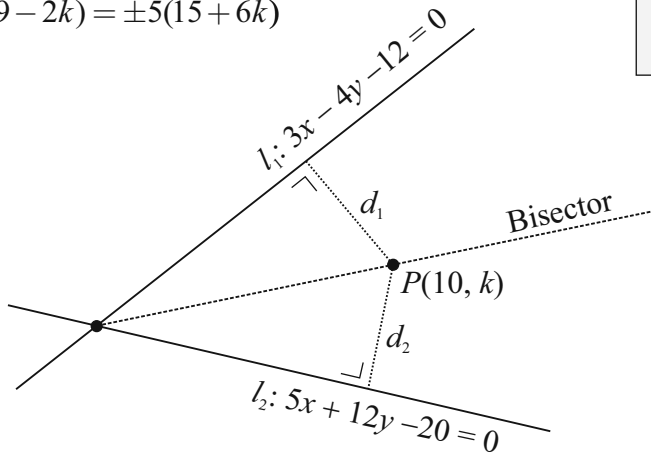
$$P(10, k)$$

$$d_2 = \frac{|5(10) + 12k - 20|}{\sqrt{5^2 + 12^2}} = \frac{|30 + 12k|}{13}$$

$$d_1 = d_2 \Rightarrow \frac{|18 - 4k|}{5} = \frac{|30 + 12k|}{13}$$

$$13(18 - 4k) = \pm 5(30 + 12k)$$

$$13(9 - 2k) = \pm 5(15 + 6k)$$



$$13(9 - 2k) = 5(15 + 6k) \quad \dots \quad 13(9 - 2k) = -5(15 + 6k)$$

$$117 - 26k = 75 + 30k \quad \dots \quad 117 - 26k = -75 - 30k$$

$$42 = 56k \quad \dots \quad 192 = -4k$$

$$\therefore k = \frac{42}{56} = \frac{3}{4} \quad \dots \quad \therefore k = -48$$

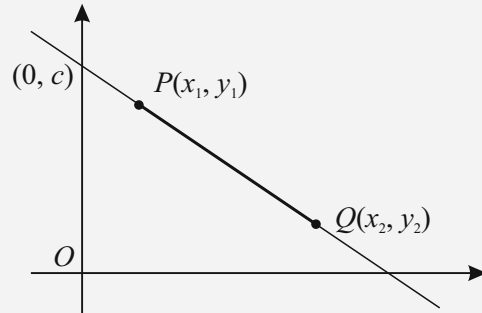
**Question 3 (c) (ii)**

$$d_1 = \frac{|18 - 4(\frac{3}{4})|}{5} = \frac{|18 - 3|}{5} = \frac{15}{5} = 3$$

**FORMULAE AND TABLES BOOK**

**Co-ordinate geometry: Line**

Distance from  $(x_1, y_1)$  to the line  $ax + by + c = 0$  [page 19]



$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

**MARKING SCHEME NOTES**

**Question 3 (c) (i) (ii) [Scale 5D (0, 2, 3, 4, 5)]**

**2:** • Relevant formula with some correct substitution

**3:** • One value for  $k$  found  
• Work indicating two values for  $k$

**4:** • Both values of  $k$   
• Positive value for  $k$  evaluated and distance calculated

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**

Call  $A$  the centre of circle  $s_1$ .

$$(x - 1)^2 + (y + 6)^2 = 360$$

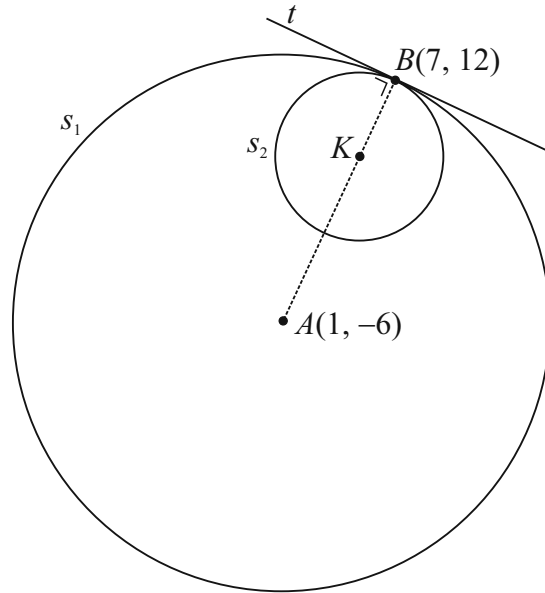
Centre  $A(1, -6)$

$$\text{Radius } r_1 = \sqrt{360} = 6\sqrt{10}$$

**MARKING SCHEME NOTES**

**Question 4 (a) [Scale 5B (0, 2, 5)]**

2: • Centre or radius

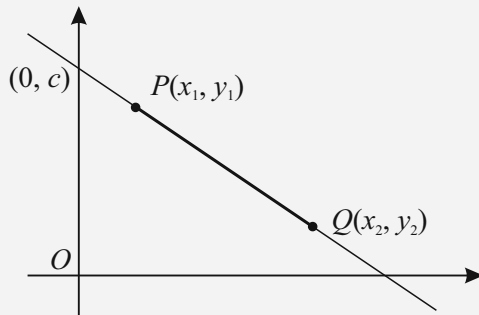


**Question 4 (b) (i)**

**FORMULAE AND TABLES BOOK**

**Co-ordinate geometry: Line [page 18]**

Point dividing  $[PQ]$  in the ratio  $a:b$



$$\left( \frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a} \right)$$

**FORMULAE AND TABLES BOOK**

**Co-ordinate geometry: Circle [page 19]**

Given centre  $(h, k)$  and radius  $r$

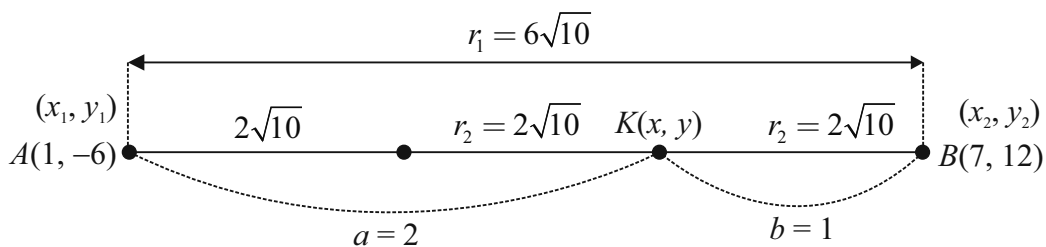
$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre  $(-g, -f)$

Radius  $\sqrt{g^2 + f^2 - c}$



$$K(x, y) = \left( \frac{1(1) + 2(7)}{1 + 2}, \frac{1(-6) + 2(12)}{1 + 2} \right) = K(5, 6)$$

**MARKING SCHEME NOTES**

**Question 4 (b) (i) [Scale 5D (0, 2, 4, 5)]**

- 2: • Formula for ratio with some correct substitution  
• Effort at setting up translation
- 4: • Substitution into ratio formula fully correct  
• One ordinate only found  
• Correct answer without supporting work

**Question 4 (b) (ii)**

Centre  $K(5, 6) = (h, k)$ ,  $r_2 = 2\sqrt{10}$

Equation of  $s_2$ :  $(x-5)^2 + (y-6)^2 = (2\sqrt{10})^2$

$$(x-5)^2 + (y-6)^2 = 40$$

**MARKING SCHEME NOTES****Question 4 (b) (ii) [Scale 10C (0, 4, 8, 10)]**

- 4: • Identifies centre  
• Identifies radius
- 8: • Equation of circle formed but error in substitution

**Question 4 (c)**

Call  $t$  the equation of the common tangent.

Line  $AB$  is perpendicular to  $t$ .

Finding the slope from the equation of a line:

$$l: ax + by + c = 0$$

$$m = -\frac{a}{b} \Rightarrow m_{\perp} = \frac{b}{a}$$

$$\text{Slope of } AB: m = \frac{12 - (-6)}{7 - 1} = 3$$

$$\text{Slope of } t: m_{\perp} = -\frac{1}{3}$$

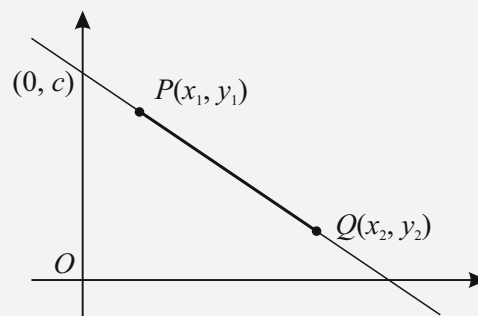
$$\text{Equation of } t: B(7, 12) = (x_1, y_1), m = -\frac{1}{3}$$

$$y - 12 = -\frac{1}{3}(x - 7)$$

$$3(y - 12) = -1(x - 7)$$

$$3y - 36 = -x + 7$$

$$x + 3y - 43 = 0$$

**FORMULAE AND TABLES BOOK**  
**Co-ordinate geometry: Line**

Slope of  $PQ$  [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of  $PQ$  [page 18]

$$y - y_1 = m(x - x_1)$$

**MARKING SCHEME NOTES****Question 4 (c) [Scale 5C (0, 2, 4, 5)]**

- 2: • Slope  $AB$  or slope of tangent  
• Some correct substitution into relevant formula
- 4: • Equation of line fully substituted

**QUESTION 5 (25 MARKS)****Question 5 (a)**

You are asked to prove an identity. You have to show that the left-hand side (LHS) is equal to the right-hand side (RHS).

Turn all trigonometric functions into sin and cos.

**FORMULAE AND TABLES BOOK****Trigonometry (page 13/14)**

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

**LHS**

$$\begin{aligned} & \tan(A + B) \\ = & \frac{\sin(A + B)}{\cos(A + B)} \\ = & \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \end{aligned}$$

**RHS**

$$\begin{aligned} & \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ = & \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A}{\cos A} \times \frac{\sin B}{\cos B}} \\ = & \frac{\left(\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}\right) \times \cos A \cos B}{\left(1 - \frac{\sin A}{\cos A} \times \frac{\sin B}{\cos B}\right) \times \cos A \cos B} \\ = & \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \end{aligned}$$

$$\therefore \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

**MARKING SCHEME NOTES****Question 5 (a) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Tan function in terms of Sine and Cosine
- 7: •  $\sin(A + B)$  or  $\cos(A + B)$  expanded  
• Numerator or denominator in fraction form
- 11: • Numerator and denominator divided by  $\cos A \cos B$   
• Both numerator and denominator expressed in form of a single fraction

**Question 5 (b)**

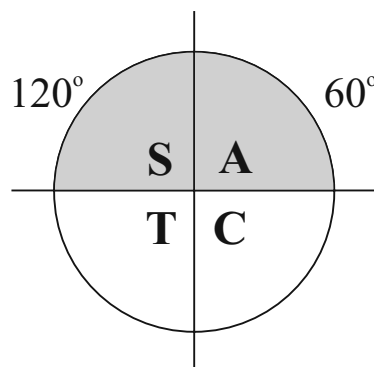
$$\sin(3x) = \frac{\sqrt{3}}{2}$$

$$\text{Reference angle } \alpha = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

$$3x = \begin{cases} 60^\circ, 420^\circ, 780^\circ \text{ [First quadrant]} \\ 120^\circ, 480^\circ, 840^\circ \text{ [Second quadrant]} \end{cases}$$

$$x = \begin{cases} 20^\circ, 140^\circ, 260^\circ \\ 40^\circ, 160^\circ, 280^\circ \end{cases}$$

$$\therefore x = 20^\circ, 40^\circ, 140^\circ, 160^\circ, 260^\circ, 280^\circ$$



**MARKING SCHEME NOTES**

**Question 5 (b) [Scale 10D (0, 2, 5, 8, 10)]**

2: • One value for  $3x$

5: • One value for  $x$

• Two or more values for  $3x$

8: • Three or more values for  $x$

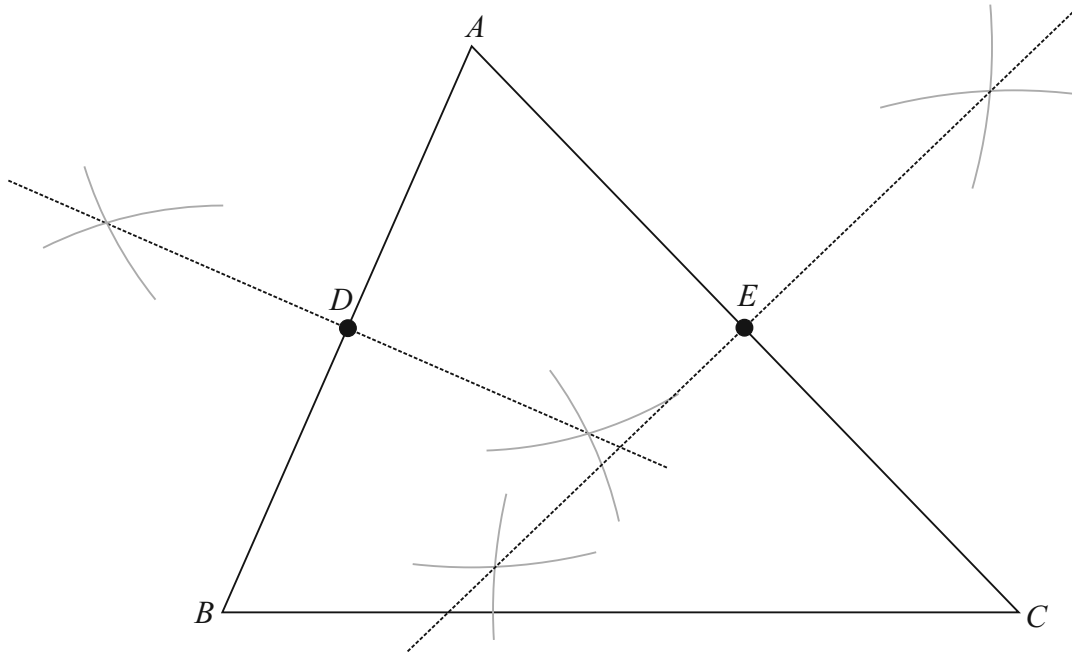
**QUESTION 6 (25 MARKS)**

**Question 6 (a)**

The centroid of a triangle is the intersection of the medians.

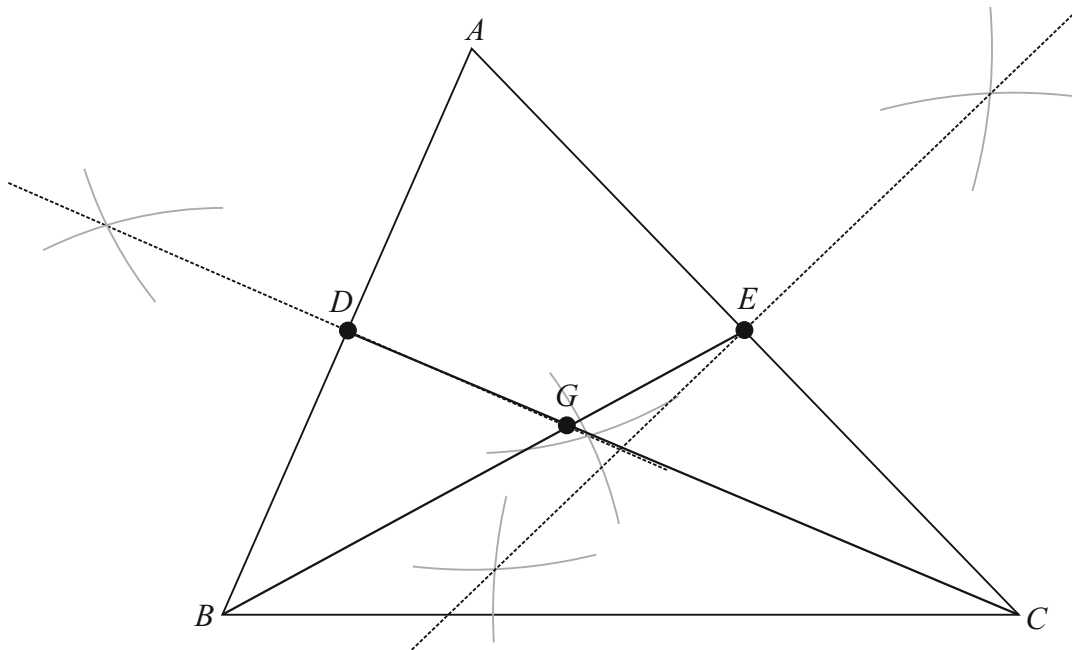
A median is a line from a vertex to the midpoint of the opposite side.

Using a compass, bisect lines  $AB$  and  $AC$  to get the midpoints  $D$  and  $E$  of these lines.



Draw medians  $BE$  and  $CD$ .

The centroid  $G$  is the intersection of these medians.



**MARKING SCHEME NOTES**

**Question 6 (a) [Scale 5C (0, 2, 4, 5)]**

- 2: • Some relevant calculation  
 • One side bisected  
 • One midpoint indicated
- 4: • One median drawn

**Question 6 (b)**

**THE TRANSVERSAL LINE THEOREM**

If three parallel lines cut off equal segments on some transversal line, then they will cut off equal intercepts on any other transversal.

**GIVEN:**  $k \parallel l \parallel m$  and  $|DE| = |EF|$ .

**PROVE:**  $|AB| = |BC|$

**CONSTRUCTION:** Draw a line  $D'F'$  through  $B$  parallel to  $DF$ .

**PROOF:**

$DEBD'$  is a parallelogram  $\Rightarrow |DE| = |D'B|$

$EFF'B'$  is a parallelogram  $\Rightarrow |EF| = |BF'|$

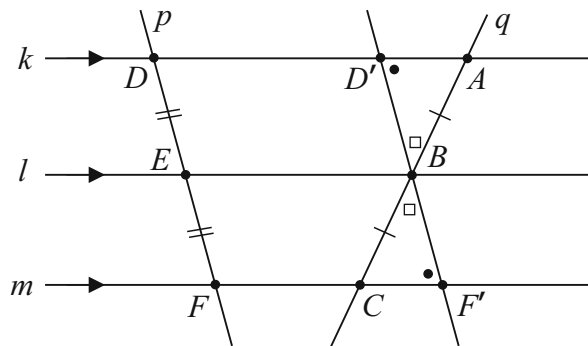
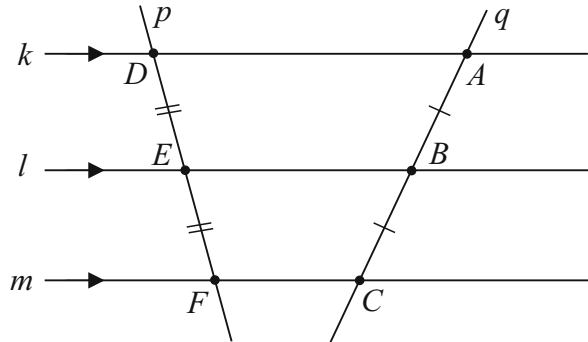
$\therefore |D'B| = |BF'|$  (because  $|DE| = |EF|$ )

$|\angle AD'B| = |\angle BF'C|$  (alternate angles)

$|\angle ABD'| = |\angle CBF'|$  (vertically opposite)

$\therefore \triangle AD'B \cong \triangle BCF'$

$\therefore |AB| = |BC|$



**MARKING SCHEME NOTES**

**Question 6 (b)**

**Diagram/Given: [Scale 5B (0, 2, 5)]**

- 2: • Effort at *Diagram* or *Given*

**Construction: [Scale 5B (0, 2, 5)]**

- 2: • Construction attempted (diagram and/or description)

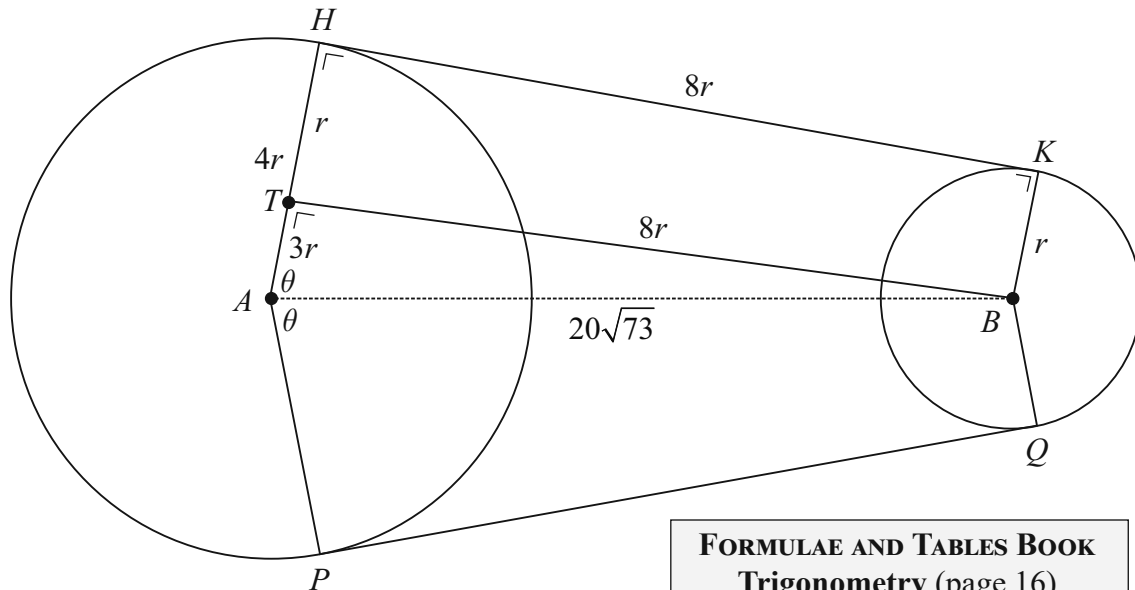
**Proof: [Scale 10C (0, 4, 8, 10)]**

- 4: • More than one critical step omitted but still some substantial work of merit
- 8: • Proof completed with one critical step omitted

**QUESTION 7 (40 MARKS)**

**Question 7 (a)**

**NOTE:** You can see from the measurements on the diagram that it is not obviously drawn to scale. To draw it to scale would make the smaller circle tiny making it difficult to see measurements.



Consider the right-angled triangle  $ATB$ .  
Apply Pythagoras.

$$(3r)^2 + (8r)^2 = (20\sqrt{73})^2$$

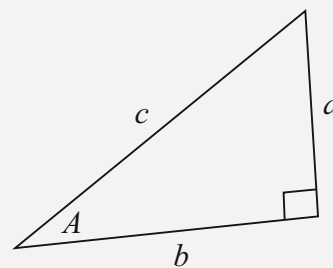
$$9r^2 + 64r^2 = 400 \times 73$$

$$73r^2 = 400 \times 73$$

$$r^2 = 400$$

$$\therefore r = \sqrt{400} = 20 \text{ cm}$$

**FORMULAE AND TABLES BOOK**  
**Trigonometry** (page 16)  
Pythagoras' theorem



$$c^2 = a^2 + b^2$$

**MARKING SCHEME NOTES**

**Question 7 (a) [Scale 15C (0, 5, 12, 15)]**

- 5:**
- $BT$  drawn correctly
  - Pythagoras formula with some correct substitution
  - Recognising  $|\angle ATB| = 90^\circ$

- 12:**
- Pythagoras formula fully substituted

**Question 7 (b)**

Area of quadrilateral  $ABKH =$  Area of rectangle  $TBKH +$  Area of right-angled triangle  $ABT$

$$\text{Area} = (8r)(r) + \frac{1}{2}(3r)(8r) = 8r^2 + 12r^2 = 20r^2 = 20(20)^2 = 8000 \text{ cm}^2$$

**MARKING SCHEME NOTES**

**Question 7 (b) [Scale 15C (0, 5, 12, 15)]**

- 5:**
- Indicates two areas
  - Effort at area of rectangle only
  - Effort at area of triangle only

- 12:**
- Area of triangle correct
  - Area of rectangle correct

**Question 7 (c) (i)**

$$\tan \theta = \frac{8r}{3r} = \frac{8}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{8}{3}\right) = 69.44^\circ$$

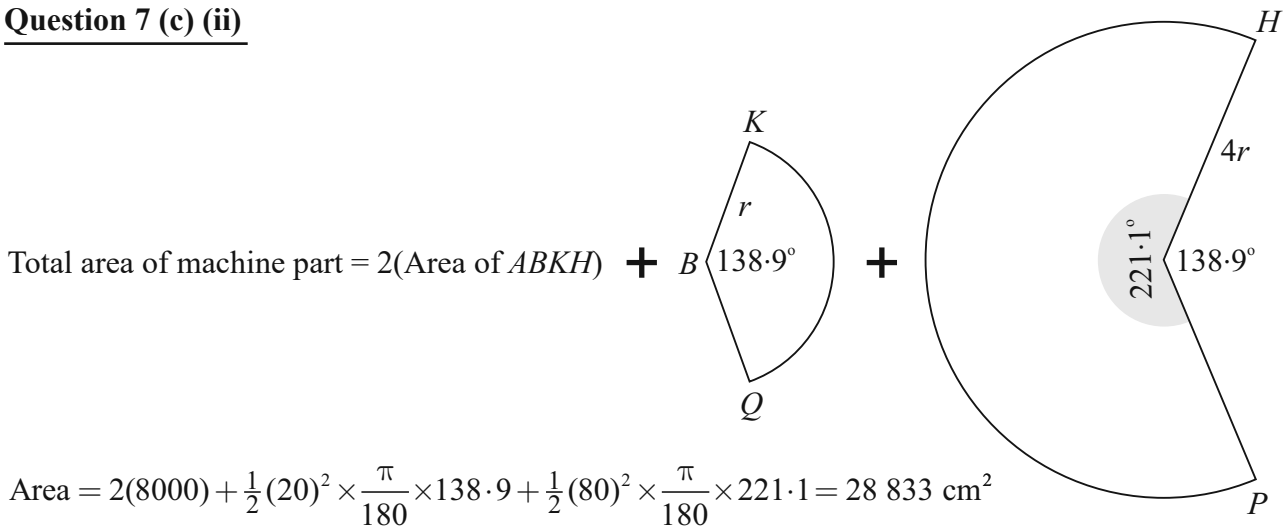
$$|\angle HAP| = 2\theta = 138.9^\circ$$

**MARKING SCHEME NOTES**

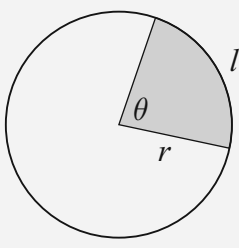
**Question 7 (c) (i) [Scale 5C (0, 2, 4, 5)]**

- 2: •  $\tan(\angle HAB) = \frac{160}{60}$  or equivalent in sin or cos  
 4: •  $|\angle HAB|$  in degrees

**Question 7 (c) (ii)**



**FORMULAE AND TABLES BOOK**  
**Length and area:**  
**Arc/sector [page 8]**



$l = r\theta$   
 $A = \frac{1}{2}r^2\theta$

when  $\theta$  is in radians.

Degrees to radians:  $\times \frac{\pi}{180^\circ}$   
 Radians to degrees:  $\times \frac{180^\circ}{\pi}$

**MARKING SCHEME NOTES**

**Question 7 (c) (ii) [Scale 5C (0, 2, 3, 4, 5)]**

- 2: • Effort at area of one region  
 3: • Area of one sector with correct substitution  
 4: • Area of two sectors with substitution correct in both

**QUESTION 8 (65 MARKS)**

$$P(\text{Successful on first free throw}) = 0.7$$

$$P(\text{Unsuccessful on first free throw}) = 0.3$$

$$P(\text{Successful if he was successful on previous free throw}) = 0.8$$

$$P(\text{Unsuccessful if he was successful on previous free throw}) = 0.2$$

$$P(\text{Successful if he was unsuccessful on previous free throw}) = 0.6$$

$$P(\text{Unsuccessful if he was unsuccessful on previous free throw}) = 0.4$$

**Question 8 (a)**

$$P(S, S, S) = 0.7 \times 0.8 \times 0.8 = 0.448 = \frac{56}{125}$$

**MARKING SCHEME NOTES****Question 8 (a) [Scale 10C (0, 4, 8, 10)]**

- 4: • One correct probability
- 8: • Identifies all three probabilities correctly  
• Three probabilities multiplied of which two are correct

**Question 8 (b)**

$$P(U, U, S) = 0.3 \times 0.4 \times 0.6 = 0.072 = \frac{9}{125}$$

**MARKING SCHEME NOTES****Question 8 (b) [Scale 10C (0, 4, 8, 10)]**

- 4: • One correct probability
- 8: • Identifies all three probabilities correctly  
• Three probabilities multiplied of which two are correct

**Question 8 (c)**

$$P(\text{Successful on third free throw})$$

$$= P(S, S, S) \text{ or } P(U, U, S) \text{ or } P(S, U, S) \text{ or } P(U, S, S)$$

$$= P(S, S, S) + P(U, U, S) + P(S, U, S) + P(U, S, S)$$

$$= \frac{56}{125} + \frac{9}{125} + 0.7 \times 0.2 \times 0.6 + 0.3 \times 0.6 \times 0.8$$

$$= \frac{56}{125} + \frac{9}{125} + \frac{21}{250} + \frac{18}{125}$$

$$= \frac{56}{125} + \frac{9}{125} + \frac{21}{250} + \frac{18}{125} = \frac{187}{250} = 0.748$$

**MARKING SCHEME NOTES****Question 8 (c) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Lists one new way
- 7: • Full listing only  
• One new probability
- 11: • Sum of three probabilities  
• Identifies all four probabilities correctly

**Question 8 (d) (i)** $p_{n+1}$  = Probability that Michael is successful on the throw after his  $n$ th throw $p_n$  = Probability that Michael is successful on his  $n$ th throwProbability that Michael is successful on the throw after his  $n$ th throw= Probability that Michael is successful on his previous throw **or** Probability that Michael is unsuccessful on his previous throw

$$\therefore p_{n+1} = 0.8 \times p_n + 0.6(1 - p_n) = 0.8p_n + 0.6 - 0.6p_n = 0.6 + 0.2p_n$$

**MARKING SCHEME NOTES****Question 8 (d) (i) [Scale 5C (0, 2, 4, 5)]**2: • Indicates  $P(S, S)$  and/or  $P(U, S)$  or equivalent4: • Substitution into equation for  $p_{n+1}$ **Question 8 (d) (ii)**

$$p_{n+1} \approx p_n \approx p$$

$$\therefore p_{n+1} = 0.6 + 0.2p_n = p_n$$

$$0.6 = 0.8p_n$$

$$\therefore p_n = 0.75$$

**MARKING SCHEME NOTES****Question 8 (d) (ii) [Scale 5C (0, 2, 4, 5)]**2: • Indicates  $P(S, S)$  and/or  $P(U, S)$  or equivalent4: • Substitution into equation for  $p_{n+1}$ **Question 8 (e) (i)**

$$a_n = p - p_n = 0.75 - p_n$$

$$\frac{a_{n+1}}{a_n} = \frac{0.75 - p_{n+1}}{0.75 - p_n} = \frac{0.75 - 0.6 - 0.2p_n}{0.75 - p_n}$$

$$= \frac{0.15 - 0.2p_n}{0.75 - p_n}$$

$$= \frac{15 - 20p_n}{75 - 100p_n}$$

$$= \frac{3 - 4p_n}{15 - 20p_n}$$

$$= \frac{3 - 4p_n}{5(3 - 4p_n)} = \frac{1}{5}$$

Therefore,  $a_n$  is a geometric sequence.**MARKING SCHEME NOTES****Question 8 (e) (i) [Scale 5C (0, 2, 4, 5)]**2: •  $a_{n+1}$  in terms of  $p$  and  $p_{n+1}$ •  $\frac{a_{n+1}}{a_n}$  in terms of  $p$ ,  $p_n$  and  $p_{n+1}$ 4: •  $\frac{a_{n+1}}{a_n}$  substituted

**Question 8 (e) (ii)**

$$\frac{a_{n+1}}{a_n} = \frac{1}{5} \Rightarrow a_{n+1} = \frac{1}{5} \times a_n$$

$$a_n = p - p_n = 0.00001$$

$n = 1: a_1 = p - p_1 = 0.75 - 0.7 = 0.05 \leftarrow a_n$  is a geometric sequence with first term  $a = 0.05$  and common ratio  $r = \frac{1}{5}$

$$\therefore a_n = a(r)^{n-1} = (0.05)\left(\frac{1}{5}\right)^{n-1} < 0.00001$$

$$\left(\frac{1}{5}\right)^{n-1} < \frac{1}{5000}$$

$$(n-1)\log\left(\frac{1}{5}\right) < \log\left(\frac{1}{5000}\right)$$

$$n-1 > \frac{\log\left(\frac{1}{5000}\right)}{\log\left(\frac{1}{5}\right)} = 5.29 \leftarrow \text{Reverse the inequality as } \log(0.2) \text{ is negative.}$$

$$n > 6.29$$

Smallest value of  $n$  is 7.

**MARKING SCHEME NOTES****Question 8 (e) (ii) [Scale 5C (0, 2, 4, 5)]**

2: •  $a_1$  in numerical form

4: •  $ar^{n-1}$  substituted

•  $a_7$  evaluated without checking  $a_6$

**Question 8 (f) (i)**

$$p \approx 0.75$$

**Question 8 (f) (ii)**

It would not be appropriate to consider Michaels's subsequent free throws in the game as sequence of Bernoulli trials as the probability changes from throw to throw. The events are not independent.

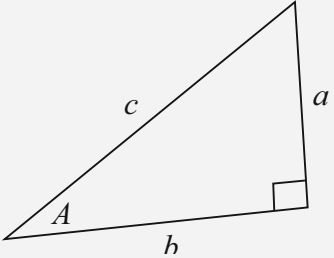
**MARKING SCHEME NOTES****Question 8 (f) (i) (ii) [Scale 5B (0, 2, 5)]**

2: • (i) correct only or (ii) correct only

**QUESTION 9 (45 MARKS)**

**Question 9 (a)**

**FORMULAE AND TABLES BOOK**  
**Trigonometry: Right-angled triangle [page 16]**

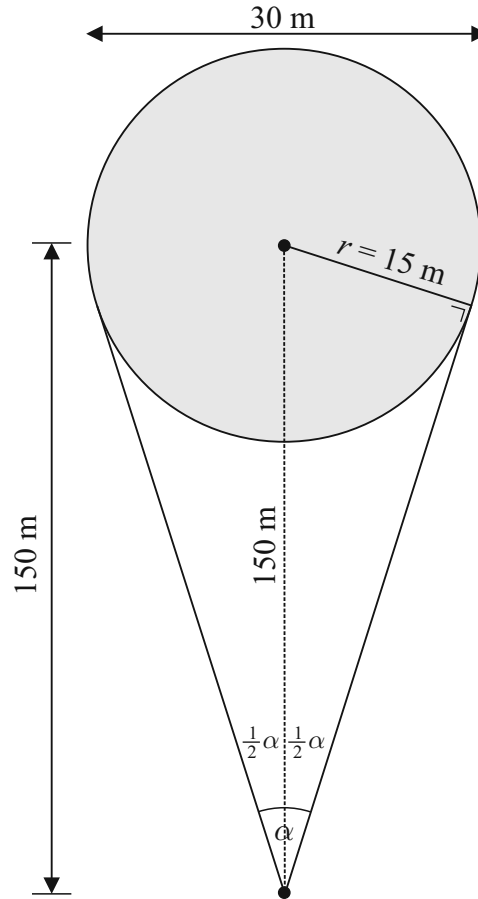


$\sin A = \frac{a}{c}$ ,  $\cos A = \frac{b}{c}$ ,  $\tan A = \frac{a}{b}$

$$\sin\left(\frac{\alpha}{2}\right) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{15}{150}$$

$$\left(\frac{\alpha}{2}\right) = \sin^{-1}\left(\frac{1}{10}\right) = 5.74^\circ$$

$$\therefore \alpha = 2 \times 5.74^\circ = 11.5^\circ$$

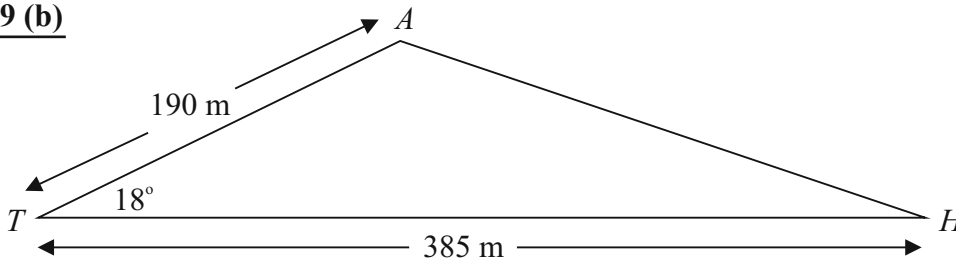


**MARKING SCHEME NOTES**

**Question 9 (a) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- Effort at expressing sine function in terms of 15 and 150
  - Finds third side of triangle and makes effort to find an angle
- 8:**
- Half angle found

**Question 9 (b)**



$$|AH|^2 = 190^2 + 385^2 - 2(190)(385)\cos 18^\circ$$

$$\therefore |AH| = \sqrt{190^2 + 385^2 - 2(190)(385)\cos 18^\circ} \approx 213 \text{ m}$$

**FORMULAE AND TABLES BOOK**  
**Trigonometry of the triangle:**  
**Cosine rule [page 16]**  
 $a^2 = b^2 + c^2 - 2bc \cos A$

**MARKING SCHEME NOTES**

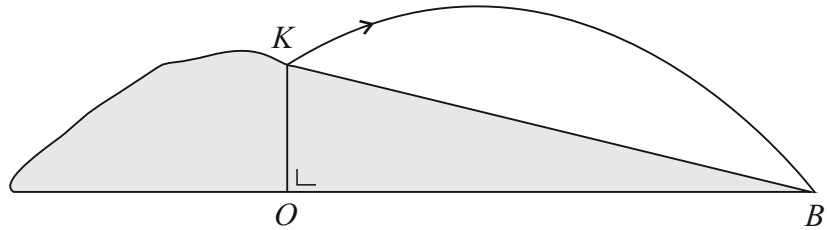
**Question 9 (b) [Scale 10C (0, 4, 8, 10)]**

- 4:**
- Cosine Rule with some correct substitution
  - Effort at calculating  $|AX|$  or  $|TX|$
- 8:**
- Cosine Rule substituted correctly
  - Finds  $|AX|$  and formulates for  $|TX|$  (or vice versa)

**Question 9 (c) (i)**

$$h = -6t^2 + 22t + 8$$

$$t = 0 : h = 8 \text{ m}$$



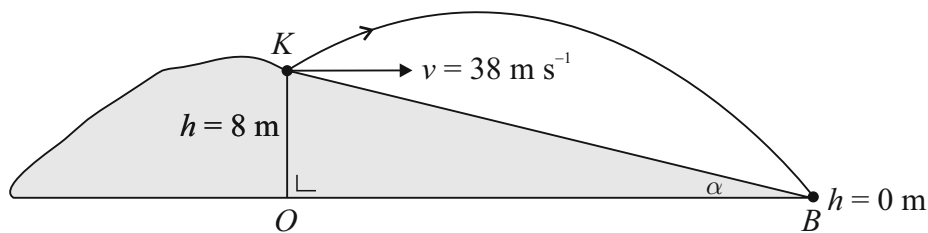
**MARKING SCHEME NOTES**

**Question 9 (c) (i) [Scale 5B (0, 2, 5)]**

2: •  $t = 0$  indicated

Note: Accept  $h = 8 \text{ m}$  without work

**Question 9 (c) (ii)**



Find the time for the ball to land by putting  $h = 0 \text{ m}$ :

$$h = 0 : h = -6t^2 + 22t + 8$$

$$\therefore -6t^2 + 22t + 8 = 0$$

$$3t^2 - 11t - 4 = 0$$

$$(3t + 1)(t - 4) = 0$$

$$\therefore t = 4 \text{ s}$$

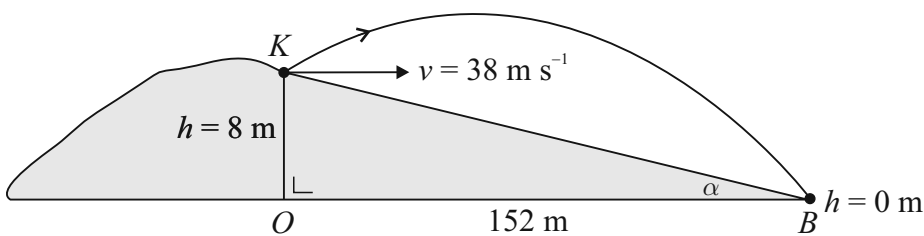
**FORMULAE AND TABLES BOOK**

**Speed:**

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

$$v = \frac{s}{t}$$

$$\text{Horizontal distance } |OB| = v \times t = 38 \times 4 = 152 \text{ m}$$



$$\tan \alpha = \frac{8}{152} \Rightarrow \alpha = \tan^{-1} \left( \frac{8}{152} \right) \approx 3^\circ$$

**MARKING SCHEME NOTES**

**Question 9 (c) (ii) [Scale 10C (0, 4, 8, 10)]**

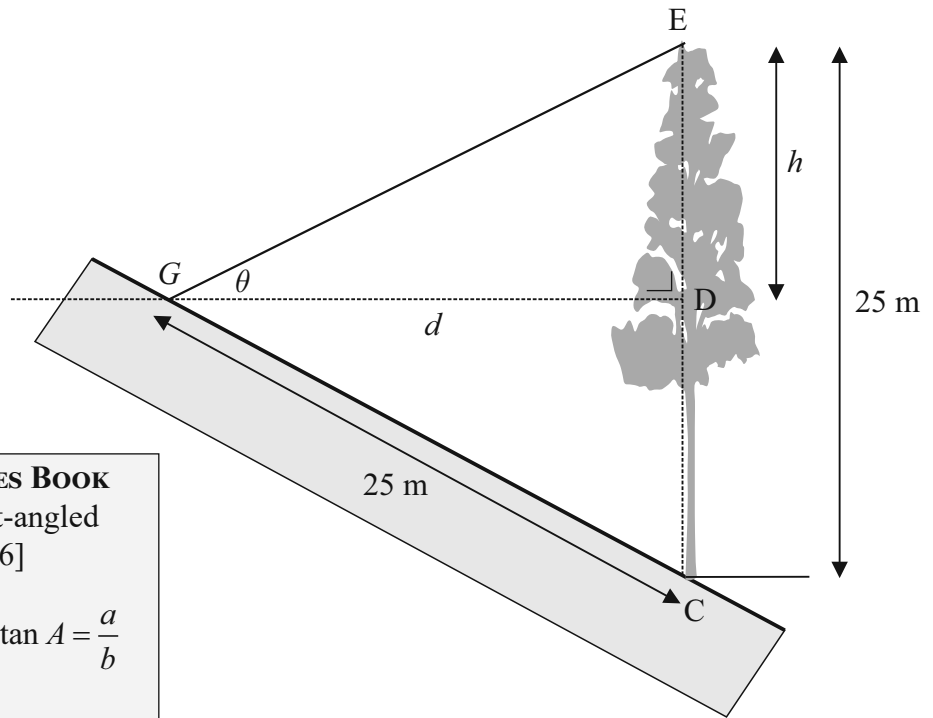
4: •  $h = 0$  indicated

8: •  $|OB|$  found for positive value for  $t$

**Question 9 (d) (i)**

$$\tan \theta = \frac{1}{2} = \frac{h}{d} \Rightarrow d = 2h$$

$$|CD| = 25 - h$$



**FORMULAE AND TABLES BOOK**

**Trigonometry:** Right-angled triangle [page 16]

$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$$

$$c^2 = a^2 + b^2$$

**MARKING SCHEME NOTES**

**Question 9 (d) (i) [Scale 5B (0, 2, 5)]**

2: •  $\frac{h}{d} = \frac{1}{2}$

•  $|CD| = 25 - h$

**Question 9 (d) (ii)**

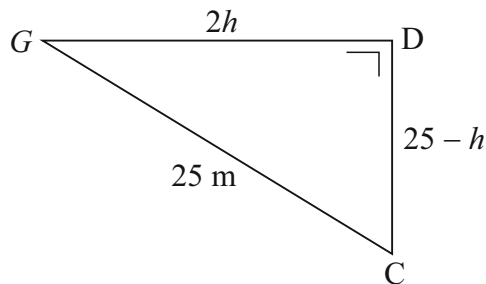
$$25^2 = (2h)^2 + (25 - h)^2$$

$$625 = 4h^2 + 625 - 50h + h^2$$

$$5h^2 - 50h = 0$$

$$5h(h - 10) = 0$$

$$\therefore h = 10 \text{ m}$$



**MARKING SCHEME NOTES**

**Question 9 (d) (ii) [Scale 5D (0, 2, 3, 4, 5)]**

2: • Pythagoras with some correct substitution

3: • Pythagoras correctly substituted

4: • Quadratic equation expanded correctly

## LC 2014 (SET E): PAPER 1

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

$x = -3$  is a root  $\Rightarrow (x + 3)$  is a factor

$x = -1$  is a root  $\Rightarrow (x + 1)$  is a factor

$x = 2$  is a root  $\Rightarrow (x - 2)$  is a factor

$$\text{Cubic} = k(\text{Linear})(\text{Linear})(\text{Linear})$$

You can find the value of  $k$  as the  $y$ -intercept  $(0, -6)$  is given.

$$y = f(x) = k(x + 3)(x + 1)(x - 2)$$

$$x = 0, y = -6: f(0) = k(3)(1)(-2) = -6$$

$$\therefore -6k = -6 \Rightarrow k = 1$$

Multiply out the brackets to obtain the cubic function.

$$f(x) = 1(x + 3)(x + 1)(x - 2)$$

$$= (x + 3)(x^2 - x - 2)$$

$$= x^3 - x^2 - 2x + 3x^2 - 3x - 6$$

$$= x^3 + 2x^2 - 5x - 6$$

**AUTHOR'S NOTE:** The official marking scheme completely ignores the step to find  $k$ . Because  $k = 1$ , they got away with this omission. However, the technique shown is the proper method for finding the expression for a function.

#### MARKING SCHEME NOTES

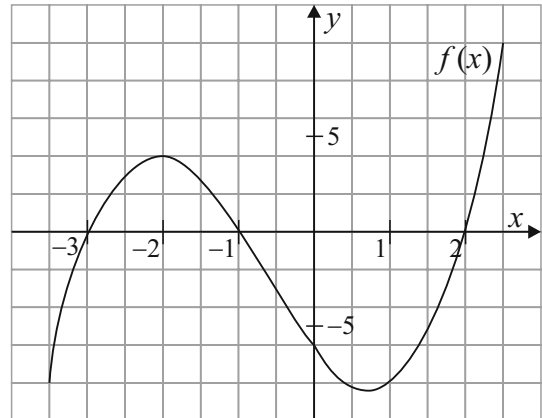
##### Question 1 (a) [Scale 15C (0, 7, 10, 15)]

7: • Only one value verified

• Recognising one factor

10: • Writing  $(x + 3)(x + 1)(x - 2)$

• Two relevant roots tested



#### Question 1 (b) (i)

$$f(x) = g(x)$$

$$x^3 + 2x^2 - 5x - 6 = -2x - 6$$

$$x^3 + 2x^2 - 3x = 0 \leftarrow \text{Bring all terms to one side}$$

$$x(x^2 + 2x - 3) = 0 \leftarrow \text{Take out } x \text{ in common}$$

$$x(x - 1)(x + 3) = 0 \leftarrow \text{Factorise the quadratic}$$

$$\therefore x = -3, 0, 1$$

Substitute the  $x$  values into either function to obtain the  $y$  co-ordinates.

$$g(x) = -2x - 6$$

$$x = -3: g(-3) = -2(-3) - 6 = 0$$

$$x = 0: g(0) = -2(0) - 6 = -6$$

$$x = 1: g(1) = -2(1) - 6 = -8$$

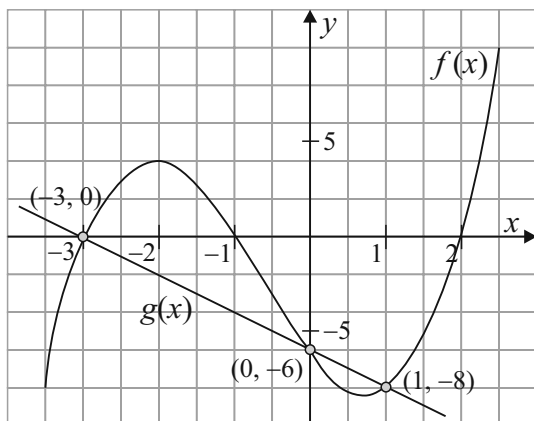
Therefore,  $(-3, 0)$ ,  $(0, -6)$  and  $(1, -8)$  are the points of intersection of the two functions.

**MARKING SCHEME NOTES**

**Question 1 (b) (i) [Scale 5C (0, 3, 4, 5)]**

- 3: • Equations correct when  $f(x) = g(x)$   
• Cubic equation not factorised
- 4: • Roots identified

**Question 1 (b) (ii)**



Use any 2 (or all 3) of the points of intersection to draw the graph of the linear function  $g(x) = -2x - 6$ .

**MARKING SCHEME NOTES**

**Question 1 (b) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: • One point found in  $g(x)$   
• Only one point indicated on graph
- 4: • Two points identified  
• Two points plotted but no graph drawn

## QUESTION 2 (25 MARKS)

### Question 2 (a)

Use the conjugate root theorem to find the other root. Therefore, if  $z_1$  is one of the roots, the conjugate of  $z_1$  is also a root. Call this root  $z_2$ .

$$z_1 = 1 - 2i \Rightarrow z_2 = 1 + 2i$$

Forming a quadratic equation from its roots.

$$\mathbf{S}: 1 - 2i + 1 + 2i = 2$$

$$\mathbf{P}: (1 - 2i)(1 + 2i) = 5$$

$$z^2 - 2z + 5 = 0$$

The cubic equation is made up of a quadratic equation multiplied by a linear equation.

$$2z^3 - 7z^2 + 16z - 15 = (z^2 - 2z + 5)(\quad) = 0$$

$$2z^3 - 7z^2 + 16z - 15 = (z^2 - 2z + 5)(2z \quad) = 0$$

$$2z^3 - 7z^2 + 16z - 15 = (z^2 - 2z + 5)(2z - 3) = 0$$

$$(2z - 3) = 0 \Rightarrow z = \frac{3}{2}$$

$$\text{Roots: } z_2 = 1 + 2i, z_3 = \frac{3}{2}$$

Other methods (not as efficient):

$1 - 2i$  is a root  $\Rightarrow (z - 1 + 2i)$  is a linear factor

$1 + 2i$  is a root  $\Rightarrow (z - 1 - 2i)$  is a linear factor

Quadratic factor:  $(z - 1 + 2i)(z - 1 - 2i)$

$$(z - 1 + 2i)(z - 1 - 2i)$$

$$= z^2 - z - 2iz - z + 1 + 2i + 2iz - 2i - 4i^2 \quad [i^2 = -1]$$

$$= z^2 - 2z + 1 + 4$$

$$= z^2 - 2z + 5$$

$$\begin{array}{r} 2z - 3 \\ z^2 - 2z + 5 \overline{) 2z^3 - 7z^2 + 16z - 15} \\ \underline{\mp 2z^3 \pm 4z^2 \mp 10z} \phantom{- 15} \\ -3z^2 + 6z - 15 \\ \underline{\pm 3z^2 \mp 6z \pm 15} \\ 0 \end{array}$$

$$(2z - 3) = 0 \Rightarrow z = \frac{3}{2}$$

$$\text{Roots: } z_2 = 1 + 2i, z_3 = \frac{3}{2}$$

If all the coefficients of a quadratic equation are real, then the roots are both real or are complex conjugates of each other, and vice versa.

Conjugate of  $z$  is represented by  $\bar{z}$

$$z = a + bi \Rightarrow \bar{z} = a - bi$$

Forming a quadratic equation from its roots.

$$z^2 - \mathbf{S}z + \mathbf{P} = 0$$

**S:** Sum of the roots

**P:** Product of the roots

Cubic = Quadratic  $\times$  Linear

The first term in the cubic equals the first term in the quadratic multiplied by the first term in the linear.

Similarly, the last term in the cubic equals the last term in the quadratic multiplied by the last term in the linear

**AUTHOR'S NOTE:** The best technique for finding the roots of the cubic equation are shown at the beginning of the solution. Only 5 marks are allocated to this so I really want my answers within the minute.



#### MARKING SCHEME NOTES

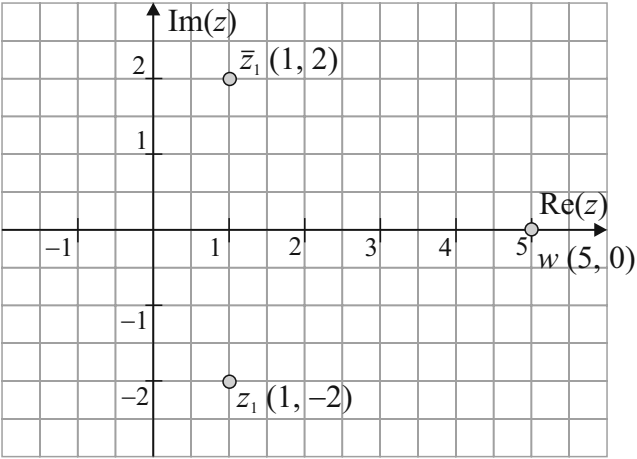
##### Question 2 (a) [Scale 5D (0, 2, 3, 4, 5)]

- 2:**
- Identifies another root
  - Forms an equation
- 3:**
- Works with correct quadratic factor
  - Indicates division of quadratic into cubic
- 4:**
- Finds third factor

**Question 2 (b) (i)**

**FORMULA: Complex Numbers**  
 Congugates  $\bar{z}$   
 $z = a + bi \Rightarrow \bar{z} = a - bi$

$z_1 = 1 - 2i$   
 $\bar{z}_1 = 1 + 2i$   
 $w = z_1 \bar{z}_1 = (1 - 2i)(1 + 2i) = 5 = 5 + 0i$



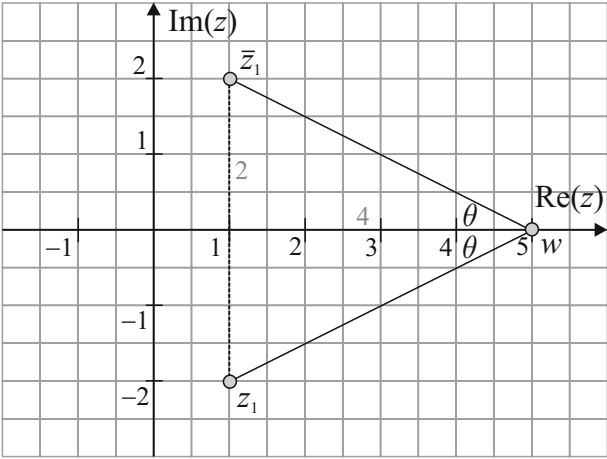
**MARKING SCHEME NOTES**  
**Question 2 (b) (i) [Scale 10C (0, 5, 7, 10)]**  
 5: • Plots one point correctly  
 • Finds  $\bar{z}_1$   
 7: • Points plotted but not labelled or labelled incorrectly  
 • Two points plotted and labelled  
 • Calculates  $w$

**Question 2 (b) (ii)**

**METHOD 1:** Divide triangle into two right-angled triangles and use tan.

**FORMULAE AND TABLES BOOK**  
**Trigonometry: Right-angled triangle [page 16]**

$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$



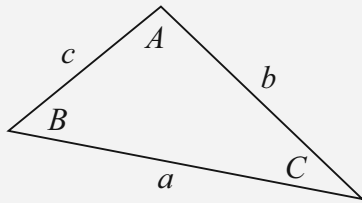
$\tan \theta = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta = 26.6^\circ$   
 $|\angle \bar{z}_1 w z_1| = 2 \times 26.6^\circ \approx 53^\circ$

**METHOD 2:** Use Cosine rule (not as efficient)

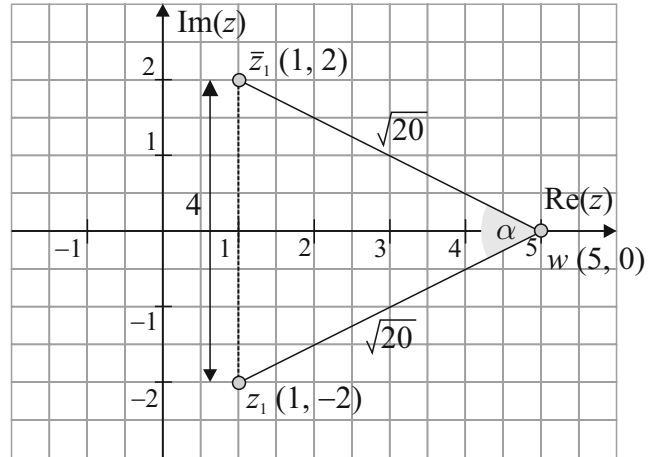
$|z_1 w| = \sqrt{(0 - (-2))^2 + (5 - 1)^2} = \sqrt{20}$   
 $|\bar{z}_1 w| = \sqrt{20}$   
 $|\bar{z}_1 z_1| = 4$

**FORMULA: Complex Numbers**  
 Modulus  $|z|$   
 $|z| = |a + bi| = \sqrt{a^2 + b^2}$

**FORMULAE AND TABLES BOOK**  
**Trigonometry of the triangle:**  
**Cosine rule [page 16]**



$$a^2 = b^2 + c^2 - 2bc \cos A$$



$$4^2 = (\sqrt{20})^2 + (\sqrt{20})^2 - 2(\sqrt{20})(\sqrt{20}) \cos \alpha$$

$$16 = 20 + 20 - 40 \cos \alpha$$

$$40 \cos \alpha = 24$$

$$\therefore \cos \alpha = \frac{24}{40} = 0.6 \Rightarrow \alpha = \cos^{-1}(0.6) = 53.13^\circ \approx 53^\circ$$

**MARKING SCHEME NOTES**

**Question 2 (b) (ii) [Scale 10C (0, 5, 7, 10)]**

- 5: • Length of any one side of triangle calculated correctly  
 • Correct definition of trig ratio  
 • Correct cos rule  
 • Recognises the half-angle
- 7: • cos value calculated but angle not found  
 • tan value of half-angle calculated

**QUESTION 3 (25 MARKS)**

**Question 3 (a)**

**STEPS TO PROOF BY INDUCTION**

1. Prove result is true for some starting value of  $n \in \mathbb{N}$ .
2. Assume result is true for  $n = k$ .
3. Prove result is true for  $n = (k + 1)$ , using step 2.

REQUIRED TO PROVE:  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

**STEP 1:** Prove the result is true for  $n = 1$ .

$$1 = \frac{1(1+1)}{2}$$

$$1 = \frac{1(2)}{2}$$

$$1 = 1 \text{ [Therefore, true for } n = 1.]$$

**STEP 2:** Assume it is true for  $n = k$ .

$$\underline{1 + 2 + 3 + \dots + k} = \frac{k(k+1)}{2}$$

**STEP 3:** Prove it is true for  $n = k + 1$ .

$$\text{Prove } \underline{(1+2+3+\dots+k) + (k+1)} = \frac{(k+1)(k+2)}{2}$$

Use the result in Step 2 to prove Step 3.

$$\begin{aligned} & (1+2+3+\dots+k) + (k+1) \\ &= \frac{k(k+1)}{2} + (k+1) \\ &= (k+1) \left[ \frac{k}{2} + 1 \right] \\ &= (k+1) \left[ \frac{k+2}{2} \right] \\ &= \frac{(k+1)(k+2)}{2} \end{aligned}$$

Therefore, assuming true for  $n = k$  means it is true for  $n = k + 1$ . So true for  $n = 1$  and true for  $n = k$  means it is true for  $n = k + 1$ . This implies it is true for all  $n \in \mathbb{N}$ .

**MARKING SCHEME NOTES**

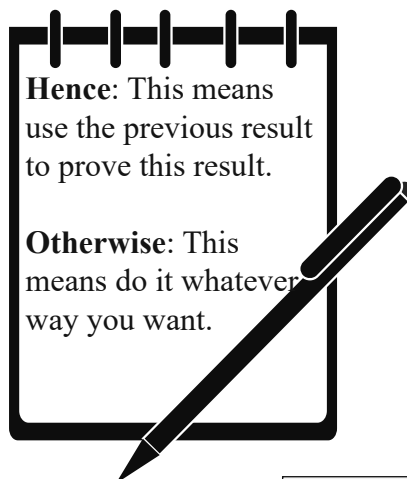
**Question 3 (a) [Scale 10D (0, 3, 7, 8, 10)]**

- 3:**
- One correct step in induction
  - Statement  $P(1)$  true
- 7:**
- Uses  $(k+1)$  term on LHS
  - Some work with  $(k+1)$  term
- 8:**
- Correct RHS
  - No conclusion

**Question 3 (b)**

Hence:

$$\begin{aligned} & 2 + 4 + 6 + \dots + 2n \\ &= 2(1 + 2 + 3 + \dots + n) \\ &= 2 \left[ \frac{n(n+1)}{2} \right] \\ &= n^2 + n \end{aligned}$$



Otherwise:

$$\begin{aligned} S_n &= 2 + 4 + 6 + \dots + 2n \quad [a = 2, d = 2, n \text{ terms}] \\ S_n &= \frac{n}{2}[2(2) + (n-1)(2)] \\ &= \frac{n}{2}[4 + 2n - 2] \\ &= \frac{n}{2}[2 + 2n] \\ &= n[n+1] \\ &= n^2 + n \end{aligned}$$

**FORMULAE AND TABLES BOOK**  
**Sequences and series: Arithmetic series**  
 [page 22]

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$a$  is the first term  
 $d$  is the common difference

**MARKING SCHEME NOTES****Question 3 (b) [Scale 10D (0, 3, 7, 8, 10)]**

- 3: • Recognising 2 as common factor  
•  $\frac{n}{2}(n+1)$
- 7: • Correct equation from (a)  
• Taking 2 out of series
- 8: • Work not completed

**OR****Question 3 (b) [Scale 10D (0, 3, 7, 8, 10)]**

- 3: • Recognition of  $a = 2$   
• Recognition of  $d = 2$   
• Correct AP formula only
- 7: • Some substitution into correct formula
- 8: • Work not fully simplified  
• Answer not in required form  
•  $S_n$  missing

**Question 3 (c)**

Sum of the first  $n$  **odd** natural numbers  $S$  + Sum of the first  $n$  **even** natural numbers  
= Sum of the first  $2n$  natural numbers

$$S + (n^2 + n) = \frac{2n(2n+1)}{2}$$

$$S = n(2n+1) - (n^2 + n)$$

$$= 2n^2 + n - n^2 - n$$

$$= n^2$$

**MARKING SCHEME NOTES****Question 3 (c) [Scale 5B (0, 3, 5)]**

- 3: •  $S_B - S_A$  indicated  
•  $S_B + S_A$   
• Use of correct series from (b)

**NOTE:** Must use result from (a) and (b) here

**QUESTION 4 (25 MARKS)****Question 4 (a)**

$$f(x) = 2x^2 - 3x - 6$$

$$\begin{aligned} f(x+h) &= 2(x+h)^2 - 3(x+h) - 6 \\ &= 2(x^2 + 2hx + h^2) - 3x - 3h - 6 \\ &= 2x^2 + 4hx + 2h^2 - 3x - 3h - 6 \end{aligned}$$

$$\begin{aligned} f(x+h) - f(x) &= 2x^2 + 4hx + 2h^2 - 3x - 3h - 6 - 2x^2 + 3x + 6 \\ &= 4hx + 2h^2 - 3h \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{4hx + 2h^2 - 3h}{h} = 4x + 2h - 3$$

$$\lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} \right] = \lim_{h \rightarrow 0} (4x + 2h - 3) = 4x - 3$$

**MARKING SCHEME NOTES****Question 4 (a) [Scale 15D (0, 5, 9, 12, 15)]**

- 5:** • Introduces  $f(x+h)$
- 9:** •  $f(x+h) - f(x)$  expressed (need not be simplified)  
• RHS only
- 12:** •  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  (need not be simplified)

**Question 4 (b)**

$$\begin{aligned} f(x) &= \frac{2x}{x+2} \\ f'(x) &= \frac{(x+2)2 - 2x(1)}{(x+2)^2} = \frac{2x+4-2x}{(x+2)^2} = \frac{4}{(x+2)^2} \end{aligned}$$

$$f'(x) = \frac{1}{4} \Rightarrow \frac{4}{(x+2)^2} = \frac{1}{4}$$

$$16 = (x+2)^2$$

$$(x+2) = \pm 4$$

$$\therefore x = -6, 2$$

$$f(x) = \frac{2x}{x+2}$$

$$x = -6: f(-6) = \frac{2(-6)}{(-6)+2} = 3$$

$$x = 2: f(2) = \frac{2(2)}{(2)+2} = 1$$

Therefore,  $(-6, 3)$  and  $(2, 1)$  are the co-ordinates of the points.

**FORMULAE AND TABLES BOOK**  
**Calculus: Quotient rule [page 25]**

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**MARKING SCHEME NOTES****Question 4 (b) [Scale 10D (0, 3, 7, 8, 10)]**

- 3: • Either  $\frac{du}{dx}$  or  $\frac{dv}{dx}$  correct  
 • No differentiation but writes  $f'(x) = \frac{1}{4}$
- 7: •  $f'(x)$  correct but not simplified
- 8: • Correct values of  $x$  from students work

**QUESTION 5 (25 MARKS)****Question 5 (a)**

$$\int 5 \cos 3x \, dx$$

$$= 5 \int \cos 3x \, dx$$

$$= \frac{5}{3} \sin 3x + c$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Integrals [page 26]**

$$\int \cos x \, dx = \sin x + c$$

$$\int \cos(ax + b) \, dx = \frac{1}{a} \sin(ax + b) + c$$

**MARKING SCHEME NOTES****Question 5 (a) [Scale 5B (0, 3, 5)]**

- 3: • Some correct integration  
 • Integrand does not contain  $c$   
 •  $c$  only

**Question 5 (b) (i)**

$$\frac{dy}{dx} = 2x - 2$$

$$\int dy = \int (2x - 2) \, dx$$

$$y = \frac{2x^2}{2} - 2x + c$$

$$= x^2 - 2x + c$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Integrals [page 26]**

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$(-2, 0)$  is on the curve.

$$y = x^2 - 2x + c$$

$$x = -2, y = 0 \Rightarrow 0 = (-2)^2 - 2(-2) + c$$

$$0 = 4 + 4 + c$$

$$\therefore c = -8$$

$$\therefore y = f(x) = x^2 - 2x - 8$$

**MARKING SCHEME NOTES****Question 5 (b) (i) [Scale 10C (0, 5, 7, 10)]**

- 5: • Some correct integration  
 • Integrand does not contain  $c$   
 •  $c$  only  
 •  $\frac{dy}{dx} = 2x - 2$  or  $\frac{dy}{dx} =$  slope of tangent
- 7: • Substitutes  $(-2, 0)$  but  $c$  not simplified  
**NOTE:** must have ' $c$ ' in equation to get high partial marks

**Question 5 (b) (ii)**

$$\begin{aligned}\bar{f} &= \frac{1}{3-0} \int_0^3 (x^2 - 2x - 8) dx \\ &= \frac{1}{3} \left[ \frac{x^3}{3} - \frac{2x^2}{2} - 8x \right]_0^3 = \frac{1}{3} \left[ \frac{x^3}{3} - x^2 - 8x \right]_0^3 \\ &= \frac{1}{3} \left[ \left( \frac{3^3}{3} - 3^2 - 8(3) \right) - 0 \right] \\ &= \frac{1}{3} [9 - 9 - 24] = -8\end{aligned}$$

**FORMULA****Average value of  $f$** 

$$\bar{f} = \frac{1}{b-a} \int_a^b f(x) dx$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Integrals [page 26]**

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

**MARKING SCHEME NOTES****Question 5 (b) (ii) [Scale 10C (0, 5, 7, 10)]**

- 5: • Correct formula only  
• Some correct integration  
• Indication of integration with correct limits  
• If only values used e.g.  $f(0), f(1), f(2)$  etc. when  $0 \leq x \leq 3$ , give Low Partial Credit for two or more values
- 7: • Limits inserted into function but not calculated  
•  $\frac{1}{(b-a)}$  missing from formula

**NOTE:** NO CREDIT – differentiation  
NO CREDIT – no integration

**QUESTION 6 (25 MARKS)****Question 6 (a) (i)**

$$T_n = \ln a^n = n \ln a$$

$$T_1 = \ln a, T_2 = 2 \ln a, T_3 = 3 \ln a$$

$$T_3 - T_2 = 3 \ln a - 2 \ln a = \ln a$$

$$T_2 - T_1 = 2 \ln a - \ln a = \ln a$$

$$\therefore T_3 - T_2 = T_2 - T_1$$

Therefore,  $T_1, T_2$  and  $T_3$  are in arithmetic sequence.

**Question 6 (a) (ii)**

$$T_n = \ln a^n = n \ln a$$

$$T_{n+1} = (n+1) \ln a$$

$$\begin{aligned}T_{n+1} - T_n &= (n+1) \ln a - n \ln a \\ &= n \ln a + \ln a - n \ln a \\ &= \ln a = \text{Constant } (d)\end{aligned}$$

Therefore, the sequence is arithmetic with common difference  $d = \ln a$ .

**FORMULAE AND TABLES BOOK**  
**Indices and logs [page 21]**

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a \left( \frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a(x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a \left( \frac{1}{x} \right) = -\log_a x$$

**MARKING SCHEME NOTES**

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

**Question 6 (a) (i) [Scale 10C (0, 5, 7, 10)]**

- 5: • Only one term correct  
 7: • Either  $(T_2 - T_1)$  or  $(T_3 - T_2)$  correct

**Question 6 (a) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: • Uses two consecutive general terms  
 • Recognition of common difference and no more  
 4: • Shows series arithmetic but does not state common difference

**Question 6 (b)**

$$T_1 + T_2 + T_3 + \dots + T_{98} + T_{99} + T_{100} = S_{100} = 10\,100$$

$$T_1 = \ln a, d = \ln a, n = 100$$

$$S_{100} = \frac{100}{2}[2 \ln a + 99 \ln a] = 10\,100$$

$$50[101 \ln a] = 10\,100$$

$$\ln a = 2$$

$$\therefore a = e^2$$

**FORMULAE AND TABLES BOOK**  
**Sequences and series: Arithmetic series**  
 [page 22]

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$a$  is the first term

$d$  is the common difference

**MARKING SCHEME NOTES**

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

**Question 6 (b) [Scale 5C (0, 3, 4, 5)]**

- 3: • Writes three or more terms in form of  $n$  and  $\ln a$   
 • Correct AP formula stated  
 • Correct  $T_n$  formula  
 4: • Correct substitution into formula  
 •  $\ln a = 2$  and does not finish

NOTE: accept  $a = e^2$  for full marks

**Question 6 (c)**

**LHS**

$$\begin{aligned} T_1 + T_2 + T_3 + \dots + T_{10} + 100d \\ = S_{10} + 100d \\ = \frac{10}{2}[2 \ln a + 9 \ln a] + 100 \ln a \\ = 5[11 \ln a] + 100 \ln a \\ = 155 \ln a \end{aligned}$$

**RHS**

$$\begin{aligned} T_{11} + T_{12} + T_{13} + \dots + T_{20} \\ = S_{20} - S_{10} \\ = \frac{20}{2}[2 \ln a + 19 \ln a] - \frac{10}{2}[2 \ln a + 9 \ln a] \\ = 10[21 \ln a] - 5[11 \ln a] \\ = 210 \ln a - 55 \ln a \\ = 155 \ln a \end{aligned}$$

**MARKING SCHEME NOTES**

NOTE: When particular values are used in ALL sections give Low Partial Credit at most each time

**Question 6 (c) [Scale 5C (0, 3, 4, 5)]**

- 3: • Recognising  $T_{11} = T_1 + 10d$  or similar work  
 4: • LHS correct in terms of  $\ln a$   
 • RHS correct in terms of  $\ln a$

NOTE: log is not needed in first solution box

**QUESTION 7 (40 MARKS)****Question 7 (a) (i)**

$$a = 2n + 1, b = 2n^2 + 2n, c = 2n^2 + 2n + 1$$

$$n = 2:$$

$$a = 2(2) + 1 = 5$$

$$b = 2(2)^2 + 2(2) = 12$$

$$c = 2(2)^2 + 2(2) + 1 = 13$$

$$a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169$$

$$c^2 = 13^2 = 169$$

$$\therefore a^2 + b^2 = c^2$$

**Question 7 (a) (ii)**

$$a = 2n + 1, b = 2n^2 + 2n, c = 2n^2 + 2n + 1$$

$$a^2 + b^2$$

$$= (2n + 1)^2 + (2n^2 + 2n)^2$$

$$= 4n^2 + 4n + 1 + 4n^4 + 8n^3 + 4n^2$$

$$= 4n^4 + 8n^3 + 8n^2 + 4n + 1$$

$$c^2 = (2n^2 + 2n + 1)^2$$

$$= (2n^2 + 2n + 1)(2n^2 + 2n + 1)$$

$$= 4n^4 + 4n^3 + 2n^2 + 4n^3 + 4n^2 + 2n + 2n^2 + 2n + 1$$

$$= 4n^4 + 8n^3 + 8n^2 + 4n + 1$$

$$\therefore a^2 + b^2 = c^2$$

**MARKING SCHEME NOTES****Question 7 (a) (i) [Scale 10B (0, 5, 10)]**

- 5: • Correct substitution of chosen value  
 • Not squaring values

NOTE: Allow 10 marks for  $n = 0$  and correct work in (a)(i)

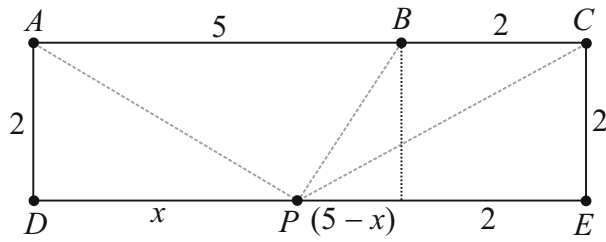
**Question 7 (a) (ii) [Scale 10D (0, 3, 7, 8, 10)]**

- 3: •  $a^2$  or  $b^2$  or  $c^2$  expressed in terms of  $n$   
 7: • Any two terms  
 8: • Three terms fully squared  
 •  $(a^2 + b^2)$  fully worked out in terms of  $n$

**Notes for (a)(i) and (a)(ii):**

- Mark particular case with scheme for (a)(i) wherever it occurs
- Mark general case with scheme for (a)(ii) wherever it occurs

**Question 7 (b) (i)**



$$|PA|^2 = x^2 + 2^2 = x^2 + 4$$

$$\begin{aligned} |PB|^2 &= (5-x)^2 + 2^2 \\ &= 25 - 10x + x^2 + 4 \\ &= x^2 - 10x + 29 \end{aligned}$$

$$\begin{aligned} |PC|^2 &= (7-x)^2 + 2^2 \\ &= 49 - 14x + x^2 + 4 \\ &= x^2 - 14x + 53 \end{aligned}$$

$$\begin{aligned} f(x) &= |PA|^2 + |PB|^2 + |PC|^2 \\ &= x^2 + 4 + x^2 - 10x + 29 + x^2 - 14x + 53 \\ &= 3x^2 - 24x + 86 \end{aligned}$$

**Question 7 (b) (ii)**

$$f(x) = 3x^2 - 24x + 86$$

$$f'(x) = 6x - 24 = 0$$

$$\therefore x = 4 = k$$

$$f(4) = 3(4)^2 - 24(4) + 86 = 38$$

**MARKING SCHEME NOTES**

**Question 7 (b) (i) [Scale 5D (0, 2, 3, 4, 5)]**

- 2: • Expression for either  $|PA|^2$  or  $|PB|^2$  or  $|PC|^2$  in terms of  $x$
- Any appropriate construction line, e.g. the line  $PM$
- 3: • Correct expression of two sides in terms of  $x$
- 4: • Correct expression of three sides in terms of  $x$
- Correct expression of function in  $x$  not simplified

**Question 7 (b) (ii) [Scale 15C (0, 7, 10, 15)]**

- 7: • Stating  $f'(x) = 0$  with no work
- Any correct differentiation

- 10: • Finding value of  $x$

**OR**

- 7: • 3 as factor

- 10: • Finding value of  $x$

**QUESTION 8 (50 MARKS)**

**Question 8 (a)**

$$y = -0.013x^2 + 0.624x$$

$$\frac{dy}{dx} = -0.026x + 0.624 = 0$$

$$0.624 = 0.026x$$

$$\therefore x = \frac{0.624}{0.026} = 24$$

$$x = 24: y = -0.013(24)^2 + 0.624(24) = 7.488$$

Therefore,  $C(24, 7.488)$ .

FIND TURNING POINTS (LOCAL MAXIMUM/MINIMUM)

Put  $\frac{dy}{dx} = 0$  and solve for  $x$

**MARKING SCHEME NOTES**

**Question 8 (a) [Scale 15C (0, 7, 10, 15)]**

- 7: • Identifies  $x = 24$   
• Any correct differentiation
- 10: • Substitutes  $x = 24$  in  $f(x)$

**Question 8 (b)**

Equation of walking deck:  $y = 5$

Equation of parabola:  $y = -0.013x^2 + 0.624x$

Find the points of intersection  $D$  and  $E$ .

$$-0.013x^2 + 0.624x = 5$$

$$-0.013x^2 + 0.624x - 5 = 0$$

$$a = -0.013, b = 0.624, c = -5$$

$$x = \frac{-0.624 \pm \sqrt{0.624^2 - 4(-0.013)(-5)}}{2(-0.013)} \approx 10, 38$$

$\therefore D(10, 5), E(38, 5)$

**FORMULAE AND TABLES BOOK**

**Algebra: Roots of the quadratic equation**

$$ax^2 + bx + c = 0 \text{ [page 20]}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**MARKING SCHEME NOTES**

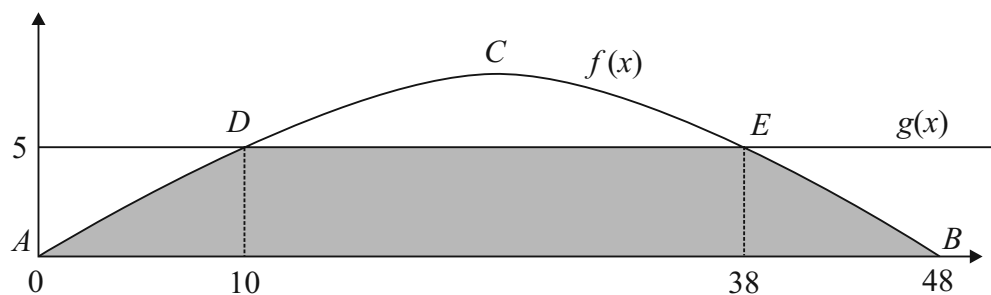
**Question 8 (b) [Scale 10C (0, 5, 7, 10)]**

- 5: • Recognition of line  $y = 5$
- 7: • Values in quadratic formula  
• Gets  $x$  values only

**NOTES:** Accept  $x = 10$  and  $x = 38$  by trial and error from correct quadratic for full marks

**NO CREDIT:** Uses  $x = 5$

### Question 8 (c)



$$f(x) = -0.013x^2 + 0.624x$$

$$g(x) = 5$$

Find the area under the  $f(x)$  curve between 0 and 10, multiply it by 2 (as it is the same as the area under the same curve between 38 and 48) and add on the area of the rectangle in the middle. The area of the rectangle is obtained by integrating  $g(x)$  between 10 and 38.

Obviously, there is an easier way to find the area of a rectangle (length by breadth =  $28 \times 5 = 140$ ), but you were asked to use integration methods. I think, either way, full marks will be given.

$$\begin{aligned} \text{Shaded area} &= 2 \int_0^{10} (-0.013x^2 + 0.624x) dx + \int_{10}^{38} 5 dx \\ &= 2 \left[ -\frac{0.013x^3}{3} + \frac{0.624x^2}{2} \right]_0^{10} + [5x]_{10}^{38} \\ &= 2 \left[ -\frac{0.013x^3}{3} + 0.312x^2 \right]_0^{10} + 5[x]_{10}^{38} \\ &= 2 \left[ \left( -\frac{0.013(10)^3}{3} + 0.312(10)^2 \right) - 0 \right] + 5(38 - 10) \\ &= 2 \left[ \frac{403}{15} \right] + 140 \approx 194 \text{ m}^2 \end{aligned}$$

#### MARKING SCHEME NOTES

#### Question 8 (c) [Scale 10D (0, 3, 7, 8, 10)]

- 3:**
- Any area formula
  - Area under curve from  $A$  to  $B$
  - Area under curve from  $D$  to  $E$
  - Correct limits
- 7:**
- Any correct integration
  - One correct area only
  - Area under curve from  $A$  to  $B$  minus area under curve from  $D$  to  $E$
- 8:**
- Limits substituted but not evaluated (must state that area of rectangle is 140)
  - Both areas
- OR**
- 3:**
- Any one area
- 7:**
- Translation
- 8:**
- 3rd area

**Question 8 (d)**

$$y = -0.013x^2 + 0.624x$$

$$y = -0.013(x^2 - 48x)$$

$$y = -0.013(x^2 - 48x + (-24)^2 - 576) \quad [\text{Completing the square}]$$

$$y = -0.013(x^2 - 48x + (-24)^2 - 576)$$

$$y = -0.013((x - 24)^2 - 576)$$

$$y = -0.013(x - 24)^2 + 7.488$$

$$y - 7.488 = -0.013(x - 24)^2$$

**Question 8 (e)**

$$y = -0.013x^2 + 0.624x$$

$$\text{Local maximum} = (24, 7.488)$$

$$y - 7.488 = -0.013(x - 24)^2$$

$$y = -2x^2 + \dots$$

$$\text{Local maximum} = (3, -4)$$

$$\therefore y - (-4) = -2(x - 3)^2$$

$$y + 4 = -2(x - 3)^2$$

$$y + 4 = -2(x^2 - 6x + 9)$$

$$y + 4 = -2x^2 + 12x - 18$$

$$y = -2x^2 + 12x - 22$$

**MARKING SCHEME NOTES****Question 8 (d) [Scale 10C (0, 5, 7, 10)]**

- 5: • Common factor of  $-0.013$  identified  
 • Attempt at equating like to like  
 • Attempt at completing square
- 7: • Values of two of the constants found  
 •  $p$  correct and completion of square correct

**Question 8 (e) [Scale 5B (0, 3, 5)]**

- 3: One correct value for the equivalent of  $k$ ,  $p$  or  $h$  in equation

**QUESTION 9 (60 MARKS)****Question 9 (a)**

$$y = 100 - 23 = 77^\circ \text{C}$$

$$t = 0, y = 77$$

$$y = Ae^{kt}$$

$$77 = Ae^{k(0)}$$

$$77 = Ae^0$$

$$\therefore A = 77$$

**Question 9 (b)**

$$t = 5, y = 88 - 23 = 65$$

$$y = Ae^{kt}$$

$$65 = 77e^{5k}$$

$$e^{5k} = \frac{65}{77} \Rightarrow 5k = \ln\left(\frac{65}{77}\right)$$

$$k = \frac{1}{5} \ln\left(\frac{65}{77}\right) = -0.0339$$

**MARKING SCHEME NOTES****Question 9 (a) [Scale 10C (0, 5, 7, 10)]**

- 5: • Value of  $y$  only  
 • Some use of 100 and/or 23
- 7: • Correct substitution into equation  
 •  $A$  calculated from incorrect  $y$

**Question 9 (b) [Scale 10C (0, 5, 7, 10)]**

- 5: • Value of  $y$  only  
 • Some use of 88 and/or 23
- 7: • Correct expression for  $k$   
 •  $k$  calculated from incorrect  $y$

**Question 9 (c)**

$$t = ?, y = 50 - 23 = 27$$

$$y = Ae^{kt}$$

$$27 = 77e^{-0.0339t}$$

$$e^{-0.0339t} = \frac{27}{77} \Rightarrow -0.0339t = \ln\left(\frac{27}{77}\right)$$

$$\therefore t = -\frac{1}{0.0339} \ln\left(\frac{27}{77}\right) = 30.91 \approx 31 \text{ minutes}$$

**MARKING SCHEME NOTES****Question 9 (c) [Scale 10C (0, 5, 7, 10)]**

- 5: • Value of  $y$  only
- 7: • Correct expression for  $t$   
 •  $t$  calculated from incorrect  $y$

**Question 9 (d)**

$$y = f(t) = 77e^{-0.0339t}$$

$$t = 0: f(0) = 77e^{-0.0339(0)} = 77$$

$$t = 10: f(0) = 77e^{-0.0339(10)} = 54.9 \approx 55$$

$$t = 20: f(0) = 77e^{-0.0339(20)} = 39.1 \approx 39$$

$$t = 30: f(0) = 77e^{-0.0339(30)} = 27.8 \approx 28$$

$$t = 40: f(0) = 77e^{-0.0339(40)} = 19.8 \approx 20$$

$$t = 50: f(0) = 77e^{-0.0339(50)} = 14.1 \approx 14$$

$$t = 60: f(0) = 77e^{-0.0339(60)} = 10.1 \approx 10$$

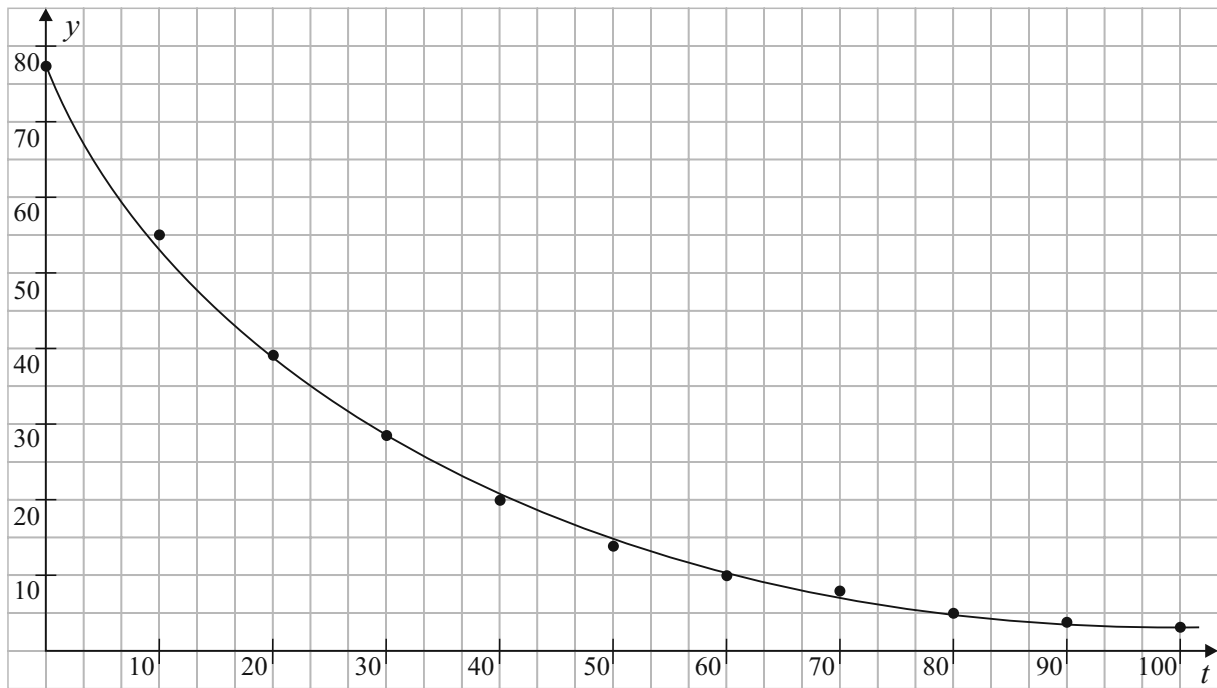
$$t = 70: f(0) = 77e^{-0.0339(70)} = 7.2 \approx 7$$

$$t = 80: f(0) = 77e^{-0.0339(80)} = 5.1 \approx 5$$

$$t = 90: f(0) = 77e^{-0.0339(90)} = 3.6 \approx 4$$

$$t = 100: f(0) = 77e^{-0.0339(100)} = 2.6 \approx 3$$

$t$	$f(t)$
0	77
10	55
20	39
30	28
40	20
50	14
60	10
70	7
80	5
90	4
100	3

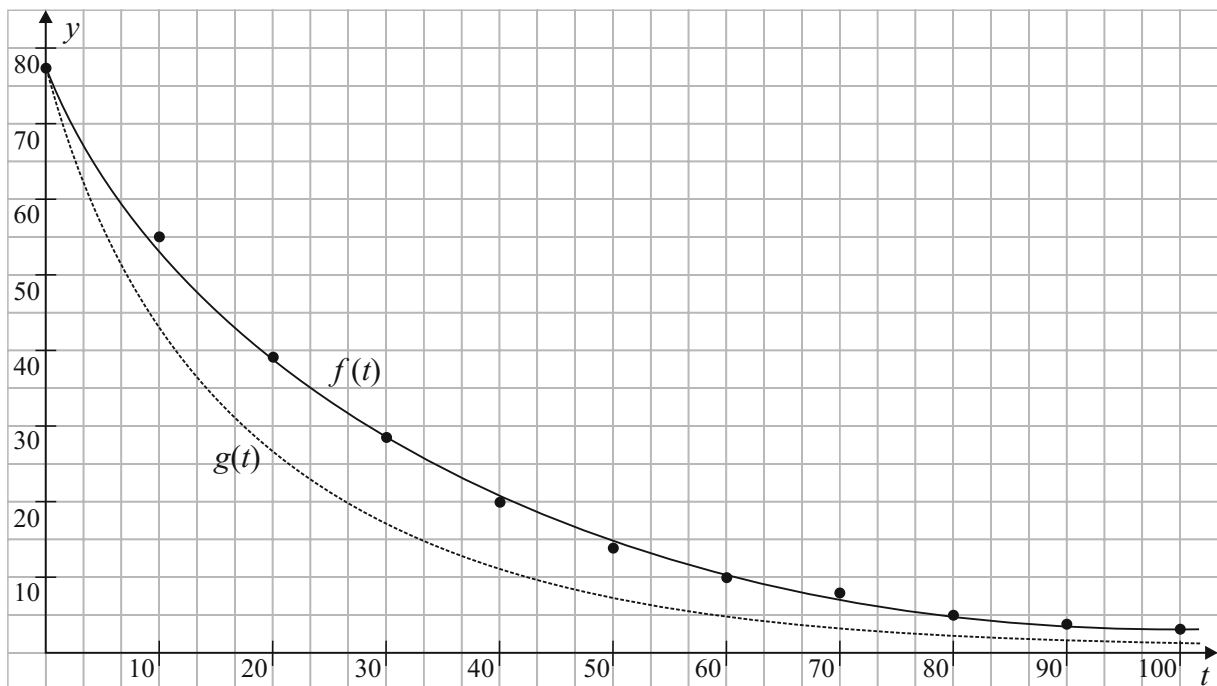


**MARKING SCHEME NOTES**

**Question 9 (d) [Scale 15C (0, 7, 10, 15)]**

- 7: • Any one point identified  
 • Graph of correct shape, even if no point correct or no point calculated  
 • Accept candidates value of  $k$   
**NOTE:** All graphs may not be the same, due to different values of  $A$  and  $k$
- 10: • Three points correctly plotted, but graph incomplete or no graph  
**NOTE:** Do not accept straight line graph

**Question 9 (e) (i)**



**Question 9 (e) (ii)**

$$g(t) = 77e^{-0.05t}$$

$$m = -0.05$$

This value of  $m$  gives a larger average rate of change for the cooling time required.

**MARKING SCHEME NOTES****Question 9 (e) (i) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: • Any attempt at similar graph  
• No graph but correct deduction
- 4: • Correct graph plotted but graph incomplete, or no graph

**Question 9 (f) (i)**

$$f(t) = 77e^{-0.0339t}$$

$$f'(t) = 77(-0.0339)e^{-0.0339t} = -2.6103e^{-0.0339t}$$

$$f'(1) = -2.6103e^{-0.0339(1)} = -2.52 \text{ }^\circ\text{C per minute}$$

$$f'(10) = -2.6103e^{-0.0339(10)} = -1.86 \text{ }^\circ\text{C per minute}$$

**Question 9 (f) (ii)**

$$f'(t) = -2.6103e^{-0.0339t}$$

$$f''(t) = -2.6103(-0.0339)e^{-0.0339t} = \frac{0.088}{e^{0.0339t}} > 0 \text{ for all } t$$

Therefore, the rate of change of  $f(t)$  is always increasing over time.

**MARKING SCHEME NOTES****Question 9 (f) (i) [Scale 5C (0, 3, 4, 5)]**

- 3: • Indication of differentiation i.e.  $\frac{dy}{dt}$ ,  $\frac{dx}{dt}$  or  $f'(t)$  (i.e. differentiation with respect to  $t$ )  
• Treats  $e$  as  $x$  in differentiation
- 4: • One value of  $\frac{dy}{dt}$  indicated

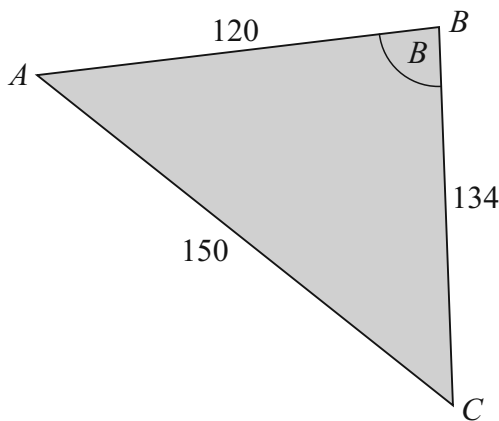
**Question 9 (f) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: • Attempt at 2nd derivative  
• Attempt at deduction from numerical values
- 4: • Shows 2nd derivative positive

## LC 2014 (SET E): PAPER 2

### QUESTION 1 (25 MARKS)

#### Question 1 (a) (i)



$$150^2 = 120^2 + 134^2 - 2(120)(134)\cos B$$

$$2(120)(134)\cos B = 120^2 + 134^2 - 150^2$$

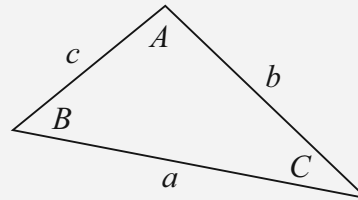
$$B = \cos^{-1}\left(\frac{120^2 + 134^2 - 150^2}{2(120)(134)}\right) = 72 \cdot 15^\circ$$

#### Question 1 (a) (ii)

$$\text{Area} = \frac{1}{2}(120)(134)\sin 72 \cdot 15^\circ = 7653 \text{ m}^2$$

#### FORMULAE AND TABLES BOOK Trigonometry of the triangle:

[page 16]



$$\text{Area: } \frac{1}{2}ab\sin C$$

$$\text{Sine Rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc\cos A$$

#### MARKING SCHEME NOTES

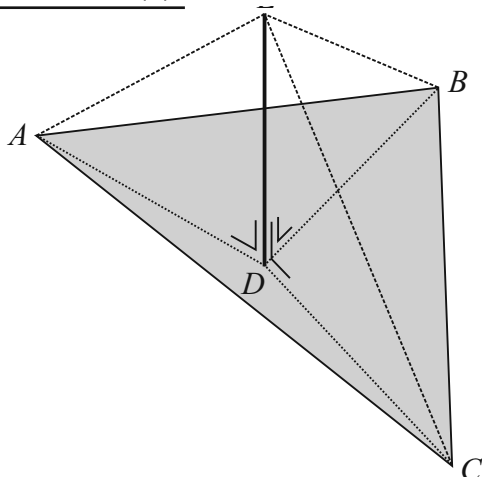
##### Question 1 (a) (i) [Scale 15D (0, 4, 7, 11, 15)]

- 4: • Identifies Cosine Rule formula
- 7: • All values correctly inserted
- 11: •  $\cos(\angle CBA)$  evaluated but angle not found  
• Substantially correct work with one non arithmetic error

##### Question 1 (a) (ii) [Scale 5C (0, 2, 3, 5)]

- 2: • Relevant area formula  
• Effort at finding a perpendicular height
- 3: • Substantially correct work with one non arithmetic error  
• Values correctly inserted

#### Question 1 (b)



Triangles  $EDA$ ,  $EDB$  and  $EDC$  are congruent because (SAS):

- $|AD| = |BD| = |CD|$  [ $D$  is the circumcentre]
- $|ED|$  is common to all 3 triangles
- $|\angle EDA| = |\angle EDB| = |\angle EDC| = 90^\circ$

Therefore,  $|AE| = |BE| = |CE|$ .

**MARKING SCHEME NOTES**

**Question 1 (b) [Scale 5C (0, 2, 3, 5)]**

- 2: • Recognises  $|AD| = |DB| = |DC|$  (any two)  
 • Recognises one relevant right angle  
 • Indicates some understanding of circumcentre of a triangle
- 3: • Recognises  $|AD| = |DB| = |DC|$  and relevant right angles but fails to conclude fully  
 • Clearly identifies two congruent triangles but does not make reference to the remaining triangle

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**

Replace angle  $B$  by  $A$  in the formula from the table book:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A + A) = \cos A \cos A - \sin A \sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

**FORMULAE AND TABLES BOOK**

**Trigonometry:**

**Compound angle formulae [page 14]**

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 15C (0, 5, 10, 15)]**

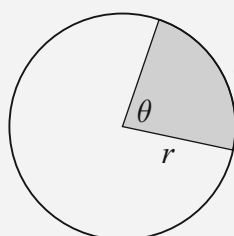
- 5: • Relevant compound angle formula  
 • Tested with one or more values for  $A$
- 10: • Expansion correct but not tidied

**Question 2 (b)**

**FORMULAE AND TABLES BOOK**

**Length and area:**

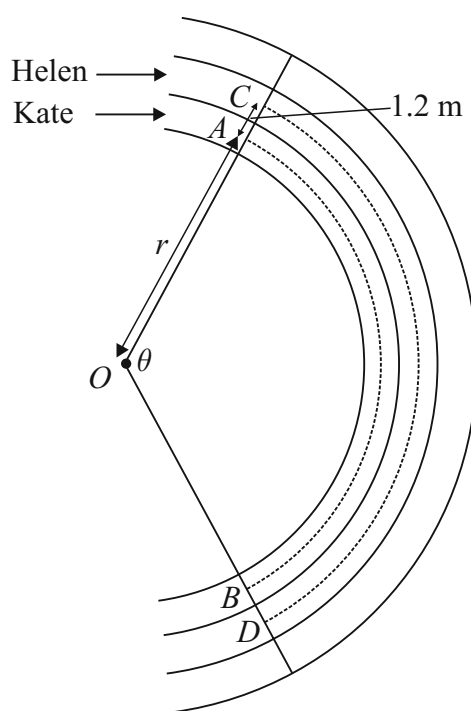
**Arc/sector [page 9]**



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

when  $\theta$  is in radians.



Call  $r$  the distance  $|OA|$ .

$$|AB| + 3 = |CD|$$

$$|AB| = r\theta \dots (1)$$

$$|CD| = (r + 1.2)\theta \Rightarrow |AB| + 3 = r\theta + 1.2\theta \dots (2)$$

$$(2) - (1) : 3 = 1.2\theta \Rightarrow \theta = \frac{3}{1.2} = 2.5 \text{ radians}$$

**MARKING SCHEME NOTES****Question 2 (b) [Scale 10D (0, 2, 5, 8, 10)]**

- 2: • Correct formula for finding either arc  
 5: • One or both arcs expressed correctly  
 8: •  $\theta$  not fully evaluated  
 •  $|CD| - |AB| = 3$  or equivalent statement  
 • Substantially correct with one non arithmetic error

**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$\text{Number of arrangements of } n \text{ objects, } p \text{ alike of one kind, } q \text{ alike of another kind} = \frac{n!}{p!q!}$$

In order to win €8 playing Game A four times, John would have hit the following segments: €0, €0, €3 and €5, but not necessarily in that order.

How many arrangements can be made of 4 objects, 2 of which are alike?

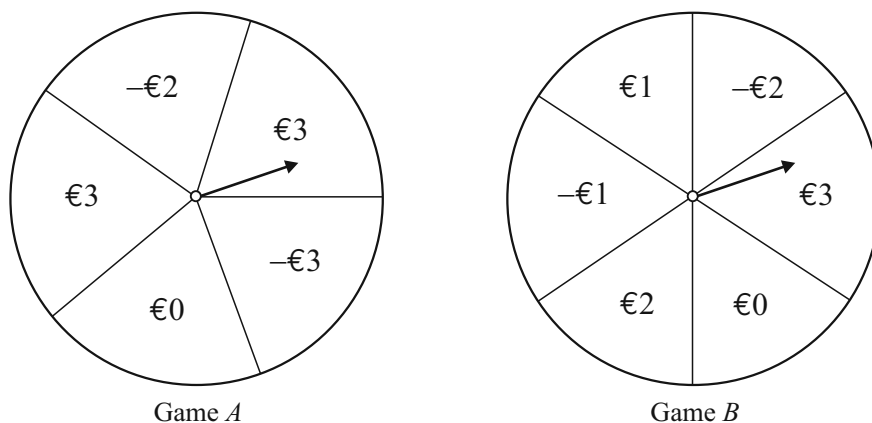
$$\text{Number of ways John could win €8} = \frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$$

**MARKING SCHEME NOTES****Question 3 (a) [Scale 10C (0, 3, 7, 10)]**

- 3: • Some reference to €3 and €5  
 7: • Listing with not more than five omitted

**Question 3 (b)**

The spinners are redrawn and the values represent the amount the charity would gain for each spin.



Calculate the expected earnings  $E(x)$  for each €3 bet placed.

$$\text{Game A: } E(x) = \frac{1}{5}(3) - \frac{1}{5}(2) + \frac{1}{5}(3) + \frac{1}{5}(0) - \frac{1}{5}(3) = \frac{1}{5} = \text{€}0.20$$

$$\text{Game B: } E(x) = \frac{1}{6}(3) - \frac{1}{6}(2) + \frac{1}{6}(1) - \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(0) = \frac{3}{6} = \text{€}0.50$$

**EXPECTED VALUE  $E(x)$** 

$$E(x) = \sum xP(x)$$

Game B would be more successful at earning money for the charity as its expected earnings are higher per spin.

**MARKING SCHEME NOTES****Question 3 (b) [Scale 5C (0, 2, 3, 5)]**

- 2: • One partially accurate statement  
• Expected outcome formula
- 3: • Correct answer but inaccurate/only partially correct supporting evidence  
• Expected outcome of both Game *A* and Game *B* calculated but incorrect or no conclusion.

**Question 3 (c)**

$$p = P(4) = \frac{1}{6}, q = P(\text{Not } 4) = \frac{5}{6}$$

$$n = 6, r = 2, p = \frac{1}{6}, q = \frac{5}{6}$$

$$P(2 \text{ successes}) = {}^6C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4 = \frac{3125}{15552} = 0.2$$

**BERNOULLI TRIALS**

$$P(r \text{ successes}) = {}^nC_r p^r q^{n-r}$$

**MARKING SCHEME NOTES****Question 3 (c) [Scale 10C (0, 3, 7, 10)]**

- 3: • Establishes probability of stopping on €4 sector once  
• Establishes probability of not stopping on €4 sector once  
• Effort to express a relevant binomial expansion

7: • Omits  $\binom{6}{2}$

- Indices incorrectly assigned

**QUESTION 4 (25 MARKS)****Question 4 (a)**

$$V = 311 \sin(100\pi t)$$

(i)  $R = [-311, 311]$

(ii)  $P = \frac{2\pi}{100\pi} = 0.02 \text{ s}$

$$f = \frac{1}{P} = \frac{1}{0.02} = 50 \text{ Hz}$$

**NOTE:**  $P = 0.02 \text{ s}$  can also be read off the graph.

**PERIOD *P* AND RANGE *R***

$$y = a \sin bx$$

$$\text{Range } R = [-a, a]$$

$$\text{Period } P = \frac{2\pi}{b}$$

$$\text{Frequency } f = \frac{1}{P}$$

**MARKING SCHEME NOTES****Question 4 (a) (i) (ii) [Scale 10C (0, 3, 7, 10)]**

- 3: • Some reference to 311 or -311  
• Some indication that term 'range' understood  
• Ranges (other than correct one) between  $\pm 300, \pm 320$  (consistent)  
• Some reference to how long it takes to complete a cycle  
• Time for one period found
- 7: • Correct range  
• Correct number of periods

**Question 4 (b) (i)**

$t$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6 = 0.01$	$t_7$	$t_8$	$t_9$	$t_{10}$	$t_{11}$	$t_{12} = 0.02$
$V$	156	269	311	269	156	0	-156	-269	-311	-269	-156	0

**Question 4 (b) (ii)**

$$\mu = 0$$

$$\sigma = \sqrt{\frac{2 \times 0^2 + 4 \times 156^2 + 4 \times 269^2 + 2 \times 311^2}{12}} = 220 \text{ V}$$

**FORMULAE AND TABLES BOOK**  
**Statistics and probability:**  
 Standard deviation from a list of  $n$   
 numbers [page 33]

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}}$$

**MARKING SCHEME NOTES****Question 4 (b) (i) (ii) [Scale 10C (0, 3, 7, 10)]**

- 3:**
- At least three further entries in table correct
  - Formula for standard deviation
- 7:**
- Table correct
  - Errors in table (with at least three additional entries) but standard deviation correct from candidates work

**Question 4 (c) (i)**

$$V_{\max} = 311 \text{ V}$$

$$\sigma = 220 \text{ V}$$

$$k\sigma = V_{\max}$$

$$\therefore k = \frac{V_{\max}}{\sigma} = \frac{311}{220} = 1.414$$

**Question 4 (c) (ii)**

$$f = 60 \text{ Hz}$$

$$\therefore P = \frac{1}{60} = \frac{2\pi}{b} \Rightarrow b = 120\pi$$

$$\sigma = 110 \text{ V}, k = 1.414$$

$$a = V_{\max} = k\sigma$$

$$\therefore a = 1.414 \times 110 = 155.54 \text{ V}$$

**PERIOD  $P$  AND RANGE  $R$** 

$$y = a \sin bx$$

$$\text{Range } R = [-a, a]$$

$$\text{Period } P = \frac{2\pi}{b}$$

$$\text{Frequency } f = \frac{1}{P}$$

**MARKING SCHEME NOTES****Question 4 (c) (i) (ii) [Scale 5C (0, 2, 3, 5)]**

- 2:**
- $k$  isolated in formula
  - Value(s) entered in formula
  - $\frac{b}{2\pi}$  written
- 3:**
- $k$  or  $b$  or  $a$  found

**QUESTION 5 (25 MARKS)**

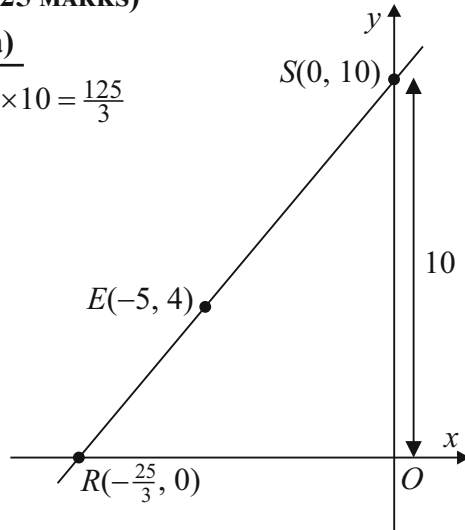
**Question 5 (a)**

$$\text{Area} = \frac{1}{2}|OR| \times 10 = \frac{125}{3}$$

$$5|OR| = \frac{125}{3}$$

$$\therefore |OR| = \frac{25}{3}$$

$$\therefore R\left(-\frac{25}{3}, 0\right)$$



**FORMULAE AND TABLES BOOK:**  
**Length and area (page 9)**  
**TRIANGLE**

$$A = \frac{1}{2}ah$$

$$A = \frac{1}{2}ab \sin C$$

**Question 5 (b)**

$$R\left(-\frac{25}{3}, 0\right), S(0, 10)$$

$$\text{Slope of } RS = \frac{10 - 0}{0 - \left(-\frac{25}{3}\right)} = \frac{6}{5}$$

$$\text{Equation of } RS: y - 10 = \frac{6}{5}(x - 0)$$

$$5y - 50 = 6x$$

$$6x - 5y + 50 = 0$$

$$E(-5, 4) \in RS: 6x - 5y + 50 = 0?$$

$$6(-5) - 5(4) + 50$$

$$= -30 - 20 + 50$$

$$= 0$$

$$\therefore E(-5, 4) \in RS$$

**FORMULAE AND TABLES BOOK**  
**Co-ordinate geometry: Line**

Slope of  $PQ$  [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of  $PQ$  [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

**MARKING SCHEME NOTES**

**Question 5 (a) [Scale 10C (0, 3, 7, 10)]**

- 3: • Relevant area of triangle formula  
 7: •  $|OR|$  found but  $x$  ordinate of  $R$  not stated  
 • Substantially correct work with one error

**Question 5 (b) [Scale 10C (0, 3, 7, 10)]**

- 3: • Effort at finding one slope  
 • Effort at finding equation of  $RS$   
 7: • Relevant conclusion not stated or implied  
 •  $E$  inserted into equation of  $RS$  but relevant conclusion not stated or implied

**Question 5 (c)**

$$y = mx + c$$

$$E(-5, 4) \in y \Rightarrow 4 = -5m + c$$

$$\therefore c = 5m + 4$$

$$y = mx + 5m + 4$$

$$x = 0: y = 5m + 4$$

$\therefore (0, 5m + 4)$  is the  $y$  intercept

$$y = 0: 0 = mx + 5m + 4$$

$$mx = -5m - 4$$

$$x = \frac{-5m - 4}{m}$$

$\therefore \left( \frac{-5m - 4}{m}, 0 \right)$  is the  $x$  intercept

$$\text{Area} = \frac{1}{2} \left( \frac{5m + 4}{m} \right) (5m + 4) = \frac{125}{3}$$

$$3(5m + 4)^2 = 250m$$

$$3(25m^2 + 40m + 16) = 250m$$

$$75m^2 + 120m + 48 = 250m$$

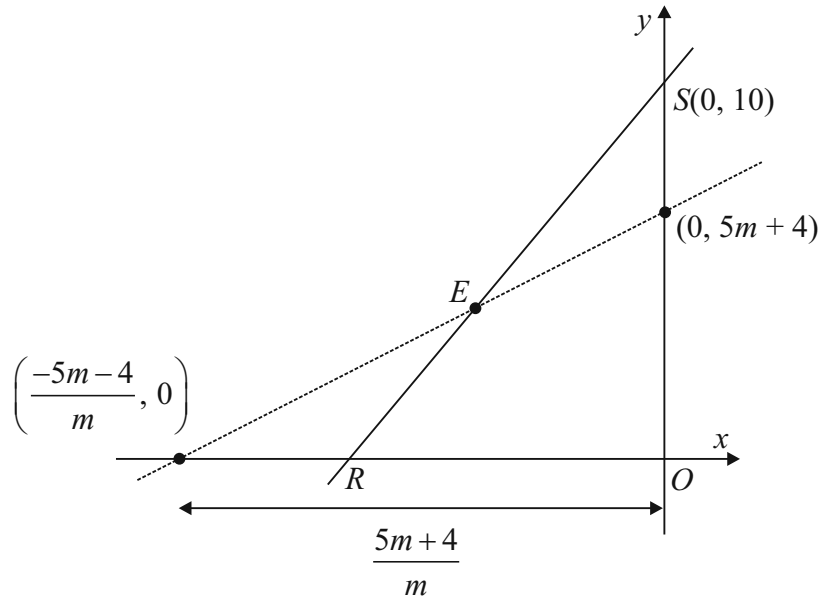
$$75m^2 - 130m + 48 = 0$$

$$(5m - 6)(15m - 8) = 0$$

$$\therefore m = \frac{6}{5}, \frac{8}{15}$$

$$c = 4 + 5m = 4 + 5\left(\frac{8}{15}\right) = \frac{20}{3}$$

ANSWERS:  $m = \frac{8}{15}, c = \frac{20}{3}$



**MARKING SCHEME NOTES**

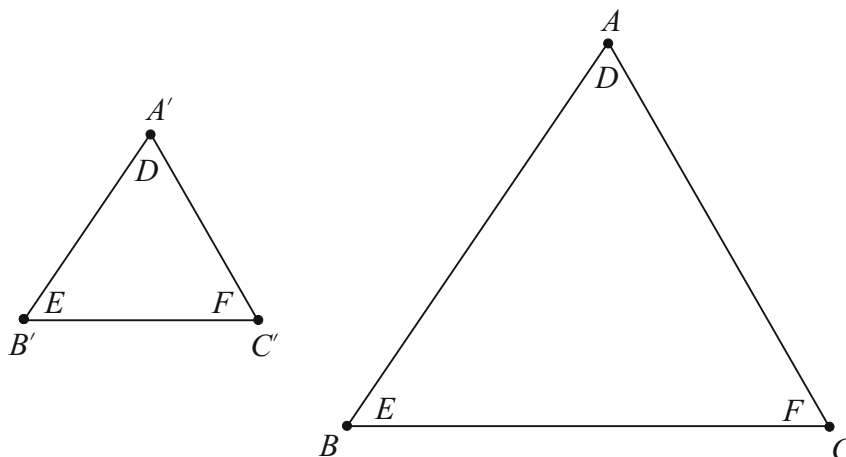
**Question 5 (c) [Scale 5C (0, 2, 3, 5)]**

- 2: • Effort at finding intercept on one or both axes  
• Effort at inserting  $(-5, 4)$  into  $y = mx + c$
- 3: • Either  $c$  or  $m$  found

**QUESTION 6A (25 MARKS)**

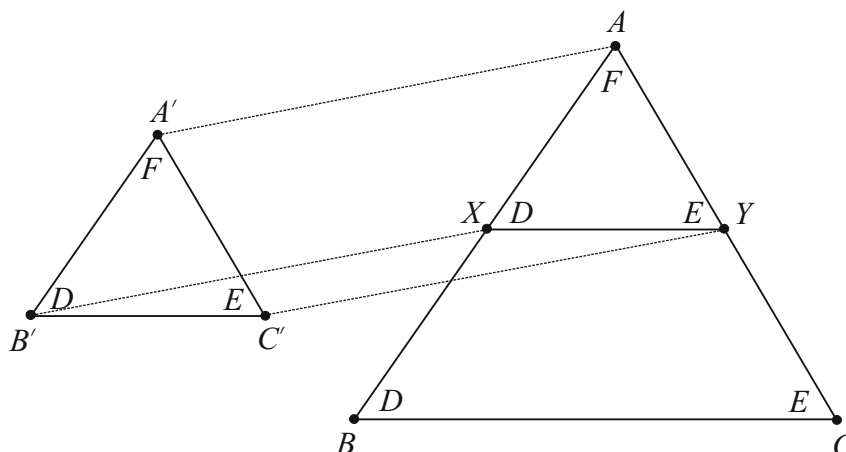
**Question 6A (a)**

**GIVEN:**  $\triangle ABC$  and  $\triangle A'B'C'$  with  $|\angle CAB| = |\angle C'A'B'| = D$ ,  $|\angle ABC| = |\angle A'B'C'| = E$  and  $|\angle ACB| = |\angle A'C'B'| = F$ .



**TO PROVE:**  $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$ .

**CONSTRUCTION:** Translate  $\triangle A'B'C'$  so that  $A' \rightarrow A$ ,  $B' \rightarrow X$  and  $C' \rightarrow Y$ .



**PROOF:**

$[XY]$  is parallel to  $[B'C']$  because  $|\angle AXY| = |\angle ABC| = \angle D$   
and  $|\angle AYX| = |\angle ACB| = \angle E$  [Corresponding angles]

$$\therefore \frac{|AB|}{|AX|} = \frac{|AC|}{|AY|} \Rightarrow \frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|}$$

Similarly  $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|}$ .

**MARKING SCHEME NOTES**

**Question 6A (a)**

**Diagram [Scale 5B (0, 2, 5)]**

2: • Effort at *Diagram* or *Given*

**Construction [Scale 5B (0, 2, 5)]**

2: • Construction attempted  
• Construction not explained or explanation incomplete

**Proof [Scale 10C (0, 3, 7, 10)]**

3: • More than one critical step omitted but still some substantial work of merit  
7: • Proof completed with one critical step omitted

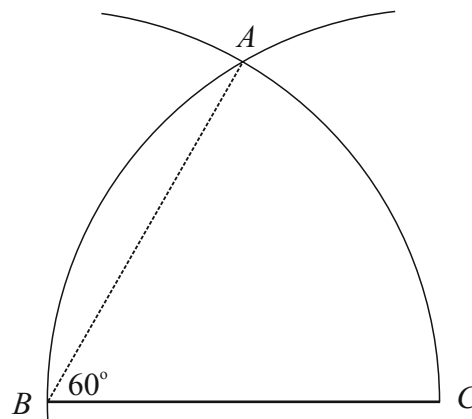
**Question 6A (b)**

Put point of compass on  $B$  and stretch it to  $C$ .  
Draw an arc as shown.

Now put the point on  $C$  and draw the arc as shown.

$A$  is where the two arcs meet. Triangle  $ABC$  is an equilateral triangle where each angle is  $60^\circ$ .

Draw the line  $BA$ . Angle  $ABC$  is a  $60^\circ$  angle.



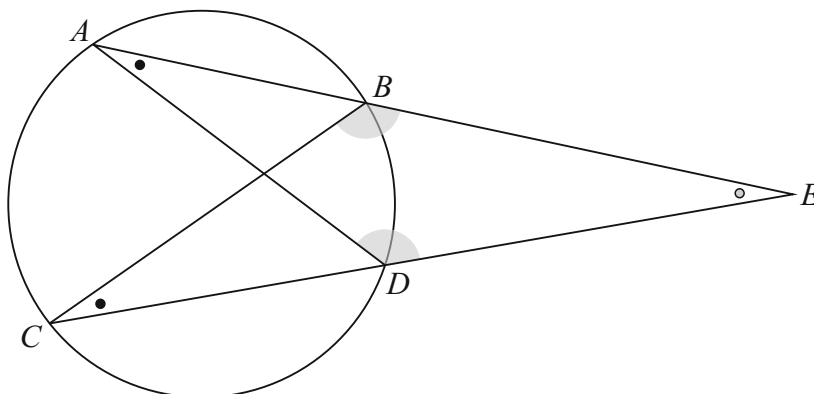
**MARKING SCHEME NOTES**

**Question 6A (b) [Scale 5B (0, 2, 5)]**

2: • Arc  $AC$  and/or arc  $AB$   
• Effort at drawing arc from  $B$

**QUESTION 6B (25 MARKS)**

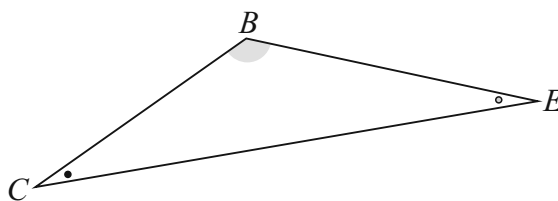
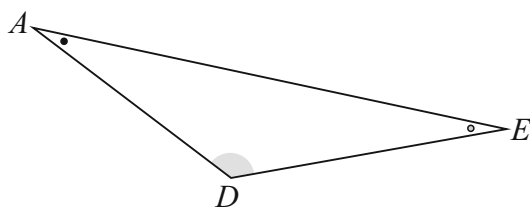
**Question 6B (a)**



$\triangle ADE$  and  $\triangle CEB$  are similar triangles because:

- $|\angle EAD| = |\angle ECB|$  [Angles standing on the same arc are equal]
- $|\angle AED| = |\angle CEB|$  [Common angle]
- $|\angle EDA| = |\angle EBC|$  [3 angles in a triangle add up to  $180^\circ$ ]

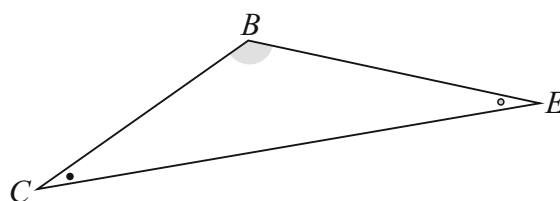
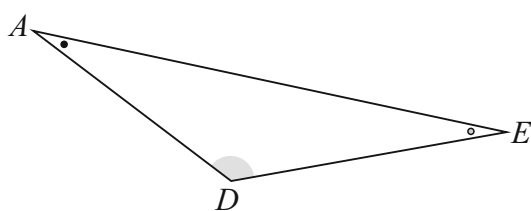
**Question 6B (b)**



$$\frac{|EA|}{|EC|} = \frac{|ED|}{|EB|} \quad [\text{The sides of similar triangles are proportional}]$$

$$\therefore |EA| \cdot |EB| = |EC| \cdot |ED|$$

**Question 6B (c)**



$$\frac{|EB|}{|ED|} = \frac{|CB|}{|AD|}$$

$$\therefore |AD| = \frac{|ED| \cdot |CB|}{|EB|} = \frac{5.94 \times 10}{6.25} = 9.504$$

**MARKING SCHEME NOTES**

**Question 6B (a) [Scale 10C (0, 3, 7, 10)]**

- 3: • Triangles named
- 7: • Two pairs of angles in relevant triangles identified but justification incomplete
- Two pairs of angles identified with justification but triangles not named

**Question 6B (b) [Scale 10C (0, 3, 7, 10)]**

- 3: • Relevant triangles identified
- Partly correct ratio
- 7: • Correct ratio established but fails to complete

**Question 6B (c) [Scale 5C (0, 2, 3, 5)]**

- 2: • Effort at establishing ratio
- 3: • Ratio established and values entered

**QUESTION 7 (45 MARKS)**

**Question 7 (a)**

Pensioners  
Students

**MARKING SCHEME NOTES**

**Question 7 (a) [Scale 10B (0, 5, 10)]**

**5:** • One correct category only

**Question 7 (b)**

Table 1										
Labour Force Statistics 2004 to 2013 - Persons aged 15 years and over (000's)										
Year	At work			Unemployed			Not in labour force			Total
	M	F	Total	M	F	Total	M	F	Total	
2004	1045.9	738.9	<b>1784.8</b>	79.6	31.6	111.2	457.1	854.2	1311.3	3207.3
2005	1087.3	779.7	<b>1867.0</b>	81.3	33.5	114.8	459.5	846.6	1306.1	3287.9
2006	1139.8	815.1	<b>1954.9</b>	80.6	38.1	118.7	457.6	844.9	1302.5	3376.1
2007	1184.0	865.6	<b>2049.6</b>	84.3	39.2	123.5	472.4	852.7	1325.1	3498.2
2008	1170.9	889.5	<b>2060.4</b>	106.3	41.0	147.3	494.8	872.5	1367.3	3575.0
2009	1039.8	863.5	<b>1903.3</b>	234.0	82.4	316.4	505.6	874.9	1380.5	3600.2
2010	985.1	843.5	<b>1828.6</b>	257.6	98.2	355.8	529.2	884.6	1413.8	3598.2
2011	970.2	843.2	<b>1813.4</b>	260.7	103.4	364.1	540.1	881.5	1421.6	3599.1
2012	949.6	823.8	<b>1773.4</b>	265.2	108.0	373.2	546.5	896.9	1443.4	3590.0
2013	974.4	829.0	<b>1803.4</b>	227.7	102.3	330.0	557.8	895.0	1452.8	3586.2

CALCULATE QUANTILES AND INTERQUARTILE RANGE FROM AN ORDERED SET OF DATA

The median divides the data into two halves. To divide the data into quarters, you then find the medians of these two halves.

If you have an **even** number of values, where the first median was the average of the two middle values, then you include the middle values in your quartile computations.

If you have an **odd** number of values, where the first median was an actual data point, then you do not include that value in your quartile computations. That is, to find the quartiles, you're only looking at the values that haven't yet been used.

Write out as an ordered list the numbers of people at work between 2004 and 2013.

1 773 400, 1 784 800, **1 803 400**, 1 813 400, **1 828 600**, **1 867 000**, 1 903 300, **1 954 900**, 2 049 600, 2 060 400



Lower quartile  $Q_1$



Median  $Q_2$



Upper quartile  $Q_3$

$$\text{Median } Q_2 = \frac{1\,828\,600 + 1\,867\,000}{2} = 1\,847\,800$$

$$\text{Upper quartile } Q_3 = 1\,954\,900$$

$$\text{Lower quartile } Q_1 = 1\,803\,400$$

$$\text{Interquartile range } Q_3 - Q_1 = 1\,954\,900 - 1\,803\,400 = 151\,500$$

**MARKING SCHEME NOTES****Question 7 (b) [Scale 5C (0, 2, 3, 5)]**

- 2: • Effort at listing numbers in order of magnitude  
 • Identifying either 1828.6 or 1867  
 • Some indication of understanding of the term ‘median’  
 • Identifying 1803.4 and/or 1954.9 or equivalent as relating to quartiles  
 • Some indication of understanding of interquartile range
- 3: • Median correct or interquartile range correct only

**Question 7 (c) (i)**

Table 1										
Labour Force Statistics 2004 to 2013 - Persons aged 15 years and over (000's)										
Year	At work			Unemployed			Not in labour force			Total
	M	F	Total	M	F	Total	M	F	Total	
2004	1045.9	738.9	1784.8	79.6	31.6	111.2	457.1	854.2	1311.3	3207.3
2005	1087.3	779.7	1867.0	81.3	33.5	114.8	459.5	846.6	1306.1	3287.9
<b>2006</b>	1139.8	815.1	<b>1954.9</b>	80.6	38.1	<b>118.7</b>	457.6	844.9	<b>1302.5</b>	<b>3376.1</b>
2007	1184.0	865.6	2049.6	84.3	39.2	123.5	472.4	852.7	1325.1	3498.2
2008	1170.9	889.5	2060.4	106.3	41.0	147.3	494.8	872.5	1367.3	3575.0
2009	1039.8	863.5	1903.3	234.0	82.4	316.4	505.6	874.9	1380.5	3600.2
2010	985.1	843.5	1828.6	257.6	98.2	355.8	529.2	884.6	1413.8	3598.2
<b>2011</b>	970.2	843.2	<b>1813.4</b>	260.7	103.4	<b>364.1</b>	540.1	881.5	<b>1421.6</b>	<b>3599.1</b>
2012	949.6	823.8	1773.4	265.2	108.0	373.2	546.5	896.9	1443.4	3590.0
2013	974.4	829.0	1803.4	227.7	102.3	330.0	557.8	895.0	1452.8	3586.2

$$\text{At work: } \frac{1813.4}{3599.1} \times 100\% = 50.4\%$$

$$\text{Unemployed: } \frac{364.1}{3599.1} \times 100\% = 10.1\%$$

$$\text{Not in the labour force: } \frac{1421.6}{3599.1} \times 100\% = 39.5\%$$

		At work	Unemployed	Not in the labour force
Persons aged 15 years and over	2006	57.9%	3.5%	38.6%
	2011	<b>50.4%</b>	<b>10.1%</b>	<b>39.5%</b>

**MARKING SCHEME NOTES****Question 7 (c) (i) [Scale 5B (0, 2, 5)]**

- 2: • One or two correct (no work shown)  
 • All incorrect but sum to 100%  
 • Correct numerator or denominator chosen for one category - work shown

**Question 7 (c) (ii)**

		At work	Unemployed	Not in the labour force
Total population	2006	46.1%	2.8%	51.1%
	2011			

Table 1										
Labour Force Statistics 2004 to 2013 - Persons aged 15 years and over (000's)										
Year	At work			Unemployed			Not in labour force			Total
	M	F	Total	M	F	Total	M	F	Total	
2004	1045.9	738.9	1784.8	79.6	31.6	111.2	457.1	854.2	1311.3	3207.3
2005	1087.3	779.7	1867.0	81.3	33.5	114.8	459.5	846.6	1306.1	3287.9
<b>2006</b>	1139.8	815.1	<b>1954.9</b>	80.6	38.1	<b>118.7</b>	457.6	844.9	<b>1302.5</b>	<b>3376.1</b>
2007	1184.0	865.6	2049.6	84.3	39.2	123.5	472.4	852.7	1325.1	3498.2
2008	1170.9	889.5	2060.4	106.3	41.0	147.3	494.8	872.5	1367.3	3575.0
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2010	985.1	843.5	1828.6	257.6	98.2	355.8	529.2	884.6	1413.8	3598.2
<b>2011</b>	970.2	843.2	<b>1813.4</b>	260.7	103.4	<b>364.1</b>	540.1	881.5	<b>1421.6</b>	<b>3599.1</b>
2012	949.6	823.8	1773.4	265.2	108.0	373.2	546.5	896.9	1443.4	3590.0
2013	974.4	829.0	1803.4	227.7	102.3	330.0	557.8	895.0	1452.8	3586.2

Persons in the population aged under 15 years of age in 2011 = 979 590

$$\text{At work: } \frac{1813.4}{3599.1 + 979.59} \times 100\% = 39.6\%$$

$$\text{Unemployed: } \frac{364.1}{3599.1 + 979.59} \times 100\% = 8.0\%$$

$$\text{Not in the labour force: } \frac{1421.6 + 979.59}{3599.1 + 979.59} \times 100\% = 52.4\%$$

		At work	Unemployed	Not in the labour force
Total population	2006	46.1%	2.8%	51.1%
	2011	<b>39.6%</b>	<b>8.0%</b>	<b>52.4%</b>

**MARKING SCHEME NOTES**

**Question 7 (c) (ii) [Scale 5B (0, 2, 5)]**

- 2:
- One or two correct (no work shown)
  - All incorrect but sum to 100%
  - Correct total population - shown
  - Correct numerator or denominator chosen for one category - work shown

**Question 7 (c) (iii)**

I agree.

REASON 1: There are more people on the dole which increases the social welfare expenditure.

REASON 2: As fewer people are in the labour force, there is less income tax available to the Government.

**MARKING SCHEME NOTES****Question 7 (c) (iii) [Scale 5B (0, 2, 5)]**

**NOTE:** Answer here depends on candidate's answers in previous sections

- 2: • One reason only  
• Reasons contradictory

**Question 7 (d) (i)**

No.

As is seen from Liam's chart, females at work suffered a decline in numbers from 2008 to 2013. This decline was less than that suffered by males.

It is seen from Niamh's chart that the number of females working as an overall percentage of the workforce increased throughout the period, even though this was a higher percentage of a declining workforce from 2008 onwards.

**Question 7 (d) (ii)**

I found Liam's chart more useful. I can see very clearly on this chart the large fall in males at work from 2009 to 2010.

**Question 7 (d) (iii)**

Year	At work			Unemployed			Not in labour force			Total
	M	F	Total	M	F	Total	M	F	Total	
2004	1045.9	738.9	1784.8	79.6	31.6	111.2	457.1	854.2	1311.3	3207.3
2005	1087.3	779.7	1867.0	81.3	33.5	114.8	459.5	846.6	1306.1	3287.9
2006	1139.8	815.1	1954.9	80.6	38.1	118.7	457.6	844.9	1302.5	3376.1
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2010	985.1	843.5	1828.6	257.6	98.2	355.8	529.2	884.6	1413.8	3598.2
2011	970.2	843.2	1813.4	260.7	103.4	364.1	540.1	881.5	1421.6	3599.1
2012	949.6	823.8	<b>1773.4</b>	265.2	108.0	373.2	546.5	896.9	1443.4	<b>3590.0</b>
2013	974.4	829.0	<b>1803.4</b>	227.7	102.3	330.0	557.8	895.0	1452.8	<b>3586.2</b>

$$\text{At work (2012): } \frac{1773.4}{3590.0} \times 100\% = 49.4\%$$

$$\text{At work (2013): } \frac{1803.4}{3586.2} \times 100\% = 50.3\%$$

Between the years 2012 to 2013, there was a 0.9% increase in employment. Assuming a similar rise the following year, 51.2% of persons, aged 15 years and over, will be at work in 2014.

**MARKING SCHEME NOTES**

**Question 7 (d) (i) [Scale 5B (0, 2, 5)]**

2: • Incomplete or contradictory statement

**Question 7 (d) (ii) [Scale 5B (0, 2, 5)]**

2: • Stating none, one or both without explanation

**Question 7 (d) (iii) [Scale 5C (0, 2, 3, 5)]**

2: • Effort at finding increase in number from 2012 to 2103

• Effort at finding % increase between 2012 and 2013

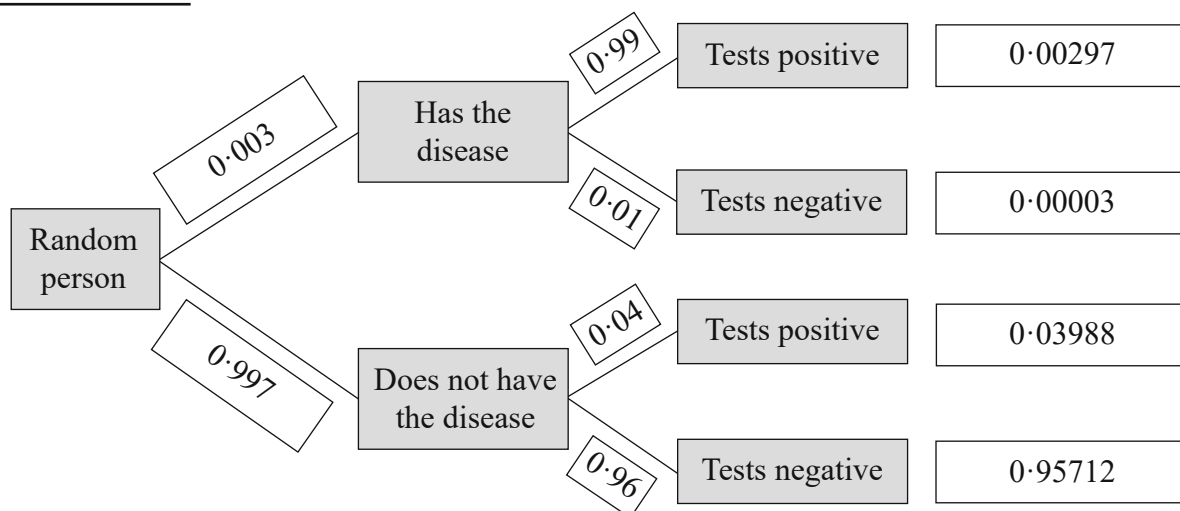
3: • Total number predicted at work in 2014

• Percentage increase calculated

**NOTE:** Candidates not required to round to any particular number of places of decimals

**QUESTION 8 (45 MARKS)**

**Question 8 (a) (i)**



**Question 8 (a) (ii)**

$$P(\text{Tests positive}) = 0.00297 + 0.03988 = 0.04285$$

**Question 8 (a) (iii)**

$$P(\text{Person has the disease}) = \frac{P(\text{Tests positive with the disease})}{P(\text{Positive test})} = \frac{0.00297}{0.04285} = 0.0693$$

**Question 8 (a) (iv)**

No.

Only 6.93% of the people who test positive have the disease. Over 93% of those who test positive do not have the disease. This would cause unnecessary alarm in the community.

**MARKING SCHEME NOTES****Question 8 (a) (i) [Scale 10C (0, 3, 7, 10)]**

- 3: • One element entered correctly  
 • One column correct  
 • Some indication that values lie between 0 and 1
- 7: • Two columns correct

**Question 8 (a) (ii) [Scale 5C (0, 2, 3, 5)]**

- 2: • One correct value chosen  
 • Addition of values indicated  
 • Configured correctly but no values entered  
 • Answer outside range
- 3: • Correct values chosen but operator incorrect

**Question 8 (a) (iii) [Scale 5C (0, 2, 3, 5)]**

- 2: • One or both correct value(s) chosen only  
 • Configured correctly but values not entered  
 • Answer outside range
- 3: • Correct values chosen but incorrect operator leading to an answer within range

**Question 8 (a) (iv) [Scale 5B (0, 2, 5)]**

- 2: • Reason incorrect or incomplete

**Question 8 (b) (i)**

$P(\text{Generic drug being successful}) = 0.51$

Null hypothesis  $H_0$ : Drug  $A$  is not more successful than the generic drug,  $P = 0.51$

Alternative hypothesis  $H_A$ : Drug  $A$  is more successful than the generic drug,  $P > 0.51$

$$P(\text{Drug } A \text{ being successful}) = \frac{296}{500}$$

$$n = 500$$

$$\text{Margin of error} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{500}}$$

$$\text{Confidence interval at the 5\% level of significance: } \left( \frac{296}{500} - \frac{1}{\sqrt{500}}, \frac{296}{500} + \frac{1}{\sqrt{500}} \right) = (0.547, 0.637)$$

There is evidence to support the company's claim that their drug is more successful than the generic drug because, based on the sample data, any values in the range 54.73% – 66.67% are possible values for the proportion of people treated successfully by the company's drug.

We can say with 95% confidence that the percentage success of drug  $A$  is between 54.73% and 66.67%. Therefore, the null hypothesis is rejected. The company claims are true.

**Question 8 (b) (ii)**

This means that drug  $B$  is less successful than the generic drug.

$$\frac{x}{500} - \frac{1}{\sqrt{500}} < 0.51 \Rightarrow x = 500 \left( 0.51 + \frac{1}{\sqrt{500}} \right) = 277.4$$

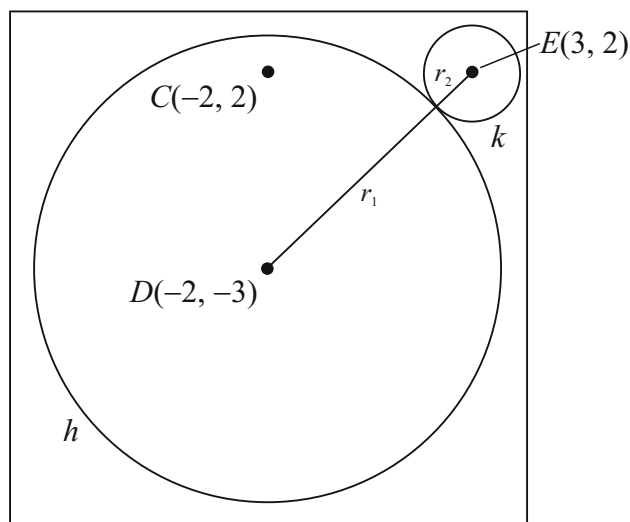
The greatest number of patients who could have been successfully treated was 277.

**MARKING SCHEME NOTES****Question 8 (b) (i) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • One relevant step e.g. null hypothesis stated only  
 • Some work towards margin of error
- 7: • Margin of error or observed proportion  
 • Margin of error and observed proportion found but fails to continue
- 11: • Failure to state null hypothesis correctly  
 • Failure to contextualise answer (e.g. Stops at reject Null Hypothesis)

**Question 8 (b) (ii) [Scale 5B (0, 2, 5)]**

- 2: •  $\frac{n}{500}$  and stops  
 • Recognises interval where result must lie  
 • Some relevant work

**QUESTION 9 (60 MARKS)****FORMULAE AND TABLES BOOK****Co-ordinate geometry: Circle [page 19]**Given centre  $(h, k)$  and radius  $r$ 

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre  $(-g, -f)$ Radius  $\sqrt{g^2 + f^2 - c}$ **Question 9 (a) (i)**

$$h: x^2 + y^2 + 4x + 6y - 19 = 0$$

Centre  $D(-2, -3)$ 

$$\begin{aligned} r_1 &= \sqrt{(-2)^2 + (-3)^2 - (-19)} \\ &= \sqrt{4 + 9 + 19} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

**Question 9 (a) (ii)**

$$E(3, 2), D(-2, -3), r_1 = 4\sqrt{2}$$

$$|ED| = r_1 + r_2$$

$$|ED| = \sqrt{(-2 - 3)^2 + (-3 - 2)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\therefore 5\sqrt{2} = 4\sqrt{2} + r_2$$

$$r_2 = \sqrt{2}$$

**MARKING SCHEME NOTES****Question 9 (a) (i) [Scale 15C (0, 5, 10, 15)]**

- 5: • Effort at relating one or more coefficients of given equation to general equation of circle  
 • Effort at completing square(s)
- 10: • Either radius or centre correct  
 • Substantive work with one critical error

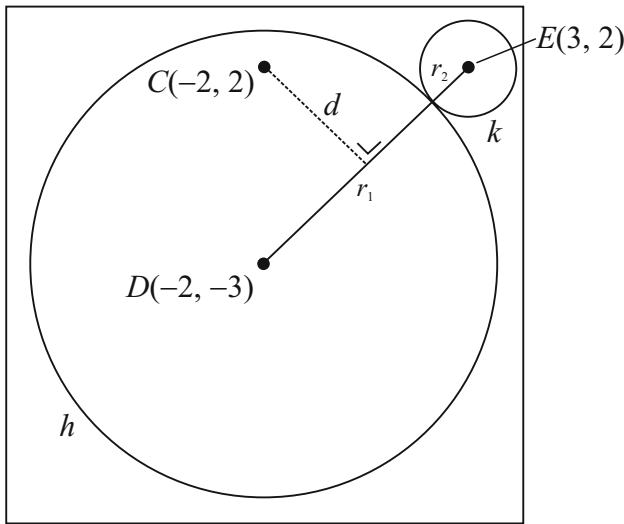
**Question 9 (a) (ii) [Scale 10C (0, 3, 7, 10)]**

- 3: • Effort at finding  $|DE|$   
 • Length of line segment formula  
 • Indicates some understanding of

$$r_1 + r_2 = |DE|$$

- 7: •  $r_1 + r_2 = |DE|$  or equivalent with known values substituted

**Question 9 (a) (iii)**



$$D(-2, -3), E(3, 2)$$

$$m = \frac{2 - (-3)}{3 - (-2)} = \frac{5}{5} = 1$$

$$y - 2 = 1(x - 3)$$

$$y - 2 = x - 3$$

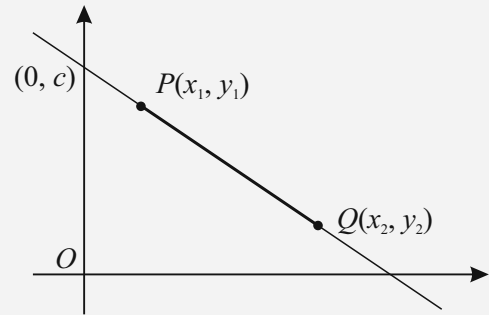
$$x - y - 1 = 0$$

$$d = \frac{|(-2) - (-2) - 1|}{\sqrt{1^2 + (-1)^2}} = \frac{|-5|}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$|DE| = 5\sqrt{2}$$

$$\therefore d = \frac{1}{2}|DE|$$

**FORMULAE AND TABLES BOOK**  
**Co-ordinate geometry: Line**



Slope of  $PQ$  [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Length of  $PQ$  [page 18]

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of  $PQ$  [page 18]

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Equation of  $PQ$  [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

Distance from  $(x_1, y_1)$  to the line  
 $ax + by + c = 0$  [page 19]

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

**MARKING SCHEME NOTES**

**Question 9 (a) (iii) [Scale 10C (0, 3, 7, 10)]**

- 3: • Slope  $DE$   
• Equation  $DE$  and stops  
• Formula for slope and/or equation of  $DE$   
• Perpendicular distance formula
- 7: • Values inserted into perpendicular distance formula  
• No conclusion stated or implied

**Question 9 (a) (iv)**

$$D(-2, -3), E(3, 2)$$

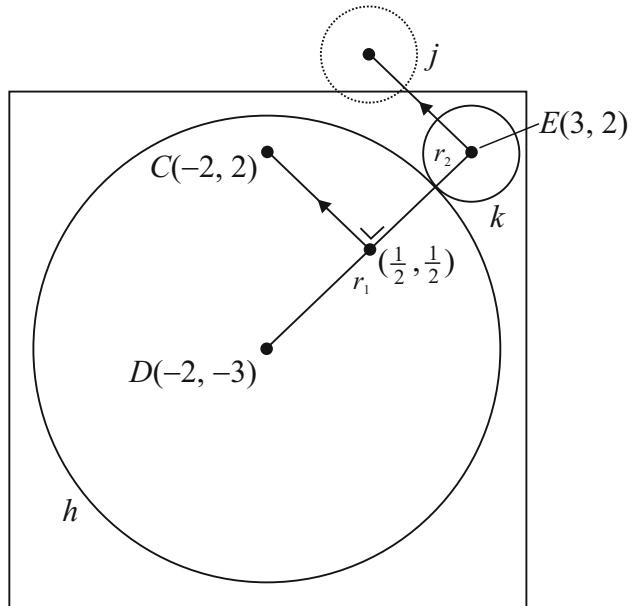
$$\text{Midpoint} = \left( \frac{-2+3}{2}, \frac{-3+2}{2} \right) = \left( \frac{1}{2}, -\frac{1}{2} \right)$$

$$\left(\frac{1}{2}, -\frac{1}{2}\right) \rightarrow C(-2, 2) \left[-\frac{5}{2}, +\frac{5}{2}\right]$$

$$E(3, 2) \rightarrow \left(3 - \frac{5}{2}, 2 + \frac{5}{2}\right) = \left(\frac{1}{2}, \frac{9}{2}\right)$$

Equation of  $j$ :  $(x - \frac{1}{2})^2 + (y - \frac{9}{2})^2 = (\sqrt{2})^2$

$$(x - \frac{1}{2})^2 + (y - \frac{9}{2})^2 = 2$$



**MARKING SCHEME NOTES**

**Question 9 (a) (iv) [Scale 5C (0, 2, 3, 5)]**

- 2: • Effort to find midpoint of  $DE$   
 • Centre found from scaled drawing
- 3: • Centre of  $j$  found and inserted into equation of circle i.e radius omitted

**Question 9 (a) (v)**

$$x^2 + x^2 = (5\sqrt{2})^2$$

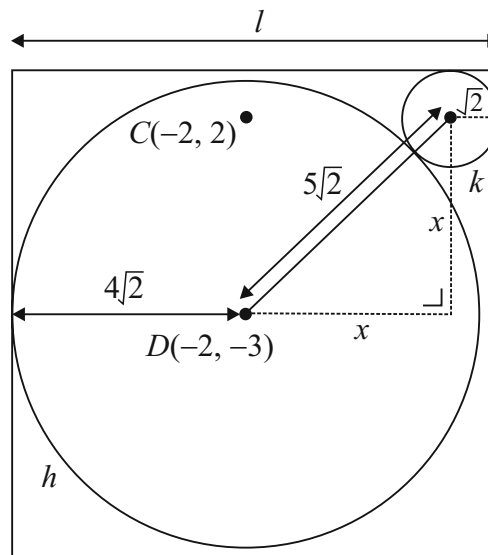
$$2x^2 = 50$$

$$x^2 = 25$$

$$\therefore x = 5$$

$$\text{Minimum } l = 4\sqrt{2} + 5 + \sqrt{2} = 12.07$$

$$\therefore l = 13$$

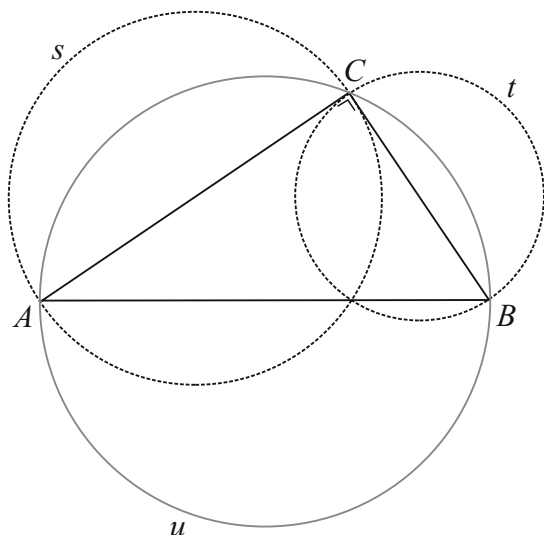


**MARKING SCHEME NOTES**

**Question 9 (a) (v) [Scale 5C (0, 2, 3, 5)]**

- 2: • Effort to find  $F$   
 • Indication length  $r_1 + r_2 + |DE|$  (or equivalent)
- 3: •  $F$  found

**Question 9 (b) (i)**

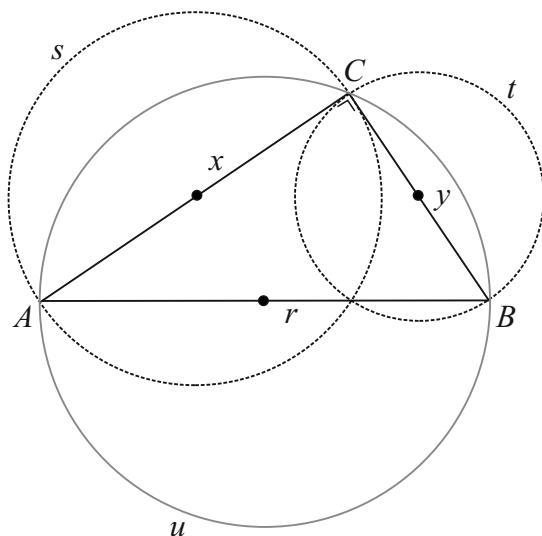


**MARKING SCHEME NOTES**

**Question 9 (b) (i) [Scale 5B (0, 2, 5)]**

- 2: • Circle containing A and B but lacking in accuracy

**Question 9 (b) (ii)**



Let  $|AB| = r$ ,  $|AC| = x$  and  $|BC| = y$ .

$$x^2 + y^2 = r^2 \text{ [Pythagoras]}$$

$$\text{Area of } s = \pi \left(\frac{1}{2}x\right)^2 = \frac{1}{4}\pi x^2$$

$$\text{Area} = \pi r^2$$

$$\text{Area of } t = \pi \left(\frac{1}{2}y\right)^2 = \frac{1}{4}\pi y^2$$

$$\text{Area of } u = \pi \left(\frac{1}{2}r\right)^2 = \frac{1}{4}\pi r^2$$

Area of  $s$  + Area of  $t$

$$= \frac{1}{4}\pi x^2 + \frac{1}{4}\pi y^2$$

$$= \frac{1}{4}\pi(x^2 + y^2)$$

$$= \frac{1}{4}\pi r^2 = \text{Area of } u$$

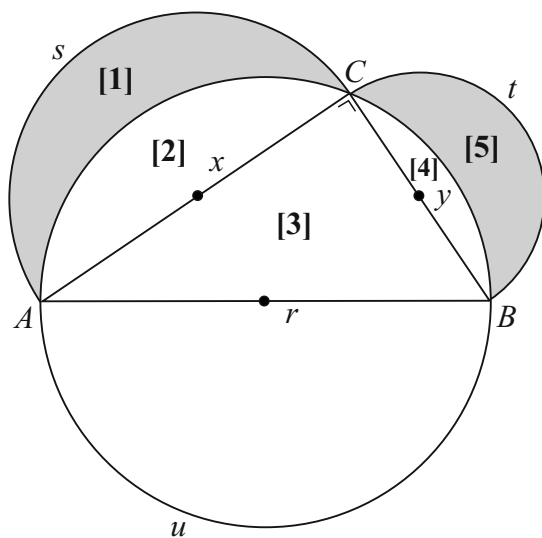
**MARKING SCHEME NOTES**

**Question 9 (b) (ii) [Scale 5C (0, 2, 3, 5)]**

- 2: • Pythagoras stated or implied  
• Effort at finding area of  $s$  or  $t$  or  $u$

- 3: • Correct expression for area of any circle e.g.  $\frac{\pi}{4} \times |AB|^2$  or  $\pi \times \frac{|AB|^2}{4}$

**Question 9 (b) (iii)**



Area of  $s$  + Area of  $t$  = Area of  $u$

$$\therefore \frac{1}{2}(\text{Area of } s + \text{Area of } t) = \frac{1}{2}(\text{Area of } u)$$

Area [1] + Area [2] + Area [4] + Area [5] = Area [2] + Area [3] + Area [4]

$$\therefore \text{Area [1]} + \text{Area [5]} = \text{Area [3]}$$

**MARKING SCHEME NOTES**

**Question 9 (b) (iii) [Scale 5C (0, 2, 3, 5)]**

- 2:**
- Statement using result from (b)(ii)
  - Recognising half the area of  $s$  or half the area  $t$  can be expressed in terms of two component areas
  - Recognising half area of  $u$  can be expressed in terms of three components
- 3:**
- Correct expression for two of the relevant areas

## LC 2013 (SET F): PAPER 1

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

$$\begin{aligned} & \frac{4}{1+\sqrt{3}i} \quad \leftarrow \text{Multiply above and below by} \\ & \quad \quad \quad \text{the conjugate of the denominator.} \\ & = \frac{4}{(1+\sqrt{3}i)} \times \frac{(1-\sqrt{3}i)}{(1-\sqrt{3}i)} \\ & = \frac{4(1-\sqrt{3}i)}{1-3i^2} \\ & = \frac{4(1-\sqrt{3}i)}{4} \\ & = 1-\sqrt{3}i \end{aligned}$$

#### FORMULA: Complex Numbers

Conjugates  $\bar{z}$

$$z = a + bi \Rightarrow \bar{z} = a - bi$$

Multiplying a complex number by its conjugate:

$$z\bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 10C (0, 3, 7, 10)]

- 3: • Does not multiply by conjugate  
• Drops  $i$ , or  $i^2 \neq -1$   
• Incomplete cross-multiplication
- 7: • Work not simplified

#### Question 1 (b)

Plotting  $z$ : Find the modulus of  $z$

$$z = 1 - \sqrt{3}i$$

$$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

#### FORMULA: Complex Numbers

Modulus  $|z|$

$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Put the point of the compass at the origin and stretch the pencil out to 2 on the Real axis. Draw an arc as shown. The point  $(1, -\sqrt{3})$  occurs at  $x = 1$  **or** use your protractor to draw a  $60^\circ$  angle as shown.

#### FORMULA: COMPLEX NUMBERS

$$z = a + bi \text{ (Cartesian form)}$$

$$z = r(\cos \theta + i \sin \theta) \text{ (Polar form)}$$

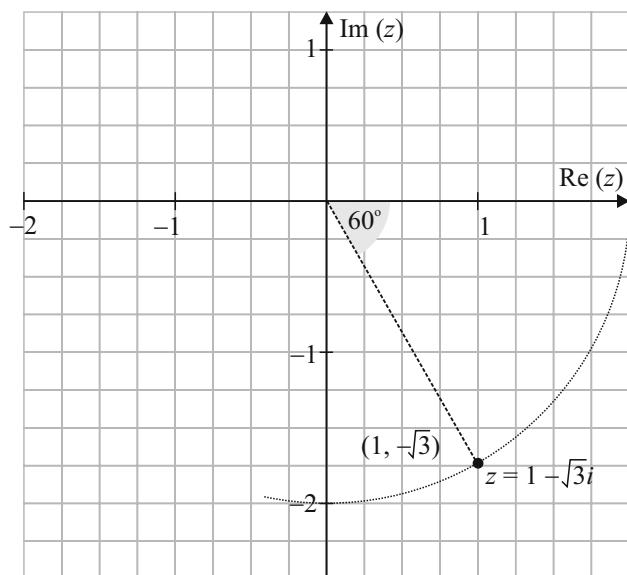
$\alpha$  is the reference angle in the first quadrant.

$$|\tan \theta| = \left| \frac{-\sqrt{3}}{1} \right| = \sqrt{3} = \tan \alpha$$

$$\therefore \alpha = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\therefore \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = 300^\circ$$

$$\therefore z = 2 \left\{ \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right\}$$



**MARKING SCHEME NOTES****Question 1 (b) [Scale 10C (0, 3, 7, 10)]**

- 3: • Work with  $\alpha$   
 • Work with  $\theta$   
 • Work with modulus  
 • Plotting  $z$
- 7: • Correct  $z$  but incorrect or no plotting  
**NOTE:** Accept  $r$ ,  $\theta$  and plot for full marks.

**Question 1 (c)**

$$z = 2 \left\{ \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right\}$$

$$\therefore z^{10} = 2^{10} \left\{ \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right\}^{10}$$

$$= 2^{10} \left\{ \cos \frac{50\pi}{3} + i \sin \frac{50\pi}{3} \right\}$$

$$= 2^{10} \left\{ \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right\}$$

$$= 2^{10} \left\{ -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right\}$$

$$= -2^9 (1 - \sqrt{3}i)$$

**FORMULAE AND TABLES BOOK:****Algebra (page 20)****DE MOIVRE'S THEOREM**

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

**MARKING SCHEME NOTES****Question 1 (c) [Scale 5C (0, 2, 4, 5)]**

- 2: • Some work with De Moivre  
 • De Moivre not used correctly
- 4: • Answer not simplified  
 •  $n$  included in answer

**NOTE:** Allow for full marks candidates incorrect angle from (b), with correct conclusion.

**0:** no use of De Moivre.

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**

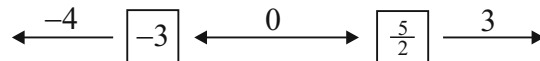
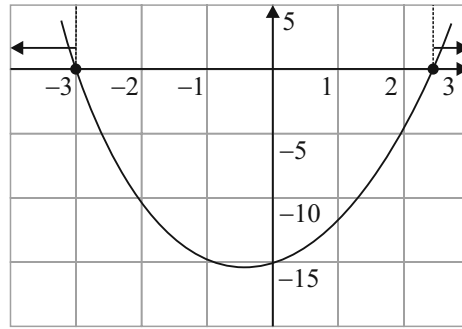
$$2x^2 + x - 15 \geq 0$$

$$2x^2 + x - 15 = 0 \quad [\text{Solve the equality.}]$$

$$(2x - 5)(x + 3) = 0$$

$$\therefore x = -3, \frac{5}{2}$$

Locate the solutions graphically (right) or using the region test (below).



$(2x - 5)(x + 3) \geq 0$	$(2x - 5)(x + 3) \geq 0$	$(2x - 5)(x + 3) \geq 0$
$(2(-4) - 5)(-4 + 3) \geq 0$	$(2(0) - 5)(0 + 3) \geq 0$	$(2(3) - 5)(3 + 3) \geq 0$
$(-13)(-1) \geq 0$	$(-5)(3) \geq 0$	$(1)(6) \geq 0$
$13 \geq 0$	$-15 \geq 0$	$6 \geq 0$

ANSWER:  $\therefore x \leq -3, x \geq \frac{5}{2}$

**MARKING SCHEME NOTES**

**Question 2 (a) [Scale 10C (0, 3, 7, 10)]**

- 3:**
- Factorises
  - Gets roots
  - Some use of quadratic root formula
- 7:**
- Wrong shape of graph, but otherwise correct
  - Incorrect deduction for correct values of  $x$
  - Correct shading on  $x$ -axis
  - Using  $x >$  only

**Question 2 (b)**

Eliminate the fractions:

$$x + y + z = 16 \dots (1)$$

$$\frac{5}{2}x + y + 10z = 40 \dots (2) (\times 2)$$

$$2x + \frac{1}{2}y + 4z = 21 \dots (3) (\times 2)$$

$$x + y + z = 16 \dots (1)$$

$$5x + 2y + 20z = 80 \dots (2)$$

$$4x + y + 8z = 42 \dots (3)$$

Eliminate  $y$  from equations (1) and (2):

$$x + y + z = 16 \dots (1) (\times 2)$$

$$5x + 2y + 20z = 80 \dots (2)$$

$$2x + 2y + 2z = 32$$

$$5x + 2y + 20z = 80$$

$$\hline 3x \quad + 18z = 48 \dots (4)$$

Eliminate  $y$  from equations (1) and (3):

$$x + y + z = 16 \dots (1)$$

$$4x + y + 8z = 42 \dots (3)$$

$$\hline 3x \quad + 7z = 26 \dots (5)$$

Eliminate  $x$  from equations (4) and (5):

$$3x + 18z = 48 \dots (4)$$

$$3x + 7z = 26 \dots (5)$$

$$\hline 11z = 22 \Rightarrow z = 2$$

$$z = 2 : 3x + 7(2) = 26 \dots (5)$$

$$3x + 14 = 26$$

$$3x = 12 \Rightarrow x = 4$$

$$4 + y + 2 = 16 \dots (1)$$

$$\therefore y = 10$$

ANSWER:  $x = 4, y = 10, z = 2$

**MARKING SCHEME NOTES****Question 2 (b) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Any relevant step to solution  
 7: • Reduces to two unknowns correctly in one equation  
 11: • Evaluates one unknown

**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$Q = e^{\frac{0.693t}{5730}} \leftarrow \text{Replace } t \text{ by 2000 and input into your calculator.}$$

$$t = 2000 : Q = e^{\frac{0.693(2000)}{5730}} = 0.7851 = 78.51\%$$

**Question 3 (b)**

$$Q = 0.3402 = e^{\frac{0.693t}{5730}} \leftarrow \text{Take the natural log of both sides.}$$

$$\ln(0.3402) = \ln\left(e^{\frac{0.693t}{5730}}\right)$$

$$\ln(0.3402) = -\frac{0.693t}{5730}$$

$$\therefore t = -\frac{5730 \ln(0.3402)}{0.693} = 8915.17 \approx 8900 \text{ years}$$

**MARKING SCHEME NOTES****Question 3 (a) [Scale 15B (0, 7, 15)]**

- 7: • Value correctly substituted into  $e$

**Question 3 (b) [Scale 10C\* (0, 3, 7, 10)]**

- 3: • Correct statement  
 • Uses logs (correctly or incorrectly)  
 • Makes an effort to isolate  $t$   
 7: • Gets correct linear equation for  $t$

**NOTE:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

**QUESTION 4 (25 MARKS)****Question 4 (a) (i)**

Annual interest rate  $r_A = 0.4\% \Rightarrow i_A = 0.04$

$(1 + i_M)^{12} = (1 + 0.04)$ , where  $i_M$  is the monthly interest rate

$$(1 + i_M)^{12} = 1.04$$

$$1 + i_M = 1.04^{\frac{1}{12}}$$

$$i_M = 1.04^{\frac{1}{12}} - 1 = 0.00327$$

$$\therefore r_M = 0.327\%$$

**MARKING SCHEME NOTES****Question 4 (a) (i) [Scale 5C\* (0, 2, 4, 5)]**

- 2:**
- Any relevant first step
  - Correct statement with no work

- 4:**
- $r$  not as a %

**NOTE:** Incomplete rounding or incorrect rounding or no rounding gets 4 marks

**Question 4 (a) (ii)**

Call  $A$  the amount of money she saves each month. She saves for 36 months at a monthly interest rate of 0.327%. Her first saving amount  $A$  earns interest for 36 months, her second saving amount  $A$  earns monthly interest for 35 months and so on.

$$15\ 000 = A \times 1.00327^{36} + A \times 1.00327^{35} + \dots + A \times 1.00327^2 + A \times 1.00327^1$$

$$15\ 000 = A(1.00327^{36} + 1.00327^{35} + \dots + 1.00327^2 + 1.00327^1)$$

$$15\ 000 = A(1.00327^1 + 1.00327^2 + \dots + 1.00327^{35} + 1.00327^{36})$$

This is a geometric series. You can work out the sum of these 36 savings.

$$a = 1.00327, r = 1.00327$$

$$S_{36} = \frac{1.00327(1 - 1.00327^{36})}{1 - 1.00327}$$

$$1500 = A \left[ \frac{1.00327(1 - 1.00327^{36})}{1 - 1.00327} \right]$$

$$\therefore A = \frac{15\ 000(1 - 1.00327)}{1.00327(1 - 1.00327^{36})} = \text{€}392$$

**FORMULAE AND TABLES BOOK****Sequences and series:****Geometric series [page 22]**

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

**MARKING SCHEME NOTES****Question 4 (a) (ii) [Scale 10D\* (0, 3, 5, 8, 10)]**

- 3:**
- Any relevant step
  - Reference to 1.00327

- 5:**
- Recognises G.P

- 8:**
- Expression for sum of G.P

**NOTE:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks

OR

**Question 4 (a) (ii)**

Find the present value  $P$  of having a future value  $F$  of €15 000 in your account after 36 months.

$$P = \frac{F}{(1+i_M)^{36}}$$
$$= \frac{15\,000}{1.00327^{36}} = \text{€}13\,336.73$$

**FORMULAE AND TABLES BOOK**

**Financial mathematics: Compound interest** [page 30]

$$F = P(1+i)^t$$

$t$  = Time period (in years)

$i$  = (Annual) rate of interest expressed as a decimal

$F$  = Final value

$P$  = Principal

**NOTE:** The time period can be months or weeks instead of years provided the interest rate is given for that time period.

You can now ask the question in reverse: How much did I have to pay back every month to repay €13 336.73? Now I can use the amortisation formula.

**FORMULAE AND TABLES BOOK**

**Financial mathematics: Amortisation - mortgages and loans**

(equal repayments at equal intervals) [page 31]

$$A = P \frac{i(1+i)^t}{(1+i)^t - 1}$$

$t$  = Time period (in years)

$i$  = (Annual) rate of interest expressed as a decimal

$A$  = (Annual) repayment amount

$P$  = Principal

**NOTE:** The time period can be months or weeks instead of years provided the interest rate is given for that time period.

$$A = 13\,336.73 \times \frac{0.00327(1.00327)^{36}}{(1.00327)^{36} - 1} = \text{€}393.30$$

**MARKING SCHEME NOTES**

**Question 4 (a) (ii) [Scale 10D\* (0, 3, 5, 8, 10)]**

**3:** • No present value or incorrect present value

**5:** • Present value correct

**8:** • Correct substitution of all values in formula

**NOTE:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

**Question 4 (b)**

$$A = P \frac{i(1+i)^t}{(1+i)^t - 1}$$
$$= 15\,000 \left[ \frac{0.00866(1+0.00866)^{36}}{(1+0.00866)^{36} - 1} \right]$$
$$= 15\,000 \left[ \frac{0.00866(1.00866)^{36}}{(1.00866)^{36} - 1} \right] = \text{€}487$$

**MARKING SCHEME NOTES****Question 4 (b) [Scale 10C\* (0, 3, 7, 10)]**

- 3: • Any relevant step
- 7: • Correct substitution of all values in formula  
• Expression for sum of G.P. in solution

**NOTE:** Incomplete rounding or incorrect rounding or no rounding gets 9 marks.

**QUESTION 5 (25 MARKS)****Question 5 (a)**

Type of function	Function	First derivative	Second derivative
Quadratic	$k$	B	I
Cubic	$f$	D	I
Trigonometric	$g$	A	III
Exponential	$h$	C	I

**Question 5 (b)**

A quadratic function differentiates to a straight line. A straight line (linear function) differentiates to a constant.

**MARKING SCHEME NOTES****Question 5 (a) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • One or two correct entries  
• Some correct work with candidates own choice of functions
- 7: • Three to five correct entries  
• Six to ten correct entries in candidate's own choice of particular quadratic, cubic, trigonometric and exponential functions
- 11: • Six to ten correct entries  
• Candidate chooses particular quadratic, cubic, trigonometric and exponential functions, with correct first and second derivatives

**Question 5 (b) [Scale 10B (0, 5, 10)]**

- 5: • Some correct statement regarding the first or second derivative of a function  
• Correct explanation of either first or second derivative

**QUESTION 6 (25 MARKS)****Question 6 (a)**

$$y = \sin x$$

$$x = 0: y = \sin 0 = 0$$

$$x = \frac{\pi}{6}: y = \sin\left(\frac{\pi}{6}\right) = 0.500 \leftarrow \text{Use your calculator in radian mode.}$$

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\pi$
$y$	0	0.500	0.866	1.000	0.866	0.500	0

**MARKING SCHEME NOTES****Question 6 (a) [Scale 5C\* (0, 2, 4, 5)]**

2: • One or two correct entries

4: • Three to six correct entries

**Question 6 (b)**

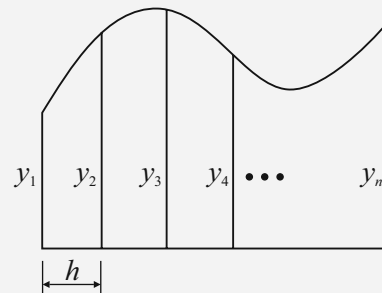
$$h = \frac{\pi}{6}$$

$$A = \frac{1}{2} \times \frac{\pi}{6} [0 + 0 + 2(0.5 + 0.866 + 1 + 0.866 + 0.5)]$$

$$= 1.95407$$

**MARKING SCHEME NOTES****Question 6 (b) [Scale 10C (0, 3, 7, 10)]**

- 3: • Correct formula.  
• Substitutes in the formula, values from the domain of the function
- 7: • Correct substitution in the formula and stops  
• Incorrect  $\frac{h}{2}$  and finishes correctly

**FORMULAE AND TABLES BOOK**  
Area approximations [page 12]**Trapezoidal Rule:**

$$A \approx \frac{h}{2} [y_1 + y_n + 2(y_2 + y_3 + y_4 + \dots + y_{n-1})]$$

**Question 6 (c)**

$$A = \int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi}$$

$$= -[\cos \pi - \cos 0]$$

$$= -[-1 - 1]$$

$$= 2$$

**FORMULAE AND TABLES BOOK****Calculus: Integrals [page 26]**

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + c$$

**MARKING SCHEME NOTES****Question 6 (c) [Scale 5C (0, 2, 4, 5)]**

- 2: • Correct limits  
• 'c' appears and no other correct work
- 4: • Correct integration

### Question 6 (d)

**FORMULA: % ERRORS**

Absolute error in a quantity = |Measured value of quantity – Accepted value of quantity|

Percentage error in a quantity =  $\frac{\text{Absolute error in quantity}}{\text{Accepted value of quantity}} \times 100\%$

$$\% \text{ error} = \frac{2 - 1.95407}{2} \times 100\% = 2.2965\% \approx 2.3\%$$

**MARKING SCHEME NOTES****Question 6 (d) [Scale 5B (0, 2, 5)]**

2: • Any correct relevant step

### QUESTION 7 (50 MARKS)

#### Question 7 (a)

Ticket price: €20, attendance: 12 000

Ticket price: €19, attendance: 13 000

Ticket price: €18, attendance: 14 000

ANSWER: 14 000 people

#### Question 7 (b)

Call  $y$  the number in the stadium.

$$y = 12\,000 \text{ when } x = \text{€}20$$

$$y = 13\,000 \text{ when } x = \text{€}19$$

$$y = 14\,000 \text{ when } x = \text{€}18$$

$$y = 15\,000 \text{ when } x = \text{€}17$$

$$\begin{aligned} y &= 12\,000 + (20 - x)1000 \\ &= 12\,000 + 20\,000 - 1000x \\ &= 32\,000 - 1000x \end{aligned}$$

#### Question 7 (c)

$f(x)$  = Number who attended  $\times$  Price

$$= y \times x$$

$$= (32\,000 - 1000x)x$$

$$= 32\,000x - 1000x^2$$

**MARKING SCHEME NOTES****Question 7 (a) [Scale 10B (0, 5, 10)]**

5: •  $(20 - 18)1000$  or equivalent

**Question 7 (b) [Scale 5B (0, 2, 5)]**

2: • Expression  $(20 - x)$

NOTE: Accept for 5 marks  $12000 + (20 - x)1000$  or equivalent

**Question 7 (c) [Scale 5B (0, 2, 5)]**

2: • Correct number of people and/or correct rate in terms of  $x$

**Question 7 (d)**

Differentiate the function with respect to  $x$ . Put the derivative equal to zero and solve for  $x$  to find the price that will give the maximum income.

$$f(x) = 32\,000x - 1000x^2$$

$$f'(x) = 32\,000 - 2000x$$

$$f'(x) = 0 \Rightarrow 32\,000 - 2000x = 0$$

$$32\,000 = 2000x$$

$$\therefore x = \frac{32\,000}{2000} = \text{€}16$$

FIND TURNING POINTS (LOCAL MAXIMUM/MINIMUM)

Put  $\frac{dy}{dx} = 0$  and solve for  $x$

**MARKING SCHEME NOTES****Question 7 (d) [Scale 10B (0, 5, 10)]**

- 5:
- Some correct differentiation of a quadratic function
  - $(32000 - 2000x) = 0$  or equivalent
  - Correct testing with incorrect deduction or no deduction
  - Possible to get full marks without use of calculus
  - Correct answer and no work

**Question 7 (e)**

$$x = \text{€}16$$

$$f(x) = 32\,000x - 1000x^2$$

$$f(16) = 32\,000(16) - 1000(16)^2 = \text{€}256\,000$$

**Question 7 (f)**

Find out the price of a ticket  $x$  that gives a full attendance of  $y = 25\,000$ .

$$y = 25\,000$$

$$y = 32\,000 - 1000x$$

$$25\,000 = 32\,000 - 1000x$$

$$1000x = 7000$$

$$\therefore x = \text{€}7$$

$$x = \text{€}7$$

$$f(x) = 32\,000x - 1000x^2$$

$$f(7) = 32\,000(7) - 1000(7)^2 = \text{€}175\,000$$

$$\text{Difference} = \text{€}256\,000 - \text{€}175\,000 = \text{€}81\,000$$

**MARKING SCHEME NOTES****Question 7 (e) [Scale 5B (0, 2, 5)]**

- 2:
- Some effort at substitution of 16 or equivalent

**Question 7 (f) [Scale 10C (0, 3, 7, 10)]**

- 3:
- Use of expression
  - Use of 25 000
  - Some use of tables
  - Equation solved
  - Price of ticket found
- 7:
- Total income from sales

**Question 7 (g)**

Call  $p$  the price of a family ticket.

If 1000 more family tickets are sold, then 4000 less single tickets will be sold. The loss overall will be €14 000.

$$\therefore 1000p - 4000 \times 16 = -14\,000$$

$$1000p = 4000 \times 16 - 14\,000$$

$$1000p = 50\,000$$

$$\therefore p = \text{€}50$$

Let  $a$  = Number of single tickets

Let  $b$  = Number of family tickets

Attendance equation:  $a + 4b = 25\,000$ ...(1)

Income equation:  $16a + 50b = 365\,000$ ...(2)

Solve equations (1) and (2):

$$a + 4b = 25\,000 \dots\dots\dots(1)(\times 16)$$

$$16a + 50b = 365\,000 \dots\dots(2)(\times -1)$$

$$16a + 64b = 400\,000$$

$$-16a - 50b = -365\,000$$

---

$$14b = 35\,000 \Rightarrow b = \frac{35\,000}{14} = 2500$$

ANSWER: 2500 family tickets

**MARKING SCHEME NOTES**

**Question 7 (g) [Scale 5D (0, 2, 3, 4, 5)]**

- 2: • Correct number of family tickets  
• One equation only  
• Income from single tickets  
• Income from family tickets
- 3: • Two correct linear equations for income in two unknowns  
•  $y = 50$  without work, or  $p = 15\,000$  without work
- 4: • Correct value for  $p$  (single ticket)  
• Correct value for  $y$  (family ticket)

**OR**

**Question 7 (g) [Scale 5D (0, 2, 3, 4, 5)]**

- 2: • One correct equation
- 3: • Three correct equations
- 4: • One unknown calculated

**QUESTION 8 (50 MARKS)**

**Question 8 (a)**

$s(t) = 6t + 0.3t^2 - 0.01t^3$  ← Replace  $t$  by 10 in  $s(t)$ .

$$\begin{aligned} s(10) &= 6(10) + 0.3(10)^2 - 0.01(10)^3 \\ &= 60 + 30 - 10 \\ &= 80 \text{ m} \end{aligned}$$

**MARKING SCHEME NOTES**

**Question 8 (a) [Scale 10C\* (0, 3, 7, 10)]**

3: • Some correct substitution into  $s(t)$

7: • Correct substitution.

•  $10s = 80 \Rightarrow s = 8$

*Full Marks:* Correct answer without work

**Question 8 (b)**

$$s = 6t + 0.3t^2 - 0.01t^3$$

$$v = \frac{ds}{dt} = 6 + 0.6t - 0.03t^2$$

$$v = 8.25 \text{ m s}^{-1}: 8.25 = 6 + 0.6t - 0.03t^2$$

$$0.03t^2 - 0.6t + 2.25 = 0$$

$$t^2 - 20t + 75 = 0$$

$$(t - 5)(t - 15) = 0$$

$$\therefore t = 5 \text{ s}, 15 \text{ s}$$

ANSWER:  $t = 5 \text{ s}$

**Note:** Ignore the other solution as the function used is specified for the first 10 s.

**FORMULAE: Speed**

Velocity  $v = \frac{\text{Change } (d) \text{ in distance } s}{\text{Change } (d) \text{ in time } t}$

$$v = \frac{ds}{dt}$$

**FORMULAE AND TABLES BOOK**

**Calculus: Derivatives [page 25]**

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

**MARKING SCHEME NOTES**

**Question 8 (b) [Scale 10D\* (0, 3, 5, 8, 10)]**

3: • Any correct differentiation

5: •  $\frac{ds}{dt}$  found correctly

8: • Fails to select correct value for  $t$

**Question 8 (c)**

$$v = 6 + 0.6t - 0.03t^2$$

$$a = \frac{dv}{dt} = 0.6 - 0.06t$$

$$a = 0.006 \text{ m s}^{-2}: 0.006 = 0.6 - 0.06t$$

$$0.06t = 0.6 - 0.006$$

$$t = \frac{0.6 - 0.006}{0.06} = 9.9 \text{ s}$$

**FORMULAE: Acceleration**

Acceleration  $a = \frac{\text{Change } (d) \text{ in velocity } v}{\text{Change } (d) \text{ in time } t}$

$$a = \frac{dv}{dt}$$

**MARKING SCHEME NOTES****Question 8 (c) [Scale 10C\* (0, 3, 7, 10)]**

- 3: • Some correct differentiation of the first derivative  
 7: • Sets up correct equation.

**Question 8 (d)**

You discovered in **part (a)** that the drop falls 80 m in the first 10 s. Therefore, it falls the remaining 540 m at constant velocity. Calculate this velocity.

$$t = 10 \text{ s: } v = 6 + 0.6t - 0.03t^2$$

$$v = 6 + 0.6(10) - 0.03(10)^2 = 9 \text{ m s}^{-1}$$

$$v = \frac{s}{t} \Rightarrow t = \frac{s}{v} = \frac{540}{9} = 60 \text{ s}$$

ANSWER: Total time = 10 + 60 = 70 s

**MARKING SCHEME NOTES****Question 8 (d) [Scale 10D\* (0, 3, 5, 8, 10)]**

- 3: • Some correct substitution  
 • Finds distance travelled after initial 10 seconds i.e. 540m  
 • Correct differentiation (relevant for terminal velocity)  
 5: • Finds  $k$   
 8: • Finds time travelling at terminal velocity and stops

**Question 8 (e)**

$$\frac{dV}{dt} = 6 \text{ mm s}^{-1}$$

$$\left(\frac{dr}{dt}\right)_{r=1.5} = ?$$

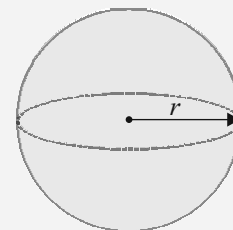
$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 6 = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{3}{2\pi r^2}$$

$$\therefore \left(\frac{dr}{dt}\right)_{r=1.5} = \frac{3}{2\pi(1.5)^2} = 0.2122 \text{ mm s}^{-1}$$

**FORMULAE AND TABLES BOOK****Surface area and volume:  
Sphere [page 8]**

$$A = 4\pi r^2$$

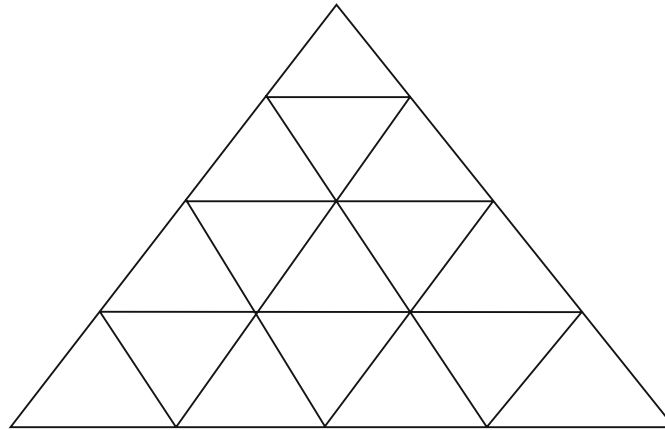
$$V = \frac{4}{3}\pi r^3$$

**MARKING SCHEME NOTES****Question 8 (e) [Scale 10D\* (0, 3, 5, 8, 10)]**

- 3: •  $V = \frac{4}{3}\pi r^3$   
 • Any relevant step  
 5: • Any two of  $\frac{dv}{dt} = 6$ ,  $\frac{dv}{dr} = 4\pi r^2$ ,  $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$   
 8: • Correct substitution into chain rule  
 • Fails to substitute for  $r$

**QUESTION 9 (50 MARKS)**

**Question 9 (a) (i)**



**Question 9 (a) (ii)**

Pattern	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
Number of small triangles	1	4	9	16
Number of matchsticks	3	9	18	30

**MARKING SCHEME NOTES**

**Question 9 (a) (i) [Scale 5B (0, 2, 5)]**

2: • Incomplete 4th line

**Question 9 (a) (ii) [Scale 5C (0, 2, 4, 5)]**

2: • One or two correct entries in empty boxes

4: • Three correct entries in empty boxes

**Question 9 (b)**

Number of triangles in the 1<sup>st</sup> pattern ( $n = 1$ ):  $T_1 = 1 = 1^2$

Number of triangles in the 2<sup>nd</sup> pattern ( $n = 2$ ):  $T_2 = 4 = 2^2$

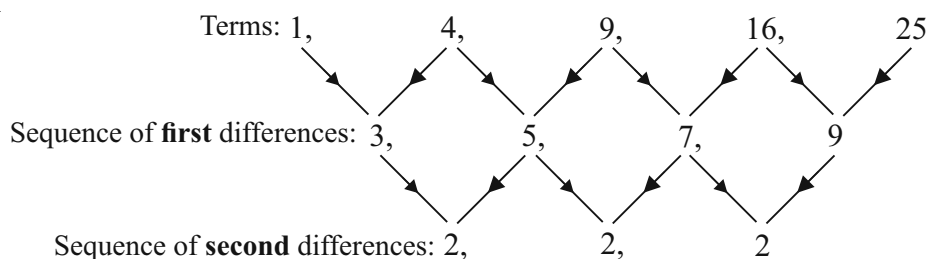
Number of triangles in the 3<sup>rd</sup> pattern ( $n = 3$ ):  $T_3 = 9 = 3^2$

Number of triangles in the 4<sup>th</sup> pattern ( $n = 4$ ):  $T_4 = 16 = 4^2$

⋮

Number of triangles in the  $n^{\text{th}}$  pattern ( $n = n$ ):  $T_n = n^2$

**OR**



A constant term of 2 in the sequence of second differences implies the number of triangles in each pattern forms a quadratic sequence.

**MARKING SCHEME NOTES****Question 9 (b) [Scale 10B (0, 5, 10)]**5: •  $1^2, 2^2, 3^2$  etc – recognising the natural numbers squared

or

**Question 9 (b) [Scale 10B (0, 5, 10)]**

5: • Second differences calculated

**Question 9 (c)**Number of matchsticks to turn 1<sup>st</sup> pattern into 2<sup>nd</sup> pattern ( $n = 2$ ):  $6 = 3(2)$ Number of matchsticks to turn 2<sup>nd</sup> pattern into 3<sup>rd</sup> pattern ( $n = 3$ ):  $9 = 3(3)$ Number of matchsticks to turn 3<sup>rd</sup> pattern into 4<sup>th</sup> pattern ( $n = 4$ ):  $12 = 3(4)$ 

⋮

Number of matchsticks to turn  $(n - 1)$ <sup>th</sup> pattern into  $n$ <sup>th</sup> pattern ( $n = n$ ):  $3n$ **MARKING SCHEME NOTES****Question 9 (c) [Scale 10B (0, 5, 10)]**

5: • Recognition of series 6, 9, 12,.... or similar

or

**Question 9 (c) [Scale 10B (0, 5, 10)]**

5: • Second differences calculated

**Question 9 (d)**

$$u_n = an^2 + bn$$

$$u_1 = a(1)^2 + b(1) = 3 \Rightarrow a + b = 3 \dots (1)$$

$$u_2 = a(2)^2 + b(2) = 9 \Rightarrow 4a + 2b = 9 \dots (2)$$

$$a + b = 3 \dots (1) (\times -2)$$

$$4a + 2b = 9 \dots (2)$$

$$-2a - 2b = -6$$

$$4a + 2b = 9$$

$$\hline 2a = 3 \Rightarrow a = \frac{3}{2}$$

$$a = \frac{3}{2} : a + b = 3 \Rightarrow b = 3 - \frac{3}{2} = \frac{3}{2}$$

$$\text{ANSWER: } a = \frac{3}{2}, b = \frac{3}{2}$$

**MARKING SCHEME NOTES****Question 9 (d) [Scale 10C (0, 3, 7, 10)]**3: • One linear equation in  $a$  and  $b$ , e.g.  $u_1: a + b = 3$ 

7: • Two correct linear equations

or

**Question 9 (d) [Scale 10C (0, 3, 7, 10)]**

3: • Recognition of A.P.

7: •  $a = d = 3$  and some use of  $S_n$  formula

**Question 9 (e)**

$$u_n = an^2 + bn = 4134$$

$$\frac{3}{2}n^2 + \frac{3}{2}n = 4134$$

$$3n^2 + 3n - 8268 = 0$$

$$n^2 + n - 2756 = 0$$

$$(n - 52)(n + 53) = 0$$

$$\therefore n = 52$$

$$\text{Number of triangles} = n^2 = 52^2 = 2704$$

**MARKING SCHEME NOTES**

**Question 9 (e) [Scale 10C (0, 3, 7, 10)]**

- 3: • Expression of  $u_n$  in one variable only  
• Quadratic equation
- 7: • Values of  $n$

## LC 2013 (SET F): PAPER 2

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

- (i) The sample space is the set of all possible outcomes of an experiment.
- (ii) Events  $E$  and  $F$  are mutually exclusive if they have no outcomes in common.  
Stated mathematically:  $P(E \cup F) = P(E) + P(F)$
- (iii) Two events are independent if the outcome of one does not depend on the outcome of the other.  
Stated mathematically:  $P(E \cap F) = P(E) \times P(F)$  [METHOD 1]

$$P(E | F) = P(E) \text{ or } P(F | E) = P(F) \text{ [METHOD 2]}$$

#### MARKING SCHEME NOTES

##### Question 1 (a) (i) [Scale 5B (0, 3, 5)]

- 3: • Incomplete statement but with some merit  
• Reference to outcomes  
• Use of # symbol

##### Question 1 (a) (ii) [Scale 5B (0, 3, 5)]

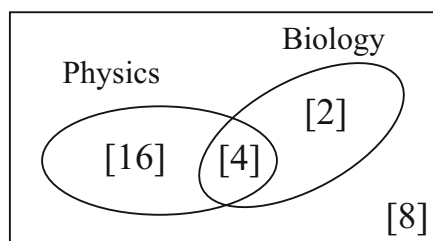
- 3: • Incomplete statement but with some merit  
• Some reference to 'common' or 'intersection' of sets  
• Venn diagram with  $E$  and  $F$  (or equivalent) shown but no reference to outcomes in common  
• Reference to  $P(E \cup F)$

##### Question 1 (a) (iii) [Scale 5B (0, 3, 5)]

- 3: • Incomplete statement but with some merit  
• Reference to comparing/contrasting the outcomes of two events  
• Reference to  $P(E \cap F)$  or  $P(E|F)$  or  $P(F|E)$

#### Question 1 (b)

- (i) Fill in the intersection of  $E$  and  $F$  first. It can be seen that 16 students study Physics only and 2 students study Biology only. 8 students study neither subject.



- (ii) [METHOD 1]

$$P(E \cap F) = \frac{\#(E \cap F)}{\#S} = \frac{4}{30} = \frac{2}{15}$$

$$P(E) = \frac{\#E}{\#S} = \frac{20}{30} = \frac{2}{3}$$

$$P(F) = \frac{\#F}{\#S} = \frac{6}{30} = \frac{1}{5}$$

$$P(E) \times P(F) = \frac{2}{3} \times \frac{1}{5} = \frac{2}{15} = P(E \cap F)$$

[METHOD 2]

$$P(E | F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{15}}{\frac{6}{30}} = \frac{2}{3}$$

$$P(E) = \frac{2}{3}$$

$$\therefore P(E | F) = P(E)$$

**MARKING SCHEME NOTES****Question 1 (b) (i) (ii) [Scale 10D (0, 3, 7, 9, 10)]**

- 3: • Venn Diagram with two or more entries correct  
 • Reference to  $P(E \cap F)$  only  
 • Reference to  $P(E)$  or  $P(F)$  only
- 7: • Venn diagram with either  $P(E \cap F)$  or  $P(E)$  or  $P(F)$  or  $P(E|F)$  or  $P(F|E)$  calculated
- 9: •  $P(E \cap F)$  and  $P(E) \times P(F)$  found but correct conclusion not stated or implied  
 • Error in sample space but correct conclusion from candidate's work

**QUESTION 2 (25 MARKS)****Question 2 (a) (i)**

$$X = 68$$

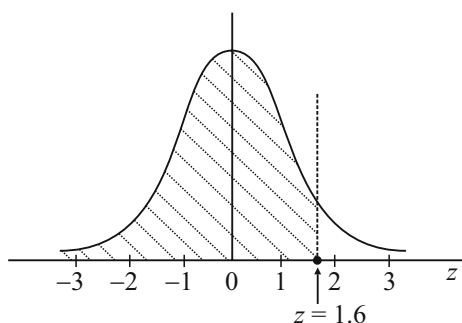
$$\mu = 60$$

$$\sigma = 5$$

$$z = ?$$

$$z = \frac{X - \mu}{\sigma} = \frac{68 - 60}{5} = 1.6$$

$$P(X \leq 68) = P(z \leq 1.6) = 0.9452$$



**FORMULAE AND TABLES BOOK**  
**Statistics and Probability: Probability**  
**distribution (standardising formula) [page 34]**

$$z = \frac{x - \mu}{\sigma}$$

$n$  = Number in the sample

$\sigma$  = standard deviation of the sample

**MARKING SCHEME NOTES****Question 2 (a) (i) [Scale 10C (0, 3, 8, 10)]**

- 3: • Any relevant step  
 • Formula written
- 8: • Reference to 1.6  
 • Incorrect reading of tables

**Question 2 (a) (ii)**

$$X = 52$$

$$\mu = 60$$

$$\sigma = 5$$

$$z = ?$$

$$z = \frac{X - \mu}{\sigma} = \frac{52 - 60}{5} = -1.6$$

$$P(52 \leq X \leq 68)$$

$$= P(-1.6 \leq z \leq 1.6)$$

$$= P(z \leq 1.6) - P(z \leq -1.6)$$

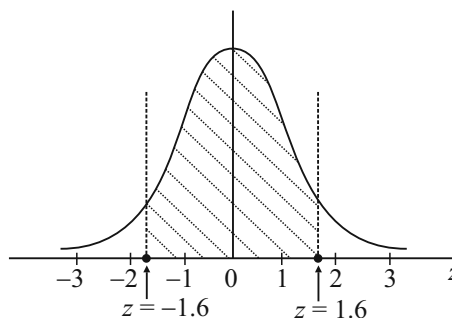
$$= P(z \leq 1.6) - P(z \geq 1.6)$$

$$= P(z \leq 1.6) - [1 - P(z \leq 1.6)]$$

$$= 2P(z \leq 1.6) - 1$$

$$= 2(0.9452) - 1$$

$$= 0.8904$$



**MARKING SCHEME NOTES**

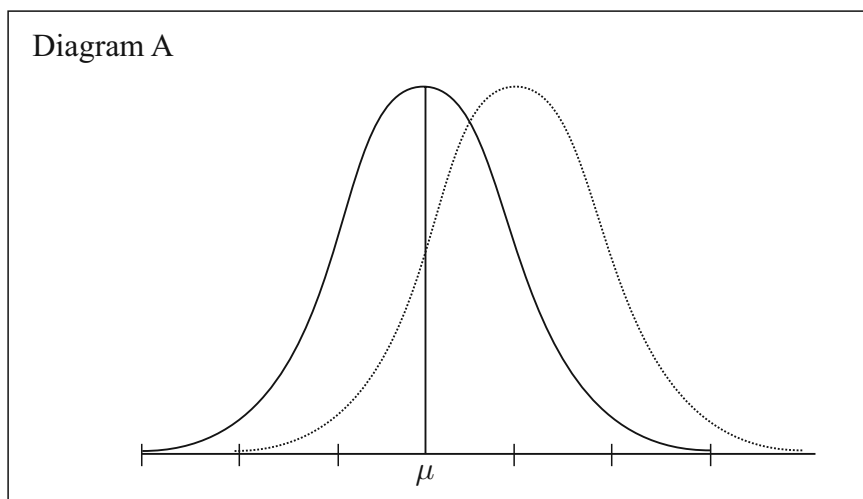
**Question 2 (a) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3:
- Any relevant step other than in a (i)
  - Diagram clearly indicating a 'new' area
  - Reference to  $-1.6$
  - $P(Z \leq -1.6) = 0.0548$  and stops
- 4:
- Probability of both situations calculated but fails to finish correctly
  - Correct method with some error

**Question 2 (b)**

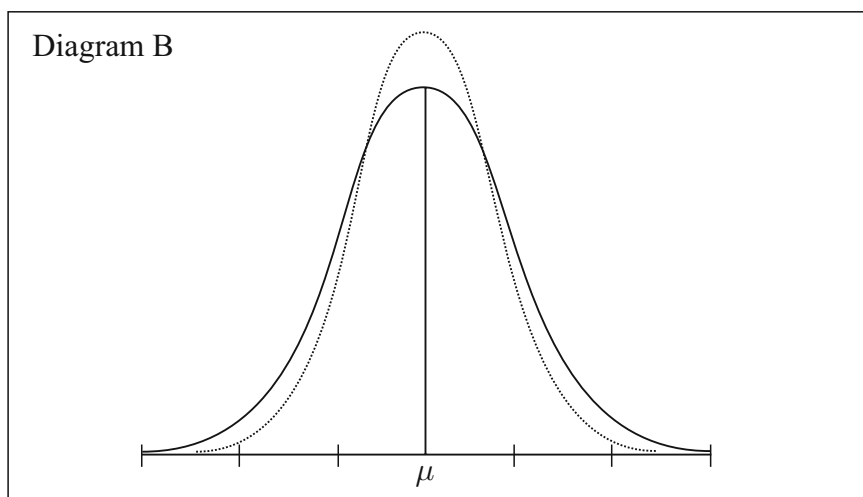
**Hormone A**

The effect of hormone A was to increase the height of all of the plants.



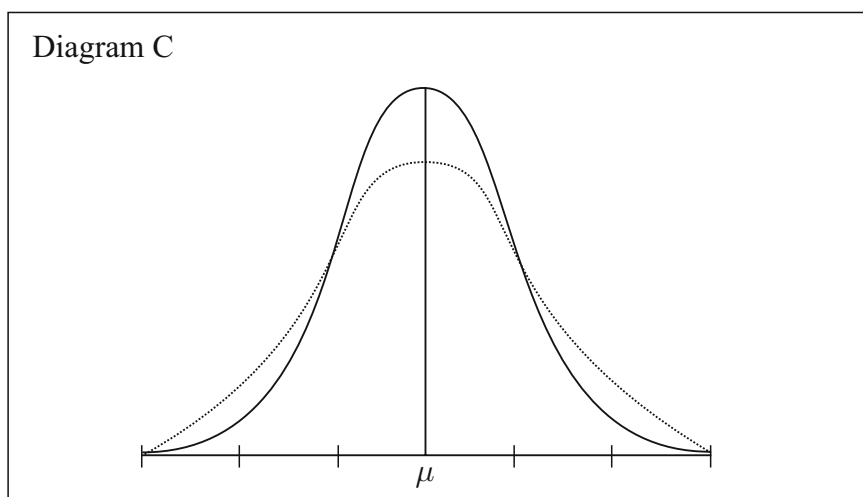
**Hormone B**

The effect of hormone B was to reduce the number of really small plants and the number of really tall plants. The mean was unchanged.



**Hormone C**

The effect of hormone C was to increase the number of small plants and the number of tall plants. The mean was unchanged.



**MARKING SCHEME NOTES**

**Question 1 (b) [Scale 10C (0, 3, 8, 10)]**

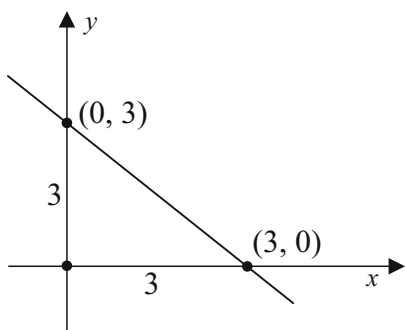
- 3: • One correct  
 • Bell shape in one or more
- 8: • Two correct

**QUESTION 3 (25 MARKS)**

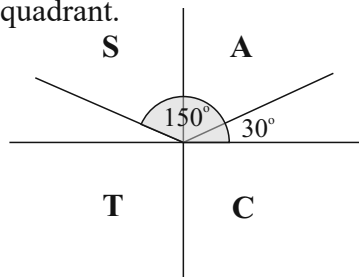
**Question 3 (a)**

Line	Equation	Slope	Intercepts
<i>h</i>	$x = 3 - y$	-1	(0, 3), (3, 0)
<i>i</i>	$2x - 4y = 3$	$\frac{1}{2}$	$(0, -\frac{3}{4}), (\frac{3}{2}, 0)$
<i>k</i>	$y = -\frac{1}{4}(2x - 7)$	$-\frac{1}{2}$	$(0, \frac{7}{4}), (\frac{7}{2}, 0)$
<i>l</i>	$4x - 2y - 5 = 0$	2	$(0, -\frac{5}{2}), (\frac{5}{4}, 0)$
<i>m</i>	$x + \sqrt{3}y - 10 = 0$	$-\frac{1}{\sqrt{3}}$	$(0, \frac{10}{\sqrt{3}}), (10, 0)$
<i>n</i>	$\sqrt{3}x + y - 10 = 0$	$-\sqrt{3}$	$(0, 10), (\frac{10}{\sqrt{3}}, 0)$

The line *h* makes equal intercepts with the *x*-axis cutting it at 3 units from the origin.



The slope is the tan of the angle with the positive *x*-axis. Tan is negative in the second quadrant.



$$\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$$

$$\therefore \theta = 180^\circ - 30^\circ = 150^\circ$$

Lines *k* and *l* are perpendicular as the product of their slopes is -1.

$$\text{Slope of } i: m_1 = \frac{1}{2}$$

$$\text{Slope of } k: m_2 = -\frac{1}{2}$$

$$m_1 \times m_2 = \frac{1}{2} \times -\frac{1}{2} = -1$$

Description	Line(s)
A line with a slope of 2.	$l$
A line which intersects the $y$ -axis at $(0, -2\frac{1}{2})$ .	$l$
A line which makes equal intercepts on the axes.	$h$
A line which makes an angle of $150^\circ$ with the positive sense of the $x$ -axis.	$m$
Two lines which are perpendicular to each other.	$k$ & $l$

**MARKING SCHEME NOTES**

**Question 3 (a) [Scale 10D (0, 3, 7, 9, 10)]**

- 3: • Any one correct  
 • Any line sketched correctly
- 7: • Any two correct
- 9: • Any four correct

**Question 3 (b)**

Slope of  $m$ :  $m_1 = -\frac{1}{\sqrt{3}}$

Slope of  $n$ :  $m_2 = -\sqrt{3}$

$$\tan \theta = \frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}}(-\sqrt{3})} = \frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1+1}$$

$$= \frac{\left(-\frac{1}{\sqrt{3}} + \sqrt{3}\right)}{1+1} \times \frac{\sqrt{3}}{\sqrt{3}} \leftarrow \text{Multiply above and below by root 3.}$$

$$= \frac{-1+3}{2\sqrt{3}} = \frac{2}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

$$\therefore \theta = \tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$$

**FORMULAE AND TABLES BOOK**

**Co-ordinate geometry: Line**

Angles between two lines of slopes  $m_1$  and  $m_2$  [page 19]

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

**MARKING SCHEME NOTES**

**Question 3 (b) [Scale 15D (0, 4, 7, 11, 15)]**

- 4: • Any reasonable step e.g. correct relevant formula  
 • Slope  $m$  or  $n$
- 7: • Both slopes
- 11: •  $\tan \theta = \frac{1}{\sqrt{3}}$  and stops

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**

$$c_1 : (h, k) = (-3, -2), r = 2$$

$$(x - (-3))^2 + (y - (-2))^2 = 2^2$$

$$(x + 3)^2 + (y + 2)^2 = 4$$

$$c_2 : x^2 + y^2 - 2x - 2y - 7 = 0$$

$$(-g, -f) = \left( -\frac{-2}{2}, -\frac{-2}{2} \right) = (1, 1)$$

$$r = \sqrt{1^2 + 1^2 - (-7)} = \sqrt{9} = 3$$

**FORMULAE AND TABLES BOOK**

**Co-ordinate geometry: Circle [page 19]**

Given centre  $(h, k)$  and radius  $r$

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre  $(-g, -f)$

Radius  $\sqrt{g^2 + f^2 - c}$

Circle	Centre	Radius	Equation
$c_1$	$(-3, -2)$	2	$(x + 3)^2 + (y + 2)^2 = 4$
$c_2$	$(1, 1)$	3	$x^2 + y^2 - 2x - 2y - 7 = 0$

**MARKING SCHEME NOTES**

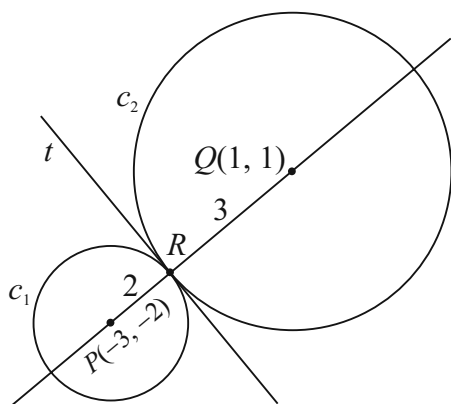
**Question 4 (a) [Scale 10C (0, 3, 8, 10)]**

- 3:**
- Any one correct
  - Any reasonable step
- 8:**
- Any two correct
  - Correct approach but error in work

**Question 4 (b) (i)**

Call the point of contact  $R$ .

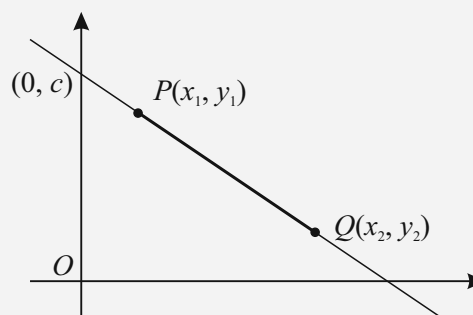
$R$  divides the line of centres,  $PQ$ , in the ratio 2: 3.



**FORMULAE AND TABLES BOOK**

**Co-ordinate geometry: Line [page 18]**

Point dividing  $[PQ]$  in the ratio  $a:b$



$$\left( \frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a} \right)$$

$$a = 2, b = 3$$

$$(x_1, y_1) = (-3, -2), (x_2, y_2) = (1, 1)$$

$$R = \left( \frac{3(-3) + 2(1)}{2 + 3}, \frac{3(-2) + 2(1)}{2 + 3} \right) = \left( \frac{-9 + 2}{5}, \frac{-6 + 2}{5} \right) = \left( -\frac{7}{5}, -\frac{4}{5} \right)$$

**MARKING SCHEME NOTES****Question 4 (b) (i) [Scale 10D (0, 3, 7, 9, 10)]**

- 3: • Some reference to ratio  
•  $c_1 \cap c_2$  and stops  
• Any reasonable step
- 7: • Some correct substitution into correct ratio formula  
• Substitution resulting in a quadratic equation in one variable
- 9: • Error in ratio formula but finishes  
• Finds one of the ordinates only

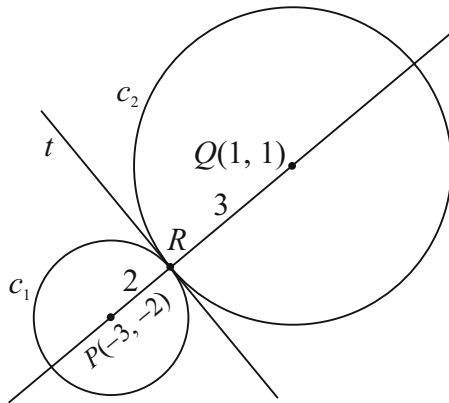
**Question 4 (b) (ii)**

Find the slope of the line  $pq$ :

$$P(-3, -2) = (x_1, y_1), Q(1, 1) = (x_2, y_2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{1 - (-3)} = \frac{3}{4}$$

$$m^\perp = -\frac{4}{3} \leftarrow \text{Tangent } t \text{ is perpendicular to the line } PQ.$$



Equation of  $t$ :

$$(x_1, y_1) = \left(-\frac{7}{5}, -\frac{4}{5}\right), m = -\frac{4}{3}$$

$$y - \left(-\frac{4}{5}\right) = -\frac{4}{3}\left(x - \left(-\frac{7}{5}\right)\right)$$

$$y + \frac{4}{5} = -\frac{4}{3}\left(x + \frac{7}{5}\right)$$

$$y + \frac{4}{5} = -\frac{4}{3}x - \frac{28}{15} \leftarrow \text{Multiply across by 15.}$$

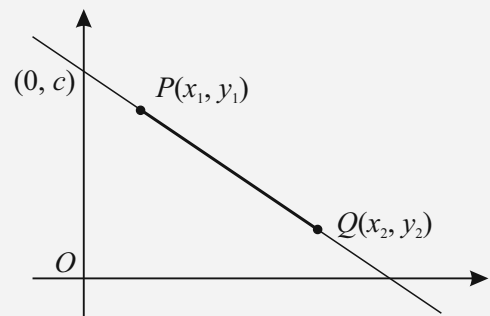
$$15y + 12 = -20x - 28$$

$$20x + 15y + 40 = 0$$

$$4x + 3y + 8 = 0$$

**MARKING SCHEME NOTES****Question 4 (b) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: • Slope of line of centres  
• Any reasonable step
- 4: • Uses point of contact with slope of diameter for equation  
•  $c_1 - c_2 = 0$  but not in linear format

**FORMULAE AND TABLES BOOK  
Co-ordinate geometry: Line**

Slope of  $PQ$  [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of  $PQ$  [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

**QUESTION 5 (25 MARKS)**

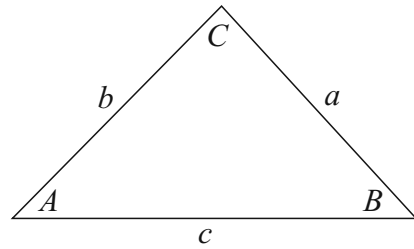
**Question 5 (a)**

$$\text{Area} = \frac{1}{2}bc \sin \angle A = \frac{1}{2}ac \sin \angle B = \frac{1}{2}ab \sin \angle C$$

$$bc \sin \angle A = ac \sin \angle B = ab \sin \angle C \quad [\text{Divide across by } abc.]$$

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

$$\therefore \frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

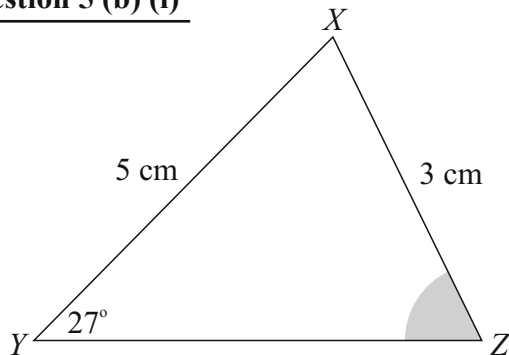


**MARKING SCHEME NOTES**

**Question 5 (a) [Scale 5C (0, 3, 4, 5)]**

- 3: • Relevant diagram  
 • One statement of area in trigonometric format  
 • Sine of a relevant angle in right angled triangle written in terms of sides  
 • Any reasonable step
- 4: • Correct approach but one error in work

**Question 5 (b) (i)**



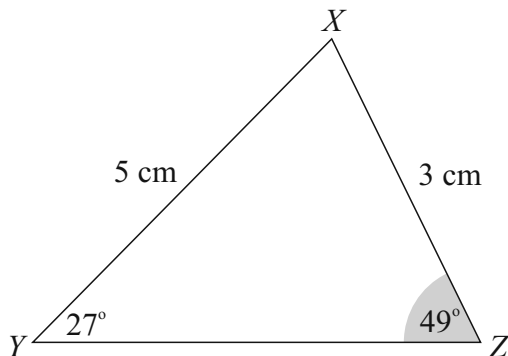
$$\frac{\sin |\angle XZY|}{5} = \frac{\sin 27^\circ}{3}$$

$$\sin |\angle XZY| = \frac{5 \sin 27^\circ}{3}$$

$$|\angle XZY| = \sin^{-1} \left( \frac{5 \sin 27^\circ}{3} \right) = 49^\circ \quad [\text{First quadrant}]$$

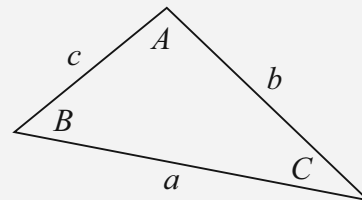
$$|\angle XZY| = 180^\circ - 49^\circ = 131^\circ \quad [\text{Second quadrant}]$$

**Question 5 (b) (ii)**



**FORMULAE AND TABLES BOOK**  
**Trigonometry of the triangle:**

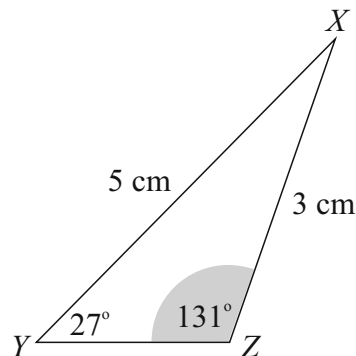
[page 16]



$$\text{Area: } \frac{1}{2}ab \sin C$$

$$\text{Sine Rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule: } a^2 = b^2 + c^2 - 2bc \cos A$$



**MARKING SCHEME NOTES****Question 5 (b) (i) [Scale 10C\* (0, 3, 8, 10)]**

- 3: • Relevant formula  
• Any reasonable step
- 8: • Error in substitution into formula but continues  
• One value only  
• Correct method but one error in work

**Question 5 (b) (ii) [Scale 5B (0, 3, 5)]**

- 3: • One position only shown  
• Triangle(s) sketched but Z not indicated

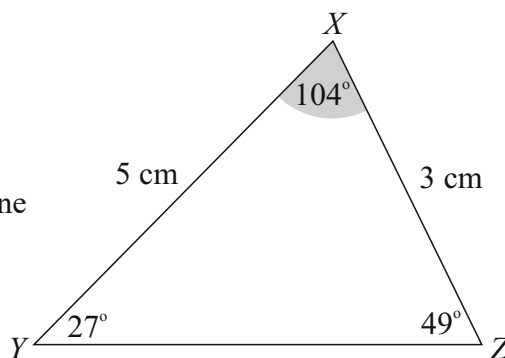
**Question 5 (c)**

$$|\angle YXZ| = 180^\circ - 27^\circ - 49^\circ = 104^\circ$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

The area of a triangle is the product of two sides by the sine of the included angle.

$$\text{Area} = \frac{1}{2}(3)(5)\sin 104^\circ = 7.3 \text{ cm}^2 \approx 7 \text{ cm}^2$$

**MARKING SCHEME NOTES****Question 5 (b) (ii) [Scale 5B\* (0, 3, 5)]**

- 3: •  $|\angle ZXY|$  only  
• Error in substitution into area formula  
• Any reasonable step

**QUESTION 6A (25 MARKS)****Question 6A (a)**

- (i) The circumcentre of a triangle is the point of intersection of *the perpendicular bisectors of the sides of the triangle*.
- (ii) The incentre of a triangle is the point of intersection of *the bisectors of the angles of the triangle*.
- (iii) The centroid of a triangle is the point of intersection of *the medians of the triangle*.

**MARKING SCHEME NOTES****Question 6A (a) [Scale 10D (0, 3, 7, 9, 10)]**

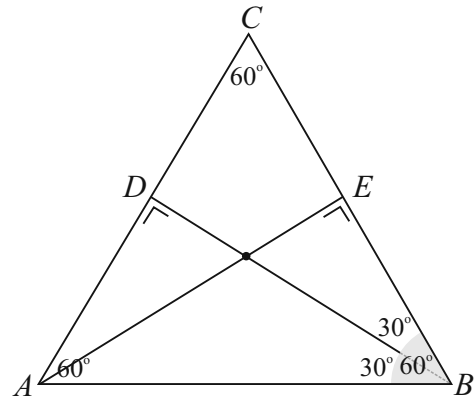
- 3: • One partially correct statement  
• One partially correct sketch  
• Any reasonable step
- 7: • One fully correct statement  
• One fully correct sketch
- 9: • Two correct statements  
• Two correct sketches

### Question 6A (b)

The lines  $BD$  and  $AE$  bisect the  $60^\circ$  angles in the equilateral triangle. Their intersection is the incentre.

Lines  $BD$  and  $AE$  are also the perpendicular bisectors of the sides as they form a  $90^\circ$  angle with the sides. Therefore, the intersection of these lines is also the circumcentre.

$D$  and  $E$  are the midpoints of the sides of the triangle.  $BD$  and  $AE$  are also medians. Therefore, the intersection of the lines  $AE$  and  $BD$  is the centroid.

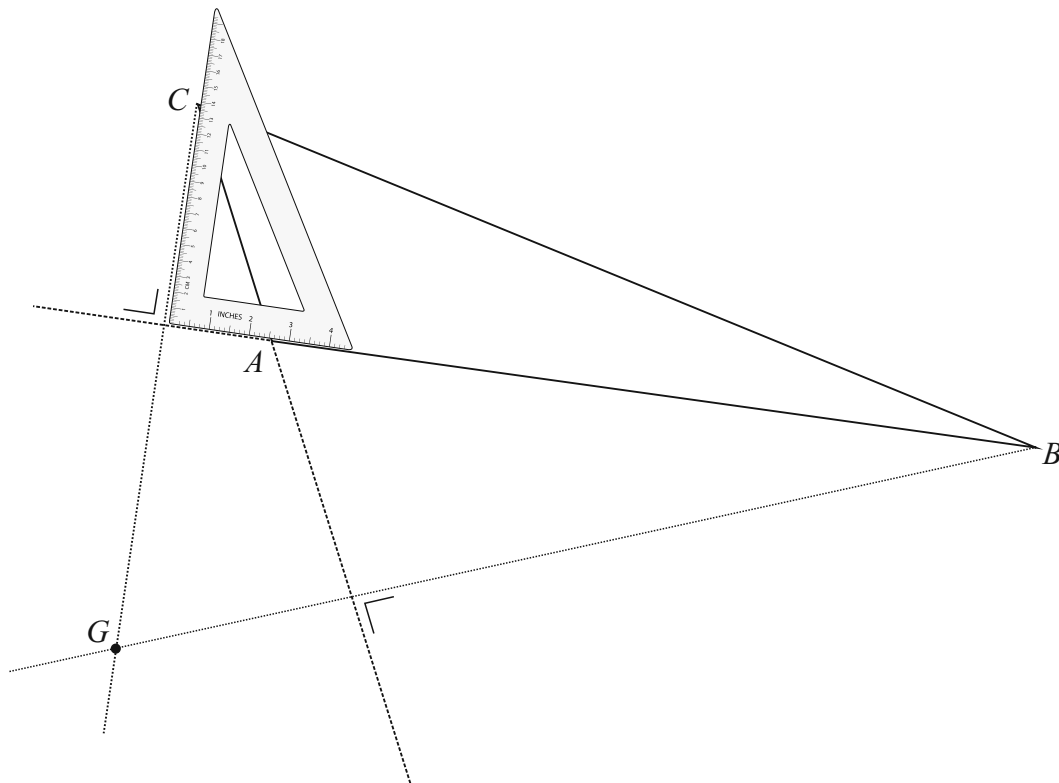


#### MARKING SCHEME NOTES

##### Question 6A (b) [Scale 5B (0, 3, 5)]

- 3:
- Some relevant reference to side(s) of triangle
  - Some relevant reference to angles of triangle
  - An incomplete 'rough' sketch

### Question 6A (c)



The orthocentre  $G$  is the intersection of the heights (altitudes).

Using a ruler, extend the lines  $AB$  and  $CA$  as shown.

Using a set square, draw a perpendicular line from  $C$  to the line  $AB$ . Similarly, draw a perpendicular line from  $B$  to the line  $AC$ .

Using a ruler, extend these two perpendicular lines till they meet at the orthocentre  $G$ .

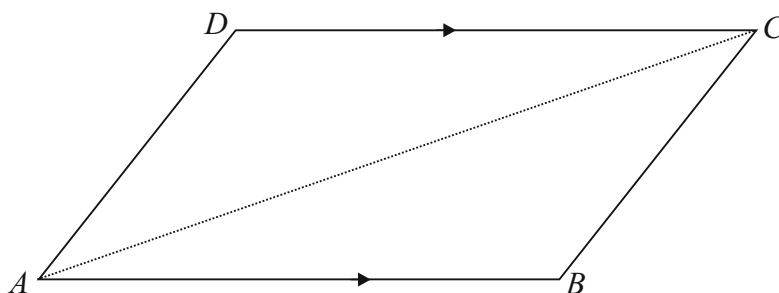
**MARKING SCHEME NOTES**

**Question 6A (c) [Scale 10D (0, 3, 7, 9, 10)]**

- 3: • Some correct elements of construction  
• Some evidence of understanding of term orthocentre  
• Any reasonable step
- 7: • One correct altitude
- 9: • Two altitudes but not intersecting

**QUESTION 6B (25 MARKS)**

**Question 6B (a)**



GIVEN: A quadrilateral  $ABCD$  with  $AB \parallel DC$  and  $|AB| = |DC|$ .

TO PROVE:  $ABCD$  is a parallelogram.

CONSTRUCTION: Draw line  $AC$ .

PROOF: Consider triangles  $ADC$  and  $ABC$ .

$|AB| = |DC|$  [Given]

$|AC| = |AC|$  [Common]

$|\angle DCA| = |\angle BAC|$  [Alternate angles]

Therefore, triangles  $ADC$  and  $ABC$  are congruent [SAS].

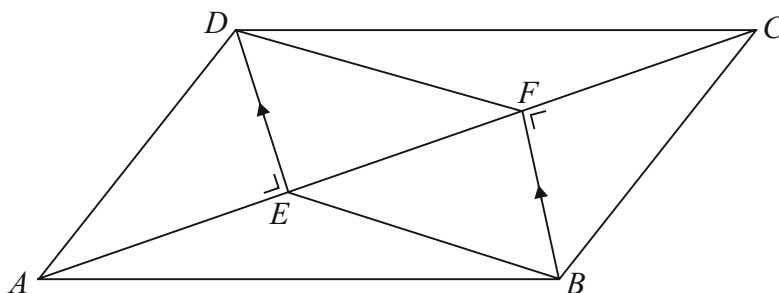
Therefore,  $AD \parallel BC$ .  $ABCD$  is a parallelogram.

**MARKING SCHEME NOTES**

**Question 6B (a) [Scale 10C (0, 3, 8, 10)]**

- 3: • Any correct step e.g. Identifies two equal sides or identifies two equal angles
- 8: • Proof with correct steps but without justification of steps  
• Congruent triangles established but fails to complete  
• No conclusion stated or implied

**Question 6B (b)**



$DE \perp AC$  and  $BF \perp AC \Rightarrow DE \parallel BF$

Area  $|\triangle ACD| = \text{Area } |\triangle ACB|$  [A diagonal bisects the area of a parallelogram.]

$$\frac{1}{2}|AC||DE| = \frac{1}{2}|AC||BF|$$

$$\therefore |DE| = |BF|$$

Therefore,  $EBFD$  is a parallelogram because  $DE \parallel BF$  and  $|DE| = |BF|$ .

**MARKING SCHEME NOTES**

**Question 6B (b) [Scale 15C (0, 5, 10, 15)]**

- 5:**
- Any correct step e.g. Establishes parallel sides or identifies two equal sides
- 10:**
- Proof with one step not fully established
  - No conclusion stated or implied

**QUESTION 7 (75 MARKS)**

**Question 7 (a)**

- (i) The population is divided into different subgroups which have common characteristics. Random samples are drawn from each subgroup according to their proportion of the population.
- (ii) Long haul economy class passengers.  
Long haul business class passengers.  
Long haul executive class passengers.  
Short haul passengers.

**MARKING SCHEME NOTES**

**Question 7 (a) (i) [Scale 5B (0, 3, 5)]**

- 3:**
- Reference to subgroups
  - Reference to sampling
  - Definition of random sample

**Question 7 (a) (ii) [Scale 5B (0, 3, 5)]**

- 3:**
- Not clearly disjoint groups
  - Incomplete number of groups (at least two)

### Question 7 (b) (i)

Was flight delayed?	Yes	No	Don't Know
	<b>231</b>	748	21

$$P(\text{His/her flight delayed}) = \frac{231}{1000} = 0.231$$

Passenger satisfaction with overall service	Satisfied	Not satisfied	Don't Know
	664	<b>238</b>	98

$$P(\text{Was not satisfied with the overall service}) = \frac{238}{1000} = 0.238$$

### Question 7 (b) (ii)

ANSWER: No

If it was, this would imply that the events were independent but this is not likely since a passenger who had his flight delayed is likely to be not satisfied with the service.

#### MARKING SCHEME NOTES

##### Question 7 (b) (i) [Scale 5B (0, 3, 5)]

3: • One correct probability

• Some evidence of relevant understanding

NOTE: Accept any other answer in the range [0.231, 0.252], provided a suitable rationale is given

##### Question 7 (b) (ii) [Scale 5B (0, 3, 5)]

3: • Correct answer without explanation

• Correct answer with incorrect or incomplete explanation

### Question 7 (c)

ANSWER: Graph (ii)

A lot of the passengers are likely to have baggage with a weight of less than the maximum 20 kg.

#### MARKING SCHEME NOTES

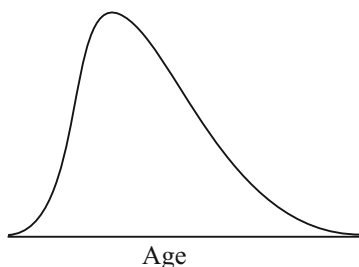
##### Question 7 (c) [Scale 5B (0, 3, 5)]

3: • Correct answer without explanation

• Correct explanation but omits to nominate correct graph

### Question 7 (d)

(i)



(ii) The median is less than the mean so the graph is skewed to the right.

**MARKING SCHEME NOTES****Question 7 (d) (i) [Scale 5B (0, 3, 5)]**

3: • A partially correct bell shape curve

**Question 7 (d) (ii) [Scale 5B (0, 3, 5)]**

3: • Mention of mean and median but interpretation not related to candidate's curve

**Question 7 (e) (i)**

Null hypothesis  $H_0$ : Satisfaction level is unchanged,  $P = 0.7$

95% margin of error of a sample of 1000:  $\frac{1}{\sqrt{1000}} = 0.0316$

Passenger satisfaction with overall service	Satisfied	Not satisfied	Don't Know
	664	238	98

Recorded satisfaction level for May:  $P = 0.664$

95% confidence interval for the population proportion:

$$[0.664 - 0.0316, 0.664 + 0.0316] = [0.6324, 0.6956]$$

There is evidence to refute the company's claims that 70% of their customers are satisfied with their overall service because, based on the sample data, any values in the range 63.24% and 69.56% are possible values for the proportion of passengers who are satisfied with the service.

Therefore, reject the null hypothesis as 0.7 lies outside this interval.

**Question 7 (e) (ii)**

ANSWER: No

$$\frac{1}{\sqrt{1000}} = 0.0316$$

$$\frac{1}{\sqrt{2000}} = 0.02236$$

$$\frac{1}{2}(0.0316) = 0.0158 \neq 0.02236$$

**IMPORTANT NOTE**

This approach to finding the confidence interval is no longer used at Higher Level.

**MARKING SCHEME NOTES****Question 7 (e) (i) [Scale 10D (0, 3, 7, 9, 10)]**

3: • One relevant step e.g. null hypothesis stated only

• Some work towards margin of error

7: • Substantive work with one or more critical omissions

• Margin of error and range found but fails to continue

9: • Failure to state null hypothesis correctly

• Failure to contextualise answer (e.g. stops at 'Reject null hypothesis')

**NOTE:** Accept candidate work based on disregarding *don't knows*, yielding an observed satisfaction rating of 664/902 and a corresponding  $n = 902$ .

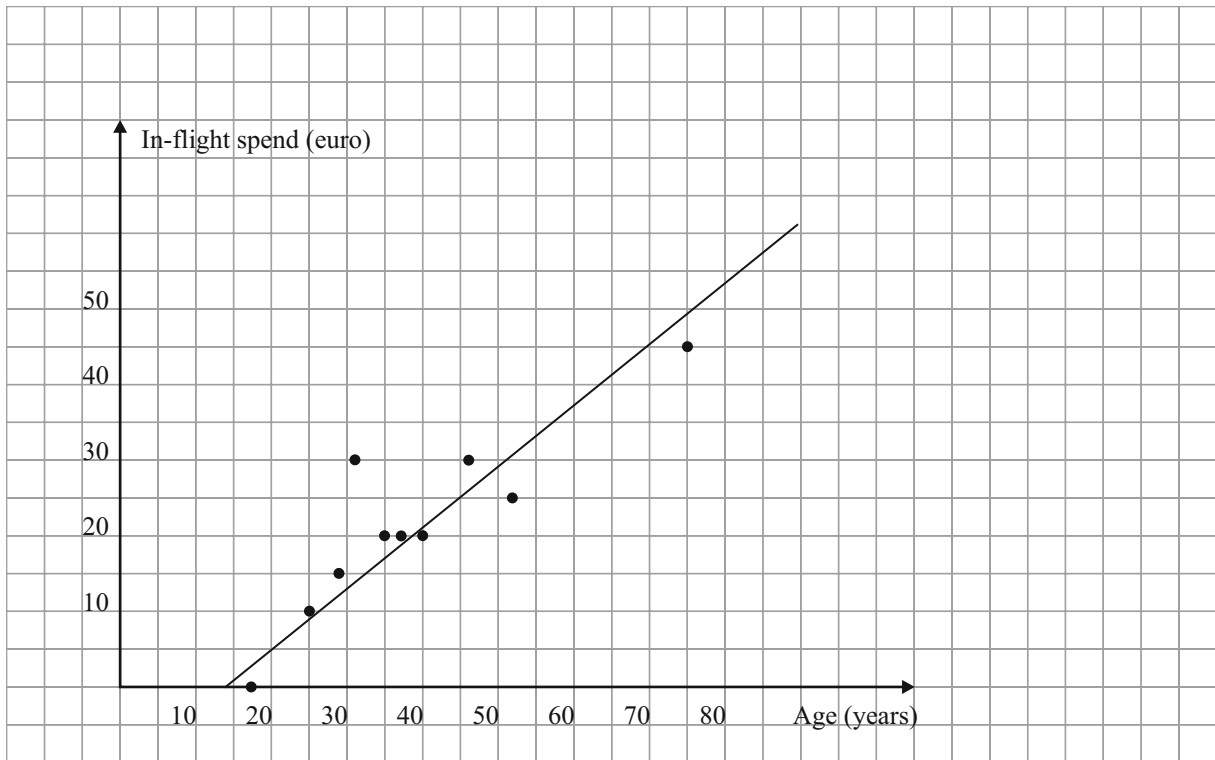
**Question 7 (e) (ii) [Scale 5B (0, 3, 5)]**

3: • Correct answer but no explanation

• Partially correct explanation

**Question 7 (f)**

(i)/(iv)



(ii)

CASIO CALCULATOR (*fx-85GT PLUS*)

Steps to find  $r$ :

Press Mode.

Press 2: Stat

Press 2:  $A + Bx$

Input your  $x$  and  $y$  values

| Press AC Button

| Press Shift followed by the Number 1

| Press 5: Reg

| Press 3:  $r$

| Press =

ANSWER:  $r = 0.8822$

(iii) Older passengers tend to spend more. There is a strong correlation between the age of a passenger and the amount they spend.

**MARKING SCHEME NOTES**

**Question 7 (f) (i) [Scale 10C (0, 3, 8, 10)]**

3: • Correct scale with at least two points plotted

8: • Correct scales but not all points plotted (one or two omissions)

• All points plotted but scales incorrect

**Question 7 (f) (ii) [Scale 5A (0, 5)]**

**Question 7 (f) (iii) [Scale 5B (0, 3, 5)]**

3: • Partially correct answer e.g. not in context

• Positive or strong positive correlation and stops

**Question 7 (f) (iv) [Scale 5B (0, 3, 5)]**

3: • Straight line but clearly not best fit

**QUESTION 8 (30 MARKS)**

THE TRIANGLE IN THIS QUESTION DOES NOT EXIST BASED ON THE VALUES GIVEN. DO NOT ATTEMPT THIS QUESTION.

**QUESTION 9 (45 MARKS)**

**Question 9 (a)**

$$\triangle XYZ : x^2 + y^2 = (4 + 8)^2 = 144$$

$$\triangle XYP : k^2 + 16 = x^2$$

$$\triangle XYP : k^2 + 64 = y^2$$

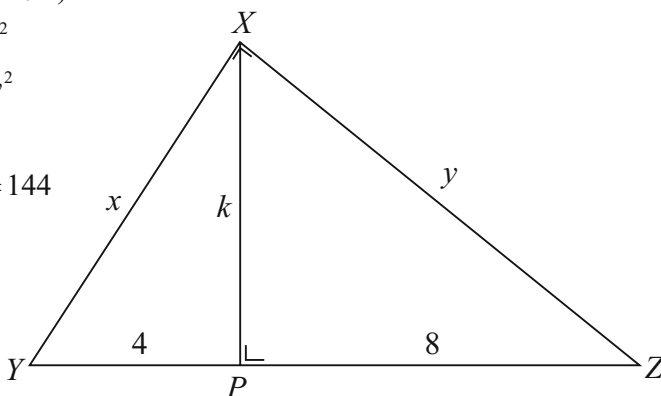
$$x^2 + y^2 = 144$$

$$k^2 + 16 + k^2 + 64 = 144$$

$$2k^2 = 64$$

$$k^2 = 32$$

$$\therefore k = \sqrt{32} = 4\sqrt{2}$$



**FORMULAE**  
Trigonometry (page 16)  
Pythagoras' theorem

$c^2 = a^2 + b^2$

**MARKING SCHEME NOTES**

**Question 9 (a) [Scale 10D (0, 3, 7, 9, 10)]**

- 3:**
- One correct statement in  $k^2$
  - One correct ratio involving  $k$
  - Some relevant use of Pythagoras
  - Any reasonable step e.g. establishing similar triangles
- 7:**
- Two correct statements in  $k^2$
  - Two correct ratios involving  $k$
- 9:**
- $k^2$  evaluated, but  $k$  not found

**Question 9 (b) (i)**

$$\text{Perimeter } P = \pi r_1 + \pi r_2 + \pi r_3$$

$$r_1 = r_2 + r_3$$

$$P = \pi r_1 + \pi(r_2 + r_3) = \pi r_1 + \pi r_1 = 2\pi r_1$$

**Question 9 (b) (ii)**

$$\text{Area of arbelos: } A = \frac{1}{2} \pi (6)^2 - \frac{1}{2} \pi (4)^2 - \frac{1}{2} \pi (2)^2 = 8\pi$$

$$\text{Area of circle with diameter } k: A = \pi \left( \frac{k}{2} \right)^2 = \pi \left( \frac{4\sqrt{2}}{2} \right)^2 = 8\pi$$

**FORMULAE AND TABLES BOOK**  
Length and area:  
Circle [page 8]

$l = 2\pi r$   
 $A = \pi r^2$

**MARKING SCHEME NOTES**

**Question 9 (b) (i) [Scale 5B (0, 3, 5)]**

- 3:**
- One relevant perimeter
  - $r_2 + r_3 = r_1$  written or implied
  - Any reasonable step

**Question 9 (b) (ii) [Scale 5B (0, 3, 5)]**

- 3:**
- Area of one semi circle
  - Area of circle with diameter  $k$
  - Area of arbelos only

**Question 9 (c)**

(i)

$r_1$	$r_2$	$r_3$	Area of arbelos
6	1	5	$\frac{1}{2}\pi(6^2 - (1^2 + 5^2)) = 5\pi \text{ cm}^2$
6	2	4	$\frac{1}{2}\pi(6^2 - (2^2 + 4^2)) = 8\pi \text{ cm}^2$
6	3	3	$\frac{1}{2}\pi(6^2 - (3^2 + 3^2)) = 9\pi \text{ cm}^2$
6	4	2	$\frac{1}{2}\pi(6^2 - (2^2 + 4^2)) = 8\pi \text{ cm}^2$
6	5	1	$\frac{1}{2}\pi(6^2 - (5^2 + 1^2)) = 5\pi \text{ cm}^2$

(ii)

$$r_1 = r_2 + r_3$$

$$r_1 = 6$$

$$r_2 = x$$

$$r_3 = r_1 - r_2 = (6 - x)$$

$$\begin{aligned} \text{Area of arbelos} &= \frac{1}{2}\pi r_1^2 - \frac{1}{2}\pi(r_2^2 + r_3^2) \\ &= \frac{1}{2}\pi(6)^2 - \frac{1}{2}\pi(x^2 + (6-x)^2) \\ &= 18\pi - \frac{1}{2}\pi(x^2 + 36 - 12x + x^2) \\ &= 18\pi - \frac{1}{2}\pi(2x^2 - 12x + 36) \\ &= 18\pi - \pi x^2 + 6\pi x - 18\pi \\ &= 6\pi x - \pi x^2 \\ &= \pi(6x - x^2) \text{ cm}^2 \end{aligned}$$

(iii)

$$A = \pi(6x - x^2)$$

$$\frac{dA}{dx} = \pi(6 - 2x) = 0$$

$$6 - 2x = 0$$

$$6 = 2x$$

$$\therefore x = 3 \text{ cm}$$

To maximise the area, find its derivative with respect to  $x$  and put the derivative equal to zero. Solve for  $x$ . Find the maximum area by substituting this value of  $x$  into the area function for the area.

$$A = \pi(6x - x^2)$$

$$x = 3 \text{ cm: } A_{\text{Max}} = \pi(6(3) - 3^2) = 9\pi \text{ cm}^2$$

**Question 9 (c) (i) [Scale 10B (0, 5, 10)]**

5: • Two correct entries

**Question 9 (c) (ii) [Scale 5B\* (0, 3, 5)]**

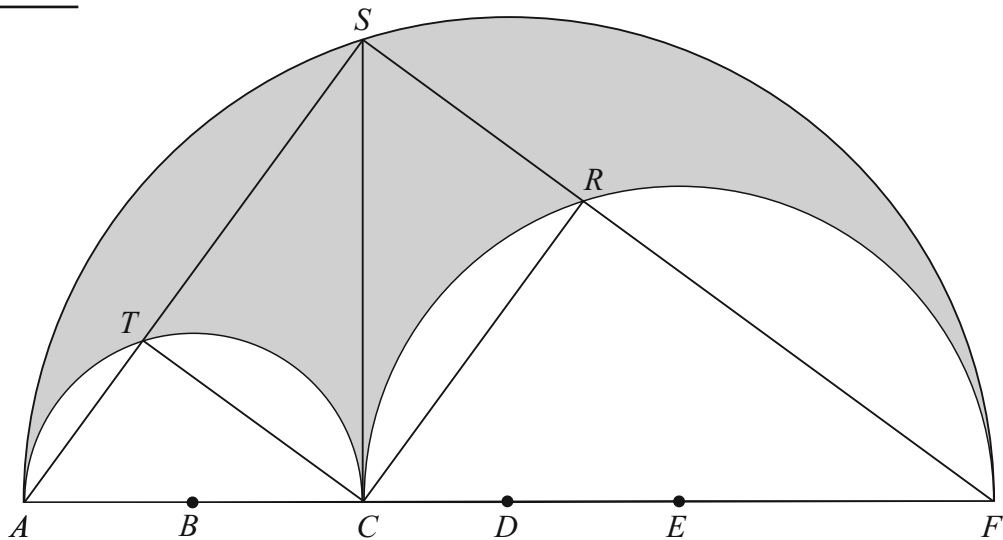
3: • Identifies  $r_3 = 6 - x$   
• Area of either semicircle in terms of  $x$

**Question 9 (c) (iii) [Scale 5C\* (0, 3, 4, 5)]**

3: • Some correct differentiation  
• Incomplete quadratic graph sketched

4: •  $x = 3$  established as maximum but area not calculated  
• Graphical result not interpreted

**Question 9 (d)**



**PROOF**

Angles  $ASF$ ,  $ATC$  and  $CRF$  are right angles. [The angle at the circle standing on a diameter is a right angle.]

Therefore, angles  $STC$  and  $SRC$  are right angles. [Straight angles]

Therefore, angle  $TCR$  is a right angle. [The 4 angles of a quadrilateral add up to  $360^\circ$ .]

Therefore,  $RSTC$  is a rectangle. [4 angles are  $90^\circ$ .]

**Question 9 (d) [Scale 5B (0, 3, 5)]**

**3:** • One relevant step

## LC 2012 (SET G): PAPER 1

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

$$a^2 - ab + b^2 = 3 \dots\dots(Q)$$

$$a + 2b + 1 = 0 \dots\dots(L)$$

$$a + 2b + 1 = 0 \Rightarrow a = -2b - 1$$

$$a^2 - ab + b^2 = 3$$

$$(-2b - 1)^2 - (-2b - 1)b + b^2 = 3$$

$$4b^2 + 4b + 1 + 2b^2 + b + b^2 = 3$$

$$7b^2 + 5b - 2 = 0$$

$$(7b - 2)(b + 1) = 0$$

$$\therefore b = -1, \frac{2}{7}$$

$$b = -1 : a = -2(-1) - 1 = 2 - 1 = 1$$

$$b = \frac{2}{7} : a = -2\left(\frac{2}{7}\right) - 1 = -\frac{4}{7} - 1 = -\frac{11}{7}$$

Solutions:  $(1, -1), \left(-\frac{11}{7}, \frac{2}{7}\right)$

#### MARKING SCHEME NOTES

##### Question 1 (a) [Scale 15C (0, 8, 14, 15)]

- 8:**
- Any attempt at trial and error
  - Writes  $a$  in terms of  $b$ , or  $b$  in terms of  $a$ , and stops
  - Any reasonable first step
- 14:**
- Solves for one variable (two values), more or less correctly
  - Substitutes  $-2b - 1$  for  $a$  in the non-linear equation and some further progress
  - Substitutes  $\frac{-a - 1}{2}$  for  $b$  in the non-linear equation and some further progress

#### Question 1 (b)

$$\frac{2x - 5}{x - 3} \leq \frac{5}{2}$$

$$\frac{2(x - 3)^2(2x - 5)}{(x - 3)} \leq \frac{5 \times 2(x - 3)^2}{2}$$

$$2(x - 3)(2x - 5) \leq 5(x - 3)^2$$

$$2(x - 3)(2x - 5) - 5(x - 3)^2 \leq 0$$

$$(x - 3)[2(2x - 5) - 5(x - 3)] \leq 0$$

$$(x - 3)[4x - 10 - 5x + 15] \leq 0$$

$$(x - 3)[5 - x] \leq 0 \leftarrow \text{You need to solve this inequality.}$$

$$(x - 3)(5 - x) = 0 \leftarrow \text{First, find the roots of the equality.}$$

$$\therefore x = 3, 5$$

Use the region test to find the regions that satisfy the inequality.

$$\begin{array}{c}
 \longleftarrow 0 \quad \boxed{3} \quad \longleftarrow 4 \quad \boxed{5} \quad \longrightarrow 6 \quad \longrightarrow \\
 (x - 3)(5 - x) \leq 0 \quad (x - 3)(5 - x) \leq 0 \quad (x - 3)(5 - x) \leq 0 \\
 (-3)(5) \leq 0 \quad (1)(1) \leq 0 \quad (3)(-1) \leq 0 \\
 \text{True} \quad \text{False} \quad \text{True}
 \end{array}$$

ANSWER:  $x < 3, x \geq 5$

**MARKING SCHEME NOTES**

**Question 1 (b) [Scale 10D (0, 3, 5, 8, 10)]**

- 3: • Any reasonable first step
- 5: • Finds particular values of  $x$  for which the inequality holds
- 8: • Solves the relevant quadratic to find the solutions,  $x = 3$  or  $x = 5$ 
  - Deals more or less correctly with one case only ( $x < 3$  or  $5 \geq x$ )
  - Solution shown on a graph
  - Substantial progress towards full solution

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**

$\mathbb{C}$  is the set of complex numbers. **Ex.**  $\sqrt{3} - i$

$$\mathbb{C} = \{a + bi \mid a \in \mathbb{R}, b \in \mathbb{R}, i^2 = -1\}$$

$G$  is the set of complex numbers where  $x, y$  are integers (whole numbers). **Ex.**  $3 + 4i, 7 + 0i$

$$G = \{x + yi \mid x, y \in \mathbb{Z}, i^2 = -1\}$$

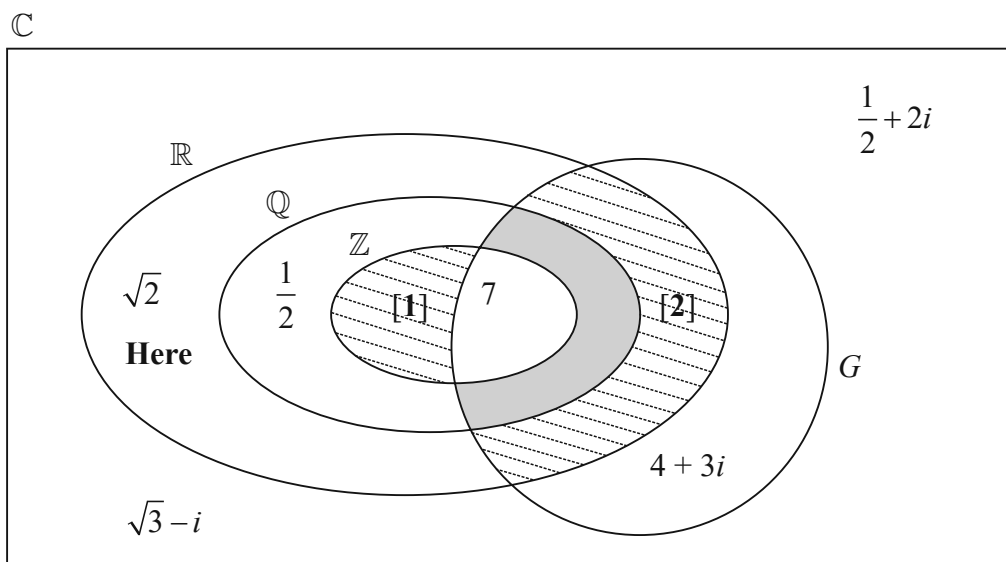
$\mathbb{R}$  is the set of real numbers. **Ex.**  $\sqrt{3} + 0i$  [ $\mathbb{R}$  is a subset of  $\mathbb{C}$ ]

$\mathbb{Q}$  is the set of rational numbers, that is any number that can be expressed as the quotient or fraction  $\frac{p}{q}$  of two integers, with the denominator  $q$  not equal to zero. **Ex.**  $\frac{1}{2} + 0i$

$$\mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$$

$\mathbb{Z}$  is the set of integers. **Ex.**  $5 + 0i, -7 + 0i$

$$\mathbb{Z} = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$$



### Question 2 (b)

Region [1] is empty as every element of  $\mathbb{Z}$  is also an element of  $G$ .

Region [2] is empty as no element in  $G$  can have values of  $x$  and  $y$  that are not integers.

### Question 2 (c)

$$ab = \frac{p}{q}(x + yi) = \frac{px}{q} + \frac{py}{q}i$$

$ab$  cannot be in the region shown as its coefficients must be rational numbers.

#### MARKING SCHEME NOTES

##### Question 2 (a) [Scale 15B (0, 11, 15)]

- 11: • One correct region shaded  
• Two regions containing no elements shaded

##### Question 2 (b) [Scale 5C (0, 3, 4, 5)]

- 3: • At least one correct  
4: • Three, four or five correct  
NOTE: Any duplicate entry counts as incorrect

##### Question 2 (c) [Scale 5B (0, 3, 5)]

- 0: • Answer in the wrong place, without explanation or work  
3: • Takes any element of  $G$  and any element of  $Q$  and finds their product  
• Some relevant work or explanation of merit  
• Writes  $\frac{p}{q}(x + iy)$   
5: • Answer in correct place, with or without explanation or work

### QUESTION 3 (25 MARKS)

#### Question 3 (a)

$$r = \frac{81}{16}, \theta = \left( \frac{4\pi}{9} + 2n\pi \right) \leftarrow \text{Write } z \text{ in polar form.}$$

$$z = r(\cos \theta + i \sin \theta)$$

$$= \frac{81}{16} \left[ \cos \left( \frac{4\pi}{9} + 2n\pi \right) + i \sin \left( \frac{4\pi}{9} + 2n\pi \right) \right]$$

$$w^4 = z \Rightarrow w = z^{\frac{1}{4}}$$

$$\therefore w = \left( \frac{81}{16} \right)^{\frac{1}{4}} \left[ \cos \left( \frac{4\pi}{9} + 2n\pi \right) + i \sin \left( \frac{4\pi}{9} + 2n\pi \right) \right]^{\frac{1}{4}} \leftarrow \text{Apply De Moivre's theorem.}$$

$$= \left( \frac{\square}{2} \right) \left[ \cos \left( \frac{\pi}{9} + \frac{n\pi}{2} \right) + i \sin \left( \frac{\pi}{9} + \frac{n\pi}{2} \right) \right]$$

The fourth root of  $z$  has 4 solutions which you find by putting  $n = 0, 1, 2, 3$ .

#### FORMULAE AND TABLES BOOK: Algebra (page 20)

##### DE MOIVRE'S THEOREM

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

$$n = 0: w_1 = \left(\frac{3}{2}\right) \left[ \cos\left(\frac{\pi}{9}\right) + i \sin\left(\frac{\pi}{9}\right) \right]$$

$$n = 1: w_2 = \left(\frac{3}{2}\right) \left[ \cos\left(\frac{11\pi}{18}\right) + i \sin\left(\frac{11\pi}{18}\right) \right]$$

$$n = 2: w_3 = \left(\frac{3}{2}\right) \left[ \cos\left(\frac{10\pi}{9}\right) + i \sin\left(\frac{10\pi}{9}\right) \right]$$

$$n = 3: w_4 = \left(\frac{3}{2}\right) \left[ \cos\left(\frac{29\pi}{18}\right) + i \sin\left(\frac{29\pi}{18}\right) \right]$$

**MARKING SCHEME NOTES**

**Question 3 (a) [Scale 20C (0, 14, 18, 20)]**

**14:** • Any reasonable first step

- $\cos \frac{4\pi}{9} + i \sin \frac{4\pi}{9}$  or  $\cos 80^\circ + i \sin 80^\circ$  appears without any further work

**18:** • Correct solution with one or two minor errors

- One fully correct value
- $\left(5 \frac{1}{16}\right)^{\frac{1}{4}} \left( \cos \frac{1}{4} \left( \frac{4\pi}{9} \right) + i \sin \frac{1}{4} \left( \frac{4\pi}{9} \right) \right)$
- Not in polar form

**Question 3 (b)**

Draw a circle with centre at the origin and radius 1.5. All 4 roots lie on this circle.

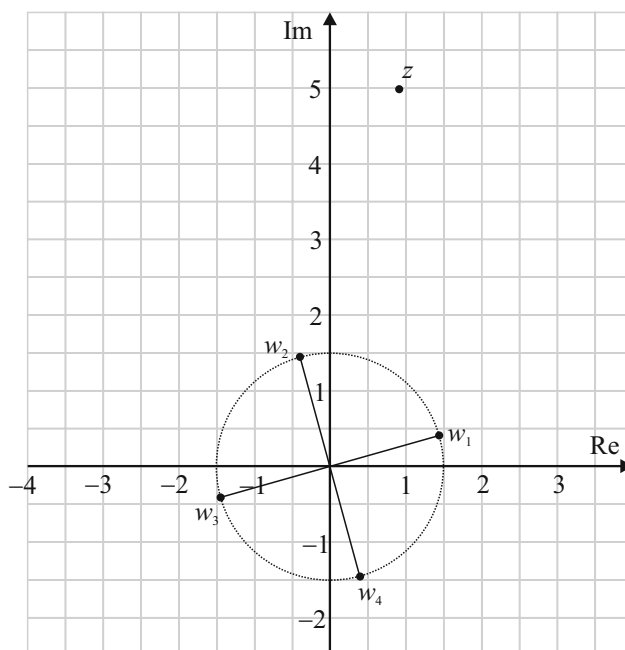
I changed the polar form of the roots to degrees to help me choose the angle the roots make with the positive x-axis.

$$w_1 = \frac{3}{2} (\cos 20^\circ + i \sin 20^\circ)$$

$$w_2 = \frac{3}{2} (\cos 110^\circ + i \sin 110^\circ)$$

$$w_3 = \frac{3}{2} (\cos 200^\circ + i \sin 200^\circ)$$

$$w_4 = \frac{3}{2} (\cos 290^\circ + i \sin 290^\circ)$$



**MARKING SCHEME NOTES**

**Question 3 (b) [Scale 5B (0, 3, 5)]**

**3:** • Some work of merit.

- If moduli are not the same, then partial credit only

**NOTE:** Need four reasonable solutions to part (a) in order to gain full credit in part (b).

## QUESTION 4 (25 MARKS)

### Question 4 (a)

STEP 1: Prove for  $n = 1$ :  $\frac{a(1-r^1)}{1-r} = \frac{a(1-r)}{(1-r)} = a$

STEP 2: Assume for  $n = k$ :  $a + ar + ar^2 + \dots + ar^{k-1} = \frac{a(1-r^k)}{(1-r)}$

STEP 3: Prove for  $n = k + 1$ :  $[a + ar + ar^2 + \dots + ar^{k-1}] + ar^k = \frac{a(1-r^{k+1})}{(1-r)}$

$$\begin{aligned} & [a + ar + ar^2 + \dots + ar^{k-1}] + ar^k \leftarrow \text{Use the result in Step 2.} \\ &= \frac{a(1-r^k)}{(1-r)} + ar^k \\ &= \frac{a(1-r^k) + (1-r)ar^k}{(1-r)} \\ &= \frac{a - ar^k + ar^k - ar^{k+1}}{(1-r)} \\ &= \frac{a - ar^{k+1}}{(1-r)} \\ &= \frac{a(1-r^{k+1})}{(1-r)} \end{aligned}$$

So the result is true for  $n = 1$  and also true for  $n = k$ . This implies it is true for  $n = k + 1$ .

This implies it is true for all  $n \in \mathbb{N}_0$ .

#### MARKING SCHEME NOTES

##### Question 4 (a) [Scale 10C (0, 5, 8, 10)]

- 5:** • Any reasonable first step  
• Correct proof without using induction
- 8:** • Solution mostly correct, including evidence of understanding the logic of proof by induction
- 10:** •  $P(k) \Rightarrow P(k+1)$

### Question 4 (b)

$$\begin{aligned} 5 \cdot 212121\dots &= 5 + \left[ \frac{21}{100} + \frac{21}{10000} + \dots \right] \\ &= 5 + \frac{21}{100} \left[ 1 + \frac{1}{100} + \frac{1}{10000} \dots \right] \\ & \quad a = 1, r = \frac{1}{100} \end{aligned}$$

#### FORMULAE AND TABLES BOOK

**Sequences and series:**  
**Geometric series** [page 22]

$$S_{\infty} = \frac{a}{1-r}, |r| < 1$$

$$\therefore 5 \cdot 212121\dots = 5 + \frac{21}{100} \left[ \frac{1}{1 - \frac{1}{100}} \right] = 5 + \frac{21}{100} \left[ \frac{1}{\frac{99}{100}} \right] = 5 + \frac{21}{100} \left[ \frac{100}{99} \right] = 5 \frac{21}{99} = 5 \frac{7}{33}$$

**MARKING SCHEME NOTES**

**Question 4 (b) [Scale 15C (0, 8, 14, 15)]**

8: •  $S_{\infty} = \frac{a}{1-r}$

- Correct answer without work
- Any reasonable first step

14: •  $5 + \frac{21}{100} + \frac{21}{10\,000} + \frac{21}{1\,000\,000} + \dots$

- Correct solution with minor arithmetic errors

**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

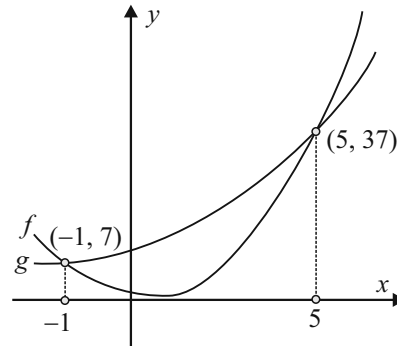
$f(x) = g(x)$

$2x^2 - 3x + 2 = x^2 + x + 7$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$\therefore x = -1, 5$



$g(-1) = (-1)^2 + (-1) + 7 = 1 - 1 + 7 = 7 \Rightarrow (-1, 7)$

$g(5) = (5)^2 + (5) + 7 = 25 + 5 + 7 = 37 \Rightarrow (5, 37)$

**MARKING SCHEME NOTES**

**Question 5 (a) [Scale 10C (0, 5, 8, 10)]**

- 5: • Constructs a table of values but fails to finish
- Any reasonable first step
- 8: • Correct x-values but no y-values (or incorrect y-values)
- Solution with one minor error

**Question 5 (b)**

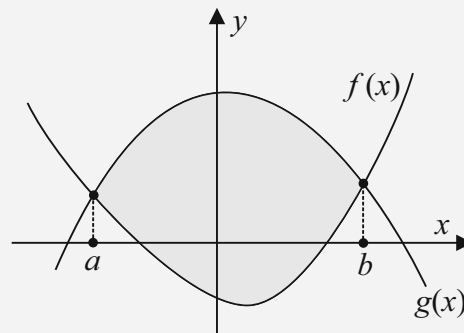
**FORMULAE AND TABLES BOOK**

**Calculus: Integrals** [page 26]

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

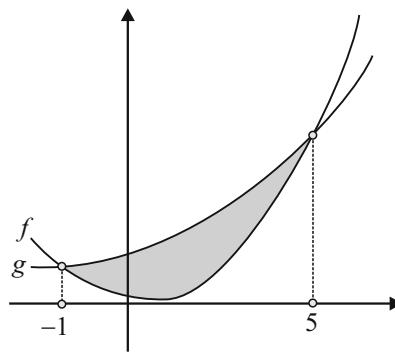
$$\begin{aligned} A &= \int_{-1}^5 [f(x) - g(x)] dx \\ &= \int_{-1}^5 [x^2 + x + 7 - 2x^2 + 3x - 2] dx \\ &= \int_{-1}^5 [-x^2 + 4x + 5] dx \\ &= \left[ -\frac{x^3}{3} + 2x^2 + 5x \right]_{-1}^5 \end{aligned}$$

**FORMULA**  
**Calculus: Area**



Area of overlap  $A = \int_a^b [f(x) - g(x)] dx$

$$\begin{aligned}
&= \left\{ -\frac{(5)^3}{3} + 2(5)^2 + 5(5) \right\} - \left\{ -\frac{(-1)^3}{3} + 2(-1)^2 + 5(-1) \right\} \\
&= \left\{ -\frac{125}{3} + 50 + 25 \right\} - \left\{ \frac{1}{3} + 2 - 5 \right\} \\
&= -\frac{125}{3} + 50 + 25 - \frac{1}{3} - 2 + 5 \\
&= -\frac{126}{3} + 78 \\
&= -42 + 78 = 36
\end{aligned}$$



### MARKING SCHEME NOTES

#### Question 5 (b) [Scale 15D (0, 5, 10, 14, 15)]

- 5:
- Area enclosed = Area under  $g$  – Area under  $f$
  - Shade the enclosed area
  - Indicate the limits on the graph
  - Any reasonable first step
- 10:
- Correct integration of  $f(x)$  or  $g(x)$
- 14:
- Correct integration of  $(f(x)$  or  $g(x))$  or  $(f(x)$  or  $g(x))$ , or both  $f(x)$  and  $g(x)$
  - Correct solution with minor arithmetic errors, and/or mishandles negative integral

## QUESTION 6 (25 MARKS)

### Question 6 (a)

$$\begin{aligned}
f(x) &= e^{-\frac{1}{2}x^2} \\
f'(x) &= e^{-\frac{1}{2}x^2} \times \left(-\frac{1}{2}(2x)\right) = -xe^{-\frac{1}{2}x^2} \\
f''(x) &= -x(-xe^{-\frac{1}{2}x^2}) + e^{-\frac{1}{2}x^2}(-1) \\
&= x^2e^{-\frac{1}{2}x^2} - e^{-\frac{1}{2}x^2} \leftarrow \text{Factorise} \\
&= e^{-\frac{1}{2}x^2}(x^2 - 1)
\end{aligned}$$

### FORMULAE AND TABLES BOOK Calculus: Derivatives [page 25]

$$y = e^x \Rightarrow \frac{dy}{dx} = e^x$$

$$y = e^{f(x)} \Rightarrow \frac{dy}{dx} = e^{f(x)} \times f'(x)$$

### MARKING SCHEME NOTES

#### Question 6 (a)

##### First derivative: [Scale 10B (0, 7, 10)]

- 7:
- Any reasonable first step

##### Second derivative: [Scale 10B (0, 7, 10)]

- 7:
- Any reasonable first step

**Question 6 (b)**

$$\frac{d^2y}{dx^2} = 0$$

POINT OF INFLECTION: Put  $\frac{d^2y}{dx^2} = 0$  and solve for  $x$ .

$$e^{-\frac{1}{2}x^2} (x^2 - 1) = 0$$

$$x^2 - 1 = 0 \leftarrow \text{Factorise the quadratic.}$$

$$(x+1)(x-1) = 0$$

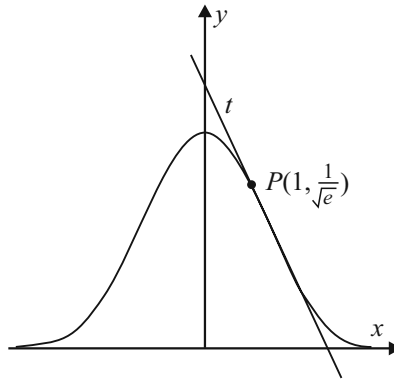
$\therefore x = -1, 1 \leftarrow$  The point  $P$  is in the first quadrant so ignore  $x = -1$ .

$$x = 1: y = e^{-\frac{1}{2}(1)^2} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \Rightarrow P\left(1, \frac{1}{\sqrt{e}}\right)$$

Finding the slope  $m$  of the tangent  $t$  at  $P$  by differentiating the function.

$$\frac{dy}{dx} = -xe^{-\frac{1}{2}x^2}$$

$$\left(\frac{dy}{dx}\right)_{x=1} = -(1)e^{-\frac{1}{2}(1)^2} = -\frac{1}{\sqrt{e}}$$



Equation of tangent  $t: P\left(1, \frac{1}{\sqrt{e}}\right), m = -\frac{1}{\sqrt{e}}$

$$y - \frac{1}{\sqrt{e}} = -\frac{1}{\sqrt{e}}(x - 1)$$

$$\sqrt{e}y - 1 = -1(x - 1)$$

$$\sqrt{e}y - 1 = -x + 1$$

$$x + \sqrt{e}y - 2 = 0$$

Cuts the  $x$ -axis: Put  $y = 0$

$$y = 0: x + \sqrt{e}(0) - 2 = 0$$

$$\therefore x = 2$$

$$\therefore (2, 0)$$

**FORMULAE AND TABLES BOOK**  
**Co-ordinate geometry: Line**

A Cartesian coordinate system with origin O. A straight line with a negative slope is drawn. Two points, P(x1, y1) and Q(x2, y2), are marked on the line. The y-intercept is labeled (0, c).

Equation of  $PQ$  [page 18]  
 $y - y_1 = m(x - x_1)$   
 $y = mx + c$

**MARKING SCHEME NOTES**

**Question 6 (b) [Scale 5C (0, 3, 4, 5)]**

- 3:**
- Any reasonable first step
  - $y - 0 = m(x - 2)$
  - States  $f''(x) = 0$  and stops
- 4:**
- Full solution with one or two errors
  - Correct tangent and stops

## QUESTION 7 (50 MARKS)

### Question 7 (a)

$$h = \left(10 - \frac{t}{200}\right)^2 \quad \leftarrow \text{Replace } t \text{ by } 10 \text{ in the formula.}$$

$$t = 0: h = \left(10 - \frac{0}{200}\right)^2 = (10)^2 = 100 \text{ m}$$

### Question 7 (b)

$$h = \left(10 - \frac{t}{200}\right)^2 \quad \leftarrow \text{Put } h \text{ equal to } 64 \text{ and solve for } t.$$

$$h = 64: 64 = \left(10 - \frac{t}{200}\right)^2$$

$$\pm 8 = 10 - \frac{t}{200}$$

$$\therefore \frac{t}{200} = 10 \mp 8$$

$$t = 200(10 \mp 8) = 400 \text{ s, } \cancel{3600} \text{ s} \quad \leftarrow \text{Reject the higher time as you are told the height of water formula applies to a certain 20-minute period.}$$

#### MARKING SCHEME NOTES

##### Question 7 (a) [Scale 15B\* (0, 11, 15)]

11: • Any reasonable first step, such as any attempt to find  $h(0)$

##### Question 7 (b) [Scale 15C\* (0, 8, 14, 15)]

- 8: • Any reasonable first step, such as setting  $h(t) = 64$   
• Attempts a systematic search (trial and improvement)  
• Attempts graphical solution
- 14: • Solution with one or two errors
- 15: • Correct answer by trial & improvement  
• Correct answer by graphical method

### Question 7 (c)

$$\frac{dV}{dt} = \frac{d}{dt}(\pi r^2 h) = \pi r^2 \frac{dh}{dt}$$

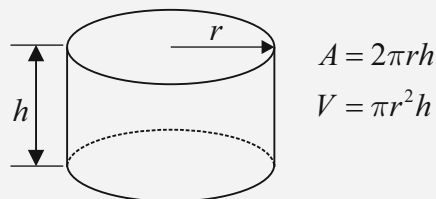
$$h = \left(10 - \frac{t}{200}\right)^2$$

$$\begin{aligned} \frac{dh}{dt} &= 2\left(10 - \frac{t}{200}\right)\left(-\frac{1}{200}\right) \\ &= -\frac{1}{100}\left(10 - \frac{t}{200}\right) \end{aligned}$$

#### FORMULAE AND TABLES BOOK

##### Surface area and volume:

Cylinder [page 10]



#### FORMULAE AND TABLES BOOK

##### Calculus: Chain rule [page 25]

$$f(x) = u(v(x)) \Rightarrow f'(x) = \frac{du}{dv} \times \frac{dv}{dx}$$

$$\begin{aligned}\frac{dV}{dt} &= \pi r^2 \frac{dh}{dt} = -\frac{\pi r^2}{100} \left(10 - \frac{t}{200}\right) \\ \left(\frac{dV}{dt}\right)_{t=400} &= -\frac{\pi(52)^2}{100} \left(10 - \frac{400}{200}\right) \\ &= -\frac{\pi(52)^2}{100} (10 - 2) \\ &= -\frac{\pi(52)^2}{100} (8) = \pi(52)^2 (0.08) \approx -680 \text{ cm}^3 \text{ s}^{-1}\end{aligned}$$

#### MARKING SCHEME NOTES

##### Question 7 (c) [Scale 5C\* (0, 3, 4, 5)]

3: • Assumes the rate of decrease is linear i.e. finds rate of decrease from a volume of  $270\,400\pi \text{ cm}^3$  to a volume of  $173\,056\pi \text{ cm}^3$

$$\bullet \frac{dv}{dt} = \frac{dh}{dt} \times \frac{dv}{dh}$$

• Any reasonable first step

4: • Finds  $\frac{dh}{dt}$  at  $t = 400$ , or comparable relevant work

• Solution by correct method, but with some errors

#### Question 7 (d)

$$\begin{aligned}-\frac{dV}{dt} &= v \times (\text{Area of hole}) = v \times (\pi(1)^2) \\ \therefore \frac{\pi(52)^2}{100} (8) &= \pi v \\ \therefore v &= \frac{(52)^2 (8)}{100} = 216.32 \text{ cm s}^{-1}\end{aligned}$$

#### MARKING SCHEME NOTES

##### Question 7 (d) [Scale 10B\* (0, 7, 10)]

7: • Any reasonable first step.

$$\bullet A = \pi r^2$$

NOTE: Ignore minus sign.

#### Question 7 (e)

$$\begin{aligned}-\frac{dV}{dt} &= v \times \pi \\ \therefore v &= \frac{-\frac{dV}{dt}}{\pi} = \frac{-\frac{\pi r^2}{100} \left(10 - \frac{t}{200}\right)}{\pi} = -\frac{r^2}{100} \left(10 - \frac{t}{200}\right) \\ \therefore v &= -\frac{52^2}{100} \sqrt{h} = 27.04 \sqrt{h}\end{aligned}$$

Use the formula given at the start of the problem to write in terms of  $h$ .

$$h = \left(10 - \frac{t}{200}\right)^2 \Rightarrow \sqrt{h} = \left(10 - \frac{t}{200}\right)$$

#### Question 7 (f)

$$\begin{aligned}v &= 27.04 \sqrt{h} \\ \therefore c \sqrt{1962h} &= 27.04 \sqrt{h} \\ c &= \frac{27.04 \sqrt{h}}{\sqrt{1962} \sqrt{h}} = \frac{27.04}{\sqrt{1962}} \approx 0.6\end{aligned}$$

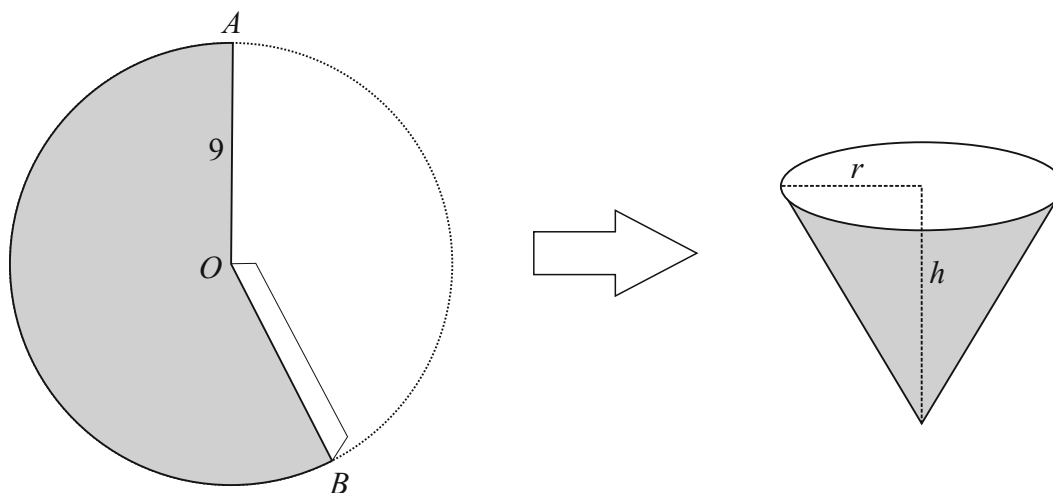
**MARKING SCHEME NOTES**

**Question 7 (e) & (f) [Scale 5C (0, 3, 4, 5)]**

- 3: • Any work of merit in either (e) or (f)  
 4: • Work of merit in both (e) and (f)  
 • One fully correct solution in either (e) or (f)

**QUESTION 8 (50 MARKS)**

**Question 8 (a)**



The radius in the circle will form the slant height  $l$  in the cup.

$$l^2 = h^2 + r^2$$

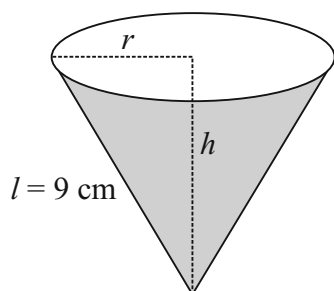
$$81 = h^2 + r^2$$

$$\therefore r^2 = 81 - h^2$$

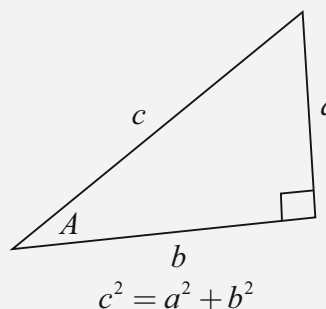
$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (81 - h^2) h$$

$$= \frac{\pi}{3} h (81 - h^2)$$



**FORMULAE AND TABLES BOOK**  
**Trigonometry** (page 16)  
 Pythagoras' theorem

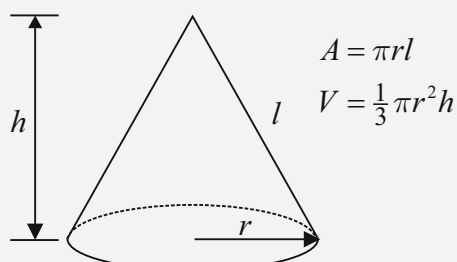


**MARKING SCHEME NOTES**

**Question 8 (a) [Scale 10B (0, 7, 10)]**

- 7: •  $r^2 = 81 - h^2$  and stops  
 • Any reasonable first step

**FORMULAE AND TABLES BOOK**  
**Surface area and volume:**  
**Cone** [page 10]



**Question 8 (b)**

$$\frac{\pi}{3}h(81-h^2) = \frac{154\pi}{3} \leftarrow \text{Put the volume } V \text{ equal to } \frac{154\pi}{3}$$

$$h(81-h^2) = 154$$

$$81h - h^3 - 154 = 0$$

$$h^3 - 81h + 154 = 0 \leftarrow \text{Solve this cubic equation using the factor theorem.}$$

**FACTOR THEOREM**

If  $k$  is a root of a polynomial equation  $P(x) = 0$ , then  $(x - k)$  is a factor of  $P(x)$  and vice versa.  
or

For a polynomial  $P(x)$ ,  $P(k) = 0 \Rightarrow P(x) = (x - k)Q(x)$ , where  $Q(x)$  is a polynomial of degree one less than  $P(x)$ .

$$h = 1: (1)^3 - 81(1) + 154 = 154 \neq 0$$

$$h = 2: (2)^3 - 81(2) + 154 = 8 - 162 + 154 = 0 \Rightarrow (h - 2) \text{ is a factor}$$

There are 2 methods to find the quadratic factor

**METHOD 1:** Lining up (A cubic is a quadratic multiplied by a linear factor)

$$h^3 - 81h + 154 = (h - 2)(h^2 + kh - 77)$$

$$h^3 + 0h^2 - 81h + 154 = h^3 + (k - 2)h^2 + (-2k - 77)h + 154$$

$$\therefore 0 = k - 2 \Rightarrow k = 2$$

$$\therefore h^3 - 81h + 154 = (h - 2)(h^2 + 2h - 77) = 0$$

Solve the quadratic equation:

$$h^2 + 2h - 77 = 0$$

$$a = 1, b = 2, c = -77$$

$$h = \frac{-2 \pm \sqrt{2^2 - 4(1)(-77)}}{2}$$

$$= \frac{-2 \pm \sqrt{312}}{2} = -1 \pm \sqrt{78}$$

$$\therefore h = -9.83, 7.83$$

ANSWERS:  $h = 2$  cm,  $7.83$  cm

**METHOD 2: DIVISION**

$$\begin{array}{r} h^2 + 2h - 77 \\ h - 2 \overline{) h^3 + 0h^2 - 81h + 154} \\ \underline{\mp h^3 \pm 2h^2} \phantom{+ 154} \\ 2h^2 - 81h + 154 \\ \underline{\mp 2h^2 \pm 4h} \phantom{+ 154} \\ -77h + 154 \\ \underline{\pm 77h \mp 154} \\ 0 \end{array}$$

**FORMULAE AND TABLES BOOK****Algebra: Roots of the quadratic equation**

$$ax^2 + bx + c = 0 \text{ [page 20]}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**MARKING SCHEME NOTES****Question 8 (b) [Scale 20D\* (0, 6, 12, 18, 20)]**

**6:** • Any reasonable first step

**12:** • Any step beyond  $\frac{154\pi}{3} = \frac{\pi}{3}h(81-h^2)$

**18:** • Substantial correct progress made (e.g. 1 root found + some further correct progress)  
• Solution by correct method, but with some errors

### Question 8 (c)

$$V = \frac{1}{3}\pi h(81 - h^2)$$

$$= 27\pi h - \frac{1}{3}\pi h^3$$

$$\frac{dV}{dh} = 27\pi - \pi h^2 = 0$$

$$\therefore 27 - h^2 = 0$$

$$27 = h^2$$

$$\therefore h = \sqrt{27} = 27^{\frac{1}{2}}$$

FIND TURNING POINTS  
(LOCAL MAXIMUM/MINIMUM)

Put  $\frac{dy}{dx} = 0$  and solve for  $x$

$$V = 27\pi h - \frac{1}{3}\pi h^3$$

$$V_{\text{Max.}} = 27\pi(27^{\frac{1}{2}}) - \frac{1}{3}\pi(27^{\frac{1}{2}})^3$$

$$= 27^{\frac{3}{2}}\pi - \frac{1}{3}(27^{\frac{3}{2}})\pi$$

$$= \frac{2}{3}(27^{\frac{3}{2}})\pi$$

$$\approx 294 \text{ cm}^3$$

#### MARKING SCHEME NOTES

##### Question 8 (c) [Scale 5C\* (0, 3, 4, 5)]

3: • Any reasonable first step

4: • Finds  $h$  correctly and stops

• Finds  $\frac{dV}{dh}$  correctly and sets  $\frac{dV}{dh} = 0$

• Error(s) in derivative, and finishes correctly or nearly correctly

### Question 8 (d)

	cups in part (b)		cup in part (c)
radius ( $r$ )	8.77 cm	4.43 cm	7.35 cm
height ( $h$ )	2 cm	7.83 cm	5.2 cm
capacity ( $V$ )	$\frac{154\pi}{3} \approx 161 \text{ cm}^3$	$\frac{154\pi}{3} \approx 161 \text{ cm}^3$	$294 \text{ cm}^3$

### Question 8 (e)

The diameters of the first and third cups are too large compared to their heights. The middle cup has the most reasonable shape ( $r = 4.43 \text{ cm}$ ,  $h = 7.83 \text{ cm}$ )

#### MARKING SCHEME NOTES

##### Question 8 (d) [Scale 5C\* (0, 3, 4, 5)]

3: • Any one correct entry

• Any correct relevant work

4: • Five or six correct entries (based on previous work)

##### Question 8 (e) [Scale 5B (0, 3, 5)]

0: • Incorrect answer, with or without explanation

3: • Correct answer, without explanation

• Cup with maximum volume

### Question 8 (f)

$$r = 4.43 \text{ cm}$$

$$h = 7.83 \text{ cm}$$

$$V = 161 \text{ cm}^3$$

The circumference of the rim of the cup is equal to the arc length of the sector that forms it.

$$\text{Circumference of the rim of the cup} = 2\pi(4.43) = 27.83 \text{ cm}$$

Arc length of circle:

$$s = r\theta$$

$$27.83 = 9\theta$$

$$\therefore \theta = \frac{27.83}{9} = 3.09 \text{ rads} = 3.09 \times \frac{180^\circ}{\pi} \approx 177^\circ$$

#### MARKING SCHEME NOTES

##### Question 8 (f) [Scale 5C\* (0, 3, 4, 5)]

- 3: • Reasonable work with any one of  $A = \pi r^2$ ,  $C = 2\pi r$ ,  $A = \pi r l$ ,  $A = \frac{1}{2} r^2 \theta$   
• Any reasonable first step
- 4: • Correct answer in radians

### QUESTION 9 (50 MARKS)

#### Question 9 (a)

(i)

altitude (km)	0	1	2	3	4	5
pressure (kPa)	101.3	89.4	79.0	69.7	61.6	54.4

Multiply each value by 0.883 to get the next value.

(ii) Result 1:  $\frac{(89.4 - 89.9)}{89.9} \times 100\% = -0.556\%$

Result 2:  $\frac{(79.0 - 79.5)}{79.5} \times 100\% = -0.628\%$

Result 3:  $\frac{(69.7 - 70.1)}{70.1} \times 100\% = -0.571\%$

Result 4:  $\frac{(61.6 - 61.6)}{61.6} \times 100\% = 0\%$

Result 5:  $\frac{(54.4 - 54.0)}{54.0} \times 100\% = 0.735\%$  [This is the maximum error.]

“Hannah’s model is accurate to within 1%”.

#### MARKING SCHEME NOTES

##### Question 9 (a) (i) [Scale 10C\* (0, 5, 8, 10)]

- 5: • One correct entry (i.e. one entry with correct ratio to previous entry)  
8: • Four correct entries

##### Question 9 (a) (ii) [Scale 5B (0, 3, 5)]

- 3: • Any correct relevant work  
• Answer outside the range [0.5%, 2%]

### Question 9 (b)

(i) Thomas:  $p = 101.3 \times e^{-0.1244(5)} = 54.385$

Hannah:  $101.3 \times 0.883^5 = 54.377$

Difference =  $54.385 - 54.377 = 0.008 < 0.01$

- (ii) Thomas may have put the 5 values for  $h$  and their corresponding  $p$  values into his formula and solved for  $c$ . He may then have averaged his  $c$  values.

$$p = 101.3e^{-ch} = \frac{101.3}{e^{ch}}$$

$$\therefore e^{ch} = \frac{101.3}{p} \leftarrow \text{Take the natural log of both sides.}$$

$$\ln e^{ch} = \ln \left( \frac{101.3}{p} \right)$$

$$ch \ln e = \ln \left( \frac{101.3}{p} \right) \leftarrow \text{NOTE: } \ln e = 1$$

$$ch = \ln \left( \frac{101.3}{p} \right) \Rightarrow c = \frac{1}{h} \ln \left( \frac{101.3}{p} \right)$$

#### MARKING SCHEME NOTES

##### Question 9 (b) (i) [Scale 5B (0, 3, 5)]

- 3: • Any correct relevant work

##### Question 9 (b) (ii) [Scale 5B (0, 3, 5)]

- 3: • Incomplete explanation, such as “solved an equation”, “used a graph”, “used two consecutive terms”  
• “He saw that 101.3 kPa was the pressure at sea level”

### Question 9 (c)

- (i) Discrete means that only a countable number of independent variables are taken.

$$h = 0, 1, 2, 3, 4, 5$$

Her model does not take account of  $h = 2.52$  km for example.

Continuous means that the independent variable  $h$  can take on all possible  $h \in \mathbb{R}^+$ .

- (ii) The continuous model can predict  $p$  for any  $h$  value, e.g.  $h = 7.62$  km. Discrete models cannot do this.

#### MARKING SCHEME NOTES

##### Question 9 (c) [Scale 5B (0, 3, 5)]

Mark both parts together, as there may be overlap between the answers:

the explanation in (i) may make the advantage clear, or the advantage in (ii) might clearly imply an understanding of the meaning

- 3: • Answer displays partial understanding of concepts and/or advantage. e.g.: “The first is for natural numbers; the second is for real numbers” (No indication of advantage)  
“You can count discrete things but not continuous things”  
• Concepts understood but reversed
- 5: • Answer displays good understanding of concepts and advantage

**Question 9 (d)**

$$h = 8848 \text{ m} = 8.848 \text{ km}$$

$$p = 101.3e^{-0.1244 \times 8.848} = 33.7 \text{ kPa}$$

**MARKING SCHEME NOTES**

**Question 9 (d) [Scale 10B\* (0, 7, 10)]**

- 7: • Any substitution into formula  
 • Any correct relevant work

**NOTE:** Accept answer rounded to nearest whole number, or to 1 or 2 decimal places  
 Otherwise, rounding penalty applies

**Question 9 (e)**

$$p_0 = 101.3 \text{ kPa}$$

$$p = \frac{p_0}{2}$$

$$101.3 \times e^{-0.1244h} = \frac{101.3}{2}$$

$$\frac{1}{e^{-0.1244h}} = \frac{1}{2}$$

$$e^{0.1244h} = 2$$

$$\ln e^{0.1244h} = \ln 2$$

$$0.1244h \ln e = \ln 2$$

$$0.1244h = \ln 2$$

$$\therefore h = \frac{\ln 2}{0.1244} = 5.57 \text{ km}$$

**MARKING SCHEME NOTES**

**Question 9 (e) [Scale 5C\* (0, 3, 4, 5)]**

- 3: • Any reasonable first step  
 4: • Solution with error(s), subject to including a correct relevant conversion from indices to logs or vice versa

**NOTE:** Accept answer rounded to 1, 2 or 3 decimal places. Otherwise, rounding penalty applies

**Question 9 (f)**

$$p = 100.3 \text{ kPa}$$

$$\therefore 100.3 = 101.3e^{-0.1244h}$$

$$\frac{100.3}{101.3} = \frac{1}{e^{0.1244h}}$$

$$e^{0.1244h} = \frac{101.3}{100.3}$$

$$0.1244h = \ln\left(\frac{101.3}{100.3}\right)$$

$$h = \frac{1}{0.1244} \ln\left(\frac{101.3}{100.3}\right) = 0.0797 \text{ km} = 79.7 \text{ m}$$

Height of a floor is say 3 m.

$$\text{Number of floors} = \frac{79.7 \text{ m}}{3 \text{ m}} \approx 27 \text{ floors}$$

**MARKING SCHEME NOTES**

**Question 9 (f) [Scale 5C\* (0, 3, 4, 5)]**

- 3: • Any reasonable first step
- 4: • Solution with error(s), subject to including a correct relevant conversion from indices to logs or vice versa
- Solves for  $h$  correctly, or nearly correctly, but then omits conversion to floors or uses estimate outside the interval  $[2, 5]$

## LC 2012 (SET G): PAPER 2

### QUESTION 1 (25 MARKS)

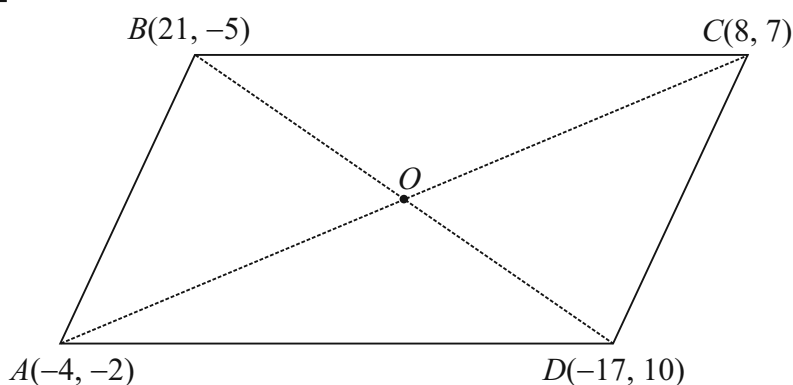
#### Question 1 (a)

**METHOD 1.** Check whether both pairs of opposite sides have the same slope (slope formula).

**METHOD 2.** Check whether both pairs of opposite sides are equal in length (distance formula).

**METHOD 3.** Check whether the midpoints of the diagonals coincide (diagonals bisecting each other).

#### Question 1 (b)



Use method 3.

$$\text{Mid-point of } [AC] = \left( \frac{-4+8}{2}, \frac{-2+7}{2} \right) = \left( 1, \frac{5}{2} \right)$$

$$\text{Mid-point of } [BD] = \left( \frac{21-17}{2}, \frac{-5+10}{2} \right) = \left( 2, \frac{5}{2} \right)$$

Therefore, the quadrilateral given is a parallelogram.

#### **MARKING SCHEME NOTES**

##### **Question 1 (a) [Scale 5B, 5B, 5B (0, 3, 5)]**

**3:** • Incomplete statement of method (with some merit)

##### **Question 1 (b) [Scale 10C (0, 4, 8, 10)]**

**4:** • Any reasonable first step

**8:** • Correct method applied with some error(s)  
• Correct method but vertices not taken in correct order (i.e ABDC or equivalent)  
• Correct method completed but with no conclusion

## QUESTION 2 (25 MARKS)

### Question 2 (a)

$$c_1 : x^2 + y^2 - 6x - 10y + 29 = 0$$

$$\text{Centre } P = (-g, -f) = (3, 5)$$

$$r_1 = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 25 - 29} = \sqrt{5}$$

$$c_2 : x^2 + y^2 - 2x - 2y - 43 = 0$$

$$\text{Centre } Q = (-g, -f) = (1, 1)$$

$$r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{1 + 1 + 43} = \sqrt{45} = 3\sqrt{5}$$

### FORMULAE AND TABLES BOOK Co-ordinate geometry: Circle [page 19]

Given centre  $(h, k)$  and radius  $r$

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre  $(-g, -f)$

Radius  $\sqrt{g^2 + f^2 - c}$

### MARKING SCHEME NOTES

#### Question 2 (a) [Scale 5B, 5B (0, 3, 5)]

- 3: • Centre or radius found  
• Incomplete statement of method (with some merit) e.g., completing square

### Question 2 (b)

#### TOUCHING CIRCLES

Circle 1: Centre  $P$ , radius  $r_1$

Circle 2: Centre  $Q$ , radius  $r_2$

External touch:  $|PQ| = r_1 + r_2$

Internal touch:  $|PQ| = r_1 - r_2$

Distance between the centres:

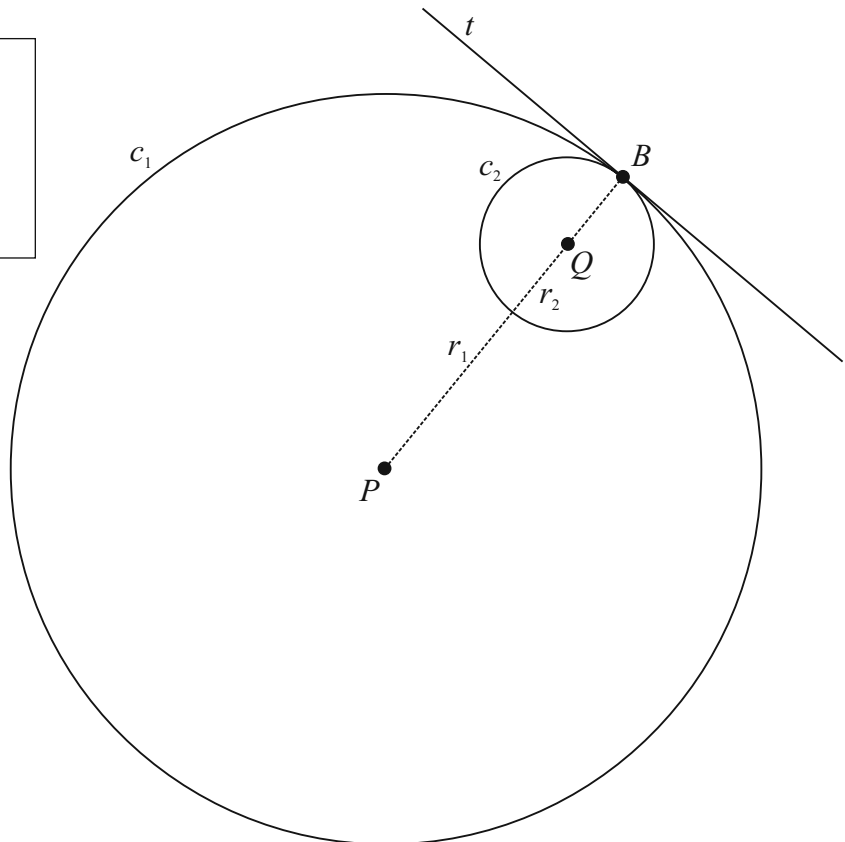
$$|PQ| = \sqrt{(3-1)^2 + (5-1)^2}$$

$$= \sqrt{4 + 16} = \sqrt{20}$$

$$= 2\sqrt{5}$$

$$r_2 - r_1 = 3\sqrt{5} - \sqrt{5} = 2\sqrt{5}$$

Therefore, the circles touch internally.



### MARKING SCHEME NOTES

#### Question 2 (b) [Scale 5C (0, 3, 4, 5)]

- 3: • Formula for distance between two centres with some substitution  
• Difference between radii found or implied
- 4: • No conclusion or incorrect conclusion

**Question 2 (c)**

Call this point  $B$ .

$$B(4, 7) \in c_1 ?$$

$$4^2 + 7^2 - 6(4) - 10(7) + 29 = 16 + 49 - 24 - 70 + 29 = 0$$

$$B(4, 7) \in c_2 ?$$

$$4^2 + 7^2 - 2(4) - 2(7) - 43 = 16 + 49 - 8 - 14 - 43 = 0$$

Therefore,  $B(4, 7)$  is on each circle.

**MARKING SCHEME NOTES**

**Question 2 (c) [Scale 5B (0, 3, 5)]**

- 3:
- $(4, 7)$  substituted into one circle
  - Subtracts both equations and stops
  - $(4, 7)$  substituted into both circles with an error
  - Finds common tangent and shows that  $(4, 7)$  is on it, without showing that it is also on one of the circles

**Question 2 (d)**

$$P(3, 5), B(4, 7)$$

$$\text{Slope of } PB = \frac{7-5}{4-3} = 2 \leftarrow \text{Line } PB \text{ is perpendicular to the tangent } t.$$

$$\text{Slope of } t = -\frac{1}{2}$$

$$\text{Equation of } t: m = -\frac{1}{2}, (x_1, y_1) = (4, 7)$$

$$y - 7 = -\frac{1}{2}(x - 4)$$

$$2y - 14 = -x + 4$$

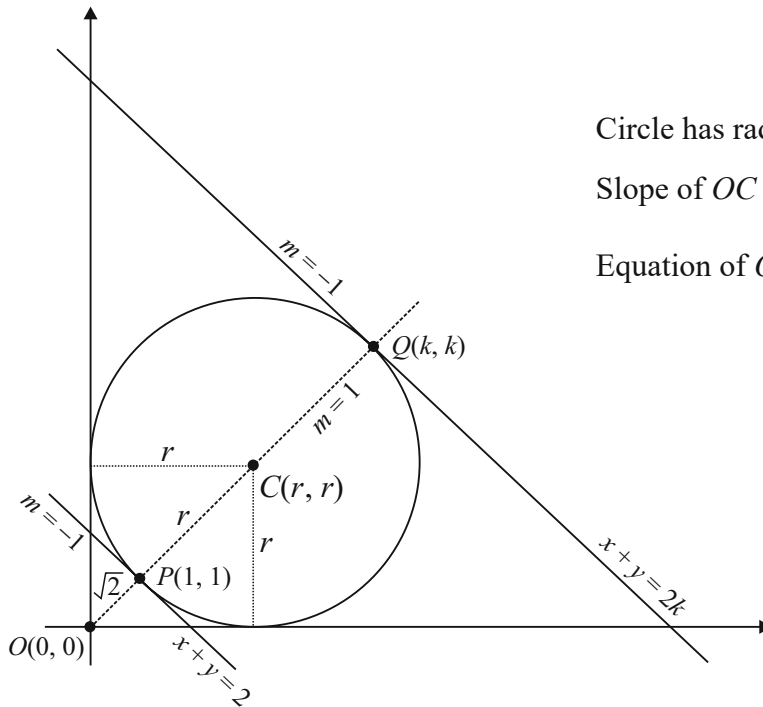
$$x + 2y = 18 = 0$$

**MARKING SCHEME NOTES**

**Question 2 (d) [Scale 5B (0, 3, 5)]**

- 3:
- Some relevant slope found
  - $(4, 7)$  inserted into equation of line formula but without slope found
  - Equation of line but  $(4, 7)$  incorrectly inserted

**QUESTION 3 (25 MARKS)**



Circle has radius  $r$ . Its centre is  $(r, r)$ .

$$\text{Slope of } OC = \frac{r-0}{r-0} = 1$$

Equation of  $OC$ :  $m = 1, (x_1, y_1) = (0, 0)$

$$y - 0 = 1(x - 0)$$

$$\therefore x - y = 0$$

Find the points of intersection of the 2 tangents with the line  $OC$  giving  $P(1, 1)$  and  $Q(k, k)$ .

Use Pythagoras to find the length of  $[OP] \Rightarrow |OP| = \sqrt{2}$

Apply Pythagoras to the right-angled triangle with hypotenuse  $[OC]$ .

$$r^2 + r^2 = (r + \sqrt{2})^2$$

$$2r^2 = r^2 + 2\sqrt{2}r + 2$$

$$r^2 - 2\sqrt{2}r - 2 = 0$$

$$r^2 - 2\sqrt{2}r + 2 = 4$$

$$(r - \sqrt{2})^2 = 4$$

$$r - \sqrt{2} = \pm 2$$

$$r = 2 + \sqrt{2}, r > 0$$

$C$  is the mid-point of  $[PQ]$ .

$$\therefore \frac{k+1}{2} = r$$

$$k = 2r - 1 = 2(2 + \sqrt{2}) - 1$$

$$= 4 + 2\sqrt{2} - 1$$

$$= 3 + 2\sqrt{2}$$

**MARKING SCHEME NOTES**

**Question 3 [Scale 25D (0, 15, 20, 22, 25)]**

**15:** • Any reasonable first step, such as:

radius / diameter indicated to one of the points of contact

intercepts of either tangent on axes indicated

$(1, 1)$  and/or  $(k, k)$  on diagram with no further work of merit

$|g| = |f|$  or  $g = \pm f$

centre  $(-g, -g)$  or equivalent

• Perpendicular distance of centre to either tangent indicated

- 20: • Centre  $(r, r)$   
 • Equation connecting  $r$  and  $k$  (i.e. work towards  $2r - 1 = k$ )  
 • Writes equation of circle  $(x - r)^2 + (y - r)^2 = r^2$   
 • Equation  $x^2 + y^2 + 2gx + 2gy + g^2 = 0$  or equivalent  
 •  $y = x$  and further work of merit  
 • States  $g = f$  and  $g^2 = f^2 = c$   
 • Perpendicular distance  $(-g, -g)$  to tangent  
 • Substantive work at finding equation of a relevant angle-bisector, other than  $y = x$
- 22: •  $r = 2 + \sqrt{2}$  or equivalent but fails to finish

## QUESTION 4 (25 MARKS)

### Question 4 (a)

Trials are independent of each other.  
 Probability of success is the same each time.

### Question 4 (b)

- (i)  $P(\text{Success}) = 0.6, P(\text{Failure}) = 0.4$   
 $P(X = 4) = {}^6C_4(0.6)^4(0.4)^2 = 0.311$

BERNOULLI TRIALS  
 $p = P(\text{Success}), q = P(\text{Failure})$   
 $P(r \text{ successes}) = {}^nC_r p^r q^{n-r}$

- (ii) She has one successful free throw in the first four free throws, followed by a successful free shot on the fifth free throw.

$$P = [{}^4C_1(0.6)^1(0.4)^3] \times (0.6) = 0.092$$

### MARKING SCHEME NOTES

#### Question 4 (a) [Scale 5C (0, 3, 4, 5)]

- 3: • One or both 'given' assumptions stated or implied  
 4: • Either 'independence' or 'probability of success the same each time' stated.

#### Question 4 (b) (i) [Scale 10C\* (0, 4, 8, 10)]

- 4: • Any first step e.g. reference to 0.4 or equivalent  
 8: • Correct expression  
 • Answer with one error in components  
 NOTE: Rounding incomplete: 9 marks

#### Question 4 (b) (ii) [Scale 10C\* (0, 4, 8, 10)]

- 4: • Reference to 0.6 or equivalent for fifth shot  
 8: • Correct expression  
 • Answer with one error in components  
 NOTE: Rounding incomplete: 9 marks

### QUESTION 5 (25 MARKS)

#### Question 5 (a)

$$\mu = 20 \text{ mm}, \sigma = 0.1 \text{ mm}$$

$$x_1 = 20 + 0.25 = 20.25 \text{ mm}$$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{20.25 - 20}{0.1} = \frac{0.25}{0.1} = 2.5$$

$$x_2 = 20 - 0.25 = 19.75 \text{ mm}$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{19.75 - 20}{0.1} = \frac{-0.25}{0.1} = -2.5$$

$P(\text{Batteries being rejected})$

$$= P(z \leq -2.5) + P(z \geq 2.5)$$

$$= P(z \geq 2.5) + P(z \geq 2.5)$$

$$= 2P(z \geq 2.5)$$

$$= 2[1 - P(z \leq 2.5)]$$

$$= 2[1 - 0.9938]$$

$$= 0.0124$$

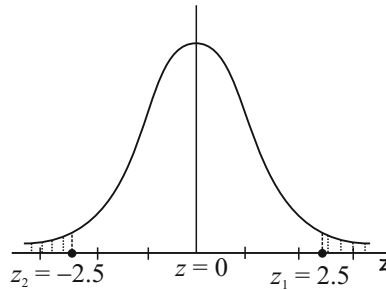
$$\text{Number rejected on average} = 10000 \times 0.0124 = 124$$

**FORMULAE AND TABLES BOOK**  
**Statistics and Probability: Probability distribution** (standardising formula) [page 34]

$$\bar{z} = \frac{\bar{x} - \mu}{\sigma}$$

$n$  = Number in the sample

$\sigma$  = standard deviation of the sample



#### MARKING SCHEME NOTES

##### Question 5 (a) [Scale 20D (0, 7, 10, 18, 20)]

- 7:**
- Any relevant step
  - Some relevant diagram
- 10:**
- Reference to 2.5
  - $P(Z > 2.5) = 0.0062$  and stops
- 18:**
- $P(|Z| > 2.5) = 0.0124$
  - Correct method with some error

#### Question 5 (b)

$$\mu = 20.05 \text{ mm}, \sigma = 0.1 \text{ mm}$$

$$x_1 = 20.25 \text{ mm}$$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{20.25 - 20.05}{0.1} = \frac{0.2}{0.1} = 2$$

$$x_2 = 19.75 \text{ mm}$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{19.75 - 20.05}{0.1} = \frac{-0.3}{0.1} = -3$$

$P(\text{Batteries being rejected})$

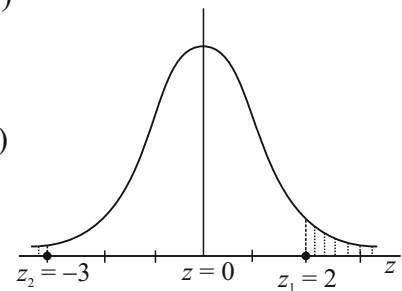
$$= P(z \leq -3) + P(z \geq 2)$$

$$= P(z \geq 3) + P(z \geq 2)$$

$$= 1 - P(z \leq 3) + 1 - P(z \leq 2)$$

$$= 2 - 0.9987 - 0.9772$$

$$= 0.0241$$



$$\% \text{ increase} = \left( \frac{0.0241 - 0.0124}{0.0124} \right) \times 100\% = 94.35\%$$

**MARKING SCHEME NOTES**

**Question 5 (b) [Scale 5C (0, 3, 4, 5)]**

- 3: • Any relevant step  
• Some relevant diagram  
• One case taken only
- 4: • Probability of both situations calculated but fails to complete fully

**QUESTION 6A (25 MARKS)**

**Question 6A (a) (i)**

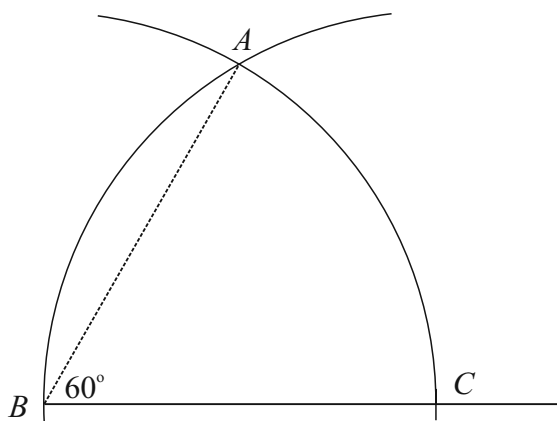
Put point of compass on  $B$  and stretch it to  $C$ .

Draw an arc as shown.

Now put the point on  $C$  and draw the arc as shown.

$A$  is where the 2 arcs meet. Triangle  $ABC$  is an equilateral triangle where each angle is  $60^\circ$ .

Draw the line  $BA$ . Angle  $ABC$  is a  $60^\circ$  angle.



**MARKING SCHEME NOTES**

**Question 6A (a) (i) [Scale 10C (0, 4, 8, 10)]**

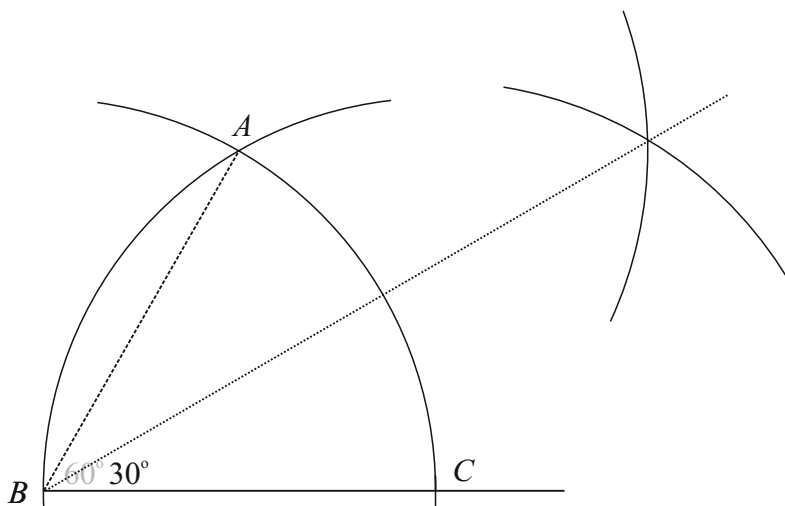
- 4: • Any correct step
- 8: • Correct method but outside the tolerance of  $2^\circ$

**Question 6A (a) (ii)**

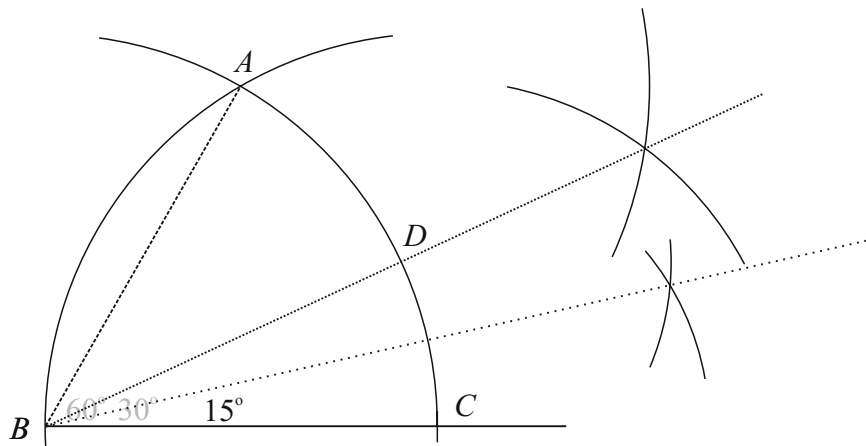
Bisect the  $60^\circ$  angle to get a  $30^\circ$  angle.

Put the point of the compass on  $A$  and draw the arc as shown.

Without changing the radius of the compass, put the point of the compass on  $B$  and draw the arc as shown. Draw a line from  $B$  through the intersection of these two arcs. The angle between this line and line  $BC$  is  $30^\circ$ .



Repeat the process to get a  $15^\circ$  angle.



**MARKING SCHEME NOTES**

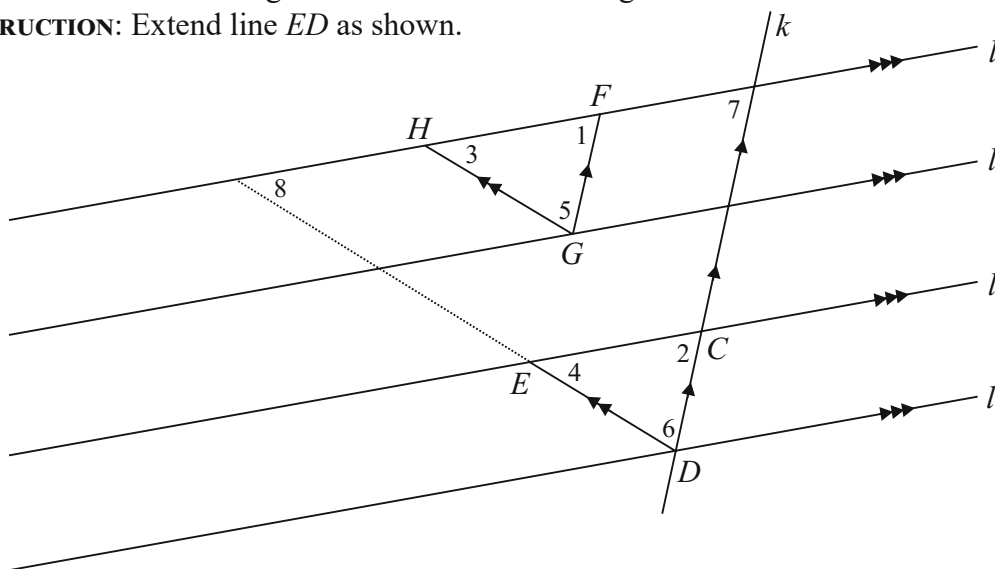
**Question 6A (a) (ii) [Scale 5C (0, 3, 4, 5)]**

- 3: • Any correct step
- 4: • Correct method but outside the tolerance of  $2^\circ$

**Question 6A (b)**

**REQUIRED TO PROVE:** Triangles  $CDE$  and  $FGH$  are congruent.

**CONSTRUCTION:** Extend line  $ED$  as shown.



**PROOF**

$\angle 1 = \angle 7 = \angle 2$  (Corresponding angles)

$\angle 3 = \angle 8 = \angle 4$  (Corresponding angles)

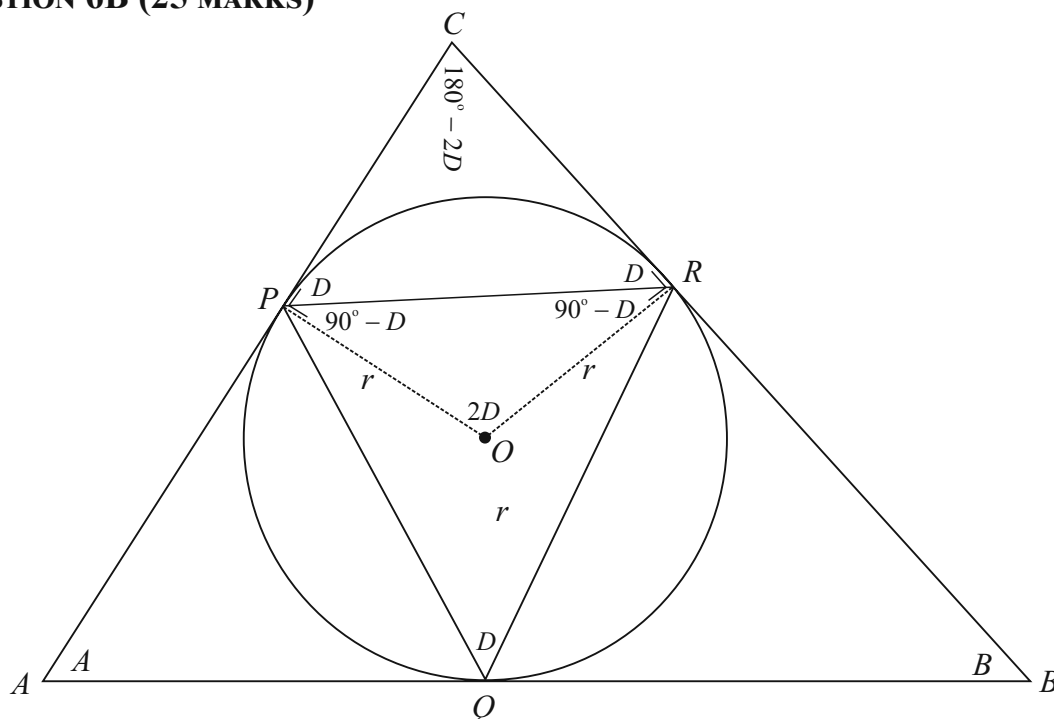
$\therefore \angle 5 = \angle 6$

$|FG| = |CD|$  and  $|HG| = |ED|$  ← If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

$\therefore \triangle CDE$  and  $\triangle FGH$  are congruent.

**MARKING SCHEME NOTES****Question 6A (b) [Scale 10C (0, 4, 8, 10)]**

- 4: • Any correct step e.g.  
 identifies two equal sides  
 identifies two equal angles  
 extends  $DE$  to intersect  $l_1$
- 8: • Proof with correct steps but without justification of steps  
 • One error in establishing congruence

**QUESTION 6B (25 MARKS)****REQUIRED TO PROVE:**

$$|\angle PQR| = \frac{1}{2}(|\angle CAB| + |\angle CBA|)$$

or

$$D = \frac{1}{2}(A + B)$$

**CONSTRUCTION:** Join  $O$  to  $R$  and  $O$  to  $P$ .  $OR$  and  $OP$  are perpendicular to the sides of the triangle. Join  $P$  to  $R$ . Triangle  $PRO$  is an isosceles triangle as  $|OP| = |OR| = r$  (radius).

**PROOF:**

$$|\angle POR| = 2D \text{ (The angle at the centre is twice the angle on the circle)}$$

$$|\angle OPR| = |\angle ORP| = (90^\circ - D) \text{ (Isosceles triangle)}$$

$$|\angle RPC| = |\angle CRP| = D \text{ (Right angle)}$$

$$|\angle PCR| = (180^\circ - 2D) \text{ (3 angles of a triangle add to } 180^\circ)$$

$$\therefore A + B + (180^\circ - 2D) = 180^\circ$$

$$A + B = 2D$$

$$\therefore D = \frac{1}{2}(A + B)$$

**MARKING SCHEME NOTES****Question 6B [Scale 25E (0, 5, 10, 15, 20, 25)]**

- 5:** • Any correct statement
- 10:** • Some substantive work towards proof e.g. at least one full step complete  
• Two distinct relevant statements
- 15:** • Substantive proof with two critical steps missing
- 20:** • Correct proof but critical step missing  
• Correct proof without justification of steps

**QUESTION 7 (75 MARKS)****Question 7 (a)**

- (i) They have gone up a lot – they were mostly between 1% and 5% in 2009, and mostly between 5% and 15% in 2011.
- (ii) They have gone up a lot too – they were mostly between 2·3% and 4·1% in 2009 and mostly between 4% and 6% in 2011.
- (iii) There appears to be a stronger relationship in 2011 than in 2009.

**MARKING SCHEME NOTES****Question 7 (a) (i) [Scale 5B (0, 3, 5)]**

- 3:** • Incomplete statement e.g. “it has changed”.

**Question 7 (a) (ii) [Scale 5B (0, 3, 5)]**

- 3:** • Incomplete or partly correct statement

**Question 7 (a) (iii) [Scale 10B (0, 5, 10)]**

- 5:** • Incomplete or partly correct statement, e.g.  
“They have changed”  
“Closer to being a line”  
Reference to positive

**Question 7 (b)**

You would need to know how many mortgage holders are represented by each point on the relevant diagram.

**MARKING SCHEME NOTES****Question 7 (b) [Scale 5B (0, 3, 5)]**

- 3:** • Some reference to the middle household  
**NOTE:** accept (for full credit) reference to needing information about mortgage holders who are not on standard variable rates.

**Question 7 (c)**

It is a question of whether higher arrears rates cause interest rates to go up, or whether higher interest rates cause arrears rates to go up, (assuming there is a causal relationship at all).

**MARKING SCHEME NOTES**

**Question 7 (b) [Scale 5C (0, 3, 4, 5)]**

- 3: • Both may be caused by something else  
 • General statement regarding correlation not implying causality – no context
- 4: • No reference to reverse situation, (e.g.: “It’s about whether high interest rates cause high arrears rates or not.”)  
 • Correct interpretation of concept, but not contextualised, (e.g. “It’s a question of which variable causes which.”)

**Question 7 (d)**

Number of properties = 475 136

Number of properties in negative equity = 145 414

Number of properties in negative equity that are also in arrears = 11 644

Number of properties neither in arrears nor negative equity = 317 355

	Arrears	Not in arrears	Total
Negative Equity	<b>11 644</b>		<b>145 414</b>
Not in Negative Equity		<b>317 355</b>	
Total			<b>475 136</b>

Number of properties in negative equity but not in arrears = 145 414 – 11 644 = 133 770

Number of properties **not** in negative equity = 475 136 – 145 414 = 329 722

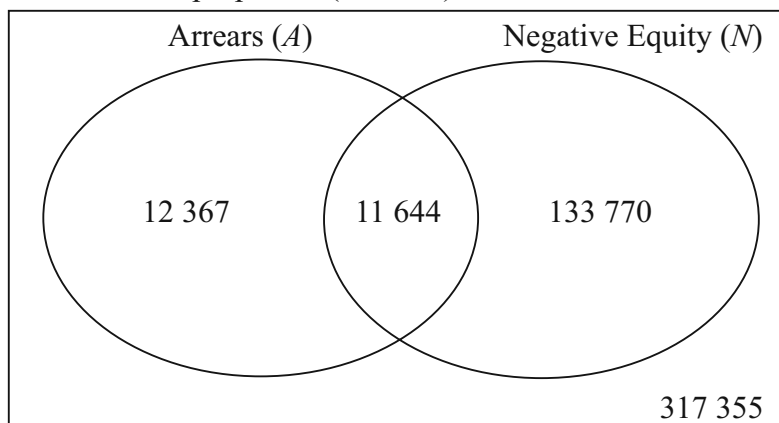
Number of properties that are in arrears but not in negative equity = 329 722 – 317 355 = 12 367

Number of properties in arrears = 11 644 + 12 367 = 24 011

Number of properties **not** in arrears = 133 770 + 317 355 = 451 125

	Arrears	Not in arrears	Total
Negative Equity	<b>11 644</b>	133 770	<b>145 414</b>
Not in Negative Equity	12 367	<b>317 355</b>	329 722
Total	24 011	451 125	<b>475 136</b>

S: Number of properties (475 136)



$$(i) \quad P(\text{Property in negative equity}) = \frac{\text{Number of properties in negative equity}}{\text{Number of properties}}$$

$$= \frac{145\,414}{475\,136} = 0.30605 \approx 0.31$$

$$(ii) \quad P(\text{Property in arrears from those in negative equity}) = \frac{11\,644}{145\,414} = 0.08$$

$$(iii) \quad P(\text{Property in negative equity from those in arrears}) = \frac{11\,644}{24\,011} = 0.48$$

#### MARKING SCHEME NOTES

##### Question 7 (d) (i) [Scale 10C\* (0, 4, 8, 10)]

4: • Uses a relevant number

• Writes  $\frac{\#E}{\#S}$  or equivalent.

• Identifies “number of outcomes of interest = ...” or “total number of outcomes = ...”.

8: • Answer in the form of a fraction

##### Question 7 (d) (ii) [Scale 5B\* (0, 3, 5)]

3: • Uses a relevant number

• Writes  $\frac{\#E}{\#S}$  or equivalent.

• Identifies “number of outcomes of interest = ...” or “total number of outcomes = ...”.

• Answer in form of a fraction

##### Question 7 (d) (iii) [Scale 10C\* (0, 4, 8, 10)]

4: • Attempt to combine (i) and (ii) for part (iii)

• Calculates total arrears and stops

8: • Answer as a fraction

•  $\frac{11\,644}{12\,367} = 0.9415 = 0.94$

#### Question 7 (e)

Null Hypothesis  $H_0$ :  $P(\text{Property is in negative equity}) = 0.31$

Alternative Hypothesis  $H_1$ : It has changed.  $P(\text{Property is in negative equity}) \neq 0.31$

At the 5% significance level the margin of error for the given sample =  $\frac{1}{\sqrt{2000}} \approx 0.0224$

Observed sample proportion =  $\frac{552}{2000} \approx 0.276$

Confidence interval =  $[0.276 - 0.0224, 0.276 + 0.0224] = [0.2536, 0.2984]$

There is evidence to support the conclusion that the situation has changed since December 2010 because, based on the sample data, any values in the range 25.36% – 29.84% are possible values for the proportion of properties in negative equity.

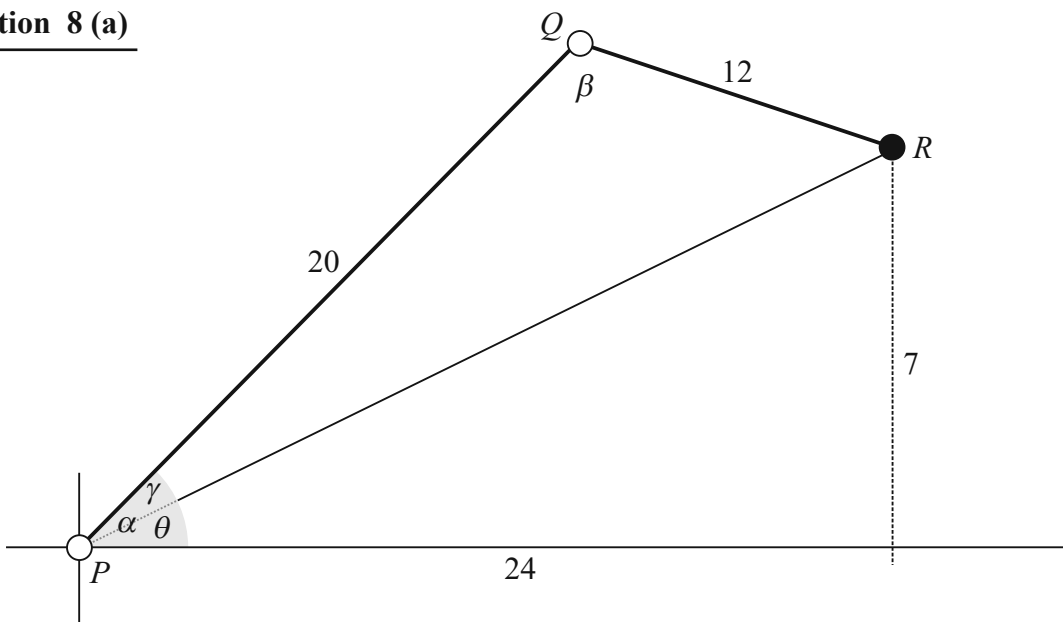
So reject null hypothesis and accept the alternative hypothesis that the proportion of properties in negative equity has changed.

**IMPORTANT NOTE**

This approach to finding the confidence interval is no longer used at Higher Level.

**MARKING SCHEME NOTES****Question 7 (e) [Scale 20D (0, 7, 10, 18, 20)]**

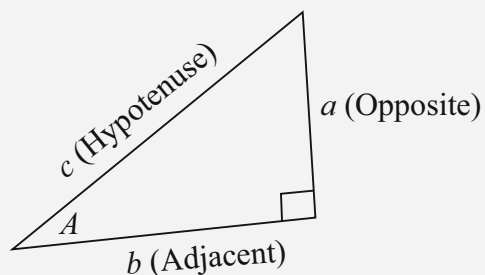
- 7: • One relevant step e.g. null hypothesis stated only  
 • Margin of error or observed proportion and does not continue
- 10: • Substantive work with one or more critical omissions  
 • Margin of error and observed proportion found but fails to continue
- 18: • Failure to state null hypothesis correctly and/or failure to contextualise answer (e.g., stops at “Reject the null hypothesis”).

**QUESTION 8 (75 MARKS)****Question 8 (a)**

$$|PR|^2 = 24^2 + 7^2 = 625$$

$$\therefore |PR| = \sqrt{625} = 25 \text{ cm}$$

$$\tan \theta = \frac{7}{24} \Rightarrow \theta = \tan^{-1}\left(\frac{7}{24}\right) = 16.26^\circ$$

**FORMULAE AND TABLES BOOK**  
**Trigonometry: Right-angled triangle [page 16]**


$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$$

$$c^2 = a^2 + b^2$$

$$25^2 = 12^2 + 20^2 - 2(12)(20) \cos \beta$$

$$\cos \beta = \frac{12^2 + 20^2 - 25^2}{2(12)(20)} = \frac{-81}{480}$$

$$\therefore \beta = \cos^{-1}\left(-\frac{81}{480}\right) = 99.7^\circ \approx 100^\circ$$

$$12^2 = 20^2 + 25^2 - 2(20)(25) \cos \gamma$$

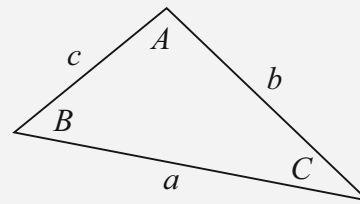
$$\cos \gamma = \frac{20^2 + 25^2 - 12^2}{2(20)(25)} = \frac{881}{1000} = 0.881$$

$$\therefore \gamma = \cos^{-1}(0.881) = 28.24^\circ$$

$$\alpha = \theta + \gamma = 16.26^\circ + 28.24^\circ = 44.4^\circ \approx 44^\circ$$

**FORMULAE AND TABLES BOOK**  
**Trigonometry of the triangle:**

[page 16]



Area:  $\frac{1}{2}ab \sin C$

Sine Rule:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule:  $a^2 = b^2 + c^2 - 2bc \cos A$

**MARKING SCHEME NOTES**

**Question 8 (a) [Scale 10B (0, 5, 10)] |PR|**

**5:** • Some use of Pythagoras

**Question 8 (a) [Scale 20C\* (0, 7, 18, 20)]  $\beta$**

**7:** • Cosine Rule with some substitution

**18:** • Cos  $\beta$  calculated

**Question 8 (a) [Scale 25D\* (0, 15, 20, 22, 25)]  $\alpha$**

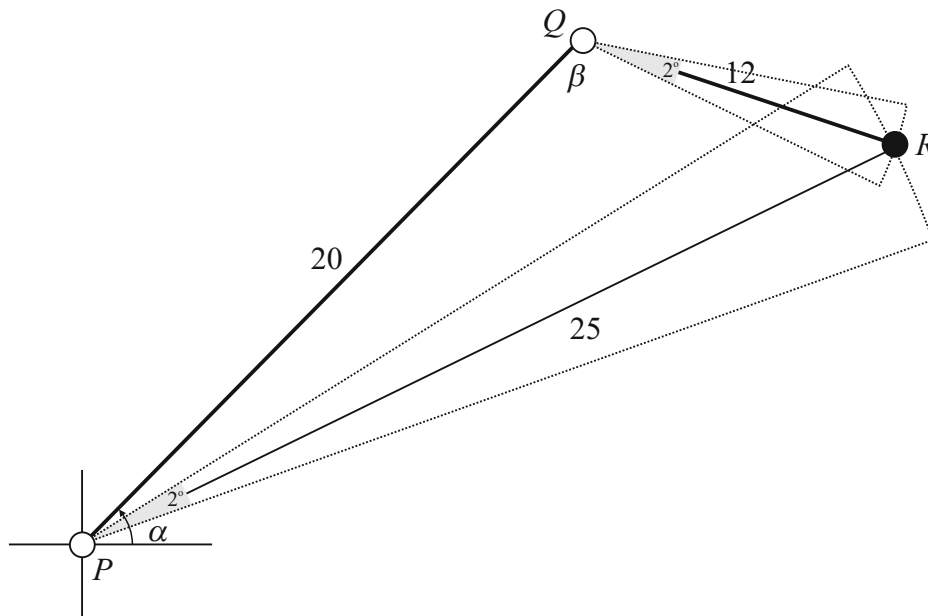
**15:** • Some work towards solving required angle with sine or cosine rule

•  $\tan \gamma = \frac{7}{24}$  with no work towards  $\alpha - \gamma$

**20:** • Cos ( $\alpha - \gamma$ ) found

**22:** •  $\alpha - \gamma$  and  $\gamma$  calculated but  $\alpha$  not evaluated

**Question 8 (b)**

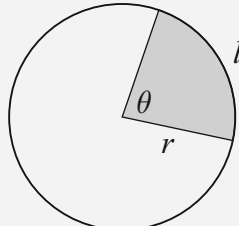


**NOTE 1:** An error of  $1^\circ$  in the angle means  $\pm 1^\circ = 2^\circ$ .

**NOTE 2:** The question states an ‘error in the location of  $R$ ’. This makes no sense. It is clearer to restate this as an ‘error in the distance moved by the point  $R$ ’.

Show the  $2^\circ$  movement in  $\alpha$  acting along the line  $PR$ . As  $\beta$  is fixed the movement of  $R$  is the same if you move arm  $PQ$  through  $2^\circ$  or arm  $PR$  through  $2^\circ$ .

**FORMULAE AND TABLES BOOK**  
**Length and area:**  
**Arc/sector [page 9]**



$l = r\theta$   
 $A = \frac{1}{2}r^2\theta$

*when  $\theta$  is in radians.*

Degrees to radians:  $\times \frac{\pi}{180^\circ}$   
 Radians to degrees:  $\times \frac{180^\circ}{\pi}$

Arc through which  $R$  moves for a  $2^\circ$  movement in alpha:  $s \approx 25 \times 2 \times \frac{\pi}{180} = 0.87 \text{ cm}$

Arc through which  $R$  moves for a  $2^\circ$  movement in beta:  $s \approx 12 \times 2 \times \frac{\pi}{180} = 0.42 \text{ cm}$

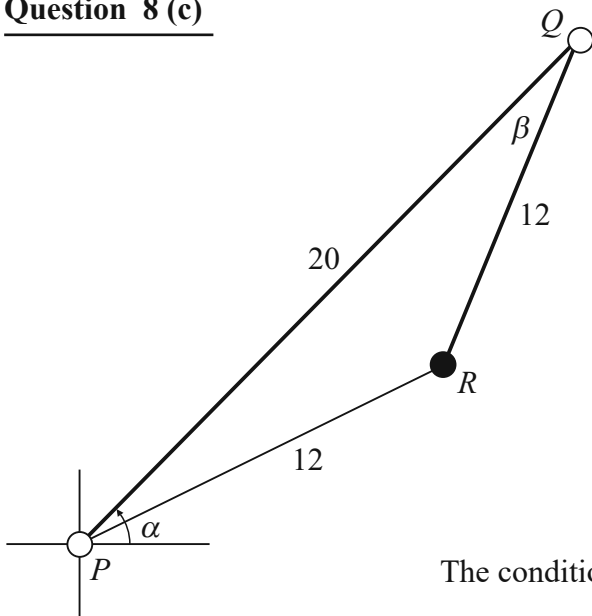
There is obviously a greater error in the distance moved by the point  $R$  for the longer arm. An error of  $1^\circ$  in  $\alpha$  causes a greater error in the distance moved by  $R$ .

**MARKING SCHEME NOTES**

**Question 8 (b) [Scale 5C (0, 3, 4, 5)]**

- 3: • Effort at working out values for angles  
 • Correct answer without justification
- 4: • Correct answer without being fully justified

**Question 8 (c)**



The distance moved by  $R$  is more sensitive to errors in  $\alpha$  when  $|PR| > 12$  and is more sensitive to errors in  $\beta$  when  $|PR| < 12$ . The position of the robotic arm shown above is the borderline case.

$$12^2 = 12^2 + 20^2 - 2(12)(20) \cos \beta$$

$$2(12)(20) \cos \beta = 20^2$$

$$24 \cos \beta = 20$$

$$\cos \beta = \frac{20}{24} = \frac{5}{6}$$

$$\therefore \beta = \cos^{-1}\left(\frac{5}{6}\right)$$

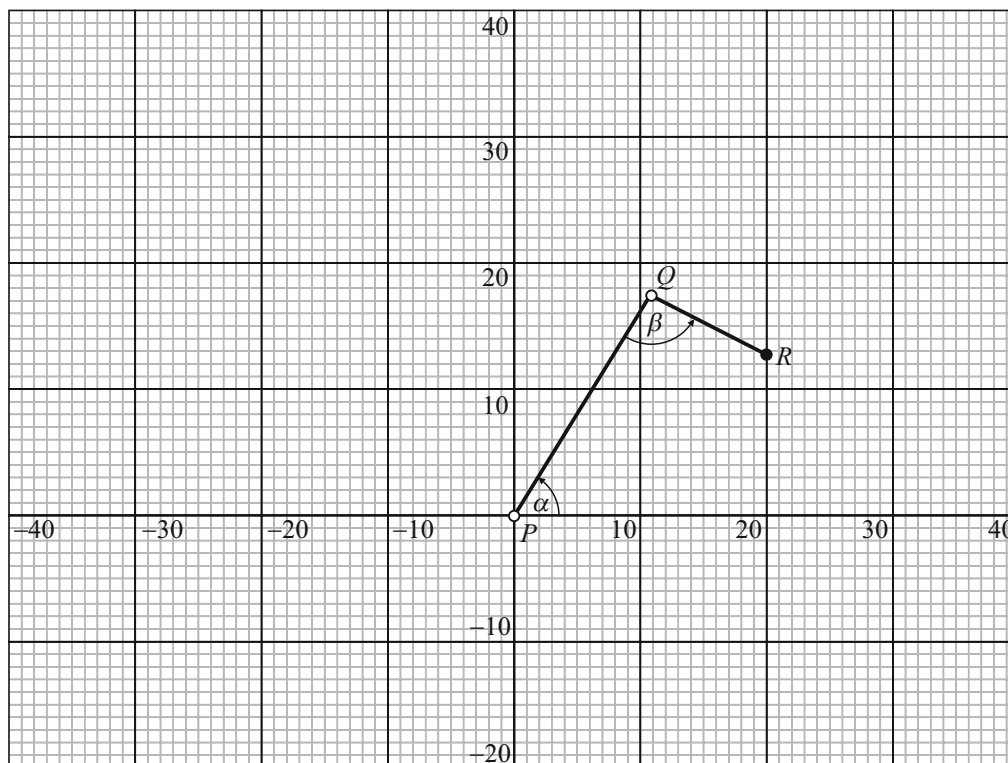
The condition  $|PR| > 12$  is true whenever  $\beta > \cos^{-1}\left(\frac{5}{6}\right) \approx 33.6^\circ$ .

**MARKING SCHEME NOTES**

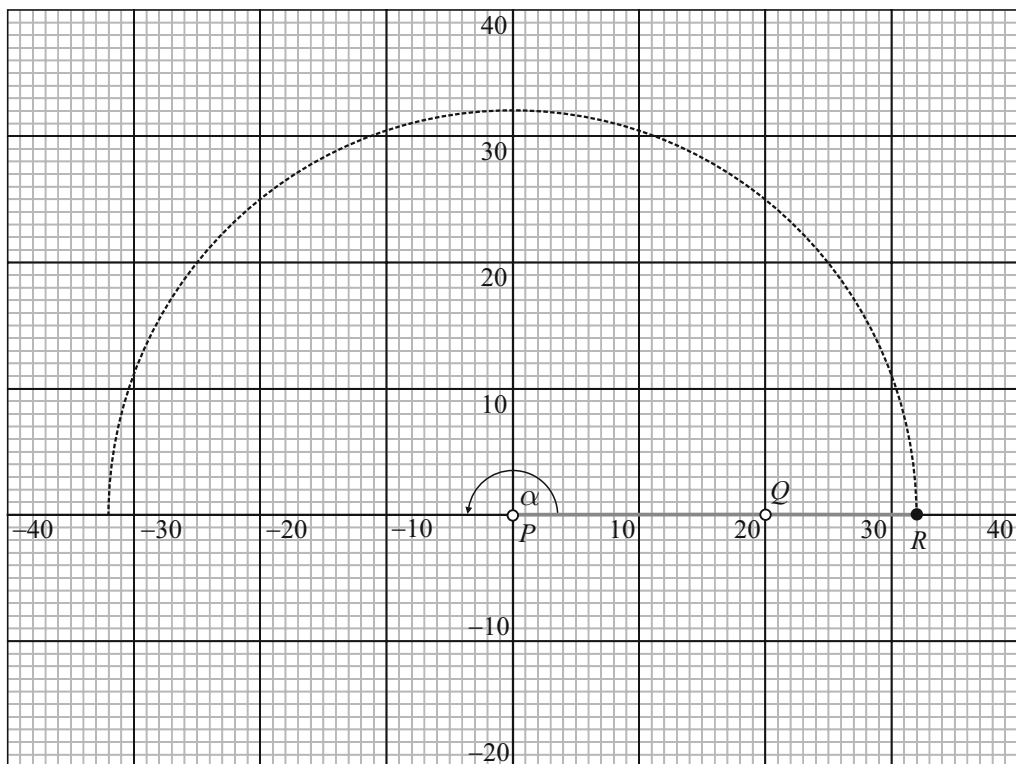
**Question 8 (c) [Scale 5C (0, 3, 4, 5)]**

- 3:
- Some reference to distance between  $P$  and  $R$
  - Treats as percentage error in angles, rather than absolute error in location. e.g. “If  $\alpha$  is smaller than  $\beta$ , then a  $1^\circ$  error in  $\alpha$  is a bigger percentage error than a  $1^\circ$  error in  $\beta$ .”
- 4:
- One situation only dealt with correctly
  - Clearly understands concept that the radius of the rotation is the determining factor, but makes error(s) in explanation (e.g. mixes up distances involved)

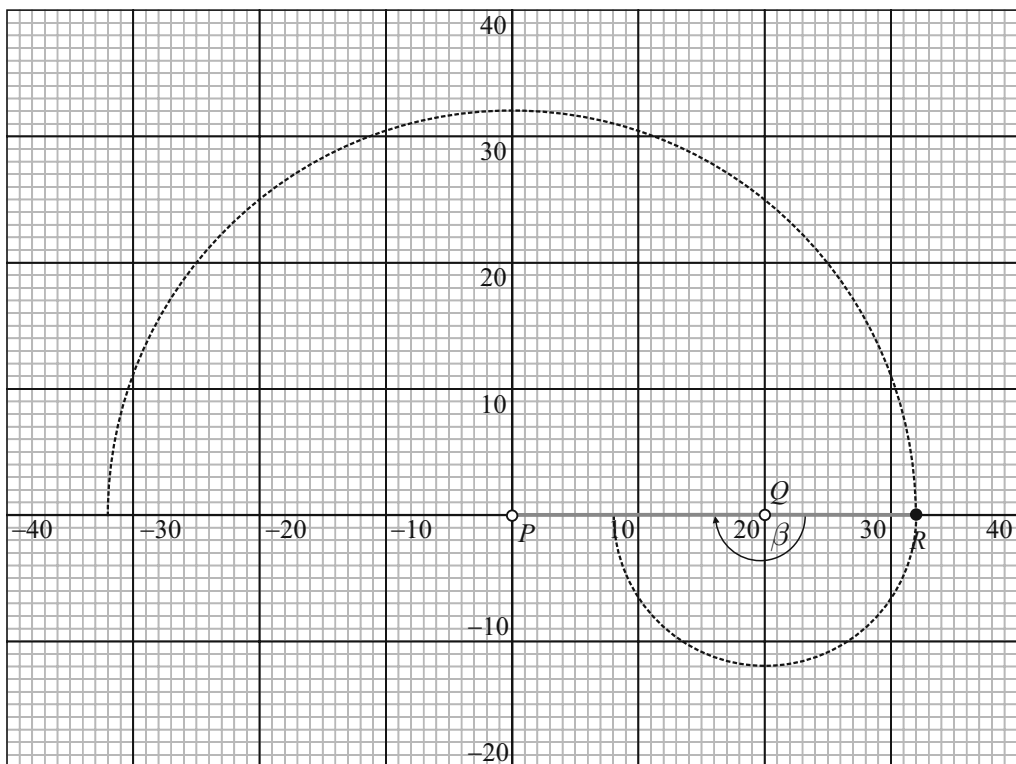
**Question 8 (d)**



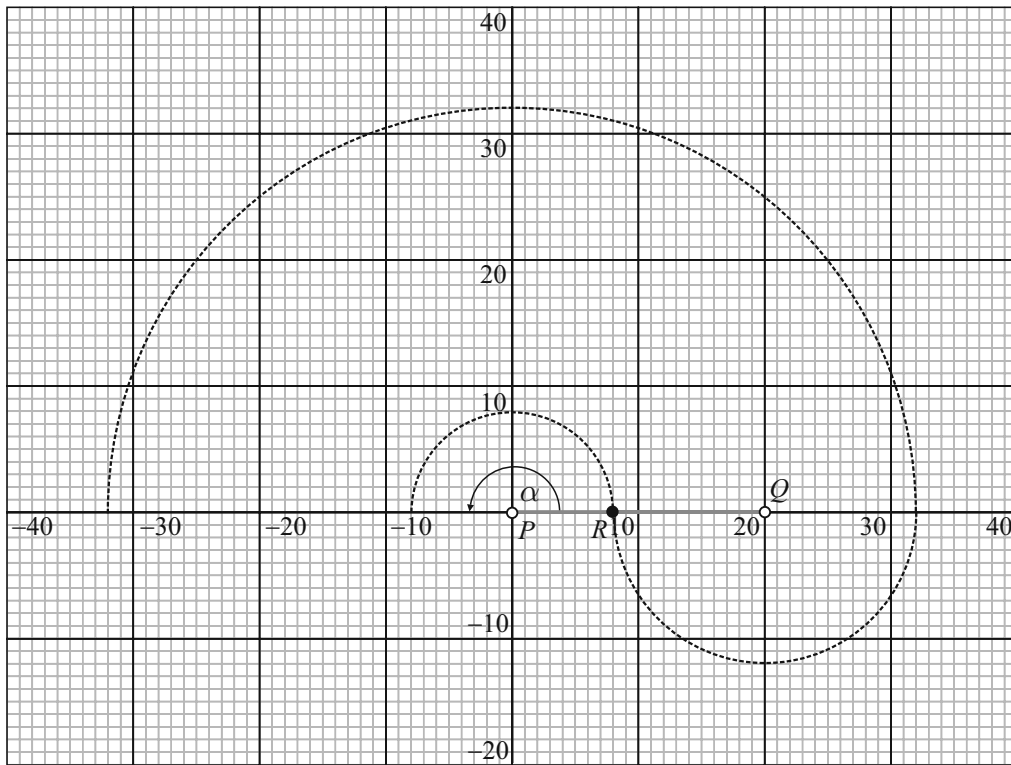
Stretch the arm  $PR$  to its full length of 32 cm. Rotate the arm about  $P$  from  $0^\circ$  to  $180^\circ$ .  $R$  traces the circle shown.



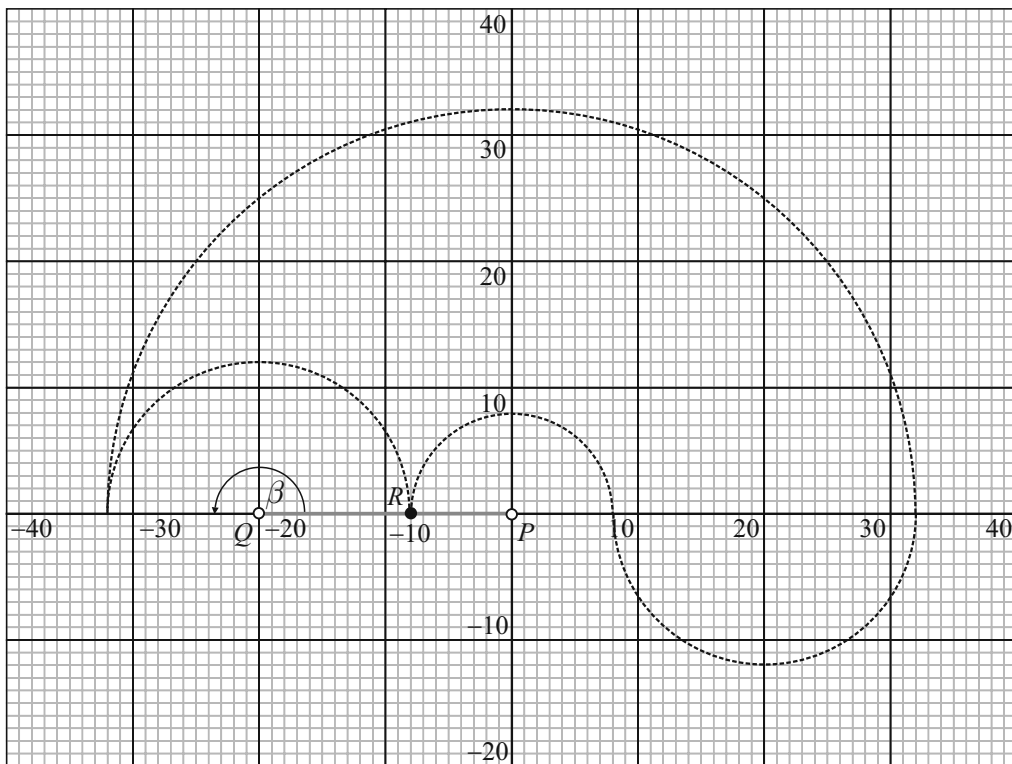
With the arm in the position shown rotate  $QR$  about  $Q$  from  $0^\circ$  to  $180^\circ$ .  $R$  traces the circle shown.



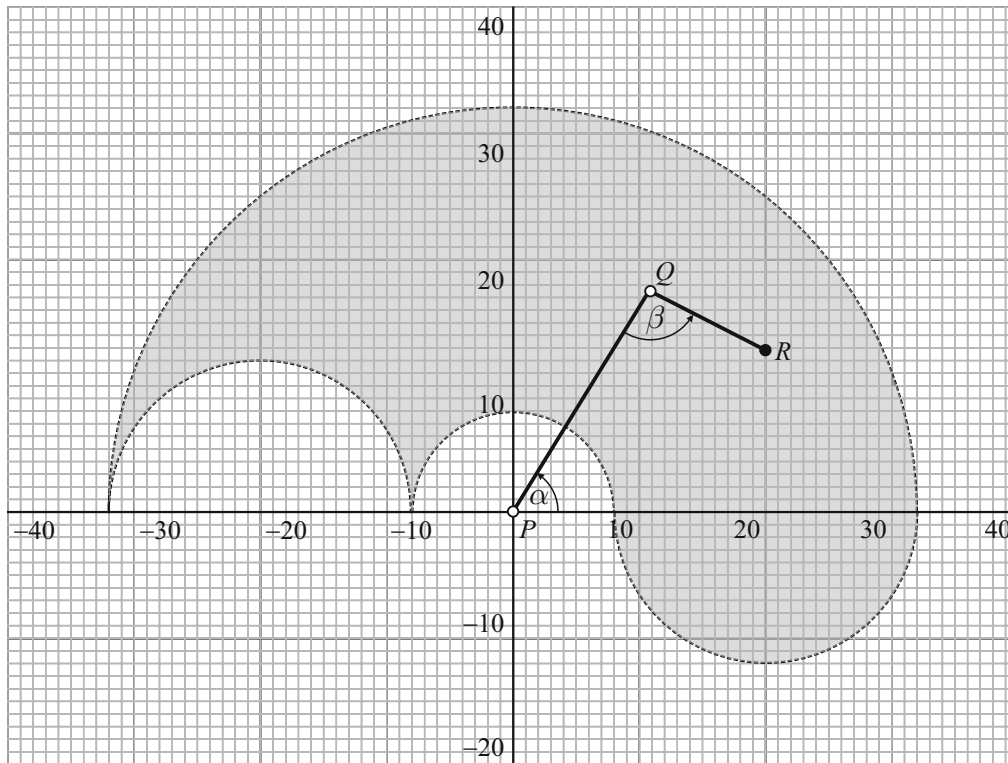
With the arm in the position shown rotate  $PQ$  about  $P$  from  $0^\circ$  to  $180^\circ$ .  $R$  traces the circle shown.



With the arm in the position shown rotate  $RQ$  about  $Q$  from  $0^\circ$  to  $180^\circ$ .  $R$  traces the circle shown.



The circles drawn were for the extreme positions of the arm. All points inside these curves can be reached by  $R$ .



**MARKING SCHEME NOTES**

**Question 8 (d) [Scale 10C (0, 4, 8, 10)]**

3: • Any relevant semi circle sketched or implied

4: • Any correct semi circle inserted in addition to semicircle centre  $P$  with radius 32.

# SAMPLE PAPER 2014 (SET H): PAPER 1

## QUESTION 1 (25 MARKS)

### Question 1 (a) (i)

$$w = -1 + \sqrt{3}i$$

$$r = |w| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

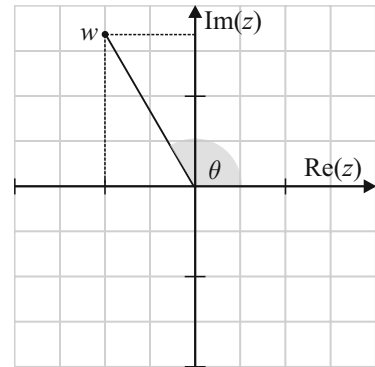
$$|\tan \theta| = \left| \frac{\sqrt{3}}{-1} \right| = \sqrt{3} = \tan \alpha, \text{ where } \alpha \text{ is the reference angle in the first quadrant.}$$

$$\therefore \alpha = \tan^{-1} \sqrt{3} = 60^\circ = \frac{\pi}{3}$$

$$\therefore \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$w = 2 \left[ \cos \left( \frac{2\pi}{3} + 2n\pi \right) + i \sin \left( \frac{2\pi}{3} + 2n\pi \right) \right] \text{ in general polar form.}$$

$$\therefore w = 2 \left[ \cos \left( \frac{2\pi + 6n\pi}{3} \right) + i \sin \left( \frac{2\pi + 6n\pi}{3} \right) \right]$$



### Question 1 (a) (ii)

$$z^2 = -1 + \sqrt{3}i \Rightarrow z = (-1 + \sqrt{3}i)^{\frac{1}{2}}$$

$$\begin{aligned} z &= 2^{\frac{1}{2}} \left[ \cos \left( \frac{2\pi + 6n\pi}{3} \right) + i \sin \left( \frac{2\pi + 6n\pi}{3} \right) \right]^{\frac{1}{2}} \\ &= \sqrt{2} \left[ \cos \left( \frac{2\pi + 6n\pi}{6} \right) + i \sin \left( \frac{2\pi + 6n\pi}{6} \right) \right] \end{aligned}$$

$$\begin{aligned} n = 0: z_1 &= \sqrt{2} \left[ \cos \left( \frac{\pi}{3} \right) + i \sin \left( \frac{\pi}{3} \right) \right] \leftarrow \text{There are 2 roots which you find by putting } n = 0 \text{ and then } n = 1. \\ &= \sqrt{2} \left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \end{aligned}$$

$$\begin{aligned} n = 1: z_2 &= \sqrt{2} \left[ \cos \left( \frac{8\pi}{6} \right) + i \sin \left( \frac{8\pi}{6} \right) \right] \\ &= \sqrt{2} \left[ \cos \left( \frac{4\pi}{3} \right) + i \sin \left( \frac{4\pi}{3} \right) \right] \\ &= \sqrt{2} \left( -\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \end{aligned}$$

#### FORMULAE AND TABLES BOOK:

Algebra (page 20)

#### DE MOIVRE'S THEOREM

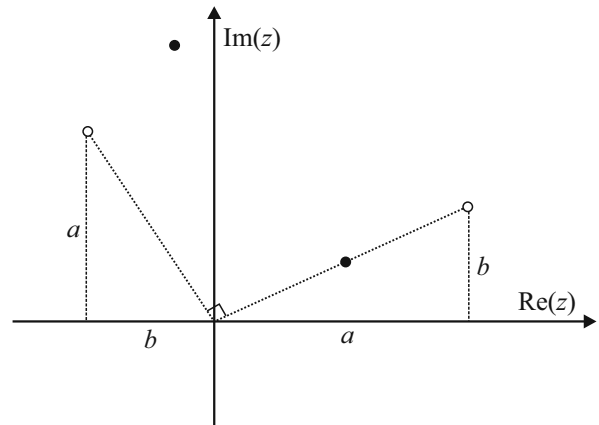
$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

**Question 1 (b)**

(i) Let  $z_1 = a + bi$ .

$$z_2 = iz_1 = i(a + bi) = ai + bi^2 = -b + ai$$

Notice that the real and imaginary parts are swapped between  $z_1$  and  $z_2$ . Can you pick out two points on the diagram where this might be the situation?



The two hollow points seem to satisfy this condition.

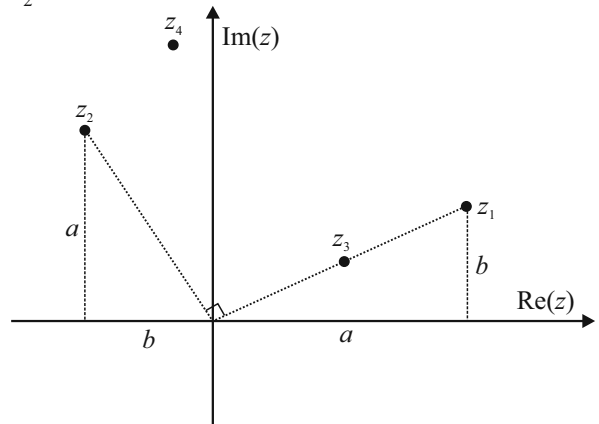
At this stage I don't know which point is  $z_1$  and  $z_2$ .

$$z_3 = kz_1 = k(a + bi) = ka + kbi$$

$z_3$  and  $z_1$  are in a straight line with the origin.

$$\begin{aligned} z_4 = z_2 + z_3 &= (-b + ai) + (ka + kbi) \\ &= (ka - b) + (a + kb)i \end{aligned}$$

You are now in a position to mark in all the points.



(ii)  $k \approx \frac{1}{2}$

**QUESTION 2 (25 MARKS)**

**Question 2 (a) (i)**

**STEPS TO PROOF BY INDUCTION**

1. Prove result is true for some starting value of  $n \in \mathbb{N}$ .
2. Assume result is true for  $n = k$ .
3. Prove result is true for  $n = (k + 1)$ .

REQUIRED TO PROVE:  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

**STEP 1:** Prove the result is true for  $n = 1$ .

$$1 = \frac{1(1+1)}{2}$$

$$1 = \frac{1(2)}{2}$$

$1 = 1$  [Therefore, true for  $n = 1$ .]

**STEP 2:** Assume it is true for  $n = k$ .

$$\underline{1 + 2 + 3 + \dots + k} = \frac{k(k+1)}{2}$$

**STEP 3:** Prove it is true for  $n = k + 1$ .

$$\text{Prove } \underline{(1 + 2 + 3 + \dots + k) + (k + 1)} = \frac{(k + 1)(k + 2)}{2}$$

Use the result in Step 2 to prove Step 3.

$$\begin{aligned} & (1 + 2 + 3 + \dots + k) + (k + 1) \\ &= \frac{k(k + 1)}{2} + (k + 1) \\ &= (k + 1) \left[ \frac{k}{2} + 1 \right] \\ &= (k + 1) \left[ \frac{k + 2}{2} \right] \\ &= \frac{(k + 1)(k + 2)}{2} \end{aligned}$$

Therefore, assuming true for  $n = k$  means it is true for  $n = k + 1$ . So true for  $n = 1$  and true for  $n = k$  means it is true for  $n = k + 1$ . This implies it is true for all  $n \in \mathbb{N}$ .

**Question 2 (a) (ii)**

$$S_n = \frac{n(n + 1)}{2}$$

$$S_{100} - S_{50} = \frac{100(101)}{2} - \frac{50(51)}{2} = 3775 \leftarrow \text{This is the sum of the numbers between 51 and 100.}$$

**QUESTION 2 (b)**

$$\begin{aligned} & \log_c \sqrt{x} + \log_c (cx) \\ &= \log_c x^{\frac{1}{2}} + \log_c (cx) \\ &= \frac{1}{2} \log_c x + \log_c c + \log_c x \\ &= \frac{3}{2} \log_c x + \log_c c \\ &= \frac{3}{2} p + 1 \end{aligned}$$

**FORMULAE AND TABLES BOOK**

**Indices and logs [page 21]**

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a \left( \frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a (x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a \left( \frac{1}{x} \right) = -\log_a x$$

### QUESTION 3 (25 MARKS)

#### Question 3 (a)

##### FACTOR THEOREM

If  $k$  is a root of a polynomial equation  $P(x) = 0$ , then  $(x - k)$  is a factor of  $P(x)$  and vice versa.  
or

For a polynomial  $P(x)$ ,  $P(k) = 0 \Rightarrow P(x) = (x - k)Q(x)$ , where  $Q(x)$  is a polynomial of degree one less than  $P(x)$ .

If  $f(-k) = 0$ , then  $-k$  is a root of  $f(x) = 0$ .

$$f(x) = x^3 + (1 - k^2)x + k$$

$$f(-k) = (-k)^3 + (1 - k^2)(-k) + k$$

$$= -k^3 - k + k^3 + k$$

$$= 0$$

#### Question 3 (b)

If  $-k$  is a root,  $(x + k)$  is a linear factor.

$$x^3 + (1 - k^2)x + k = (x + k)(x^2 + ax + 1) \leftarrow \text{A cubic is a linear multiplied by a quadratic.}$$

$$x^3 + 0x^2 + (1 - k^2)x + k = x^3 + (a + k)x^2 + (ak + 1)x + k$$

$$\therefore 0 = a + k \Rightarrow a = -k$$

$$\therefore x^3 + (1 - k^2)x + k = (x + k)(x^2 - kx + 1)$$

$x^2 - kx + 1 = 0$   $\leftarrow$  Solve the quadratic equation using the formula.

$$a = 1, b = -k, c = 1$$

$$x = \frac{-(-k) \pm \sqrt{(-k)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{k \pm \sqrt{k^2 - 4}}{2}$$

##### FORMULAE AND TABLES BOOK Algebra: Roots of the quadratic equation

$$ax^2 + bx + c = 0 \text{ [page 20]}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Question 3 (c)

To have exactly one real root, the quadratic equation will give complex roots. This means the expression inside the square root must be negative.

$$k^2 - 4 < 0$$

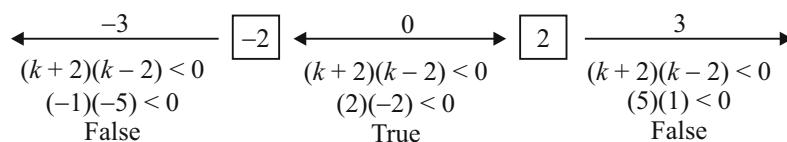
$$(k + 2)(k - 2) < 0$$

Solve the inequality above. First, solve the equality to locate the roots.

$$k^2 - 4 = 0$$

$$k^2 = 4$$

$$k = \pm 2$$



ANSWER:  $-2 < k < 2$

### QUESTION 4 (25 MARKS)

#### Question 4 (a)

$2x + 8y - 3z = -1 \dots (1)$  ← Eliminate the  $x$ 's by subtracting pairs of equations.

$$2x - 3y + 2z = 2 \dots (2)$$

$$2x + y + z = 5 \dots (3)$$

$(1) - (2) : 11y - 5z = -3 \dots (4) (\times 4)$  ← Eliminate the  $z$ 's.

$(1) - (3) : 7y - 4z = -6 \dots (5) (\times -5)$

$$\begin{array}{r} 44y - 20z = -12 \\ -35y + 20z = 30 \\ \hline 9y = 18 \Rightarrow y = 2 \end{array}$$

$11(2) - 5z = -3 \dots (4)$  ← Substitute the value of  $y$  into equation (4) to find  $z$ .

$$22 - 5z = -3$$

$$-5z = -25$$

$$\therefore z = 5$$

$2x + 8(2) - 3(5) = -1 \dots (1)$  ← Substitute the values of  $y$  and  $z$  into equation (1) to find  $x$ .

$$2x + 16 - 15 = -1$$

$$2x = -2$$

$$\therefore x = -1$$

**ANSWER:**  $(-1, 2, 5)$

#### QUESTION 4 (b) (i)

$f(x) = g(x)$  ← Find the points where the graphs of the 2 functions intersect.

$$|x - 3| = 2$$

$$x - 3 = \pm 2$$

$$\therefore x = 1, 5$$

Find where the  $f(x)$  cuts the axes.

Cuts  $x$ -axis: Put  $f(x) = 0$

$$f(x) = 0$$

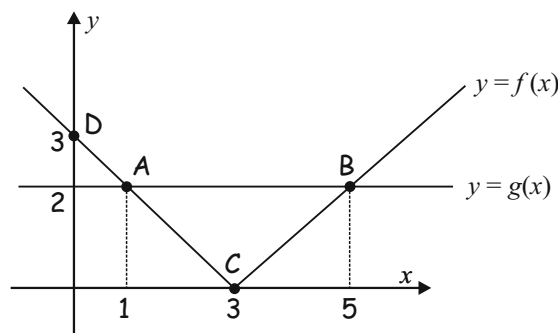
$$|x - 3| = 0$$

$$x - 3 = 0$$

$$x = 3$$

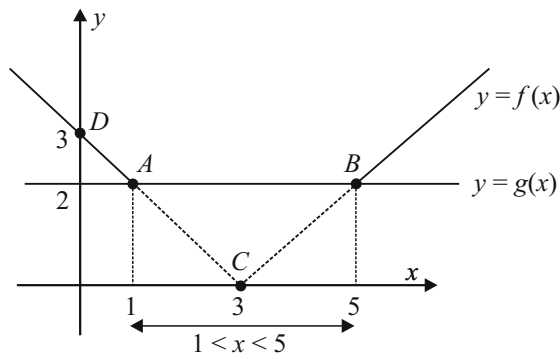
Cuts  $y$ -axis: Put  $x = 0$

$$f(0) = |0 - 3| = |-3| = 3$$



**ANSWERS:**  $A = (1, 2), B = (5, 2), C = (3, 0), D = (0, 3)$

**QUESTION 4 (b) (ii)**



You can see from the graph the values of  $x$  where  $f(x) < g(x)$ .

**ANSWER:**  $1 < x < 5$

**QUESTION 5 (25 MARKS)**

**QUESTION 5 (a)**

Sketch the function by finding its turning points and end points.

$$f(x) = x^3 - 5x^2 + 3x + 5$$

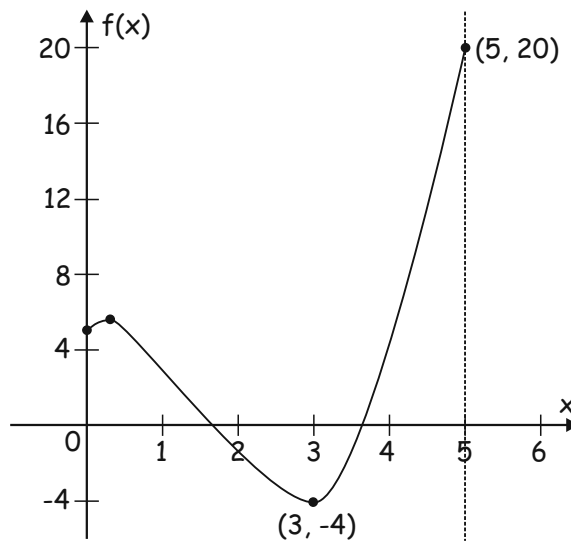
$$f'(x) = 3x^2 - 10x + 3$$

$$\text{Turning points: } f'(x) = 0$$

$$3x^2 - 10x + 3 = 0$$

$$(3x - 1)(x - 3) = 0$$

$$x = \frac{1}{3}, 3$$



$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 5\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 5 = 0 \Rightarrow \left(\frac{1}{3}, \frac{148}{27}\right) \text{ is a turning point.}$$

$$f(3) = (3)^3 - 5(3)^2 + 3(3) + 5 = 0 \Rightarrow (3, -4) \text{ is a turning point.}$$

$$x = 0: f(0) = (0)^3 - 5(0)^2 + 3(0) + 5 = 5 \Rightarrow (0, 5) \text{ is the starting point.}$$

$$x = 5: f(5) = (5)^3 - 5(5)^2 + 3(5) + 5 = 20 \Rightarrow (5, 20) \text{ is the finishing point.}$$

**ANSWER:** Maximum value of  $f = 20$

Minimum value of  $f = -4$

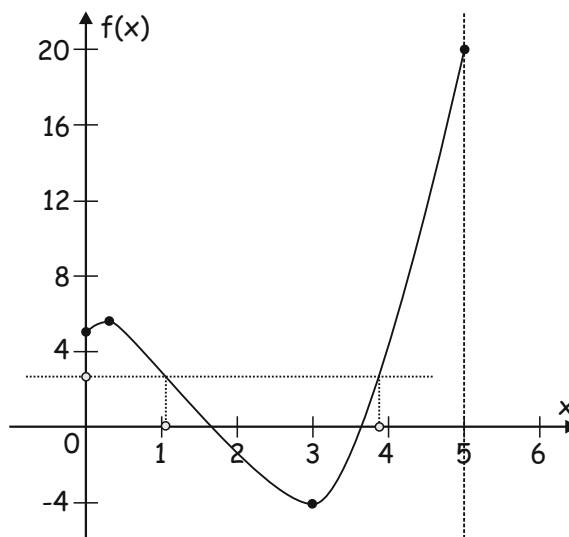
### QUESTION 5 (b)

The function  $f$  is **not** injective.

An injective function never maps distinct elements of its domain to the same element of its range. Each element in the domain must map on to a unique element in the range.

Look at the sketch of a diagram of the function  $f$ .

You can see from the sketch that the function is not injective as two points are shown on the  $x$ -axis which map on to the same point on the  $f(x)$  axis.



### QUESTION 6 (25 MARKS)

#### Question 6 (a)

- (i) 1.  $I_1 = \frac{1}{4}x^4 + x^3 + 3x + 1$   
2.  $I_2 = \frac{1}{4}x^4 + x^3 + 3x + 2$   
3.  $I_3 = \frac{1}{4}x^4 + x^3 + 3x + 3$

**FORMULAE AND TABLES BOOK**  
**Calculus: Integrals** [page 26]

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

(ii)  $\int f(x) dx = h(x)$

$h(x)$  is an indefinite integral which means that when you differentiate it with respect to  $x$  you get  $h'(x)$

or

$$h'(x) = f(x).$$

(iii)  $I = \frac{1}{4}x^4 + x^3 + 3x + c$

#### QUESTION 6 (b)

(i)  $h(x) = x \ln x$

$$h'(x) = x \times \frac{1}{x} + (\ln x) \times 1 = 1 + \ln x$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Product rule** [page 25]

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

(ii)  $h'(x) = 1 + \ln x \Rightarrow \ln x = h'(x) - 1$

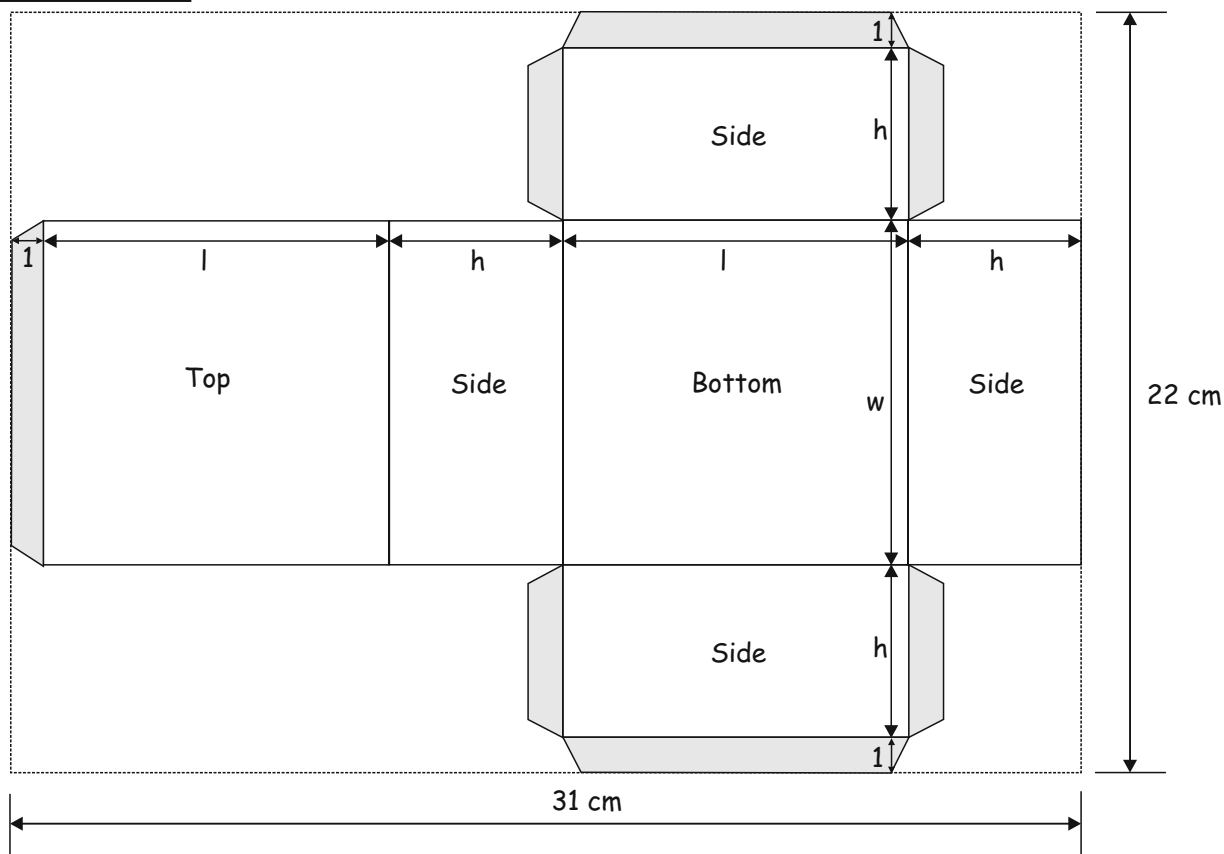
$$\int \ln x dx = \int (h'(x) - 1) dx = h(x) - x + c = x \ln x - x + c$$

**FORMULAE AND TABLES BOOK**  
**Calculus: Derivatives** [page 25]

$$y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

**QUESTION 7 (50 MARKS)**

**Question 7 (a)**



$$2h + 2l + 1 = 31$$

$$2l + 2h = 30$$

$$l + h = 15$$

$$l = (15 - h) \text{ cm}$$

$$2h + w + 2 = 22$$

$$2h + w = 20$$

$$w = (20 - 2h) \text{ cm}$$

**Question 7 (b)**

$$V = l \times b \times h = (15 - h)(20 - 2h)h$$

**Question 7 (c)**

Square bottom:  $l = w$

$$15 - h = 20 - 2h$$

$$2h - h = 20 - 15$$

$$\therefore h = 5 \text{ cm}$$

$$V = (15 - (5))(20 - 2(5))(5)$$

$$= (10)(10)(5)$$

$$= 500 \text{ cm}^3$$

**Question 7 (d)**

$(15 - h)(20 - 2h)h = 500$  ← Form a cubic equation by putting the volume of the box equal to 500.

$$(300 - 50h + 2h^2)h = 500$$

$$2h^3 - 50h^2 + 300h - 500 = 0$$

$$h^3 - 25h^2 + 150h - 250 = 0$$

$h = 5$  is a solution of this cubic. Therefore,  $(h - 5)$  is a linear factor. The other factor is a quadratic. Find the quadratic by lining up.

$$h^3 - 25h^2 + 150h - 250 = (h - 5)(h^2 + kh + 50)$$

$$h^3 - 25h^2 + 150h - 250 = h^3 + (k - 5)h^2 + (50 - 5k)h - 250$$

$$\therefore -25 = k - 5 \Rightarrow k = -20$$

$$h^3 - 25h^2 + 150h - 250 = (h - 5)(h^2 - 20h + 50) = 0$$

$h^2 - 20h + 50 = 0$  ← Solve the quadratic using the formula.

$$a = 1, b = -20, c = 50$$

$$h = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(50)}}{2(1)}$$

$$= \frac{20 \pm \sqrt{400 - 200}}{2}$$

$$= \frac{20 \pm \sqrt{200}}{2}$$

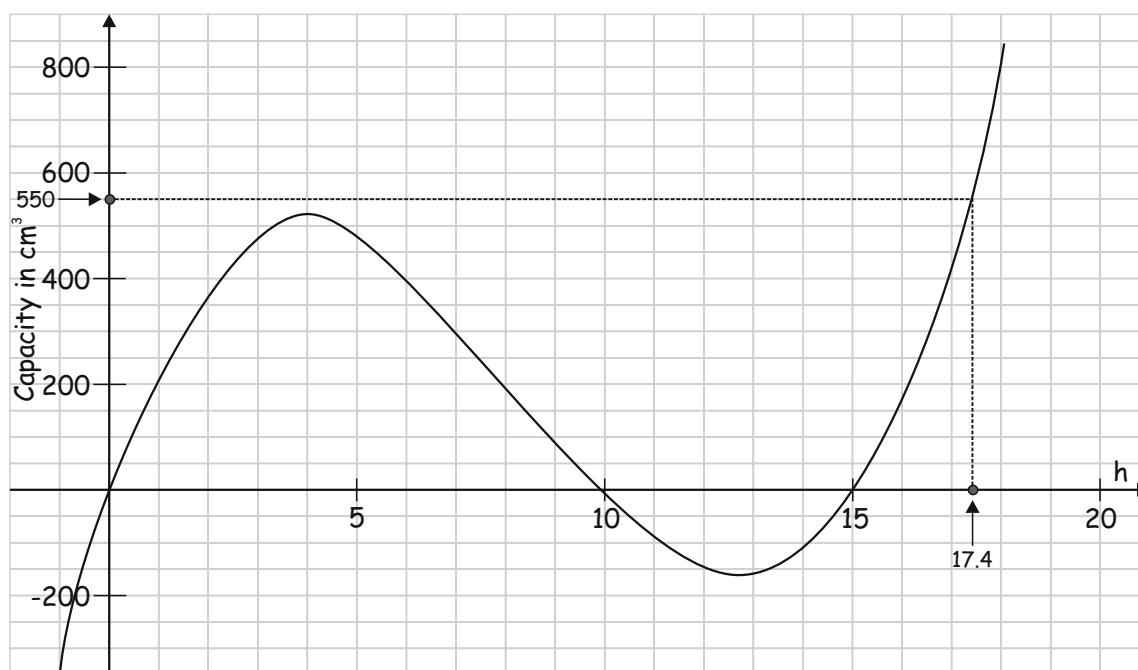
$$= \frac{20 \pm 10\sqrt{2}}{2}$$

$$= 10 \pm 5\sqrt{2}$$

$= 17.1, 2.9$  cm [Discard the solution of  $h = 17.1$  cm. The length  $l$  of the box is equal to  $(15 - h)$ . The length is not long enough to accommodate a value of  $17.1$  cm.]

**ANSWER:**  $h = 2.9$  cm

### Question 7 (e)



**EXPLANATION:**

10% extra volume:  $500 \times 1.1 = 550 \text{ cm}^3$   
Go to  $550 \text{ cm}^3$  on the  $y$ -axis and read off the  $h$  value.  
 $\therefore h \approx 17.4 \text{ cm}$

A height  $h = 17.4 \text{ cm}$  is too long to make a box from this piece of cardboard. The length  $l$  of the box is equal to  $(15 - h)$ . The length is not long enough to accommodate a value of  $17.4 \text{ cm}$ .

**QUESTION 8 (50 MARKS)**

**Question 8 (a)**

$P = ?$   
 $F = \text{€}20\,000$   
 $t = 1 \text{ year}$   
 $i = 0.03$   
$$P = \frac{F}{(1+i)^t} = \frac{20\,000}{(1+0.03)^1} = \text{€}19\,417.48$$

**Question 8 (b)**

$$P = \frac{20\,000}{(1.03)^t}$$

**Question 8 (c)**

I need to calculate the retirement fund that he has saved for and is available on the day of his retirement. €20 000 is drawn down immediately. The next €20 000 will not be drawn down for another year so its present value on the day of retirement is  $\frac{20\,000}{(1.03)^1}$ . The next €20 000 will not

be drawn from the retirement fund for 2 years so its present value is  $\frac{20\,000}{(1.03)^2}$ . And so on.

$$20\,000 + \frac{20\,000}{(1.03)^1} + \frac{20\,000}{(1.03)^2} + \dots + \frac{20\,000}{(1.03)^{24}}$$
$$= 20\,000 \left[ 1 + \frac{1}{1.03} + \frac{1}{1.03^2} + \dots + \frac{1}{1.03^{24}} \right]$$
$$a = 1, r = \frac{1}{1.03}, n = 25$$

$$S_n = 20\,000 \left[ \frac{1 - \left(\frac{1}{1.03}\right)^{25}}{1 - \frac{1}{1.03}} \right] = \text{€}358\,710.84$$

**FORMULAE AND TABLES BOOK**  
**Financial mathematics: Compound interest [page 30]**

$$F = P(1 + i)^t$$

$t$  = Time period (in years)  
 $i$  = (Annual) rate of interest expressed as a decimal  
 $F$  = Final value  
 $P$  = Principal  
**NOTE:** The time period can be months or weeks instead of years provided the interest rate is given for that time period.

**FORMULAE AND TABLES BOOK**  
**Sequences and series:**  
**Geometric series [page 22]**

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

### Question 8 (d)

(i)  $(1+i)^{12} = 1.03$

$$1+i = 1.03^{\frac{1}{12}} = 1.002466$$

$$\therefore i = 0.002466 = 0.2466\%$$

(ii)  $F = P(1.002466)^n$

His first payment  $P$  will be compounded 480 times at an interest rate of 0.2466%. His second payment  $P$  will be compounded 479 times at an interest rate of 0.2466%. And so on.

(iii)  $358\,710.84 = P(1.002466)^{480} + \dots + P(1.002466)^1$

$$= \frac{P(1.002466)(1 - (1.002466)^{480})}{1 - 1.002466}$$

$$\therefore P = \text{€}390.17$$

### Question 8 (e)

10 years less:  $30 \times 12 = 360$  months

$$358\,710.84 = \frac{P(1.002466)(1 - (1.002466)^{360})}{1 - 1.002466}$$

$$\therefore P = \text{€}618.35$$

---

### **QUESTION 9 (50 MARKS)**

#### Question 9 (a)

(i)  $f(x) = -0.5x^2 + 5x - 0.98$

$$f(0.2) = -0.5(0.2)^2 + 5(0.2) - 0.98 = 0$$

(ii)  $f(x) = -0.5x^2 + 5x - 0.98$

$$f'(x) = -x + 5 = 0 \Rightarrow x = 5$$

$$f(5) = -0.5(5)^2 + 5(5) - 0.98 = 11.52 \Rightarrow (5, 11.52) \text{ is a turning point.}$$

$$f''(x) = -1 < 0 \Rightarrow (5, 11.52) \text{ is a local maximum.}$$

#### Question 9 (b) (i)

$$f(x) = -0.5x^2 + 5x - 0.98$$

$$f(1) = -0.5(1)^2 + 5(1) - 0.98 = 3.52$$

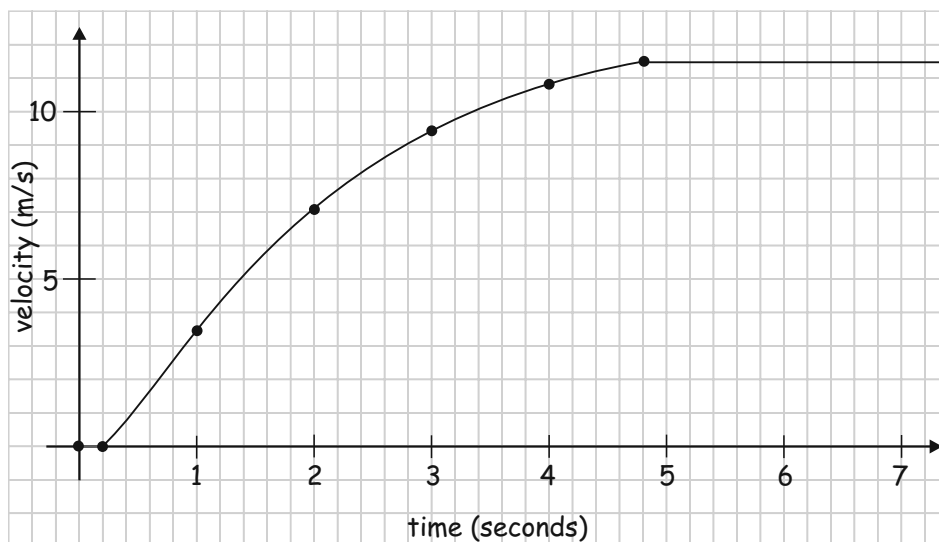
$$f(2) = -0.5(2)^2 + 5(2) - 0.98 = 7.02$$

$$f(3) = -0.5(3)^2 + 5(3) - 0.98 = 9.52$$

$$f(4) = -0.5(4)^2 + 5(4) - 0.98 = 11.02$$

$$f(5) = -0.5(5)^2 + 5(5) - 0.98 = 11.52$$

[Work out a number of points for the middle part of the graph and then plot it.]



### Question 9 (b) (ii)

In the first 0.2 s the sprinter does not move. He/she is still in the blocks. To find the distance travelled over the first 5 s find the area under the velocity curve from 0.2 s to 5 s.

$$\begin{aligned}
 s &= \int v \, dt \\
 &= \int (-0.5t^2 + 5t - 0.98) \, dt \\
 &= \frac{-0.5t^3}{3} + \frac{5t^2}{2} - 0.98t + c
 \end{aligned}$$

$$v = \frac{ds}{dt} \Rightarrow s = \int v \, dt$$

To find the distance  $s$  travelled replace  $t$  by 5 and then  $t$  by 0.2 and subtract the answers.

$$s = \left[ \frac{-0.5(5)^3}{3} + \frac{5(5)^2}{2} - 0.98(5) + c \right] - \left[ \frac{-0.5(0.2)^3}{3} + \frac{5(0.2)^2}{2} - 0.98(0.2) + c \right] = 36.9 \text{ m}$$

### Question 9 (b) (iii)

The sprinter runs 36.9 m over the first 5 seconds.

He/she runs the rest of the race (63.1 m) at the maximum speed of 11.52 m/s.

$$11.52 = \frac{63.1}{t} \Rightarrow t = \frac{63.1}{11.52} = 5.48 \text{ s}$$

$$v = \frac{s}{t}$$

**ANSWER:** Total time = 10.48 s

### Question 9 (c)

(i)  $\frac{dV}{dt} \propto -A \Rightarrow \frac{dV}{dt} = -kA$

$$\frac{d\left(\frac{4}{3}\pi r^3\right)}{dt} = -k(4\pi r^2)$$

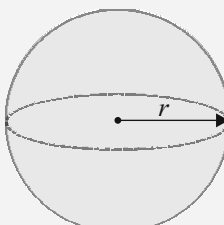
$$\frac{4}{3}\pi r^2 \frac{dr}{dt} = -k(4\pi r^2)$$

$$\frac{dr}{dt} = -k$$

**FORMULAE AND TABLES BOOK**

**Surface area and volume:**

**Sphere [page 8]**



$A = 4\pi r^2$   
 $V = \frac{4}{3}\pi r^3$

(ii)  $\frac{dr}{dt} = -k \Rightarrow \int dr = -k \int dt$

$$r = -kt + c$$

$$t = 0 : r = r_0 \Rightarrow c = r_0$$

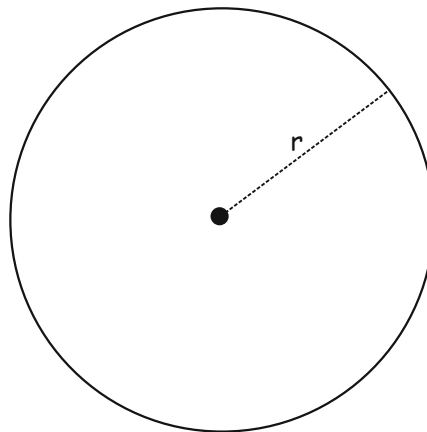
$$\therefore r = -kt + r_0$$

$$t = 1 : \frac{2}{3} \pi r_0^2 = \frac{4}{3} \pi r^3 \Rightarrow r = \frac{r_0}{2^{\frac{1}{3}}}$$

$$\therefore \frac{r_0}{2^{\frac{1}{3}}} = -k(1) + r_0 \Rightarrow k = \left(1 - \frac{1}{2^{\frac{1}{3}}}\right) r_0$$

$$t = T : r = 0 \Rightarrow 0 = -kT + r_0$$

$$\therefore T = \frac{r_0}{k} = \frac{r_0}{\left(1 - \frac{1}{2^{\frac{1}{3}}}\right) r_0} = 4.847 \text{ hours} \approx 291 \text{ minutes}$$



## SAMPLE PAPER 2014 (SET H): PAPER 2

### QUESTION 1 (25 MARKS)

#### Question 1 (a)

$$1 - 0.383 - 0.575 - 0.004 = 0.038$$

$x$	13	14	15	16
$P(X=x)$	0.383	0.575	<b>0.038</b>	0.004

$$E(X) = \sum xP(x) = 13 \times 0.383 + 14 \times 0.575 + 15 \times 0.038 + 16 \times 0.004 = 13.663$$

#### Question 1 (b)

$E(X)$  represents the mean value of the age of all the second year students on 1 January 2010.

#### Question 1 (c)

$$n = 10$$

$$r = 6$$

$$p(14) = 0.575$$

$$q(\text{Not } 14) = 0.425$$

BERNOULLI TRIALS

$p = P(\text{Success}), q = P(\text{Failure})$

$$P(r \text{ successes}) = {}^n C_r p^r q^{n-r}$$

$$P(6 \text{ out of } 10 \text{ are } 14 \text{ years of age}) = {}^{10} C_6 \times (0.575)^6 (0.425)^4 = 0.248$$

---

### QUESTION 2 (25 MARKS)

#### Question 2 (a)

**STRATIFIED SAMPLING:** A probability sampling technique where the entire population is divided into non-overlapping subgroups (strata) and the final subjects are randomly selected proportionally from the different strata.

An advantage of this method over simple random sampling is greater precision using samples of the same size.

**CLUSTER SAMPLING:** A probability sampling technique in which the sampler takes several steps in choosing the sample population. Firstly, the population is divided into clusters. A simple random sample of clusters is then selected from all of these clusters.

Finally, individuals are selected randomly from each cluster. An advantage over a simple random sample is that it is much cheaper.

#### Question 2 (b) (i)

$$\text{Margin of error} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1111}} = 0.03$$

**Question 2 (b) (ii)**Null hypothesis  $H_0: P = 0.23$ Alternative hypothesis  $H_1: P \neq 0.23$ Sample proportion  $P = \frac{234}{1111} = 0.2106$ Confidence interval =  $[0.2106 - 0.03, 0.2106 + 0.03] = [0.1806, 0.2406]$ 

There is evidence to support the party's claim that it has the support of 23% of the electorate because, based on the sample data, any values in the range 18% – 24% are possible values for the proportion of the electorate who support the party.

23% is in this confidence interval. Therefore, you cannot reject the null hypothesis.

**QUESTION 3 (25 MARKS)****Question 3 (a)**

To see if a point is on a line, substitute the point into the equation of the line and show it satisfies the equation.

Is  $P(4k - 2, 3k + 1) \in l_1: 3x - 4y + 10 = 0$ ?

$$\begin{aligned} & 3(4k - 2) - 4(3k + 1) + 10 \\ &= 12k - 6 - 12k - 4 + 10 \\ &= 0 \end{aligned}$$

**Question 3 (b)**

$l_1: 3x - 4y + 10 = 0 \Rightarrow m_1 = \frac{3}{4}$

$l_2: m_2 = -\frac{4}{3}$

Equation of  $l_2$ : Point  $P(4k - 2, 3k + 1)$ ,  $m_2 = -\frac{4}{3}$ 

$$\begin{aligned} y - (3k + 1) &= -\frac{4}{3}(x - (4k - 2)) \\ 3(y - 3k - 1) &= -4(x - 4k + 2) \\ 3y - 9k - 3 &= -4x + 16k - 8 \\ 4x + 3y - 25k + 5 &= 0 \end{aligned}$$

**Question 3 (c)**

$Q(3, 11) \in l_2: 4x + 3y - 25k + 5 = 0$

$\therefore 4(3) + 3(11) - 25k + 5 = 0$

$12 + 33 - 25k + 5 = 0$

$50 = 25k$

$\therefore k = 2$

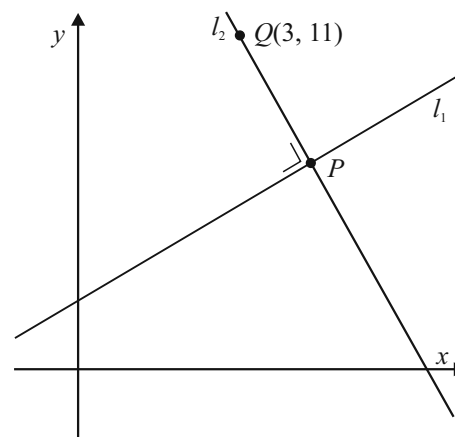
**Question 3 (d)** $P$  is the point at the foot of the perpendicular.

$P(4k - 2, 3k + 1) = P(4(2) - 2, 3(2) + 1) = P(6, 7)$

Finding the slope from the equation of a line:

$l: ax + by + c = 0$

$m = -\frac{a}{b} \Rightarrow m_{\perp} = \frac{b}{a}$

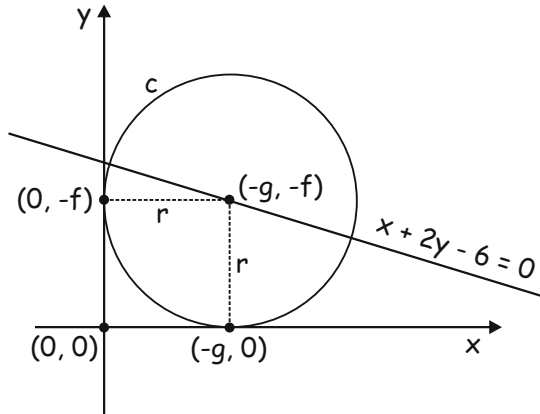


**QUESTION 4 (25 MARKS)**

Sketch the situation.

The circle  $c$  touches the  $x$ -axis at  $(-g, 0)$  and touches the  $y$ -axis at  $(0, -f)$ .

The line passes through the centre of the circle  $c$ .



**FORMULAE AND TABLES BOOK**  
**Co-ordinate geometry: Circle [page 19]**

Given centre  $(h, k)$  and radius  $r$

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre  $(-g, -f)$

Radius  $\sqrt{g^2 + f^2 - c}$

$(-g, -f) \in x + 2y - 6 = 0$  [If a point is on a line, substitute the point into the equation of the

$\therefore (-g) + 2(-f) - 6 = 0$  line and it satisfies the equation.]

$$-g - 2f - 6 = 0$$

$$g + 2f = -6 \dots \dots (1)$$

The points  $(-g, 0)$  and  $(0, -f)$  satisfy the equation of the circle.

$$(-g, 0) \in c : x^2 + y^2 + 2gx + 2fy + c = 0$$

$$(0, -f) \in c : x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\therefore (-g)^2 + (0)^2 + 2g(-g) + 2f(0) + c = 0$$

$$\therefore (0)^2 + (-f)^2 + 2g(0) + 2f(-f) + c = 0$$

$$g^2 - 2g^2 + c = 0$$

$$f^2 - 2f^2 + c = 0$$

$$c = g^2$$

$$c = f^2$$

$$\therefore c = g^2 = f^2 \dots \dots (2)$$

$$\therefore g^2 = f^2 \Rightarrow g = \pm f \dots \dots (3)$$

To find the equation of each circle, choose  $g = +f$  for the first circle equation and  $g = -f$  for the second circle equation.

$$g = +f \text{ [Substitute into Eqn. (1)]}$$

$$g = -f$$

$$g + 2g = -6$$

$$-f + 2f = -6$$

$$3g = -6$$

$$f = -6$$

$$g = -2$$

$$g = 6$$

$$f = -2 \text{ [Substitute into Eqn. (2)]}$$

$$c = g^2 = (6)^2 = 36$$

$$c = g^2 = (-2)^2 = 4$$

**EQUATION OF CIRCLES**

$$x^2 + y^2 + 2(-2)x + 2(-2)y + 4 = 0 \Rightarrow x^2 + y^2 - 4x - 4y + 4 = 0$$

$$x^2 + y^2 + 2(6)x + 2(-6)y + 36 = 0 \Rightarrow x^2 + y^2 + 12x - 12y + 36 = 0$$

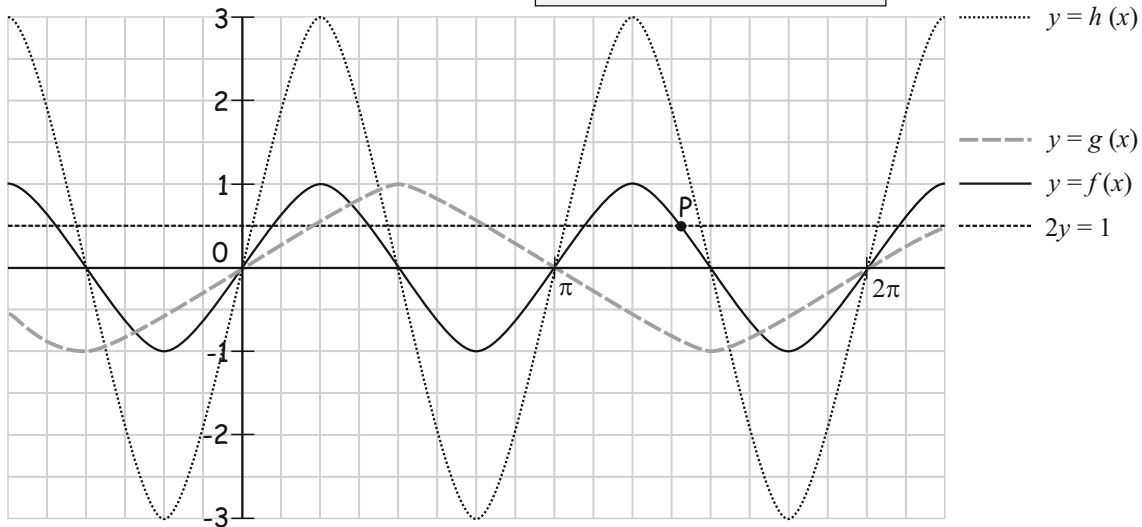
**QUESTION 5 (25 MARKS)**

**QUESTION 5 (a)**

$g: x \mapsto \sin x \Rightarrow P = 2\pi, R = [-1, 1]$

$h: x \mapsto 3 \sin 2x \Rightarrow P = \frac{2\pi}{2} = \pi, R = [-3, 3]$

**PERIOD  $P$  AND RANGE  $R$**   
 $y = a \sin bx$   
 Range  $R = [-a, a]$   
 Period  $P = \frac{2\pi}{b}$



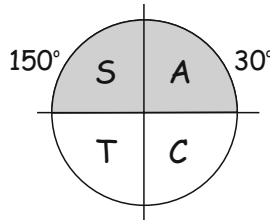
**QUESTION 5 (b)**

$y = f(x) = \sin 2x$

$2y = 1 \Rightarrow y = \frac{1}{2}$

$\therefore \sin 2x = \frac{1}{2}$

$2x = \sin^{-1}(\frac{1}{2}) = 30^\circ = \frac{\pi}{6}$



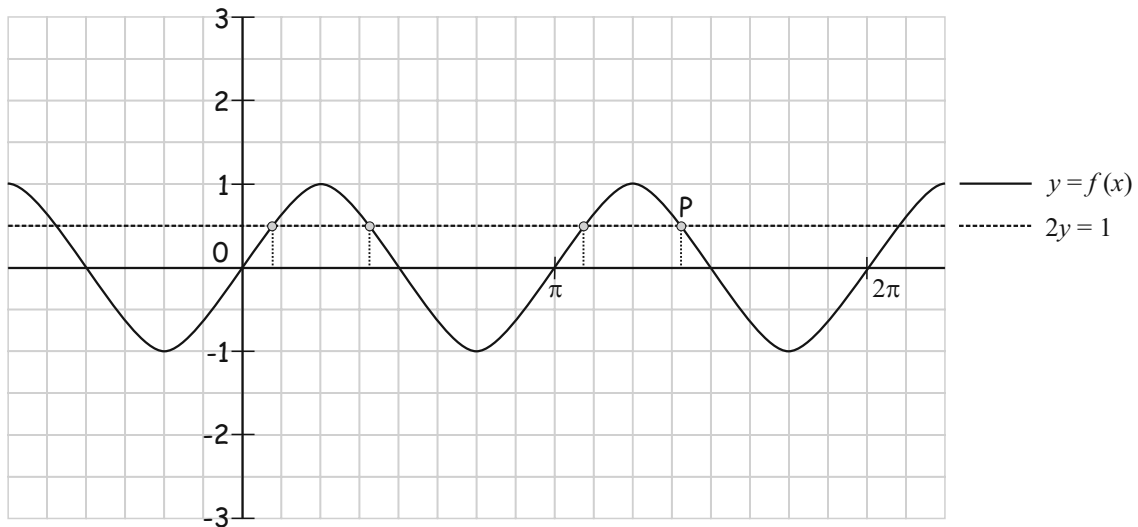
$2x = \frac{\pi}{6}, \frac{13\pi}{6}$  [First quadrant]

$2x = \frac{5\pi}{6}, \frac{17\pi}{6}$  [Second quadrant]

$x = \frac{\pi}{12}, \frac{13\pi}{12}$

$x = \frac{5\pi}{12}, \frac{17\pi}{12}$

$\therefore x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$  [These are all the solutions between 0 and  $2\pi$ .]



ANSWER:  $P\left(\frac{17\pi}{12}, \frac{1}{2}\right)$

### QUESTION 6A (25 MARKS)

*Explanation:*

Proof by contradiction is a form of proof that establishes the truth or validity of a proposition by showing that the proposition being false would imply a contradiction.

*Example:*

Prove  $x + \frac{1}{x} \geq 2$  for all  $x > 0, x \in \mathbb{R}$ .

To prove this let's assume it is false, i.e.  $x + \frac{1}{x} < 2$  for all  $x > 0, x \in \mathbb{R}$ .

$$x + \frac{1}{x} < 2$$

$$x^2 + 1 < 2x$$

$$x^2 - 2x + 1 < 0$$

$$(x-1)^2 < 0 \text{ [This statement is false for all values of } x.]$$

This is a contradiction.

### QUESTION 6B (25 MARKS)

$OECD$  is a cyclic quadrilateral because its opposite angles add up to  $180^\circ$ .

$$|\angle OEC| + |\angle ODC| = 90^\circ + 90^\circ = 180^\circ$$

It follows that the other pair of opposite angles also add up to  $180^\circ$  as the four angles in a quadrilateral add up to  $360^\circ$ .

#### CYCLIC QUADRILATERALS

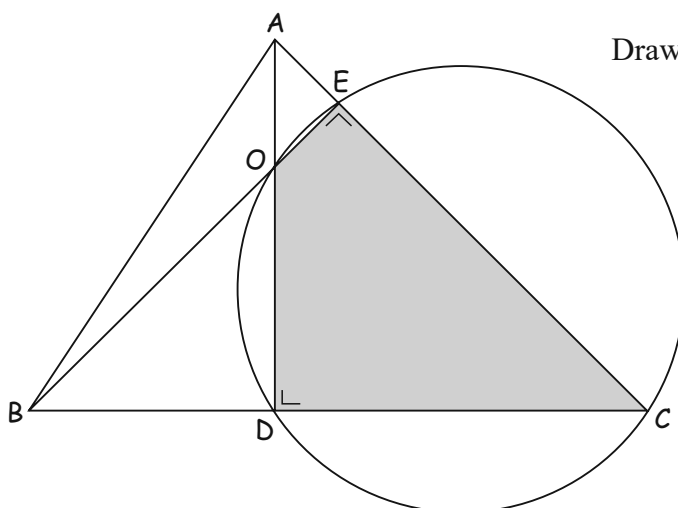
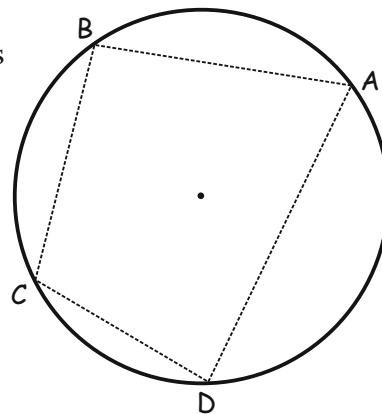
A cyclic quadrilateral is a four sided figure whose vertices lie on a circle.

Opposite angles of a cyclic quadrilateral add up to  $180^\circ$ .

$$\angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$

Conversely, if the opposite angles of a quadrilateral add up to  $180^\circ$  then it is a cyclic quadrilateral.



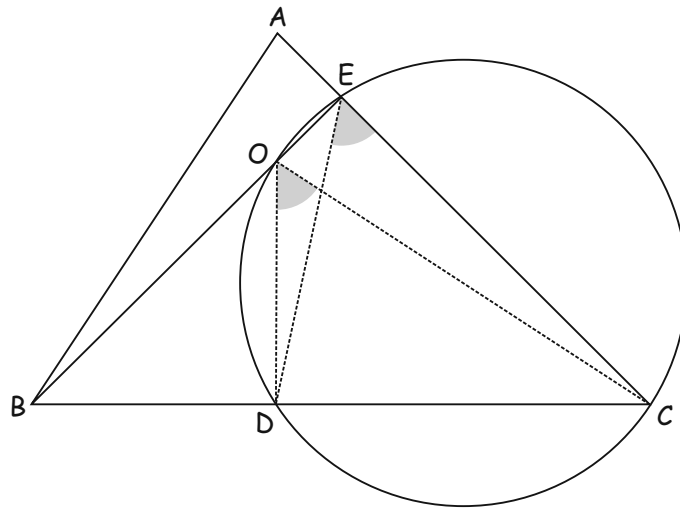
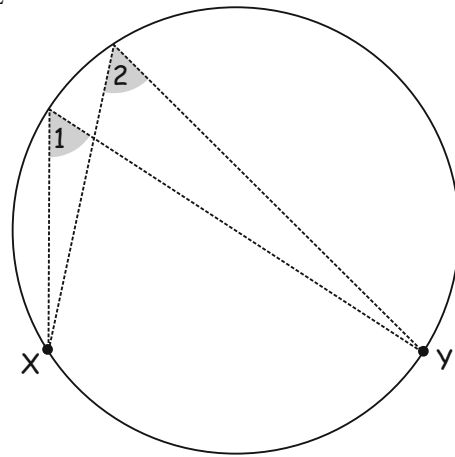
Draw a circle around the quadrilateral.

ANGLES STANDING ON THE SAME ARC IN A CIRCLE

Angles 1 and 2 are standing on the same arc [AB].

Angles standing on the same arc are equal.

$$\therefore |\angle 1| = |\angle 2|.$$



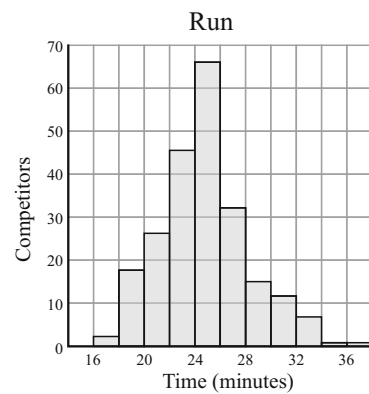
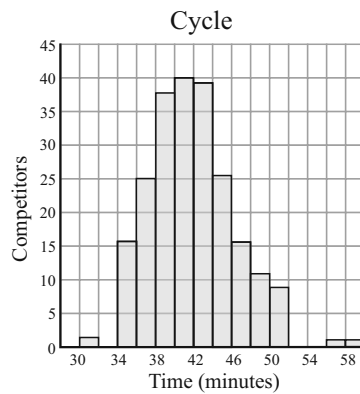
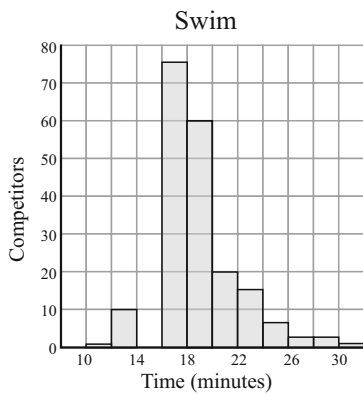
$$|\angle DOC| = |\angle DEC| \text{ [Both angles are standing on arc [DC]]}$$

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**QUESTION 7 (75 MARKS)**

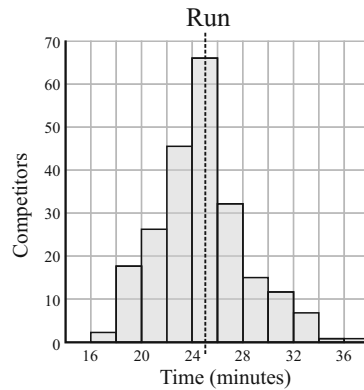
**QUESTION 7 (a)**

(i)	Swim	Cycle	Run
Mean	18.329	41.927	?
Median	17.900	41.306	?
Mode	#N/A	#N/A	#N/A
Standard Deviation	?	4.553	3.409
Sample Variance	10.017	20.729	11.622
Skewness	1.094	0.717	0.463
Range	19.226	27.282	20.870
<b>Minimum</b>	<b>11.350</b>	<b>31.566</b>	<b>16.466</b>
<b>Maximum</b>	<b>30.576</b>	<b>58.847</b>	<b>37.336</b>
Count	224	224	224



You can match the histograms to the events by looking at the maximum and minimum times to complete each event.

- (ii) The median is a line on the histogram that bisects its area. The area to the left of it is equal to the area to the right of it. Using your eye this appears to lie along the class interval of 24–26 minutes.  
 mean  $\approx$  median  $\approx$  25 minutes



- (iii) Swim: Sample Variance  $s = 10.017$

$$s = \sigma^2$$

$$\sigma = \sqrt{s} = \sqrt{10.017} = 3.16 \text{ minutes}$$

- (iv) There was probably no discrete modal result as all times were different or there were lots of the same times. Therefore, there is no modal time. There is a modal class alright but no modal time.

### QUESTION 7 (b)

Cycle vs. Swim: Moderate positive correlation  
 Run vs. Swim: Moderately strong positive correlation  
 Run vs. Cycle: Strong positive correlation

### QUESTION 7 (c)

$$\text{Run/Swim: } y = 0.53x + 15.2$$

$$\text{Brian: } x = 17.6 \text{ mins}$$

$$y = 0.53(17.6) + 15.2 = 24.528 \text{ mins}$$

$$\text{Run/Cycle: } y = 0.58x + 0.71$$

$$\text{Brian: } x = 35.7 \text{ mins}$$

$$y = 0.58(35.7) + 0.71 = 21.416 \text{ mins}$$

Take the average of the 2 run times:  $\frac{24.528 + 21.416}{2} = 22.972 \text{ mins}$

The mean finishing time for the overall event was 88.1 minutes and the standard deviation was 10.3 minutes.

### QUESTION 7 (d)

In any normal distribution with mean  $\mu$  and standard deviation  $\sigma$ .

1. 68.26% of the data falls within  $1\sigma$  of the mean  $\mu$ .
2. 95.46% of the data falls within  $2\sigma$  of the mean  $\mu$ .
3. 99.74% of the data falls within  $3\sigma$  of the mean  $\mu$ .

$$\mu = 88.1 \text{ mins}, \sigma = 10.3 \text{ mins}$$

$$\mu - 2\sigma = 88.1 - 2(10.3) = 67.5 \text{ mins}$$

$$\mu + 2\sigma = 88.1 + 2(10.3) = 108.7 \text{ mins}$$

“95% of the athletes took between **67.5** and **108.7** minutes to complete the race.”

### QUESTION 7 (e)

$$z = \frac{100 - 88.1}{10.3} = 1.155$$

$$P(x < 100) = P(z < 1.155) = 0.877$$

$$\text{Number of athletes} = 224 \times 0.877 \approx 196$$

**FORMULAE AND TABLES BOOK**  
**Statistics and Probability: Probability distribution** (standardising formula) [page 34]

$$\bar{z} = \frac{\bar{x} - \mu}{\bar{\sigma}}$$

$n$  = Number in the sample

$\sigma$  = standard deviation of the sample

### QUESTION 7 (f)

Let  $p$  = Probability of completing the race in less than 100 minutes =  $p(0.877)$

Let  $q$  = Probability of completing the race in more than 100 minutes =  $q(1 - 0.877) = q(0.123)$

This is the order in which she interviews the athletes:

Fixed

$q \quad p \quad p \quad p \quad \boxed{p} \quad q$

$$\text{No. of permutations} = \frac{5!}{4!} = 5$$

$$P = (0.123)^2 \times (0.877)^4 \times 5 = 0.0447$$

**BERNOULLI TRIALS**

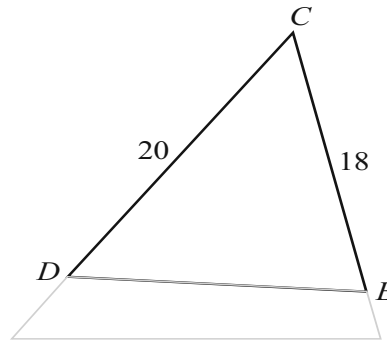
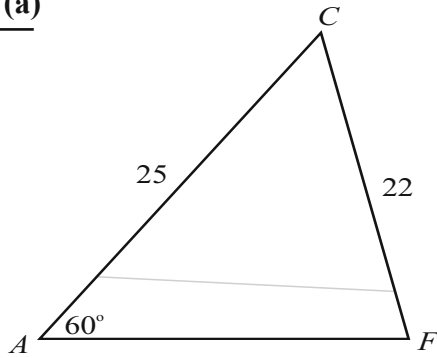
$p = P(\text{Success}), q = P(\text{Failure})$

$$P(r \text{ successes}) = {}^n C_r p^r q^{n-r}$$

The probability that the second person she interviews will be the sixth person she approaches is about 4.5%.

**QUESTION 8 (50 MARKS)**

**QUESTION 8 (a)**



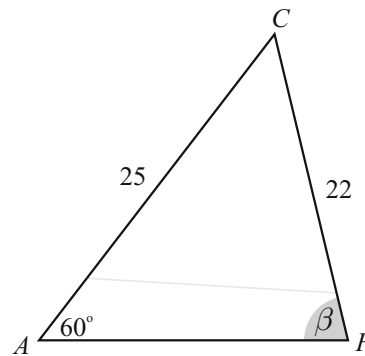
Call  $\beta$  the measure of  $|\angle CFA|$ . Use the Sine Rule to find this angle.

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b}} \quad [\text{Use the Sine Rule anytime you are given 2 sides and a non-included angle.}]$$

$$\frac{\sin \beta}{25} = \frac{\sin 60^\circ}{22}$$

$$\therefore \sin \beta = \frac{25 \sin 60^\circ}{22}$$

$$\beta = \sin^{-1}\left(\frac{25 \sin 60^\circ}{22}\right) = 79.8^\circ$$

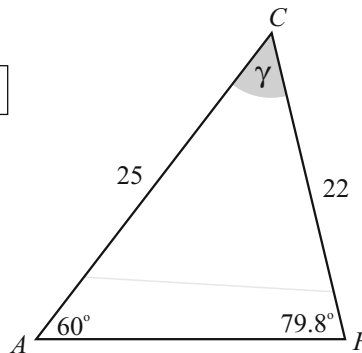


Call  $\gamma$ , the measure of  $|\angle ACF|$ .

**The 3 angles of a triangle add up to  $180^\circ$ .**

$$\gamma + 60^\circ + 79.78^\circ = 180^\circ$$

$$\gamma = 180^\circ - 60^\circ - 79.8^\circ = 40.2^\circ$$

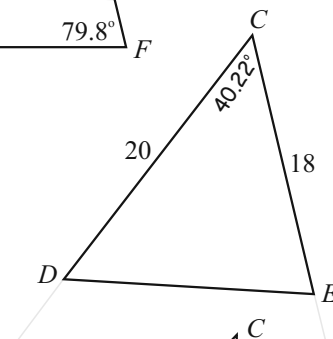


Use the Cosine Rule to find  $|DE|$ .

$$\boxed{a^2 = b^2 + c^2 - 2bc \cos A} \quad [\text{Use the Cosine Rule anytime you are given 2 sides and an included angle.}]$$

$$|DE|^2 = 20^2 + 18^2 - 2(20)(18) \cos 40.2^\circ$$

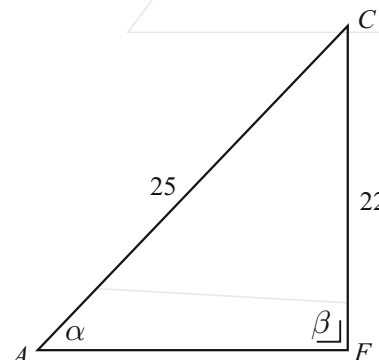
$$|DE| = \sqrt{20^2 + 18^2 - 2(20)(18) \cos 40.2^\circ} = 13.2 \text{ cm}$$



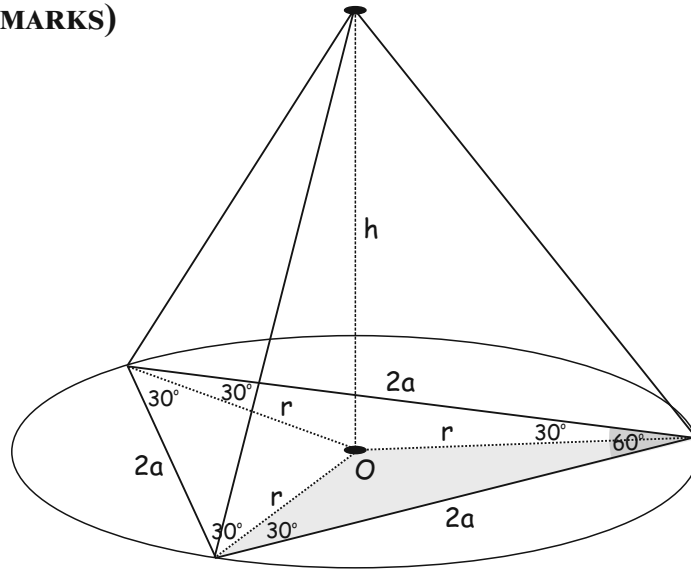
**QUESTION 8 (b)**

$$\sin \alpha = \frac{22}{25} \Rightarrow \alpha = \sin^{-1}\left(\frac{22}{25}\right) = 62^\circ$$

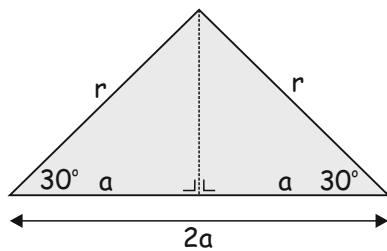
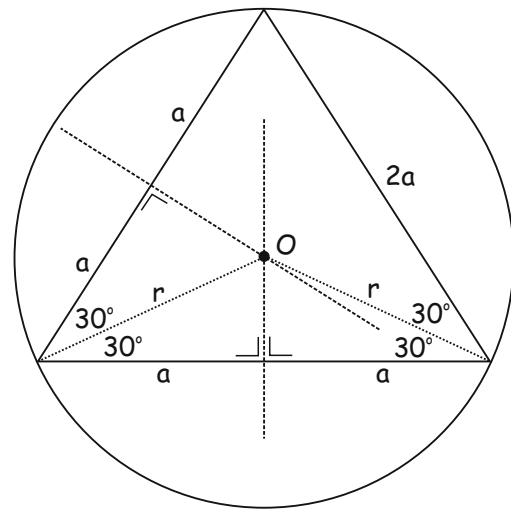
The maximum of the angle  $\beta$  is  $90^\circ$ . If it goes beyond  $90^\circ$  the solar panel will topple over.



**QUESTION 9 (25 MARKS)**



An equilateral triangle fits into the base of the cylinder exactly. The circle at the base of the cylinder is a circumcircle with centre  $O$ . The centre of a circumcircle  $O$  is found by intersecting the perpendicular bisectors of the sides of the triangle. Lift out the highlighted triangle to find an expression for  $r$ .

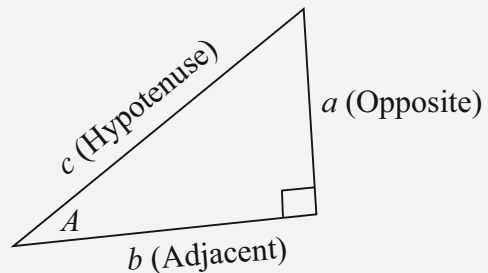


$$\cos 30^\circ = \frac{a}{r}$$

$$\frac{\sqrt{3}}{2} = \frac{a}{r}$$

$$\therefore r = \frac{2a}{\sqrt{3}}$$

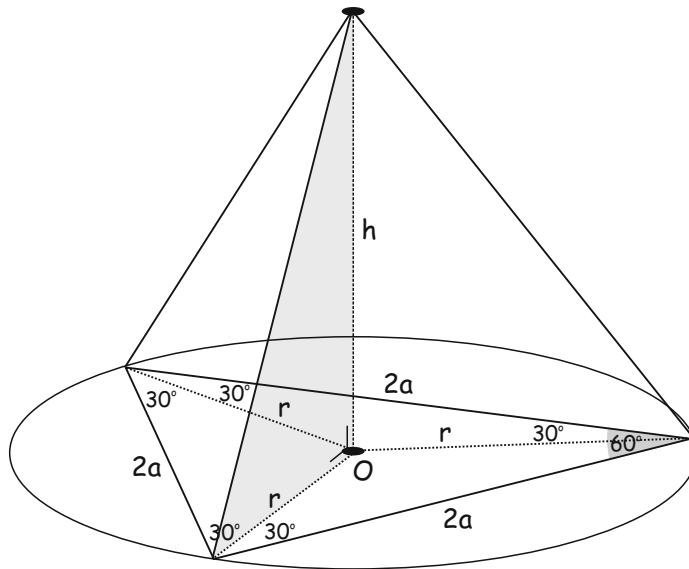
**FORMULAE AND TABLES BOOK**  
**Trigonometry: Right-angled triangle [page 16]**



$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$$

$$c^2 = a^2 + b^2$$

Lift out the highlighted triangle to find an expression for  $h$ . It is a right-angled triangle.

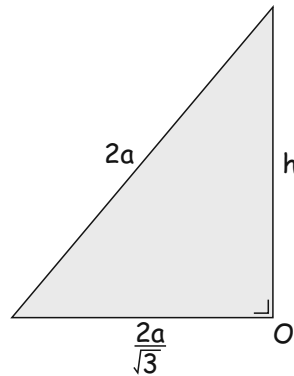


$$h^2 + \left(\frac{2a}{\sqrt{3}}\right)^2 = (2a)^2$$

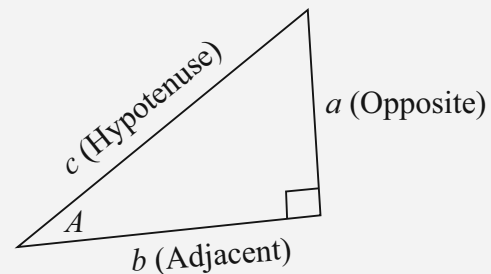
$$h^2 + \frac{4a^2}{3} = 4a^2$$

$$h^2 = 4a^2 - \frac{4a^2}{3} = \frac{8a^2}{3}$$

$$h = \sqrt{\frac{8a^2}{3}} = \frac{2\sqrt{2}a}{\sqrt{3}}$$



**FORMULAE AND TABLES BOOK**  
**Trigonometry: Right-angled triangle** [page 16]



$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$$

$$c^2 = a^2 + b^2$$

$$V = \pi r^2 h$$

$$= \pi \left(\frac{2a}{\sqrt{3}}\right)^2 \left(\frac{2\sqrt{2}a}{\sqrt{3}}\right)$$

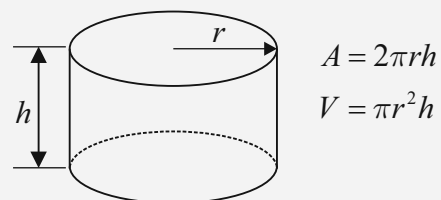
$$= \pi \left(\frac{4a^2}{3}\right) \left(\frac{2\sqrt{2}a}{\sqrt{3}}\right)$$

$$= \frac{8\pi\sqrt{2}a^3}{3\sqrt{3}}$$

$$= \frac{8\pi\sqrt{2}a^3}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \left(\frac{8\sqrt{6}}{9}\right) \pi a^3$$

**FORMULAE AND TABLES BOOK**  
**Surface area and volume:**  
**Cylinder** [page 10]



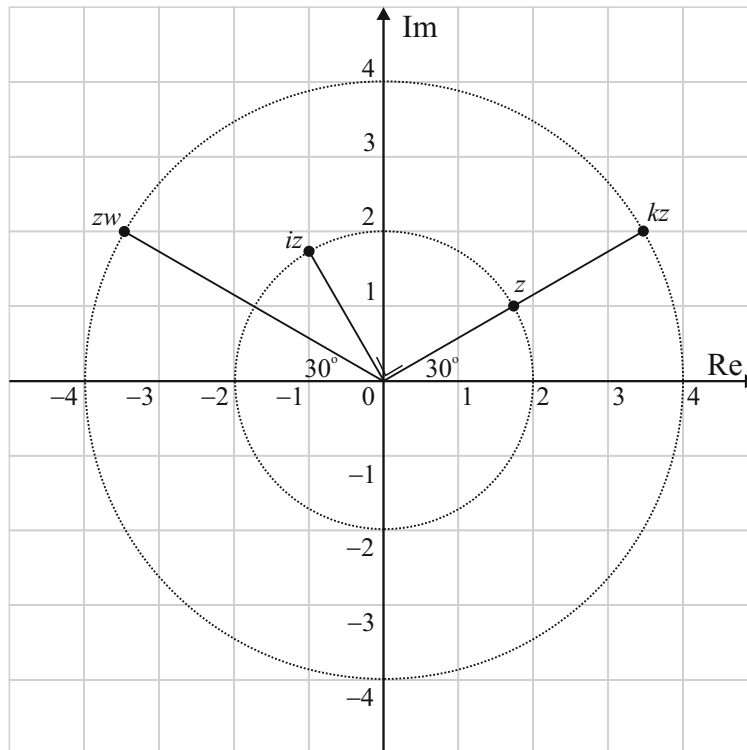
**SAMPLE PAPER 1: PAPER 1**

**QUESTION 1 (25 MARKS)**

**Question 1 (a)**

$$z = 2 \left\{ \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right\} \quad \boxed{z = r(\cos \theta + i \sin \theta)}$$

$$|z| = 2, \arg z = \frac{\pi}{6} = 30^\circ \quad \boxed{|z| = r, \arg z = \theta}$$



**Question 1 (b)**

$$kz = 4 \left\{ \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right\} \Rightarrow |kz| = 4, \arg kz = \frac{\pi}{6} = 30^\circ$$

**Question 1 (c)**

Write  $i$  as a complex number in polar form:  $i = 0 + 1i = 1 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$

$$w = iz = 1 \left\{ \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right\} 2 \left\{ \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right\} \quad \text{[When you multiply complex numbers in polar form you add their arguments.]}$$

$$= 2 \left\{ \cos \left( \frac{\pi}{2} + \frac{\pi}{6} \right) + i \sin \left( \frac{\pi}{2} + \frac{\pi}{6} \right) \right\}$$

$$= 2 \left\{ \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right\}$$

$$|w| = 2, \arg w = \frac{2\pi}{3} = 120^\circ$$

Multiplying by  $i$  rotates a complex number by  $90^\circ$  anticlockwise.

**Question 1 (d)**

When you multiply complex numbers in polar form you add their arguments.

$$zw = 2 \left\{ \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right\} 2 \left\{ \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right\} = 4 \left\{ \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right\}$$

$$|zw| = 4, \arg zw = \frac{5\pi}{6} = 150^\circ$$

**Question 1 (e)**

$$\begin{aligned} z^{12} &= 2^{12} \left\{ \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right\}^{12} \quad [r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) \quad (\text{De Moivre's Theorem}) \\ &= 2^{12} \left\{ \cos \frac{12\pi}{6} + i \sin \frac{12\pi}{6} \right\} \\ &= 2^{12} \{ \cos 2\pi + i \sin 2\pi \} \\ &= 2^{12} \{ 1 + i(0) \} = 2^{12} \end{aligned}$$

**QUESTION 2 (25 MARKS)** [THEME: All about irrationals, root 2 and root 3]

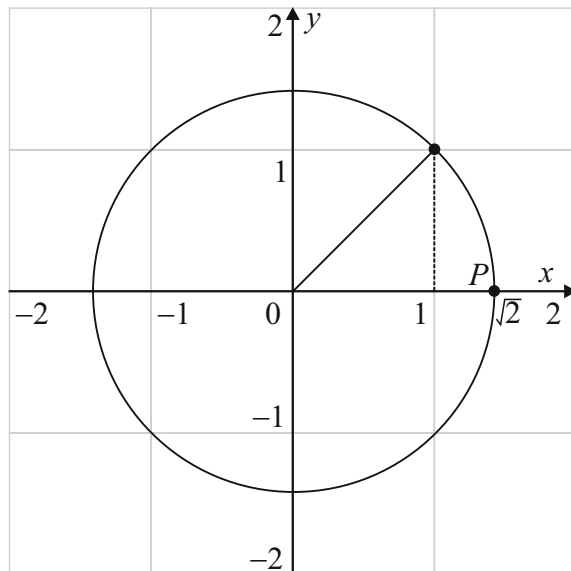
**Question 2 (a)**

Draw a circle with centre (0, 0) through the point (1, 1).

The radius of this circle is

$$r = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}.$$

$\sqrt{2}$  is the distance of (0, 0) to P on this scale.



**Question 2 (b) (i)**

$$\begin{aligned} &(\sqrt{3} + \sqrt{2})^2 \\ &= (\sqrt{3})^2 + 2\sqrt{3}\sqrt{2} + (\sqrt{2})^2 \\ &= 3 + 2\sqrt{6} + 2 \\ &= 5 + 2\sqrt{6} \end{aligned}$$

**Question 2 (b) (ii)**

$$\begin{aligned} &(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) \\ &= 3 + \sqrt{6} - \sqrt{6} - 2 = 1 \end{aligned}$$

Or (use the difference of 2 squares)

$$\begin{aligned} &(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2}) \\ &= (\sqrt{3})^2 - (\sqrt{2})^2 \\ &= 3 - 2 = 1 \end{aligned}$$

**Question 2 (b) (iii)**

$$\begin{aligned} \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} &= \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})} \\ &= \frac{5 + 2\sqrt{6}}{1} = 5 + 2\sqrt{6} \end{aligned}$$

**Question 2 (c) (i)**

$$\log_{\sqrt{2}} x + \log_{\sqrt{2}}(x+4) = 4$$

$$\log_{\sqrt{2}} x(x+4) = 4$$

$$x^2 + 4x = (\sqrt{2})^4 = (2^{\frac{1}{2}})^4 = 2^2 = 4$$

$$x^2 + 4x - 4 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-4)}}{2(1)} = \frac{-4 \pm \sqrt{16+16}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 4\sqrt{2}}{2} = -2 \pm 2\sqrt{2}$$

$$x > 0 \Rightarrow x = 2\sqrt{2} - 2$$

**Question 2 (c) (ii)**

$$x^2 - 2x > 1$$

$$x^2 - 2x - 1 > 0$$

$$x^2 - 2x - 1 = 0 \text{ [Solve the equality using the quadratic formula.]}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Carry out the region test, or whatever test you use, to locate the regions that satisfy the inequality:

$\longleftarrow$	$\longleftarrow$	$\longrightarrow$
$-1$	$0$	$3$
$\longleftarrow$	$\longleftarrow$	$\longrightarrow$
$1 - \sqrt{2}$	$1 + \sqrt{2}$	
$\approx 0.41$	$\approx 2.41$	
$x^2 - 2x - 1 > 0$	$x^2 - 2x - 1 > 0$	$x^2 - 2x - 1 > 0$
$(-1)^2 - 2(-1) - 1$	$(0)^2 - 2(0) - 1$	$(3)^2 - 2(3) - 1$
$= 2 > 0$ (True)	$= -1 > 0$ (False)	$= 2 > 0$ (True)

SOLUTIONS:  $x < 1 - \sqrt{2}$ ,  $x > 1 + \sqrt{2}$ ,  $x \in \mathbb{R}$

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**QUESTION 3 (25 MARKS)**

**Question 3 (a) (i)**

$$\lim_{n \rightarrow \infty} \frac{2n}{3n+4} = \lim_{n \rightarrow \infty} \frac{2n}{n(3+\frac{4}{n})} = \lim_{n \rightarrow \infty} \frac{2}{(3+\frac{4}{n})} = \frac{2}{3} \quad \text{[You can use the technique shown for finding the limit or you can find the limit by inspection.]}$$

**Question 3 (a) (ii)**

$$\lim_{n \rightarrow \infty} 2 \left(\frac{1}{3}\right)^n = 2 \lim_{n \rightarrow \infty} \left(\frac{1}{3}\right)^n = 2 \times 0 = 2$$

$$\lim_{n \rightarrow \infty} r^n = 0, |r| < 1$$

**Question 3 (a) (iii)**

$$\lim_{n \rightarrow \infty} \frac{1}{3} (2)^n = \frac{1}{3} \lim_{n \rightarrow \infty} 2^n = \infty$$

$$\lim_{n \rightarrow \infty} r^n = \infty, |r| > 1$$

**Question 3 (b) (i)**

**STEPS FOR PROOF BY INDUCTION**

1. Prove result is true for some starting value of  $n \in \mathbb{N}$ .
2. Assume result is true for  $n = k$ .
3. Prove result is true for  $n = (k+1)$ .

Terms of a geometric sequence:  $a, ar, ar^2, ar^3, \dots, ar^{n-1}$

1. Prove true for  $n = 1$ :  $S_1 = \frac{a(1-r^1)}{1-r} = a$  [Therefore, true for  $n = 1$ .]

2. Assume true for  $n = k$ : Assume  $S_k = a + ar + \dots + ar^{k-1} = \frac{a(1-r^k)}{1-r}$

3. Prove true for  $n = k + 1$ : Prove  $S_{k+1} = a + ar + \dots + ar^{k-1} + ar^k = \frac{a(1-r^{k+1})}{1-r}$

**Proof:**

$$\begin{aligned} (a + ar + \dots + ar^{k-1}) + ar^k &= \frac{a(1-r^k)}{1-r} + ar^k \\ &= \frac{a(1-r^k) + ar^k(1-r)}{1-r} = \frac{a - ar^k + ar^k - ar^{k+1}}{1-r} \\ &= \frac{a(1-r^{k+1})}{1-r} \end{aligned}$$

Therefore, assuming true for  $n = k$  means it is true for  $n = k + 1$ . So true for  $n = 1$  and true for  $n = k$  means it is true for  $n = k + 1$ . This implies it is true for all  $n \in \mathbb{N}$ .

**Question 3 (b) (ii)**

$$\begin{aligned} \lim_{n \rightarrow \infty} S_n &= \lim_{n \rightarrow \infty} \frac{a(1-r^n)}{1-r} = \frac{a}{1-r} \lim_{n \rightarrow \infty} (1-r^n) = \frac{a}{1-r} (1 - \lim_{n \rightarrow \infty} r^n) \\ &= \frac{a}{1-r} (1-0) = \frac{a}{1-r} \end{aligned}$$

$$\lim_{n \rightarrow \infty} r^n = 0, |r| < 1$$

**Question 3 (c)**

$$\begin{aligned}
 0.5\dot{2}3 &= 0.5232323\dots = \frac{1}{2} + \left( \frac{23}{1000} + \frac{23}{100\,000} + \dots \right) \left[ a = \frac{23}{1000}, r = \frac{1}{100} \right] \\
 &= \frac{1}{2} + \frac{\frac{23}{1000}}{1 - \frac{1}{100}} \\
 &= \frac{1}{2} + \frac{\frac{23}{1000}}{\frac{99}{100}} \\
 &= \frac{1}{2} + \frac{23}{990} \\
 &= \frac{259}{990}
 \end{aligned}$$

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**

$$\begin{aligned}
 kx^3 - 6x^2 + bx - 6 &= (x+1)(x-3)(kx+2) \\
 kx^3 - 6x^2 + bx - 6 &= (x^2 - 2x - 3)(kx+2) \\
 kx^3 - 6x^2 + bx - 6 &= kx^3 + 2x^2 - 2kx^2 - 4x - 3kx - 6 \\
 kx^3 - 6x^2 + bx - 6 &= kx^3 + (2-2k)x^2 + (-4-3k)x - 6
 \end{aligned}$$

$$\begin{array}{l|l}
 -6 = 2 - 2k & b = -4 - 3k \\
 2k = 8 & b = -4 - 3(4) = -16 \\
 \therefore k = 4 &
 \end{array}$$

[A cubic equation equals a linear factor by a linear factor by a linear factor. The coefficients of  $x$  in each linear factor must multiply together to give  $k$  and the constants in each linear factor must multiply together to give  $-6$ . Find  $k$  and  $b$  by lining up the coefficients.]

$$\begin{aligned}
 (x+1)(x-3)(4x+2) &= 0 \\
 \therefore x &= -1, -\frac{1}{2}, 3
 \end{aligned}$$

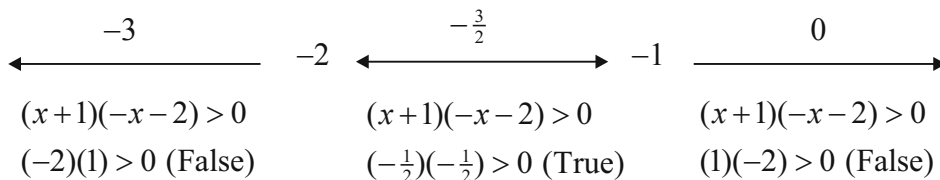
**Question 4 (b)**

$$\frac{x}{x+1} > 2 \quad \text{[Multiply across by } (x+1)^2 \text{. This is a positive value and so will not reverse the inequality.]}$$

$$\begin{aligned}
 \frac{x}{x+1}(x+1)^2 &> 2(x+1)^2 \\
 x(x+1) - 2(x+1)^2 &> 0 \\
 (x+1)[x - 2(x+1)] &> 0 \\
 (x+1)[x - 2x - 2] &> 0 \\
 (x+1)[-x - 2] &> 0
 \end{aligned}$$

Solve the equality:  
 $(x+1)(-x-2) = 0$   
 $\therefore x = -2, -1$

Carry out the region test, or whatever test you use, to locate the regions that satisfy the inequality:



SOLUTIONS:  $-2 < x < -1, x \in \mathbb{R}$

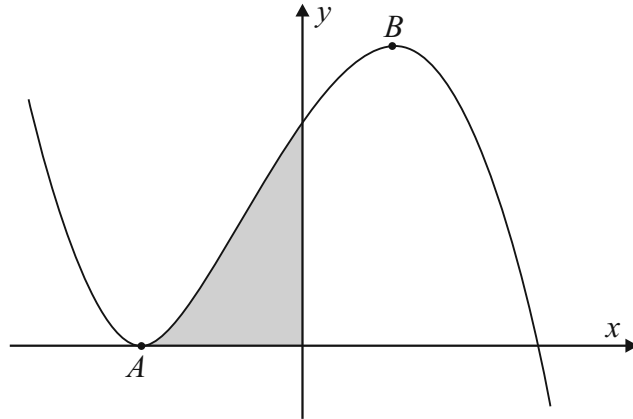
**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

$$f(x) = (x+2)^2(5-2x)$$

$$\begin{aligned} f'(x) &= (x+2)^2(-2) + (5-2x)2(x+2)^1 \\ &= 2(x+2)[-(x+2) + (5-2x)] \\ &= 2(x+2)[-x-2+5-2x] \\ &= 2(x+2)[-3x+3] \\ &= 6(x+2)(1-x) \end{aligned}$$

$$\begin{aligned} f''(x) &= 6\{(x+2)(-1) + (1-x)(1)\} \\ &= 6\{-x-2+1-x\} \\ &= 6\{-2x-1\} \\ &= -6(2x+1) \end{aligned}$$



$$\begin{aligned} f'(x) = 0 &\Rightarrow 6(x+2)(1-x) = 0 \\ \therefore x &= -2, 1 \end{aligned}$$

TO FIND TURNING POINTS (TP):  
Put  $\frac{dy}{dx} = f'(x) = 0$  and solve for  $x$ .

$$\begin{aligned} f''(-2) &= -6(-3) = 18 > 0 \Rightarrow \text{Local Minimum} \\ f''(1) &= -6(3) = -18 < 0 \Rightarrow \text{Local Maximum} \end{aligned}$$

$$\left(\frac{d^2y}{dx^2}\right)_{TP} < 0 \Rightarrow \text{Local Maximum}$$

$$\left(\frac{d^2y}{dx^2}\right)_{TP} > 0 \Rightarrow \text{Local Minimum}$$

$$f(x) = (x+2)^2(5-2x)$$

$$f(-2) = (-2+2)^2(5-2(-2)) = 0 \Rightarrow A(-2, 0) \text{ is the local minimum.}$$

$$f(1) = (1+2)^2(5-2(1)) = (9)(3) = 27 \Rightarrow B(1, 27) \text{ is the local maximum.}$$

**Question 5 (b)**

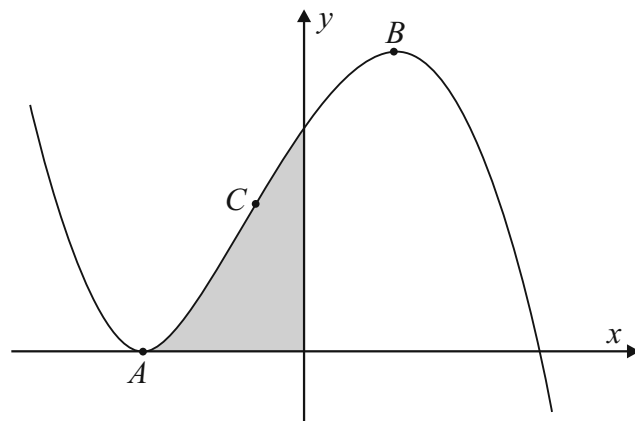
TO FIND POINT OF INFLECTION:  
Put  $\frac{d^2y}{dx^2} = f''(x) = 0$  and solve for  $x$ .

$$\begin{aligned} f''(x) = 0 &\Rightarrow -12x - 6 = 0 \\ \therefore x &= -\frac{1}{2} \end{aligned}$$

$$f(x) = (x+2)^2(5-2x)$$

$$\begin{aligned} f\left(-\frac{1}{2}\right) &= \left(-\frac{1}{2}+2\right)^2\left(5-2\left(-\frac{1}{2}\right)\right) \\ &= \left(\frac{3}{2}\right)^2(5+1) \\ &= \left(\frac{9}{4}\right)(6) = \frac{27}{2} \end{aligned}$$

$$\therefore C\left(-\frac{1}{2}, \frac{27}{2}\right) \text{ is point of inflection.}$$



**Question 5 (c)**

$$m = \frac{27}{2}, (x_1, y_1) = \left(-\frac{1}{2}, \frac{27}{2}\right)$$

$$y - \frac{27}{2} = \frac{27}{2}\left(x + \frac{1}{2}\right)$$

$$y - \frac{27}{2} = \frac{27}{2}x + \frac{27}{4}$$

$$4y - 54 = 54x + 27$$

$$54x - 4y + 81 = 0$$

**Question 5 (d)**

$$\begin{aligned} A &= \int_{-2}^0 ((x+2)^2(5-2x)) dx \\ &= \int_{-2}^0 ((x^2+4x+4)(5-2x)) dx \\ &= \int_{-2}^0 (-2x^3-3x^2+12x+20) dx \\ &= \left[ -\frac{2x^4}{4} - \frac{3x^3}{3} + \frac{12x^2}{2} + 20x \right]_{-2}^0 \\ &= \left[ -\frac{1}{2}x^4 - x^3 + 6x^2 + 20x \right]_{-2}^0 \\ &= 0 - \left\{ -\frac{1}{2}(-2)^4 - (-2)^3 + 6(-2)^2 + 20(-2) \right\} \\ &= +8 - 8 - 24 + 40 \\ &= 16 \end{aligned}$$

**QUESTION 6 (25 MARKS)****Question 6 (a) (i)**

$$\begin{aligned} &\int \sin^2 x \, dx \\ &= \frac{1}{2} \int (1 - \cos 2x) \, dx \\ &= \frac{1}{2} \left( x - \frac{1}{2} \sin 2x \right) + c \end{aligned}$$

**Question 6 (a) (ii)**

$$\begin{aligned} &\int_0^{\ln 2} 3e^{2x} \, dx \\ &= \frac{3}{2} [e^{2x}]_0^{\ln 2} \\ &= \frac{3}{2} (e^{2 \ln 2} - e^0) \\ &= \frac{3}{2} (e^{\ln 2^2} - e^0) \\ &= \frac{3}{2} (4 - 1) \\ &= \frac{9}{2} \end{aligned}$$

**Question 6 (a) (iii)**

$$\boxed{x^3 + 1 = (x+1)(x^2 - x + 1)}$$

$$\begin{aligned} &\int \frac{x^3 + 1}{x+1} \, dx \\ &= \int \frac{(x+1)(x^2 - x + 1)}{(x+1)} \, dx \\ &= \int (x^2 - x + 1) \, dx \\ &= \frac{1}{3}x^3 - \frac{1}{2}x^2 + x + c \end{aligned}$$

**Question 6 (a) (iv)**

$$\begin{aligned} &\int_1^9 \frac{x+1}{\sqrt{x}} \, dx \\ &= \int_1^9 \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right) \, dx \\ &= \int_1^9 (x^{\frac{1}{2}} + x^{-\frac{1}{2}}) \, dx \\ &= \left[ \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right]_1^9 \\ &= \left\{ \frac{2}{3}(9)^{\frac{3}{2}} + 2(9)^{\frac{1}{2}} \right\} - \left\{ \frac{2}{3}(1)^{\frac{3}{2}} + 2(1)^{\frac{1}{2}} \right\} \\ &= \frac{2}{3}(27) + 2(3) - \frac{2}{3}(1) - 2(1) \\ &= 18 + 6 - \frac{2}{3} - 2 \\ &= \frac{64}{3} \end{aligned}$$

**Question 6 (a) (v)**

$$\begin{aligned} &\int_0^1 \sqrt{2^x} \, dx \\ &= \int_0^1 2^{\frac{1}{2}x} \, dx \\ &= \left[ \frac{2 \times 2^{\frac{1}{2}x}}{\ln 2} \right]_0^1 \\ &= \frac{2}{\ln 2} \{ 2^{\frac{1}{2}(1)} - 2^{\frac{1}{2}(0)} \} \\ &= \frac{2}{\ln 2} (\sqrt{2} - 1) \end{aligned}$$

**Question 6 (b)**

$$f'(x) = 9x^2 - 4x + 5$$

$$\begin{aligned} f(x) &= \int f'(x) dx = \int (9x^2 - 4x + 5) dx \\ &= 3x^3 - 2x^2 + 5x + c \end{aligned}$$

$$f(1) = 1$$

$$f(1) = 3(1)^3 - 2(1)^2 + 5(1) + c = 1$$

$$3 - 2 + 5 + c = 1$$

$$\therefore c = -5$$

$$\therefore f(x) = 3x^3 - 2x^2 + 5x - 5$$

**QUESTION 7 (50 MARKS)****Question 7 (a)**

$a$  is the initial amount of Carbon-14

$$Q = ae^{bt}$$

$t = 0$ :  $Q = ae^0 = a$  [At  $t = 0$  years the amount of Carbon-14 present is  $a$ . After a half-life of

$t = 5730$  years:  $Q = \frac{1}{2}a$  5730 years, the amount of Carbon-14 present will fall by a half of  $a$ .]

$$\therefore \frac{1}{2}a = ae^{b(5730)}$$

$$\frac{1}{2} = e^{b(5730)}$$

$$\ln\left(\frac{1}{2}\right) = b(5730)$$

$$-\ln 2 = 5730b$$

$$\therefore b = -\left(\frac{\ln 2}{5730}\right) \text{ yr}^{-1}$$

**Question 7 (b)**

$$Q = ae^{bt} \Rightarrow \frac{dQ}{dt} = abe^{bt}$$

$$\frac{\left(\frac{dQ}{dt}\right)_{t=4000}}{\left(\frac{dQ}{dt}\right)_{t=1000}} = \frac{abe^{-\left(\frac{\ln 2}{5730}\right)4000}}{abe^{-\left(\frac{\ln 2}{5730}\right)1000}} = \frac{e^{-\left(\frac{\ln 2}{5730}\right)4000}}{e^{-\left(\frac{\ln 2}{5730}\right)1000}} = e^{-\left(\frac{\ln 2}{5730}\right)3000} = 0.696$$

**Question 7 (c) (i)**

$$\frac{Q}{a} = e^{-\left(\frac{\ln 2}{5730}\right)2000} = 0.785 = 78.5\% \quad [Q \text{ divided by } a \text{ (the initial amount) will give you the fraction of Carbon-14 present at any time.}]$$

**Question 7 (c) (ii)**

$$\frac{Q}{a} = 0.93 = e^{-\left(\frac{\ln 2}{5730}\right)t}$$

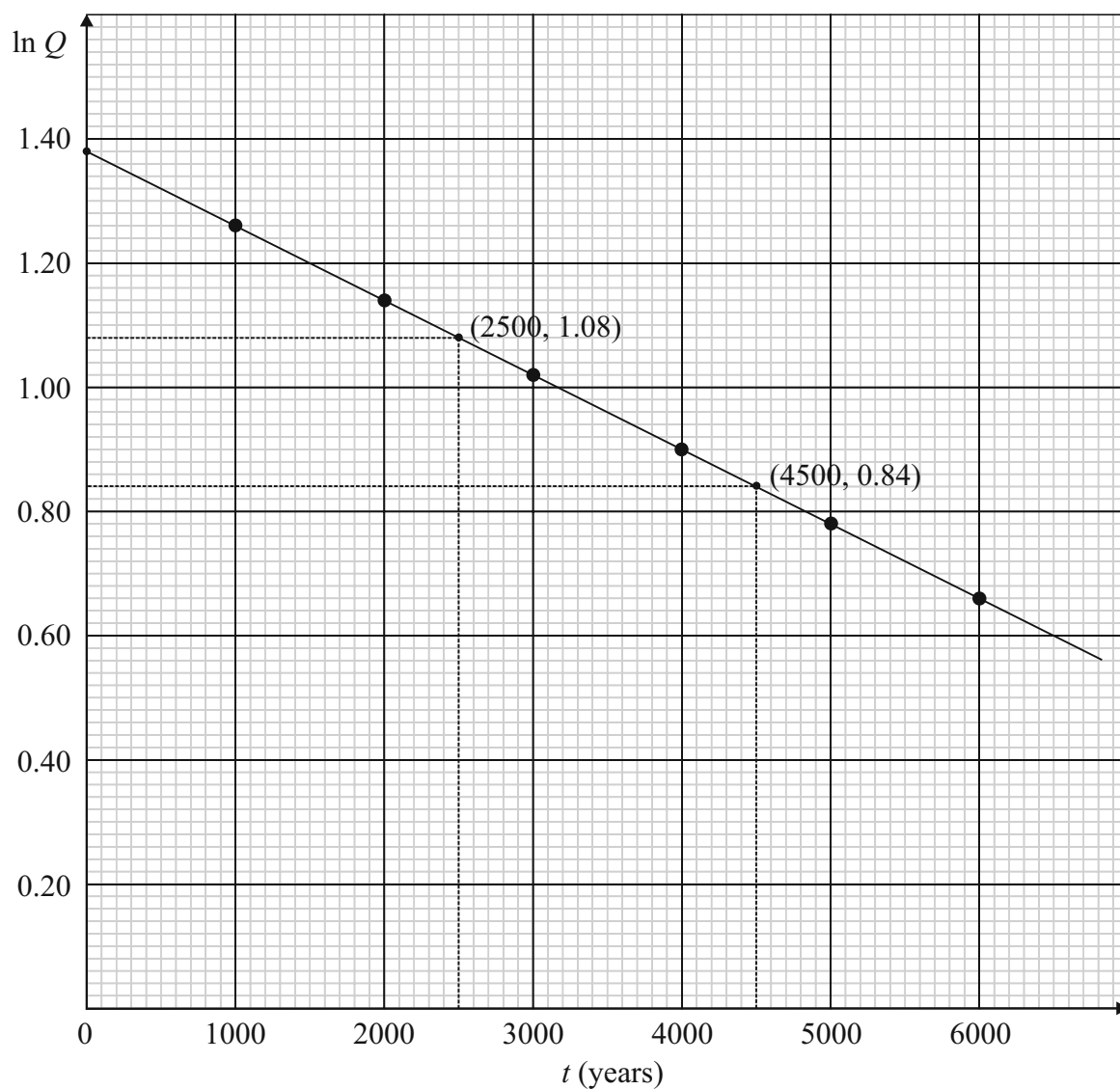
$$\ln(0.93) = -\left(\frac{\ln 2}{5730}\right)t$$

$$\therefore t = -\frac{5730 \ln(0.93)}{\ln 2} = 600 \text{ years}$$

**Question 7 (d) (i)**

$t$ (years)	1000	2000	3000	4000	5000	6000
$Q$ (mg)	3.54	3.14	2.78	2.47	2.18	1.94
$\ln Q$	1.26	1.14	1.02	0.90	0.78	0.66

**Question 7 (d) (ii)**



**Question 7 (e)**

$$Q = ae^{bt}$$

$$\ln Q = \ln(ae^{bt})$$

$$\ln Q = \ln a + \ln(e^{bt})$$

$$\ln Q = bt + \ln a \quad (\rightarrow y = mx + c)$$

$b$  is the slope of the graph and  $\ln a$  is the  $y$ -intercept.

$$b = \frac{1.08 - 0.84}{2500 - 4500} = -1.2 \times 10^{-4} \text{ year}^{-1}$$

$$b = -\frac{\ln 2}{5730} = -1.21 \times 10^{-4} \text{ year}^{-1}$$

$$\ln a = 1.38 \Rightarrow a = e^{1.38} = 3.975 \text{ mg}$$

**Question 7 (f)**

$$Q = ae^{bt}$$

$$Q - 0.05Q = ae^{bt_1}$$

$$Q + 0.05Q = ae^{bt_2}$$

$$0.95Q = ae^{bt_1}$$

$$1.05Q = ae^{bt_2}$$

$$\frac{0.95Q}{1.05Q} = \frac{e^{bt_1}}{e^{bt_2}} = e^{b(t_1 - t_2)}$$

$$\therefore \ln\left(\frac{0.95}{1.05}\right) = b(t_1 - t_2)$$

$$(t_1 - t_2) = \frac{\ln\left(\frac{0.95}{1.05}\right)}{b} = \frac{\ln\left(\frac{0.95}{1.05}\right)}{-\left(\frac{\ln 2}{5730}\right)} = 827 \text{ years}$$

**QUESTION 8 (50 MARKS)**

**Question 8 (a)**

$$x = 20\,000 - 100p$$

$$100p = 20\,000 - x$$

$$p = 200 - 0.01x \geq 0$$

$$\therefore 20\,000 \geq x$$

$$x \leq 20\,000$$

**Question 8 (b) (ii)**

It costs €40 to produce one more phone at any level of production.

**Question 8 (c) (ii)**

$$R = 200x - 0.01x^2 \geq 0$$

$$20\,000x - x^2 \geq 0$$

$$x(20\,000 - x) \geq 0$$

Solve the equality:

$$x(20\,000 - x) = 0$$

$$\therefore x = 0, 20\,000$$

SOLUTION:  $0 \leq x \leq 20\,000$

**Question 8 (b) (i)**

$$C = 120\,000 + 40x$$

$$\frac{dC}{dx} = 40 = \text{constant}$$

**Question 8 (c) (i)**

$$R = x \times p$$

$$p = 200 - 0.01x$$

$$\therefore R = x(200 - 0.01x) = 200x - 0.01x^2$$

**Question 8 (d)**

$$R = 200x - 0.01x^2$$

$$\frac{dR}{dx} = 200 - 0.02x$$

$$\frac{dR}{dx} = 0 \Rightarrow 200 - 0.02x = 0$$

$$200 = 0.02x$$

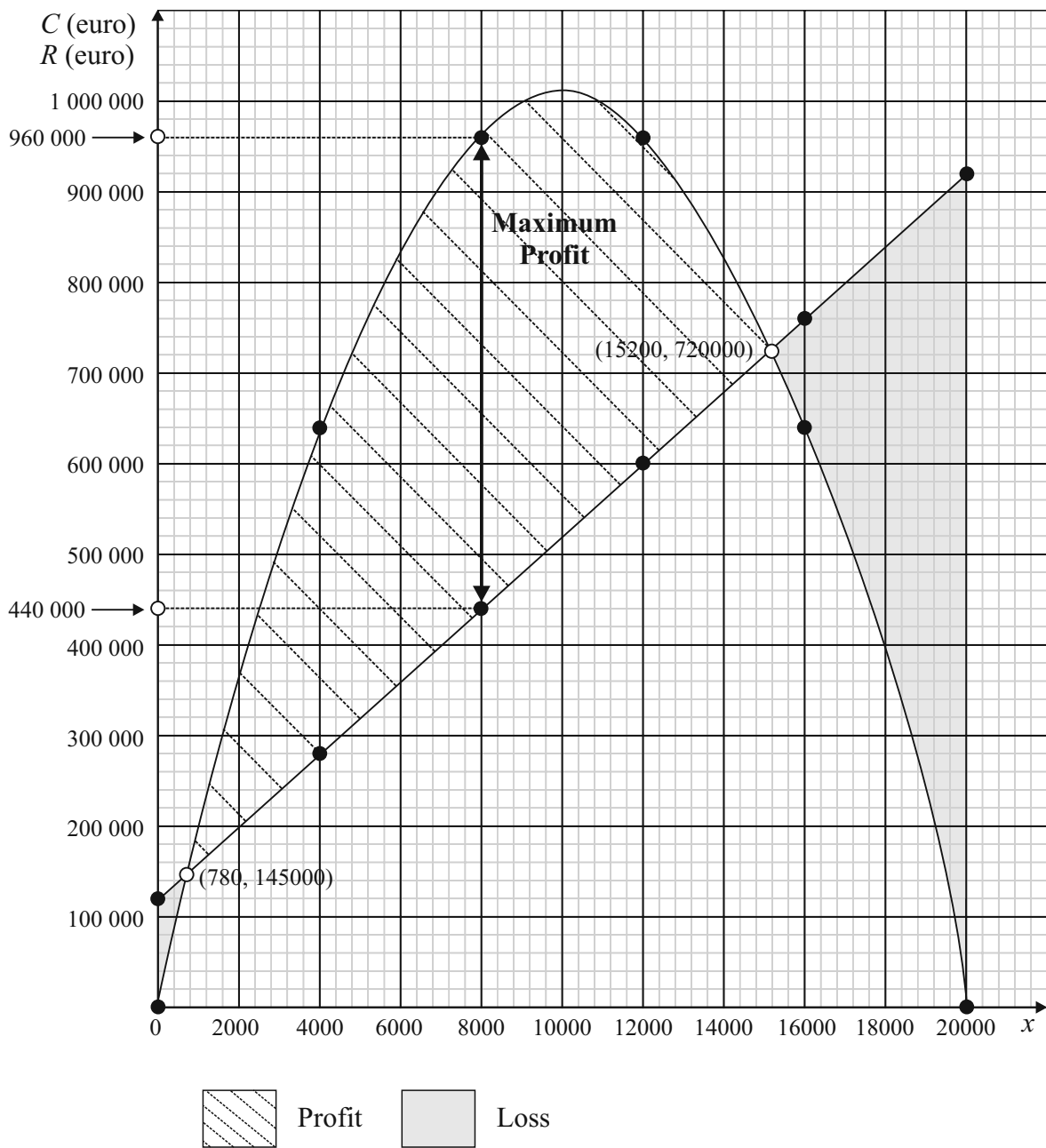
$$\therefore x = 10\,000$$

$$R_{\text{Max.}} = 200(10\,000) - 0.01(10\,000)^2 = \text{€}1\,000\,000$$

**Question 8 (e) (i)**

$x$	$R$ (€)	$C$ (€)
0	0	120 000
4000	640 000	280 000
8000	960 000	440 000
12 000	960 000	600 000
16 000	640 000	760 000
20 000	0	920 000

**Question 8 (e) (ii)**



**Question 8 (f)** (See graph above)

**Question 8 (g)**

$$R = C$$

$$200x - 0.01x^2 = 120\,000 + 40x$$

$$x^2 - 16\,000x + 12\,000\,000 = 0$$

$$x = \frac{-(-16\,000) \pm \sqrt{(-16\,000)^2 - 4(12\,000\,000)}}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = 789, 15\,211$$

Breakeven points from graph: (780, 145 000), (15 200, 720 000)

**Question 8 (h) (i)**

$$\text{CALCULUS: } P = R - C = 200x - 0.01x^2 - 120\,000 - 40x$$

$$= -0.01x^2 + 160x - 120\,000$$

$$\frac{dP}{dx} = -0.02x + 160$$

$$\frac{dP}{dx} = 0 \Rightarrow -0.02x + 160 = 0$$

$$160 = 0.02x$$

$$\therefore x = 8000$$

$$P_{\text{Max.}} = -0.01(8000)^2 + 160(8000) - 120\,000 = \text{€}520\,000$$

GRAPH: The maximum profit is the greatest distance between the  $R$  and  $C$  graphs.

$$P_{\text{Max.}} = \text{€}960\,000 - \text{€}440\,000 = \text{€}520\,000$$

**Question 8 (h) (ii)**

$$x = 8000$$

$$p = 200 - 0.01x$$

$$\therefore p = 200 - 0.01(8000) = \text{€}120$$

**QUESTION 9 (50 MARKS)****Question 9 (a)**

$$\frac{dp}{dt} = kp \text{ [Separate the variables.]}$$

$$\int \frac{dp}{p} = k \int dt$$

$$\ln p = kt + c$$

$$\text{At } t = 0, p = N: \ln N = 0 + c \Rightarrow c = \ln N$$

$$\ln p = kt + \ln N$$

$$\ln p - \ln N = kt$$

$$\ln\left(\frac{p}{N}\right) = kt$$

$$\frac{p}{N} = e^{kt}$$

$$\therefore p = Ne^{kt}$$

**Question 9 (b)**

$$p = Ne^{3t}$$

$$p = 2N \Rightarrow 2N = Ne^{3t}$$

$$2 = e^{3t}$$

$$\ln 2 = 3t$$

$$t = \frac{\ln 2}{3} = 0.231 \text{ days} = 333 \text{ minutes}$$

**Question 9 (f)**

$$p = 1000e^{\sin 3t}$$

$$t = 0: p = 1000e^{\sin 3(0)} = 1000e^0 = 1000$$

$$t = \frac{\pi}{6}: p = 1000e^{\sin 3(\frac{1}{6}\pi)} = 1000e \approx 2700$$

$$t = \frac{\pi}{3}: p = 1000e^{\sin 3(\frac{1}{3}\pi)} = 1000e^0 = 1000$$

$$t = \frac{\pi}{2}: p = 1000e^{\sin 3(\frac{1}{2}\pi)} = 1000e^{-1} = \frac{1000}{e} \approx 370$$

$$t = \frac{2\pi}{3}: p = 1000e^{\sin 3(\frac{2}{3}\pi)} = 1000e^0 = 1000$$

$$t = \frac{5\pi}{6}: p = 1000e^{\sin 3(\frac{5}{6}\pi)} = 1000e \approx 2700$$

**Question 9 (c)**

$$\int \cos mx \, dx = \frac{1}{m} \sin mx + c$$

**Question 9 (d)**

$$\frac{dp}{dt} = mp \cos mt$$

$$\int \frac{dp}{p} = \int m \cos mt \, dt$$

$$\ln p = \frac{m \sin mt}{m} + c$$

$$\ln p = \sin mt + c$$

$$t = 0, p = N: \ln N = \sin 0 + c \Rightarrow c = \ln N$$

$$\ln p = \sin mt + \ln N$$

$$\ln\left(\frac{p}{N}\right) = \sin mt$$

$$p = Ne^{\sin mt}$$

**Question 9 (e)**

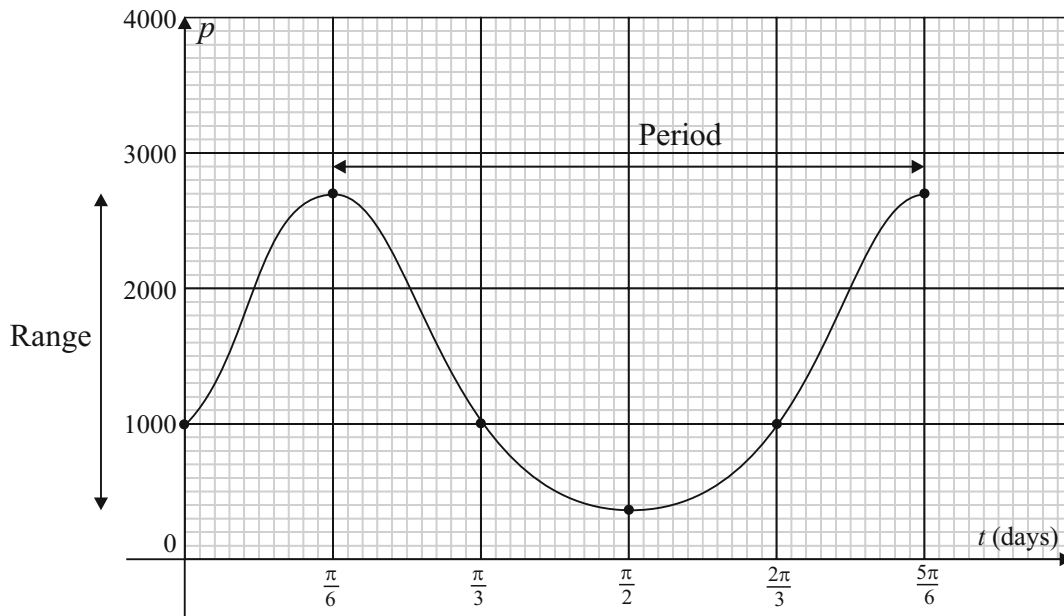
$$p = Ne^{\sin 3t}$$

$$p = 2N: 2N = Ne^{\sin 3t}$$

$$\sin 3t = \ln 2$$

$$t = \frac{1}{3} \sin^{-1}(\ln 2) = 0.255 \text{ days} = 368 \text{ minutes}$$

$t$ (days)	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
$p$	1000	$1000e$	1000	$\frac{1000}{e}$	1000	$1000e$
$p$	1000	2700	1000	370	1000	2700



**Question 9 (f) (i)**

$$\text{Period} = \frac{2\pi}{3} = 2.1 \text{ days}$$

This is the time for the population to return to its original value at any point in the cycle.

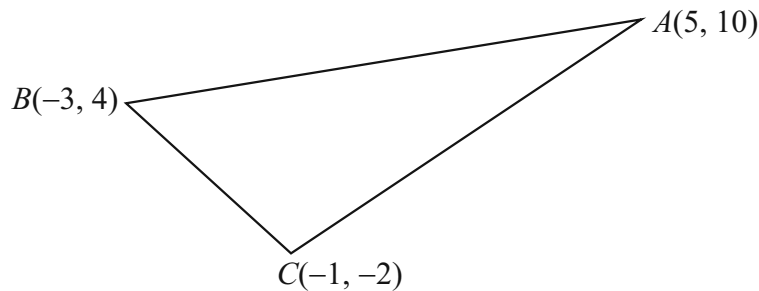
**Question 9 (f) (ii)**

$$\text{Range} = \left[ \frac{1000}{e}, 1000e \right]$$

This represents the population in a cycle from its maximum to minimum values.

**SAMPLE PAPER 1: PAPER 2**

**QUESTION 1 (25 MARKS)**



**Question 1 (a)**

$$A(5, 10), B(-3, 4)$$

$$\text{Mid-point} = \left( \frac{5-3}{2}, \frac{10+4}{2} \right) = (1, 7)$$

$$m = \frac{4-10}{-3-5} = \frac{-6}{-8} = \frac{3}{4} \Rightarrow m_{\perp} = -\frac{4}{3}$$

$$k: (y-7) = -\frac{4}{3}(x-1)$$

$$3(y-7) = -4(x-1)$$

$$3y-21 = -4x+4$$

$$4x+3y-25=0$$

$$A(5, 10), C(-1, -2)$$

$$\text{Mid-point} = \left( \frac{5-1}{2}, \frac{10-2}{2} \right) = (2, 4)$$

$$m = \frac{-2-10}{-1-5} = \frac{-12}{-6} = 2 \Rightarrow m_{\perp} = -\frac{1}{2}$$

$$l: (y-4) = -\frac{1}{2}(x-2)$$

$$2(y-4) = -1(x-2)$$

$$2y-8 = -x+2$$

$$x+2y-10=0$$

**Question 1 (b)**

$$4x+3y-25=0$$

$$x+2y-10=0(\times -4)$$

[Solve the equations simultaneously for  $k$  and  $l$  to find their point of intersection  $D$ .]

$$4x+3y-25=0$$

$$-4x-8y+40=0$$

$$\hline -5y+15=0 \Rightarrow y=3$$

$$x+2(3)-10=0$$

$$x+6-10=0$$

$$\therefore x=4$$

Circumcentre  $D(4, 3)$

$$r = |AD| = \sqrt{(5-4)^2 + (10-3)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$\text{Equation of circumcircle: } (x-4)^2 + (y-3)^2 = (\sqrt{50})^2$$

$$(x-4)^2 + (y-3)^2 = 50$$

## QUESTION 2 (25 MARKS)

### Question 2 (a)

$$s_1 : x^2 + y^2 - 9 = 0$$

$$x^2 + y^2 = 9$$

Centre  $(0, 0)$ ,  $r = 3$

$$s_2 : x^2 + y^2 - 6y + 8 = 0$$

Centre  $(-g, -f) = (0, 3)$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{0 + 9 - 8} = \sqrt{1} = 1$$

$$\sqrt{(h-0)^2 + (k-3)^2} = (r+1) \Rightarrow h^2 + (k-3)^2 = (r+1)^2$$

$$\sqrt{(h-0)^2 + (k-0)^2} = (r+3) \Rightarrow h^2 + k^2 = (r+3)^2$$

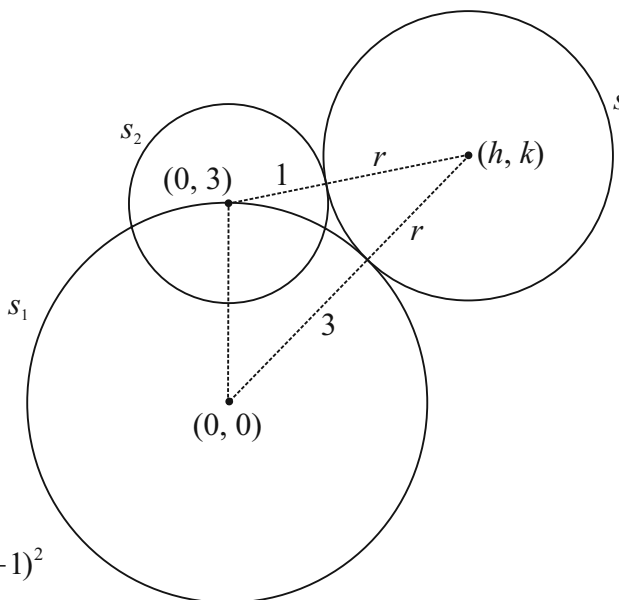
$$(k-3)^2 - k^2 = (r+1)^2 - (r+3)^2$$

$$(k-3+k)(k-3-k) = (r+1+r+3)(r+1-r-3) \quad \boxed{a^2 - b^2 = (a-b)(a+b)}$$

$$(2k-3)(-3) = (2r+4)(-2)$$

$$-6k+9 = -4r-8$$

$$4r+17 = 6k$$



### Question 2 (b)

$$h^2 + k^2 = (r+3)^2$$

$$4r+17 = 6k \Rightarrow r = \frac{6k-17}{4}$$

$$h^2 + k^2 = \left( \frac{6k-17}{4} + 3 \right)^2$$

$$h^2 + k^2 = \left( \frac{6k-17+12}{4} \right)^2 = \left( \frac{6k-5}{4} \right)^2$$

$$h^2 = \frac{(6k-5)^2}{16} - k^2$$

$$16h^2 = (6k-5)^2 - 16k^2 \quad \boxed{a^2 - b^2 = (a-b)(a+b)}$$

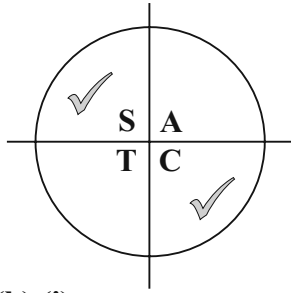
$$16h^2 = (6k-5+4k)(6k-5-4k)$$

$$16h^2 = (10k-5)(2k-5)$$

$$\therefore 16h^2 = 5(2k-1)(2k-5)$$

**QUESTION 3 (25 MARKS)**

**Question 3 (a) (i)**



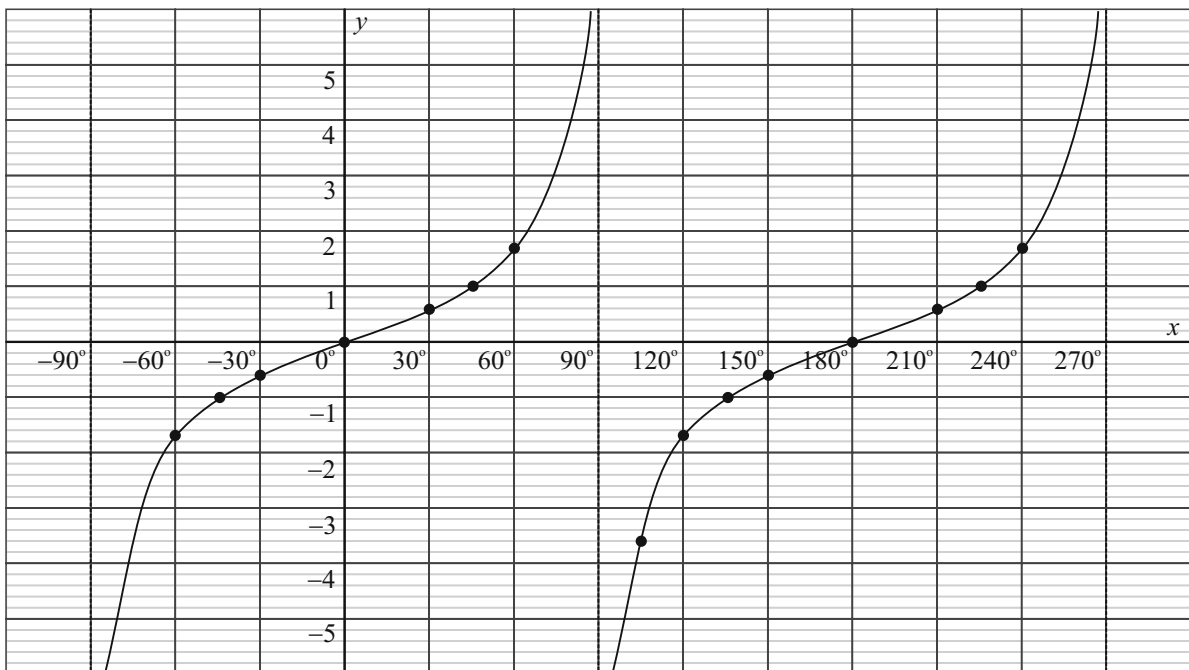
**Question 3 (a) (ii)**

$$\begin{aligned} \tan 330^\circ &= \tan(360^\circ - 30^\circ) \\ &= -\tan 30^\circ \\ &= -\frac{1}{\sqrt{3}} \end{aligned}$$

**Question 3 (b) (i)**

$x^\circ$	-90	-60	-45	-30	0	30	45	60	90
$y$	$-\infty$	-1.7	-1	-0.6	0	0.6	1	1.7	$\infty$

$x^\circ$	105	120	135	150	180	210	225	240	270
$y$	-3.7	-1.7	-1	-0.6	0	0.6	1	1.7	$-\infty$



**Question 3 (b) (ii)**

Yes, because each value of  $x$  only gives one value of  $y$ .

**Question 3 (b) (iii)**

Period =  $180^\circ$  (function repeats itself every  $180^\circ$ )

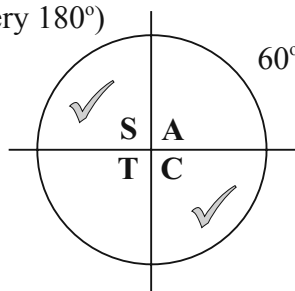
Range =  $[-\infty, \infty]$

**Question 3 (b) (iv)**

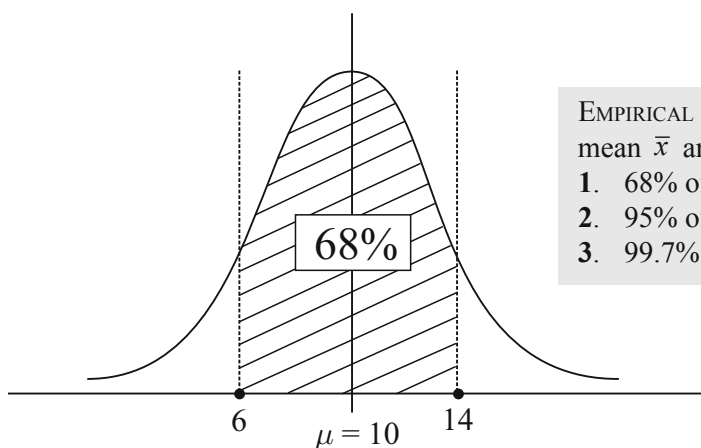
$$\tan x = \sqrt{3} \Rightarrow x = \tan^{-1}(\sqrt{3}) = 60^\circ$$

$$\text{Second quadrant: } x = 180^\circ - 60^\circ = 120^\circ$$

$$\text{Fourth quadrant: } x = 360^\circ - 60^\circ = 300^\circ$$



### QUESTION 4 (25 MARKS)



EMPIRICAL RULE: In any normal distribution with mean  $\bar{x}$  and standard deviation  $\sigma$ .

1. 68% of the data falls within  $1\sigma$  of the mean  $\bar{x}$ .
2. 95% of the data falls within  $2\sigma$  of the mean  $\bar{x}$ .
3. 99.7% of the data falls within  $3\sigma$  of the mean  $\bar{x}$ .

#### Question 4 (a) (i)

Standard deviation = 4  
Median = 10

#### Question 4 (a) (ii)

$$x = 10 : z = \frac{10 - 10}{4} = 0$$

$$x = 14 : z = \frac{14 - 10}{4} = 1$$

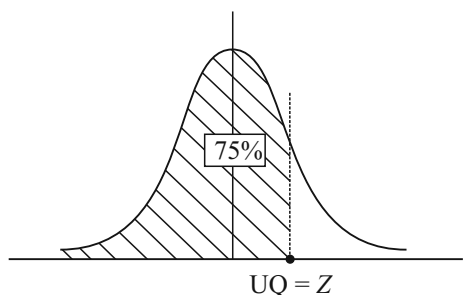
$$x = 6 : z = \frac{6 - 10}{4} = -1$$

$$z = \frac{x - \mu}{\sigma}$$

#### Question 4 (b)

The standard normal distribution has a mean of **0** and standard deviation of **1** and has the same shape and area distribution as the original normal distribution.

#### Question 4 (c) (i)



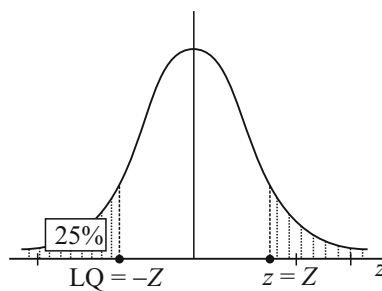
$$P(z < Z) = 0.75$$

$$\therefore z = 0.68 = \frac{x - 10}{4}$$

$$2.72 = x - 10$$

$$\therefore x = 12.72 \text{ (Upper Quartile)}$$

#### Question 4 (c) (ii)



$$z = -0.68 = \frac{x - 10}{4}$$

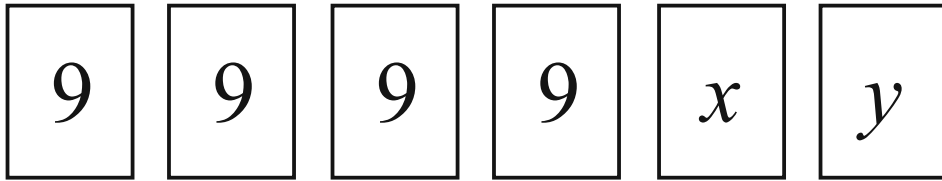
$$-2.72 = x - 10$$

$$\therefore x = 7.28 \text{ (Lower Quartile)}$$

#### Question 4 (c) (iii)

$$\text{Interquartile range} = \text{UQ} - \text{LQ} = 12.72 - 7.28 = 5.44$$

**QUESTION 5 (25 MARKS)**



**Question 5 (a)**

(i) Median =  $\frac{9+9}{2} = 9$

(ii) Mode = 9

(iii) Range cannot be worked out without knowing the values of  $x$  and  $y$ .

Possibilities for arranging the numbers in ascending order:  
 9, 9, 9, 9,  $x$ ,  $y$   
 $x$ ,  $y$ , 9, 9, 9, 9  
 $x$ , 9, 9, 9, 9,  $y$   
 $y$ , 9, 9, 9, 9,  $x$   
 9, 9, 9, 9, 9, 9

**Question 5 (b)**

Mean = 9

Therefore sum =  $9 \times 6 = 54$

$x + y = 54 - 36 = 18$

Possibilities for  $x$  and  $y$ :

$x$	1	2	3	4	5	6	7	8	9
$y$	17	16	15	14	13	12	11	10	9

OR

$x$	17	16	15	14	13	12	11	10	9
$y$	1	2	3	4	5	6	7	8	9

Greatest range = [1, 17]

$R = 17 - 1 = 16$

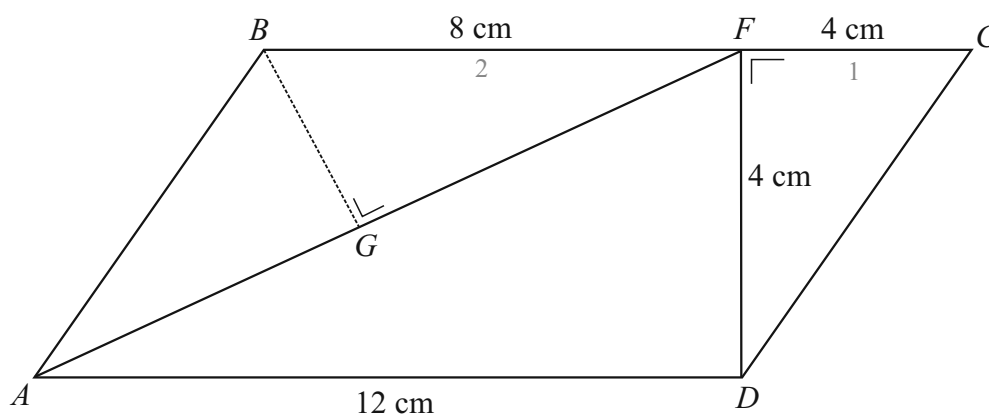
**Question 5 (c)**

$R = x - y = 6$

$\therefore 2x = 24 \Rightarrow x = 12$

$\therefore y = 6$

**QUESTION 6 (25 MARKS)**



**Question 6 (a)**

$$|AD| = |BC| = 12 \text{ cm}$$

$$|BF| = 2|FC|$$

$$\therefore |BF| = 8 \text{ cm}, |FC| = 4 \text{ cm}$$

**Question 6 (b)**

$$\text{Triangle } FCD: A = \frac{1}{2}(4)(4) = 8 \text{ cm}^2$$

$$\text{Triangle } AFD: A = \frac{1}{2}(12)(4) = 24 \text{ cm}^2$$

$$\text{Parallelogram } ABCD: A = (12)(4) = 48 \text{ cm}^2$$

$$\text{Triangle } BFA: A = 48 - 8 - 24 = 16 \text{ cm}^2$$

**Question 6 (c)**

$$|AF|^2 = 4^2 + 12^2 = 16 + 144 = 160 \Rightarrow |AF| = \sqrt{160} = 4\sqrt{10}$$

$$\frac{1}{2}(4\sqrt{10})|BG| = 16$$

$$\therefore |BG| = \frac{4\sqrt{10}}{5} \text{ cm}$$

**QUESTION 7 (75 MARKS)**

**Question 7 (a) (i)**

$$\mu = 120$$

$$\sigma = 15$$

$$135 = 120 + 15 = \mu + 1\sigma$$

$$105 = 120 - 15 = \mu - 1\sigma$$

EMPIRICAL RULE: 68% of young adults have a blood pressure reading of between 105 mm of Hg and 135 mm of Hg.

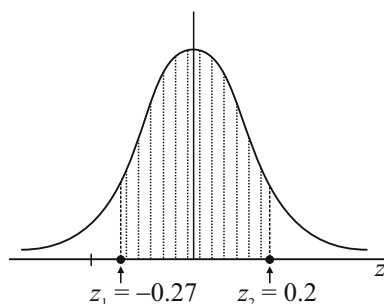
**Question 7 (a) (ii)**

$$\mu = 120$$

$$\sigma = 15$$

$$x_1 = 116: z_1 = \frac{116 - 120}{15} = -0.27$$

$$x_2 = 123: z_2 = \frac{123 - 120}{15} = 0.2$$



$$\begin{aligned}
P &= P(-\infty \text{ to } 0.2) - P(-\infty \text{ to } -0.27) \\
&= P(-\infty \text{ to } 0.2) - P(z > 0.27) \\
&= P(-\infty \text{ to } 0.2) - \{1 - P(z < 0.27)\} \\
&= 0.5793 - 1 + 0.6064 \\
&= 0.1857
\end{aligned}$$

From a sample of 100 people the number will be  $100 \times 0.1857 = 18$  or 19 people

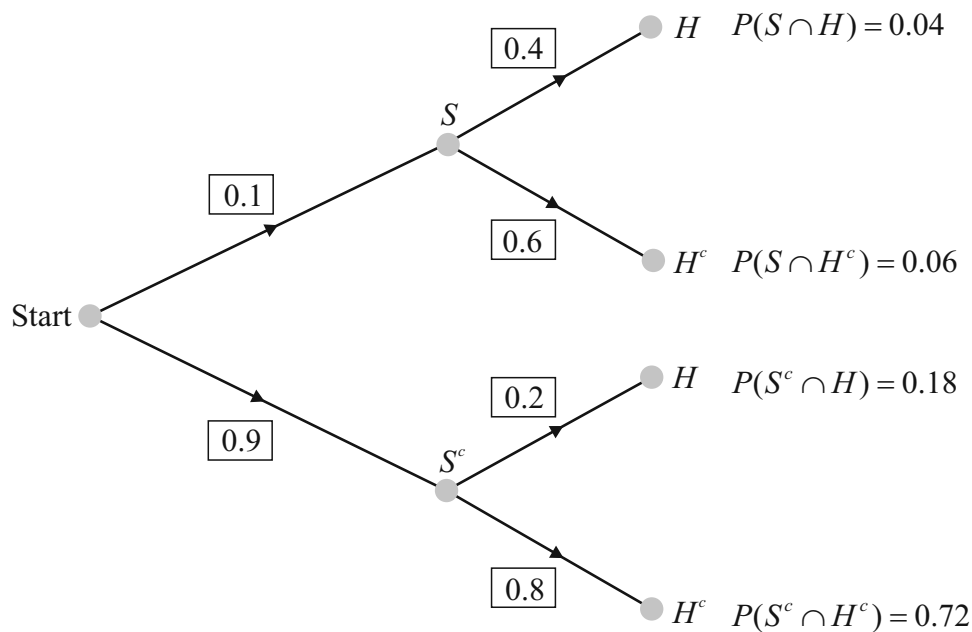
**Question 7 (b)**

- (i)  $P(S) = 0.1$
- (ii)  $P(H|S) = 0.4$
- (iii)  $P(H|S^c) = 0.2$

**Question 7 (c)**

$$P(S|H) = \frac{P(H \cap S)}{P(H)}$$

**Question 7 (d)**



**Question 7 (e)**

$$\begin{aligned}
P(H) &= P(S \cap H) + P(S^c \cap H) \\
&= 0.04 + 0.18 \\
&= 0.22
\end{aligned}$$

**Question 7 (f)**

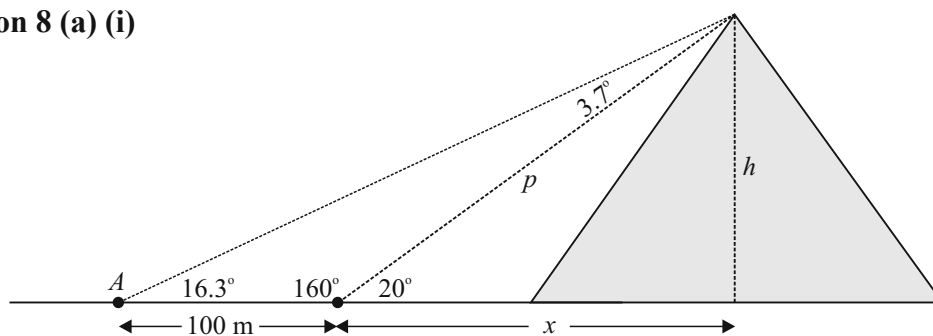
$$P(S|H) = \frac{P(S \cap H)}{P(H)} = \frac{0.04}{0.22} = 0.182$$

**Question 7 (g)**

$$\begin{aligned}
P(\text{Having a stroke}) &= 0.182 \\
P(\text{Not having a stroke}) &= 0.818 \\
P(\text{At least 2 out of 10 people will suffer a stroke}) \\
&= 1 - P(2 \text{ out of 10 people will suffer a stroke}) \\
&= 1 - \{ {}^{10}C_0 (0.182)^0 (0.818)^{10} + {}^{10}C_1 (0.182)^1 (0.818)^9 \} \\
&= 0.57
\end{aligned}$$

**QUESTION 8 (75 MARKS)**

**Question 8 (a) (i)**



**Question 8 (a) (ii)**

$$\frac{100}{\sin 3.7^\circ} = \frac{p}{\sin 16.3^\circ} \quad \boxed{\frac{a}{\sin A} = \frac{b}{\sin B}}$$

$$\therefore p = \frac{100 \sin 16.3^\circ}{\sin 3.7^\circ} = 434.92 \text{ m}$$

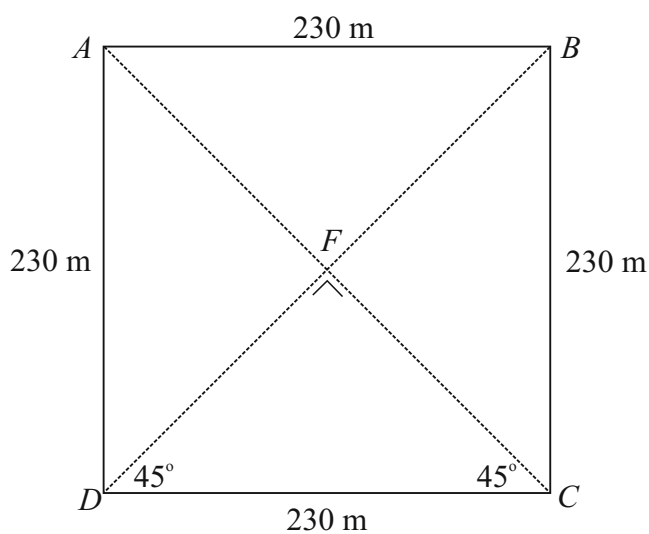
$$\sin 20^\circ = \frac{h}{434.92}$$

$$\therefore h = 434.92 \sin 20^\circ = 148.75 \approx 149 \text{ m}$$

**Question 8 (b)**

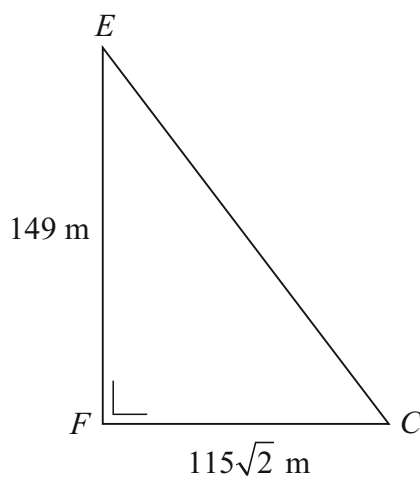
$$\cos 45^\circ = \frac{|FC|}{230}$$

$$|FC| = 230 \cos 45^\circ = 115\sqrt{2} \text{ m}$$

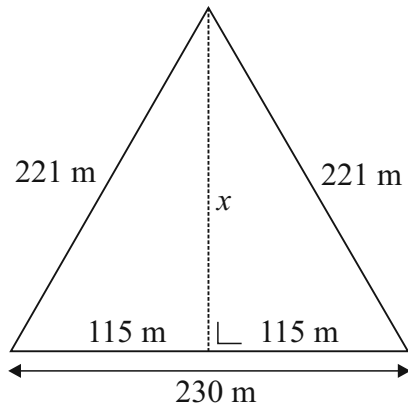


**Question 8 (c)**

$$|EC| = \sqrt{149^2 + (115\sqrt{2})^2} = 221 \text{ m}$$

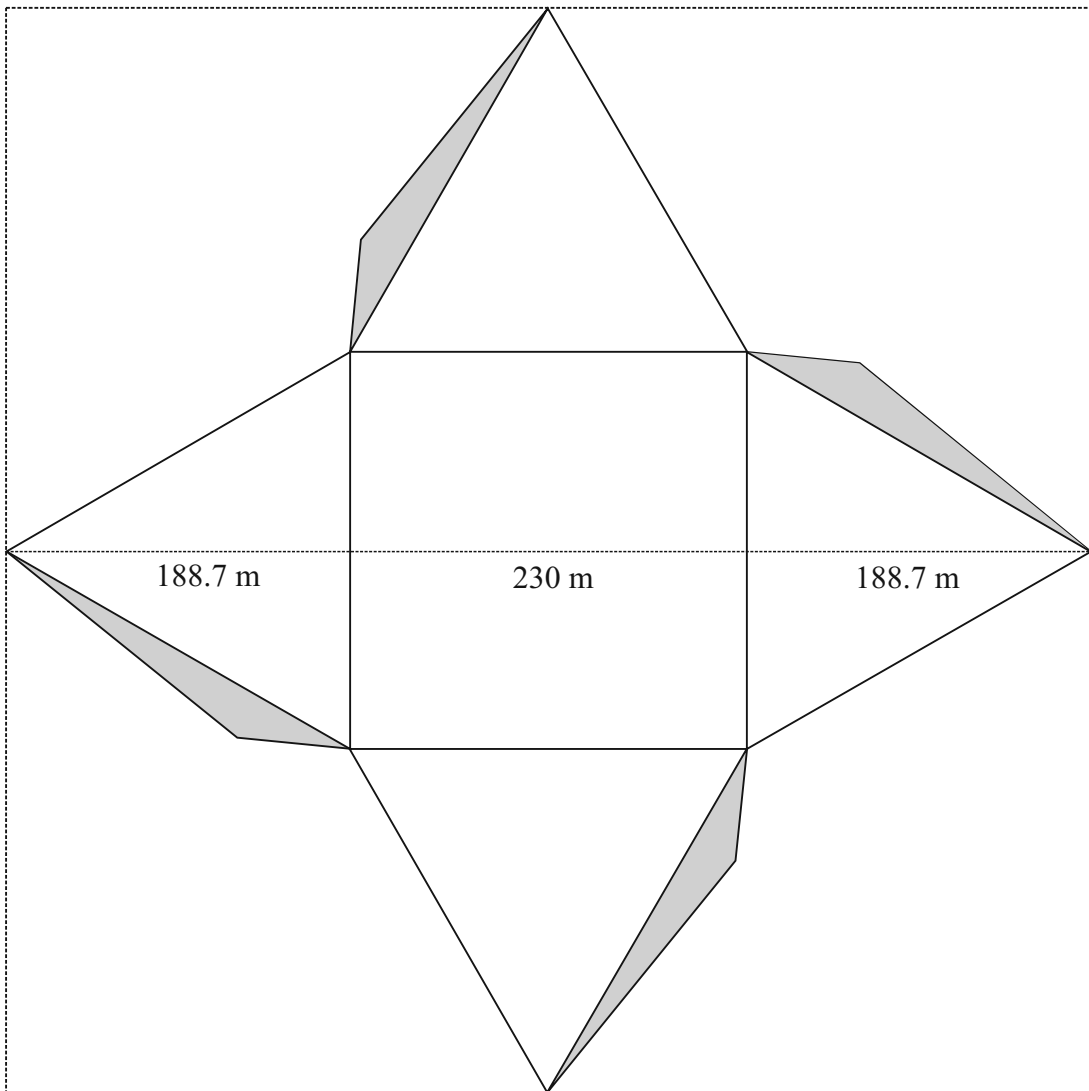


Question 8 (d)



$$115^2 + x^2 = 221^2$$

$$x = \sqrt{221^2 - 115^2} = 188.7 \text{ m}$$



$$\text{Length of square} = \frac{2(188.7) + 230}{50} = 12.15 \text{ m}$$

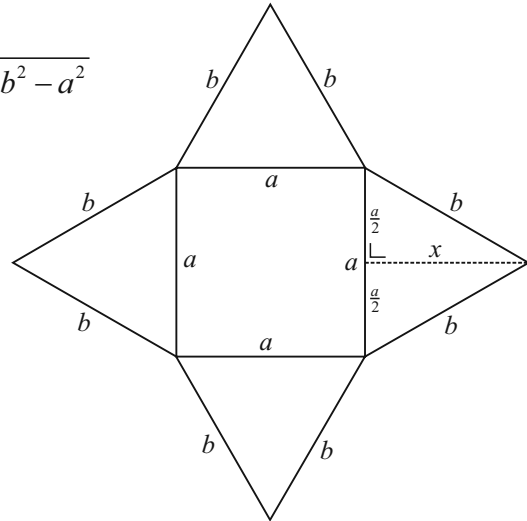
**Question 8 (e)**

$$\left(\frac{a}{2}\right)^2 + x^2 = b^2 \Rightarrow x = \sqrt{b^2 - \frac{a^2}{4}} = \sqrt{\frac{4b^2 - a^2}{4}} = \frac{1}{2}\sqrt{4b^2 - a^2}$$

Area of 4 triangles:

$$A = 4\left(\frac{1}{2} \times \text{base} \times \text{height}\right) = 2 \times \text{base} \times \text{height}$$

$$= 2a \times \frac{1}{2}\sqrt{4b^2 - a^2} = a\sqrt{4b^2 - a^2}$$



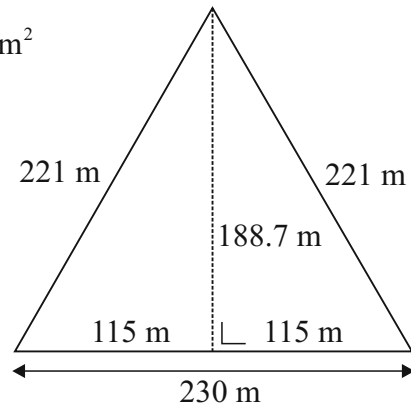
**Question 8 (f) (i)**

Surface area:  $A = a\sqrt{4b^2 - a^2} = 230\sqrt{4(221)^2 - 230^2} = 86\,812 \text{ m}^2$

**Question 8 (f) (ii)**

Area of a casing stone =  $(0.86 \text{ m})(0.86 \text{ m}) = 0.7396 \text{ m}^2$

Number of stones =  $\frac{86\,812}{0.7396} \approx 117\,000$



**SAMPLE PAPER 2: PAPER 1**

**QUESTION 1 (25 MARKS)**

**Question 1 (a)**

$$\log_9(x+2) - \log_3 x = 0 \quad \boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

$$\log_9(x+2) - \frac{\log_9 x}{\log_9 3} = 0$$

$$\log_9(x+2) - \frac{\log_9 x}{\frac{1}{2}} = 0$$

$$\log_9(x+2) - 2\log_9 x = 0 \quad \boxed{\log_a \left( \frac{x}{y} \right) = \log_a x - \log_a y}$$

$$\log_9 \frac{x+2}{x^2} = 0$$

$$\frac{x+2}{x^2} = 9^0 = 1 \quad \boxed{a^x = y \Leftrightarrow \log_a y = x}$$

$$x+2 = x^2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad [x = -1 \text{ is not allowed as } x > 0.]$$

**Question 1 (b) (i)**

$$\binom{5}{3} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$

$$\binom{5}{2} = \frac{5 \times 4}{2 \times 1} = 10$$

**Question 1 (b) (ii)**

$$\begin{aligned} (p+q)^5 &= \binom{5}{0} p^5 q^0 + \binom{5}{1} p^4 q^1 + \binom{5}{2} p^3 q^2 + \binom{5}{3} p^2 q^3 + \binom{5}{4} p^1 q^4 + \binom{5}{5} p^0 q^5 \\ &= p^5 + 5p^4 q + 10p^3 q^2 + 10p^2 q^3 + 5p^1 q^4 + q^5 \end{aligned}$$

**Question 1 (b) (iii)**

$$T_3 = 10p^3 q^2 = 10(0.6)^3 (0.4)^2 = 0.3456$$

## QUESTION 2 (25 MARKS)

### Question 2 (a)

$$z = 2 + 2\sqrt{3}i$$

$$|z| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4 \quad \boxed{z = r(\cos \theta + i \sin \theta)}$$

$$\tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3} \quad \boxed{|z| = r, \arg z = \theta}$$

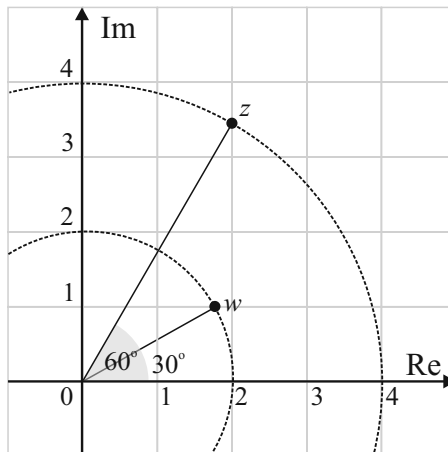
$$\therefore z = 4 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$w = \sqrt{3} + i$$

$$|w| = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$$

$$\therefore w = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$



### Question 2 (b) (i)

$$\begin{aligned} zw &= 4 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \\ &= 8 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \\ &= 8(0 + i) \\ &= 8i \end{aligned}$$

### Question 2 (c) (i)

$$\begin{aligned} \frac{z}{w} &= \frac{4 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}{2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)} = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \\ &= 2 \left( \frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt{3} + i \end{aligned}$$

### Question 2 (c) (iii)

$$\begin{aligned} \left( \frac{z}{w} \right)^{14} &= 2^{14} \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)^{14} \quad \boxed{[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)} \quad (\text{De Moivre's Theorem}) \\ &= 2^{14} \left( \cos \frac{14\pi}{6} + i \sin \frac{14\pi}{6} \right) \\ &= 2^{14} \left( \cos \frac{7\pi}{3} + i \sin \frac{7\pi}{3} \right) = 2^{14} \left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = 2^{13} (1 + \sqrt{3}i) \end{aligned}$$

### Question 2 (b) (ii)

‘According to De Moivre’s theorem, when two complex numbers are multiplied together, the arguments are added and the moduli are multiplied to form the new complex number.’

### Question 2 (c) (ii)

‘According to De Moivre’s theorem, when two complex numbers are divided, the arguments are subtracted and the moduli are divided to form the new complex number.’

### QUESTION 3 (25 MARKS)

#### Question 3 (a)

100, 90, 81,.....

$$a = 100, r = 0.9$$

Which term is equal to 50, half of its original mass?

$$50 = 100(0.9)^{n-1}$$

$$0.5 = 0.9^{n-1}$$

$$\log_{10}(0.5) = (n-1)\log_{10}(0.9)$$

$$\therefore n = \frac{\log_{10}(0.5)}{\log_{10}(0.9)} + 1 = 7.58 \text{ days}$$

Seven days must elapse before it is less than half its original mass.

#### Question 3 (b)

$$S_n = \frac{108(1 - (\frac{1}{3})^n)}{\frac{2}{3}} = 162(1 - (\frac{1}{3})^n)$$

$$S_\infty = \frac{108}{\frac{2}{3}} = 162$$

$$S_n - S_\infty = 0.05$$

$$162(1 - (\frac{1}{3})^n) - 162 = 0.05$$

$$162(\frac{1}{3})^n = 0.05$$

$$3^n = 3240$$

$$n \log_{10} 3 = \log_{10} 3240$$

$$\therefore n = \frac{\log_{10} 3240}{\log_{10} 3} = 7.36$$

Eight terms must be added together so that the sum differs from the sum to infinity by less than 0.05.

### QUESTION 4 (25 MARKS)

#### Question 4 (a)

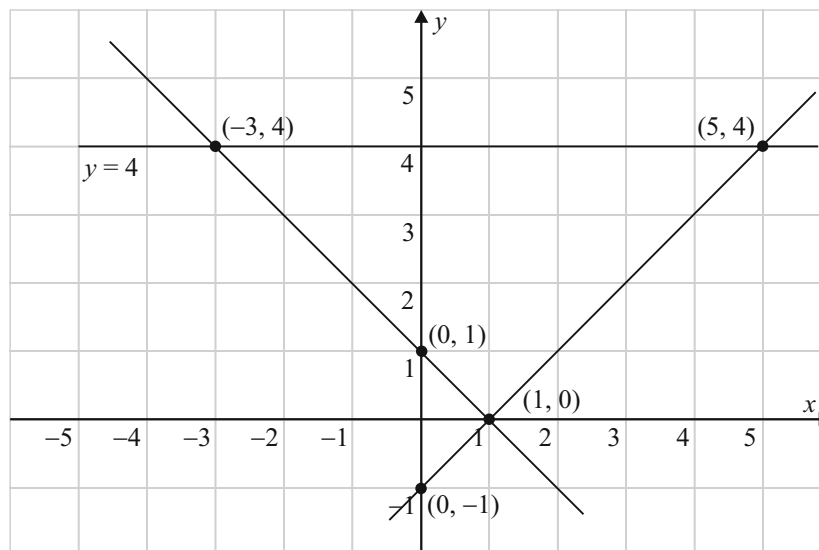
Draw each of the straight line graphs by plotting their intercepts.

$$y = |x - 1| \Rightarrow y = \pm(x - 1)$$

$$y = x - 1 \rightarrow \text{Intercepts: } (0, -1), (1, 0)$$

$$y = -x + 1 \rightarrow \text{Intercepts: } (0, 1), (1, 0)$$

Point of intersection: (1, 0)



#### Question 4 (b)

Points of intersection: (-3, 4), (5, 4)

#### Question 4 (c)

$$x < -3, x > 5$$

**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

Yes. Each value of  $y$  corresponds to only one value of  $x$ .

**Question 5 (b)**

$$f(x) = y = 2^x$$

$$\log_2 y = \log_2 2^x$$

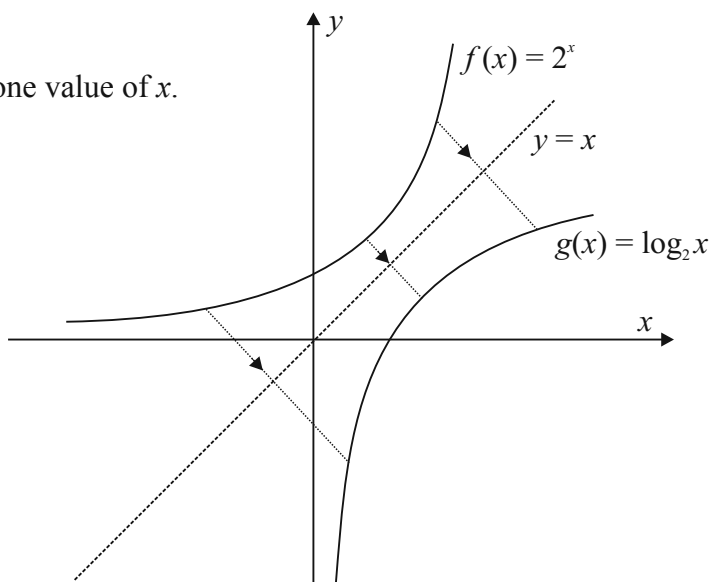
$$\log_2 y = x \log_2 2$$

$$\log_2 y = x$$

$$\therefore f^{-1}(x) = \log_2 y = g(x)$$

$g(x)$  is the inverse function of  $f(x)$ .

Reflect the curve for  $f(x)$  by an axial symmetry through the line  $y = x$  to get its inverse function.



**Question 5 (c)**

It is not injective because certain values of  $y$  correspond to two values of  $x$ .

**Question 5 (d)**

$$x = 0 \Rightarrow y = 2^0 = 1$$

$$\therefore A(0, 1)$$

$$(0, 1) \in h(x) \Rightarrow 1 = -(-1)^2 + a$$

$$\therefore a = 2$$

$$h(x) = 2 - (x-1)^2$$

**Question 5 (e)**

Local maximum is at  $y = 2$ .

$$h(x) = 2 - (x-1)^2$$

$$\therefore 2 = 2 - (x-1)^2$$

$$(x-1)^2 = 0$$

$$x-1 = 0$$

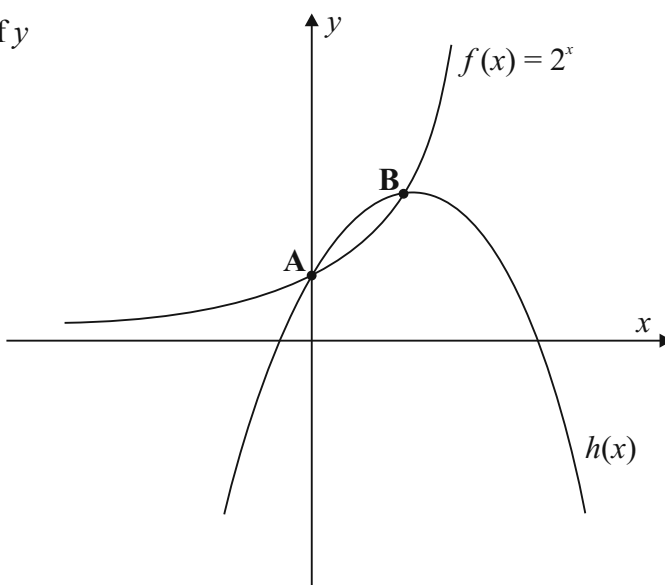
$$\therefore x = 1$$

Local Maximum  $B(1, 2)$

$B(1, 2) \in f(x)$ ?

$$f(x) = 2^x$$

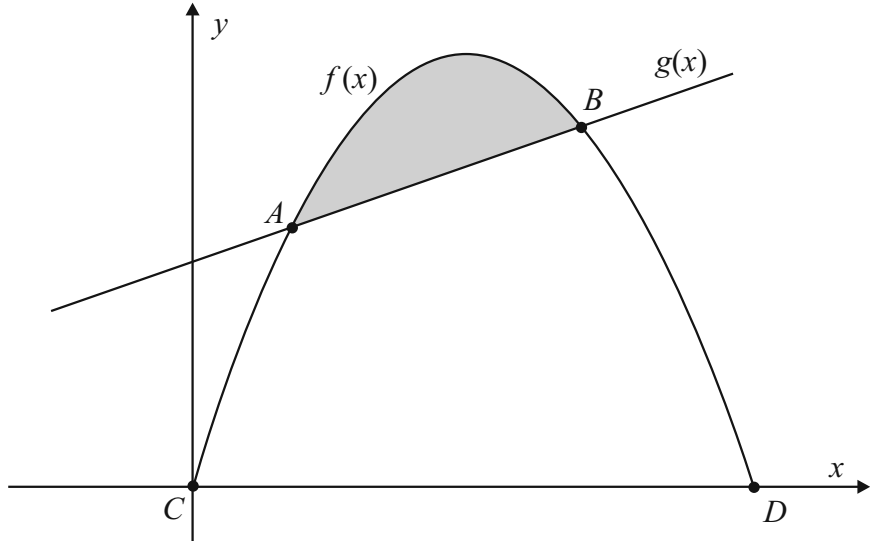
$$2 = 2^1 \text{ (True)}$$



**QUESTION 6 (25 MARKS)**

**Question 6 (a)**

$$\begin{aligned}
 f(x) &= g(x) \\
 5x - x^2 &= x + 3 \\
 0 &= x^2 - 4x + 3 \\
 0 &= (x-1)(x-3) \\
 \therefore x &= 1, 3 \\
 g(1) &= 1 + 3 = 4 \\
 g(3) &= 3 + 3 = 6 \\
 A(1, 4), B(3, 6)
 \end{aligned}$$



**Question 6 (b)**

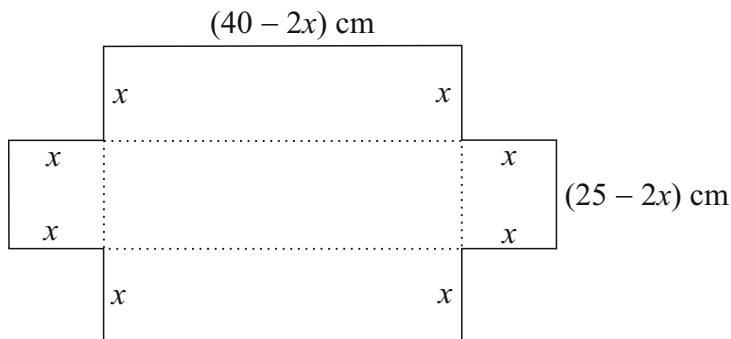
$$\begin{aligned}
 |A| &= \int_1^3 (f(x) - g(x)) dx \\
 &= \int_1^3 (-x^2 + 4x - 3) dx \\
 &= \left[-\frac{1}{3}x^3 + 2x^2 - 3x\right]_1^3 \\
 &= \left\{-\frac{1}{3}(3)^3 + 2(3)^2 - 3(3)\right\} - \left\{-\frac{1}{3}(1)^3 + 2(1)^2 - 3(1)\right\} \\
 &= -9 + 18 - 9 + \frac{1}{3} - 2 + 3 \\
 &= \frac{4}{3}
 \end{aligned}$$

**Question 6 (c)**

$$\begin{aligned}
 C(0, 0), D(5, 0) \\
 \text{Average value} &= \frac{1}{5-0} \int_0^5 f(x) dx \\
 &= \frac{1}{5} \int_0^5 (5x - x^2) dx \\
 &= \frac{1}{5} \left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5 \\
 &= \frac{1}{5} \left\{\frac{5}{2}(5)^2 - \frac{1}{3}(5)^3\right\} - \{0\} \\
 &= \frac{1}{5} \left(\frac{125}{2} - \frac{125}{3}\right) = \frac{25}{6}
 \end{aligned}$$

**QUESTION 7 (50 MARKS)**

**Question 7 (a) (i)**



$$l = 40 - 2x, b = 25 - 2x, h = x$$

**Question 7 (b) (i)**

$$\begin{aligned}
 S &= 2(40 - 2x)x + 2(25 - 2x)x + (40 - 2x)(25 - 2x) \\
 &= 80x - 4x^2 + 50x - 4x^2 + 1000 - 80x - 50x + 4x^2 \\
 &= 1000 - 4x^2
 \end{aligned}$$

**Question 7 (a) (ii)**

$$\begin{aligned}
 V &= (40 - 2x)(25 - 2x)x \\
 &= (1000 - 80x - 50x + 4x^2)x \\
 &= (1000 - 130x + 4x^2)x \\
 &= 1000x - 130x^2 + 4x^3
 \end{aligned}$$

**Question 7 (b) (ii)**

$$\begin{aligned}
 600 &= 1000 - 4x^2 \\
 4x^2 &= 400 \\
 x^2 &= 100 \\
 \therefore x &= 10 \text{ cm}
 \end{aligned}$$

**Question 7 (c)**

$$V = 4x^3 - 130x^2 + 1000x$$

$$4x^3 - 130x^2 + 1000x = 1512$$

$$2x^3 - 65x + 500x - 756 = 0$$

$$x = 1: 2(1)^3 - 65(1) + 500(1) - 756 = -319 \neq 0$$

$$x = 2: 2(2)^3 - 65(2) + 500(2) - 756 = 0$$

**Question 7 (d)**

$$V = 4x^3 - 130x^2 + 1000x$$

$$\frac{dV}{dx} = 12x^2 - 260x + 1000$$

Maximum Volume:  $\frac{dV}{dx} = 0$

$$\frac{dV}{dx} = 0 \Rightarrow 12x^2 - 260x + 1000 = 0$$

$$3x^2 - 65x + 250 = 0$$

$$(3x - 50)(x - 5) = 0$$

$$x = 5, \frac{50}{3} \text{ cm} \quad [\text{Ignore the second solution as it will cause one of the sides to be negative.}]$$

$$V_{\text{Max.}} = 4(5)^3 - 130(5)^2 + 1000(5) = 500 - 3250 + 5000 = 2250 \text{ cm}^3$$

$$l = 40 - 2(5) = 30 \text{ cm}, b = 25 - 2(5) = 15 \text{ cm}, h = 5 \text{ cm}$$

**Question 7 (e)**

$$\frac{dV}{dt} = -(2t + 5)$$

$$V = -\int (2t + 5) dt = -t^2 - 5t + c$$

$$V = 2250 \text{ when } t = 0: 2250 = c$$

$$\therefore V = -t^2 - 5t + 2250$$

$$-t^2 - 5t + 2250 = 0$$

$$t^2 + 5t - 2250 = 0$$

$$(t - 45)(t + 50) = 0$$

$$\therefore t = 45 \text{ s}$$

**Question 7 (f) (i)**

$$\tan 45^\circ = \frac{r}{h} \Rightarrow 1 = \frac{r}{h}$$

$$\therefore r = h$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^3$$

**Question 7 (f) (ii)**

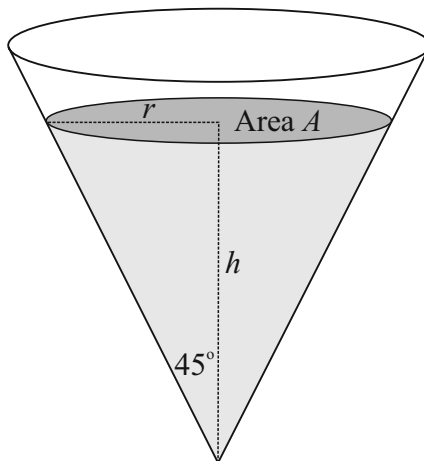
$$V = \frac{1}{3} \pi r^3$$

$$\frac{dV}{dt} = \pi r^2 \times \frac{dr}{dt} = 3 \Rightarrow \frac{dr}{dt} = \frac{3}{\pi r^2}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt} = 2\pi r \times \frac{3}{\pi r^2} = \frac{6}{r}$$

$$\left( \frac{dA}{dt} \right)_{r=2} = \frac{6}{2} = 3 \text{ cm}^2/\text{s}$$



## QUESTION 8 (50 MARKS)

### Question 8 (a)

$$M = \log_{10} \left( \frac{s}{s} \right) = \log_{10} 1 = 0$$

### Question 8 (b)

$$M_1 = \log_{10} \left( \frac{A_1}{s} \right) = 8.3$$

$$M_2 = \log_{10} \left( \frac{4A_1}{s} \right)$$

$$M_2 - M_1 = \log_{10} \left( \frac{4A_1}{s} \right) - \log_{10} \left( \frac{A_1}{s} \right)$$

$$M_2 - 8.3 = \log_{10} \left( \frac{4A_1 \times s}{s A_1} \right)$$

$$M_2 - 8.3 = \log_{10} 4$$

$$\therefore M_2 = \log_{10} 4 + 8.3 = 8.9$$

### Question 8 (d)

$$M_1 = 5.9, A = A_1$$

$$M_1 = \log_{10} \left( \frac{A_1}{s} \right) = 5.9 \Rightarrow \frac{A_1}{s} = 10^{5.9}$$

$$M_2 = 6.5, A = A_2$$

$$M_2 = \log_{10} \left( \frac{A_2}{s} \right) = 6.5 \Rightarrow \frac{A_2}{s} = 10^{6.5}$$

$$\text{Fractional change in amplitude} = \frac{A_2 - A_1}{A_1} = \frac{A_2}{A_1} - 1$$

$$= \frac{10^{6.5} \times s}{10^{5.9} \times s} - 1 = 10^{0.6} - 1 = 2.98$$

$$\% \text{ change} = 298\%$$

### Question 8 (e)

$$6.5 = \log_{10} \left( \frac{A}{10^{-4}} \right)$$

$$10^{6.5} = \frac{A}{10^{-4}}$$

$$\therefore A = 10^{6.5} \times 10^{-4} = 10^{2.5} = 316 \text{ cm}$$

### Question 8 (f)

$$E = 1.74 \times 10^{(5+1.44 \times 4.3)} = 2.7 \times 10^{11} \text{ J}$$

### Question 8 (c)

$$M_1 = \log_{10} \left( \frac{A_1}{s} \right) = 8.3$$

$$M_2 = \log_{10} \left( \frac{A_2}{s} \right) = 4$$

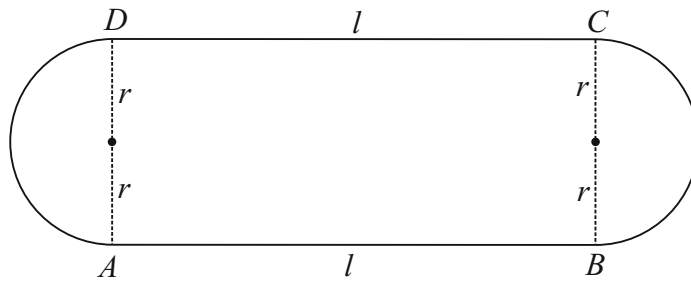
$$M_1 - M_2 = \log_{10} \left( \frac{A_1}{s} \right) - \log_{10} \left( \frac{A_2}{s} \right)$$

$$4.3 = \log_{10} \left( \frac{A_1 \times s}{s A_2} \right) = \log_{10} \left( \frac{A_1}{A_2} \right)$$

$$\therefore \frac{A_1}{A_2} = 10^{4.3} = 19953$$

Modified Mercalli Scale		Richter Magnitude Scale
I	Detected only by sensitive instruments	1.5
II	Felt by few persons at rest, especially on upper floors; delicately suspended objects may swing.	2
III	Felt noticeably indoors, but not always recognised as earthquake; standing autos rock slightly, vibration like passing truck.	2.5
IV	Felt indoors by many, outdoors by few, at night some may awaken; dishes, windows, doors disturbed; autos rock noticeably	3
V	Felt by most people; some breakage of dishes, windows, and plaster; disturbance of tall objects	3.5
VI	Felt by all, many frightened and run outdoors; falling plaster and chimneys; sand and mud ejected; drivers of autos disturbed	4
VII	Everybody runs outdoors; damage to buildings varies depending on quality of construction; noticed by drivers of autos	4.5
VIII	Panel walls thrown out of frames; fall of walls, monuments, chimneys; sand and mud ejected; drivers of autos disturbed	5
IX	Buildings shifted off foundations, cracked, thrown out of plumb; ground cracked; underground pipes broken	5.5
X	Most masonry and frame structures destroyed; ground cracked, rails bent, landslides	6
XI	Few structures remain standing; bridges destroyed, fissures in ground, pipes broken, landslides, rails bent	6.5
XII	Damage total; waves seen on ground surface, lines of sight and level distorted, objects thrown up in air	7
		7.5
		8

**QUESTION 9 (50 MARKS)**



**Question 9 (a)**

$$P = 2l + 2\pi r$$

**Question 9 (b)**

$$400 = 2l + 2\pi r$$

$$200 = l + \pi r$$

$$r = \frac{200 - l}{\pi}$$

**Question 9 (c)**

$$A = 2rl = \frac{2l(200 - l)}{\pi} = \frac{400}{\pi}l - \frac{2}{\pi}l^2$$

**Question 9 (d)**

$$\frac{dA}{dl} = \frac{400}{\pi} - \frac{4}{\pi}l$$

$$\frac{dA}{dl} = 0 \Rightarrow \frac{400}{\pi} - \frac{4}{\pi}l = 0$$

$$\frac{400}{\pi} = \frac{4}{\pi}l$$

$$\therefore l = 100 \text{ m}$$

**Question 9 (e) (i)**

$$A: s = ut \Rightarrow 100 = 9 \cdot 1t$$

$$\therefore t = 10.99 \text{ s}$$

$$B: s = 5t + 0.56t^2$$

$$100 = 5t + 0.56t^2$$

$$\therefore 0.56t^2 + 5t - 100 = 0$$

$$t = \frac{-5 \pm \sqrt{25 + 224}}{2 \times 0.56} = 9.62 \text{ s}$$

**Question 9 (e) (ii)**

B wins the race.

**Question 9 (e) (iii)**

$$9 \cdot 1t = 5t + 0.56t^2$$

$$0.56t^2 - 4 \cdot 1t = 0$$

$$t(0.56t - 4 \cdot 1) = 0$$

$$t = \frac{4 \cdot 1}{0.56} = 7.32 \text{ s}$$

$$s = 9 \cdot 1t = 9 \cdot 1(7.32) = 66.6 \text{ m}$$

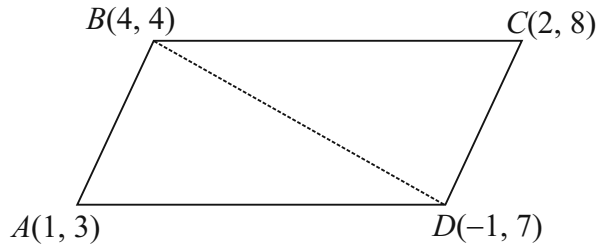
**SAMPLE PAPER 2: PAPER 2**

**QUESTION 1 (25 MARKS)**

**Question 1 (a)**

The diagonal bisects the area of a parallelogram. Find the area of triangle  $ABD$  and multiply the answer by 2.

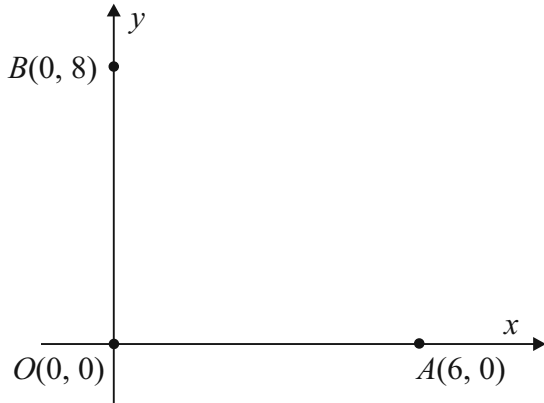
$$\begin{aligned} A(1, 3) &\rightarrow (0, 0) \\ B(4, 4) &\rightarrow (3, 1) \\ D(-1, 7) &\rightarrow (-2, 4) \end{aligned} \quad \left\| \begin{aligned} A &= \frac{1}{2}|x_1y_2 - x_2y_1| \\ &= \frac{1}{2}|3(4) - (-2)(1)| \\ &= \frac{1}{2}|12 + 2| \\ &= 7 \end{aligned} \right.$$



Area of parallelogram  $ABCD = 14$

**NOTE:** The diagram in the question is a rough sketch showing the relative positions of the points. A grid is drawn whenever we wish to display the absolute positions of the points.

**Question 1 (b)**



Intercepts of  $2x + 3y = c$ :  $D(\frac{1}{2}c, 0)$ ,  $E(0, \frac{1}{3}c)$

$$\begin{aligned} &O(0, 0), D(\frac{1}{2}c, 0), E(0, \frac{1}{3}c) \\ \text{Area } |\Delta ODE| &= \frac{1}{2}|(0)(0) - (\frac{1}{2}c)(\frac{1}{3}c)| = \frac{1}{2}|\frac{1}{6}c^2| \\ &O(0, 0), A(6, 0), B(0, 8) \\ \text{Area } |\Delta OAB| &= \frac{1}{2}|(0)(0) - (6)(8)| = \frac{1}{2}|48| \\ \\ \text{Area } |\Delta ODE| &= \frac{1}{2} \text{Area } |\Delta OAB| \\ \therefore \frac{1}{2}|\frac{1}{6}c^2| &= \frac{1}{2} \times \frac{1}{2}|48| \\ \frac{1}{6}c^2 &= 24 \\ c^2 &= 144 \\ \therefore c &= \pm\sqrt{144} = \pm 12 \end{aligned}$$

## QUESTION 2 (25 MARKS)

Slope of  $t$ :  $m = -\frac{4}{3}$

Circle  $s$ : Centre  $O(3, 0)$ ,  $r = 5$

Equation of  $OQ$ : Point  $O(3, 0)$ ,  $m = \frac{3}{4}$

$$m = \frac{3}{4}$$

$$y - 0 = \frac{3}{4}(x - 3)$$

$$4y = 3x - 9$$

$$3x - 4y - 9 = 0$$

Find the point of intersection of lines  $t$  and  $OQ$ .

$$4x + 3y + 13 = 0 \quad (\times 3)$$

$$3x - 4y - 9 = 0 \quad (\times -4)$$

$$12x + 9y + 39 = 0$$

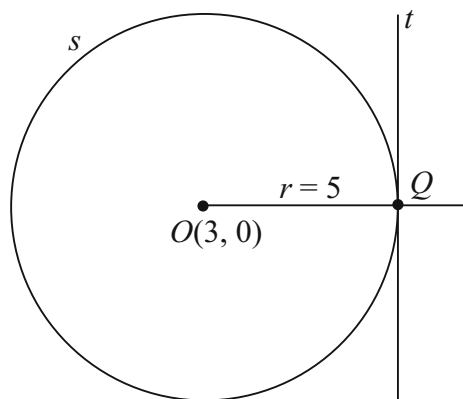
$$-12x + 16y + 36 = 0$$

$$\hline 25y + 75 = 0 \Rightarrow y = -3$$

$$y = -3: 4x + 3(-3) + 13 = 0 \Rightarrow 4x = -4$$

$$\therefore x = -1$$

Point of contact  $Q(-1, -3)$



The perpendicular line from  $O$  bisects the chord.

The distance from the centre to the chord is 3.

The chord passing through  $Q$  is called  $l$ .

$$l: mx - y + k = 0$$

$$Q(-1, -3) \in l: -m + 3 + k = 0 \Rightarrow k = m - 3$$

$$l: mx - y + m - 3 = 0$$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d = 3, (x_1, y_1) = (3, 0)$$

$$l: mx - y + m - 3 = 0$$

$$3 = \frac{|m(3) - (0) + m - 3|}{\sqrt{m^2 + 1}} \quad [\text{The perpendicular distance of the chord } l \text{ from the centre } O \text{ is 3.}]$$

$$3\sqrt{m^2 + 1} = |3m + m - 3|$$

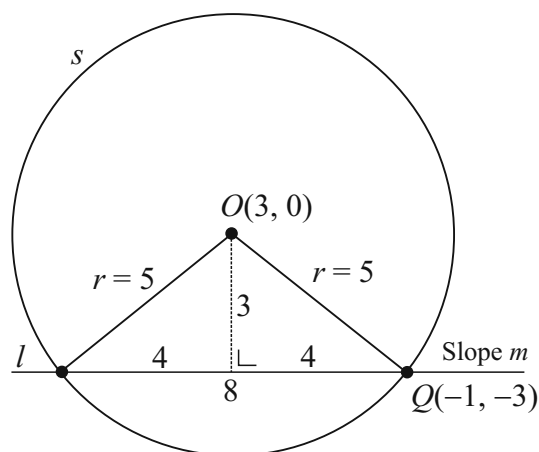
$$3\sqrt{m^2 + 1} = |4m - 3|$$

$$9m^2 + 9 = 16m^2 - 24m + 9$$

$$7m^2 - 24m = 0$$

$$m(7m - 24) = 0$$

$$m = 0, \frac{24}{7}$$



**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$\sin 30^\circ = \frac{h}{10} \Rightarrow h = 10 \sin 30^\circ = 5 \text{ m}$$

**Question 3 (b)**

$$\tan 30^\circ = \frac{5}{x} \Rightarrow x = \frac{5}{\tan 30^\circ} = 5\sqrt{3} \text{ m}$$

$$d = 5\sqrt{3} - 3 = 5.66 \text{ m}$$

**Question 3 (c)**

$$\tan \alpha = \frac{5}{5.66} \Rightarrow \alpha = \tan^{-1}\left(\frac{5}{5.66}\right) = 41.46^\circ$$

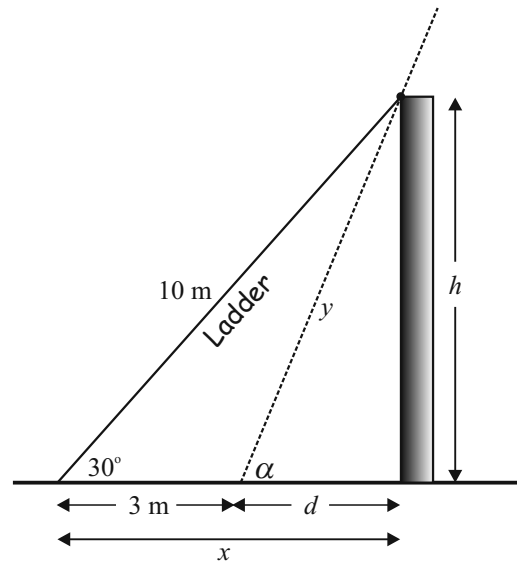
**Question 3 (d)**

$$\sin \alpha = \frac{5}{y}$$

$$\sin 41.46^\circ = \frac{5}{y}$$

$$y = \frac{5}{\sin 41.46^\circ} = 7.55 \text{ m}$$

Length of ladder protruding beyond the top of the wall =  $10 - 7.55 = 2.45 \text{ m}$

**QUESTION 4 (25 MARKS)****Question 4 (a)**

Condition: There are only two possible outcomes (success or failure) in each trial.

Condition: There is a fixed number of trials  $n$ .

Condition: The probability of success  $p$  is fixed from trial to trial.

Condition: The trials are independent.

Condition: The binomial random variable is the number of successes in  $n$  trials.

**Question 4 (b) (i)**

$$P(\text{Income} < \text{€}32\,000) = \frac{1}{2}, P(\text{Income} > \text{€}32\,000) = \frac{1}{2}$$

$$P(x \geq 8) = {}^{10}C_8 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 + {}^{10}C_9 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^9 + {}^{10}C_{10} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} = 0.055$$

**Question 4 (b) (ii)**

$$P(x \geq 2) = 1 - P(\text{None or 1}) = 1 - {}^{10}C_0 \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0 - {}^{10}C_1 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 = 0.99$$

**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

Life expectancy (years)	Number of countries	Life expectancy (years)	Cumulative number of countries
30–35	2	< 35	2
35–40	3	< 40	5
40–45	9	< 45	14
45–50	12	< 50	26
50–55	22	< 55	48
55–60 (LQ)	46	< 60	94
60–65 (M)	58	< 65	152
65–70	50	< 70	202
70–75 (UQ)	40	< 75	242
75–80	32	< 80	274
80–85	16	< 85	290
	290		

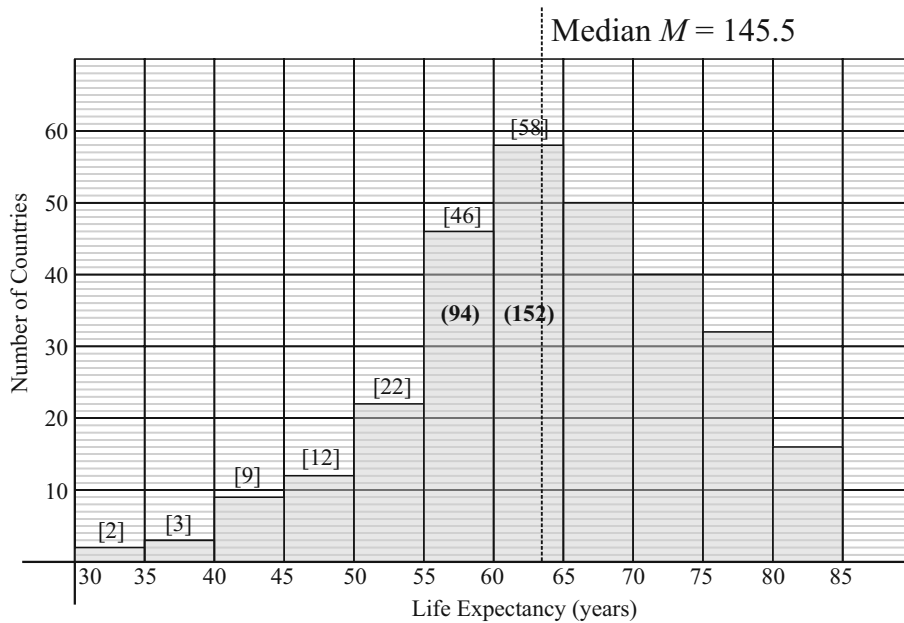
**Question 5 (b)**

Number of countries = 290

Median class:  $290(0.5) + 0.5 = 145.5$  (Class 60–65)

Upper quartile class:  $290(0.75) + 0.75 = 218.25$  (Class 70–75)

Lower quartile class:  $290(0.25) + 0.25 = 72.75$  (Class 55–60)



**Question 5 (c)**

$$M = 60 + \left( \frac{145.5 - 94}{58} \right) 5 = 64.4$$

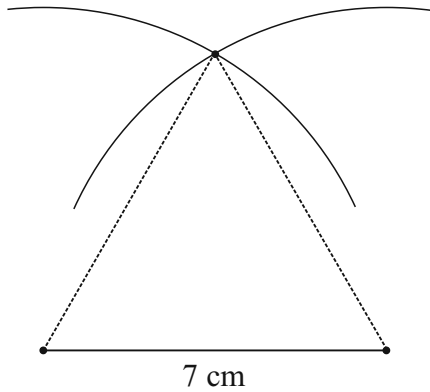
**Question 5 (d)**

Modal class: 60–65

Shape: Skewed left

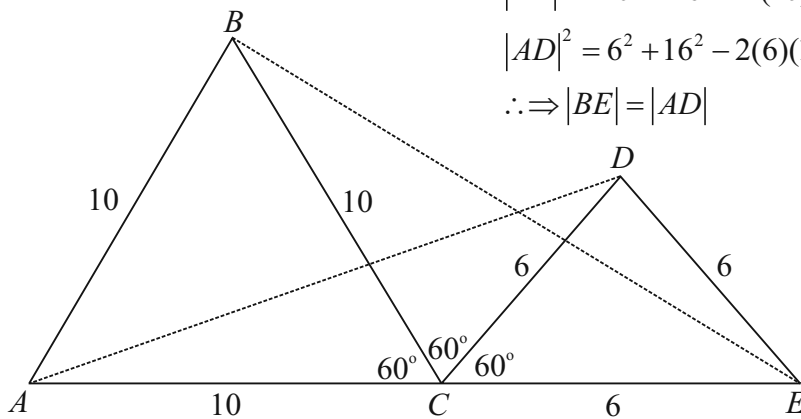
**QUESTION 6 (25 MARKS)**

**Question 6 (a)**



Draw a line of length 7 cm with your ruler.  
 Open your compass to a radius of 7 cm using this line.  
 Placing the point of the compass on each end point of the line draw arcs of radius 7 cm.  
 Draw lines from the end points of the line to the point of intersection of the two arcs.  
 An equilateral triangle of side 7 cm has been constructed.

**Question 6 (b)**



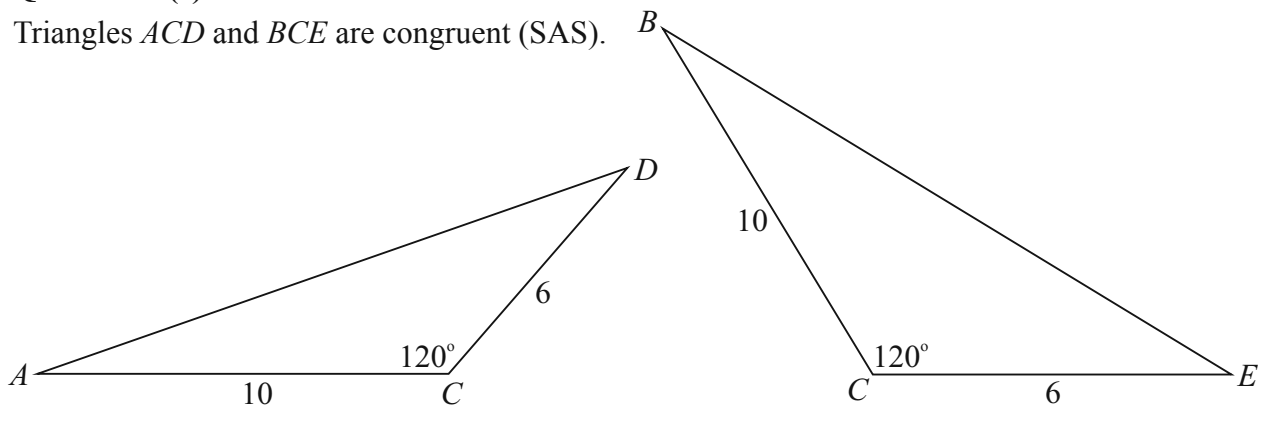
$$|BE|^2 = 10^2 + 16^2 - 2(10)(16)\cos 60^\circ = 196 \Rightarrow |BE| = 14$$

$$|AD|^2 = 6^2 + 16^2 - 2(6)(16)\cos 60^\circ = 196 \Rightarrow |AD| = 14$$

$$\therefore \Rightarrow |BE| = |AD|$$

**Question 6 (c)**

Triangles  $ACD$  and  $BCE$  are congruent (SAS).



**QUESTION 7 (75 MARKS)**

**Question 7 (a) (i)**

$$R = \frac{100I}{\text{GDP}}$$

$R = 10.1\%$ ,  $\text{GDP} = \text{€}159 \text{ billion}$ ,  $I = ?$

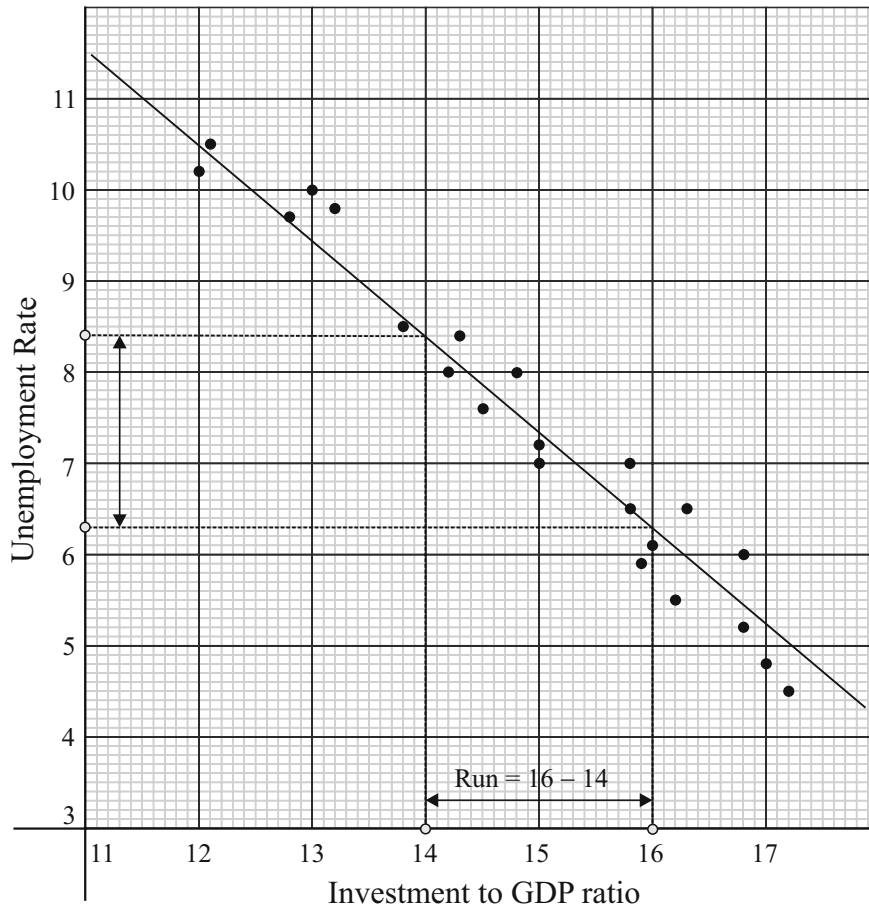
$$10.1 = \frac{100I}{159}$$

$$\therefore I = \frac{159 \times 10.1}{100} = \text{€}16.1 \text{ billion}$$

**Question 7 (a) (ii)**

$$\text{GDP per capita} = \frac{\text{€}159\,000\,000\,000}{4\,588\,252} = \text{€}34\,654$$

**Question 7 (b)**



**Question 7 (c) (i)**

The data shows that the most effective way to reduce *the unemployment rate* is to *increase investment to GDP ratio*.

**Question 7 (c) (ii)**

- A. Linear,
- B. Negative shape,
- C. Very strong correlation

**Question 7 (d)**

CASIO CALCULATOR (*fx-85GT PLUS*)  
 Steps to find  $r$ :  
 Press Mode.  
 Press 2: Stat  
 Press 2:  $A + Bx$   
 Input your  $x$  and  $y$  values  
 Press AC Button  
 Press Shift followed by the Number 1  
 Press 5: Reg  
 Press 3:  $r$   
 Press =

$$r = -0.9767$$

**Question 7 (e) (i)**

(14, 8.4), (16, 6.3)

$$m = \frac{8.4 - 6.3}{14 - 16} = -1.1$$

**Question 7 (e) (ii)**

$m = -1.1$ ,  $(x_1, y_2) = (14, 8.4)$

$$(y - 8.4) = -1.1(x - 14)$$

$$y - 8.4 = -1.1x + 15.4$$

$$1.1x + y - 23.8 = 0$$

**Question 7 (e) (iii)**

$$y = 3 : 1.1x + (3) - 23.8 = 0$$

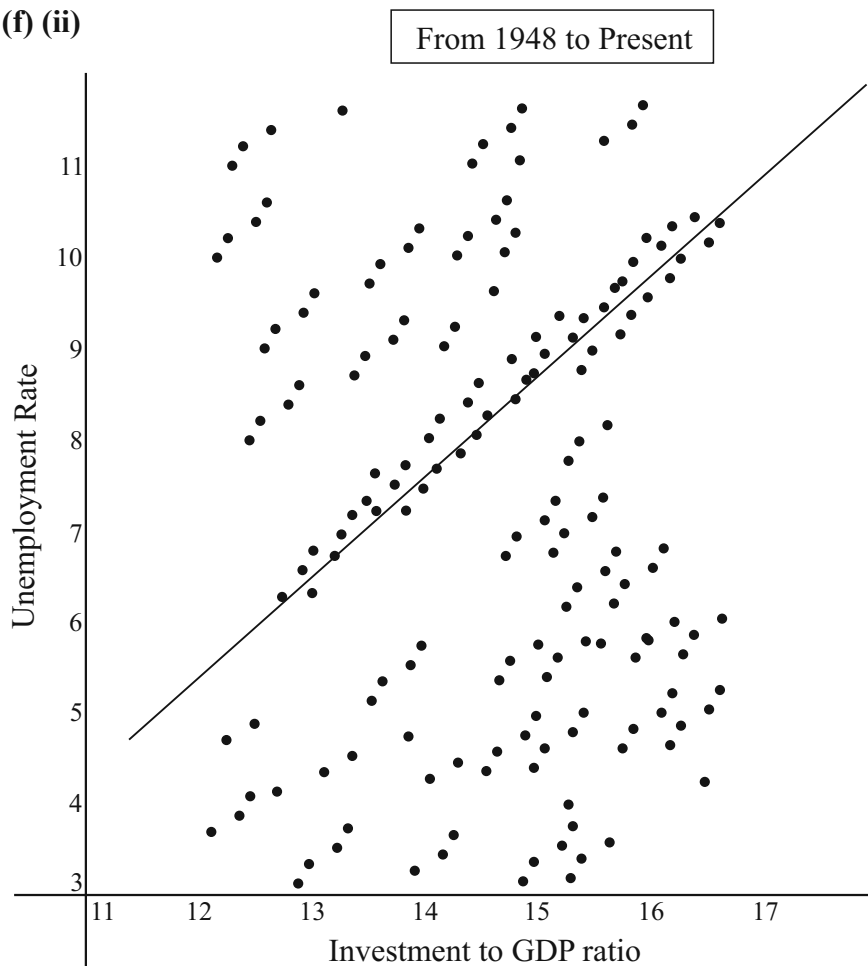
$$\therefore x = 18.9$$

**Question 7 (f) (i)**

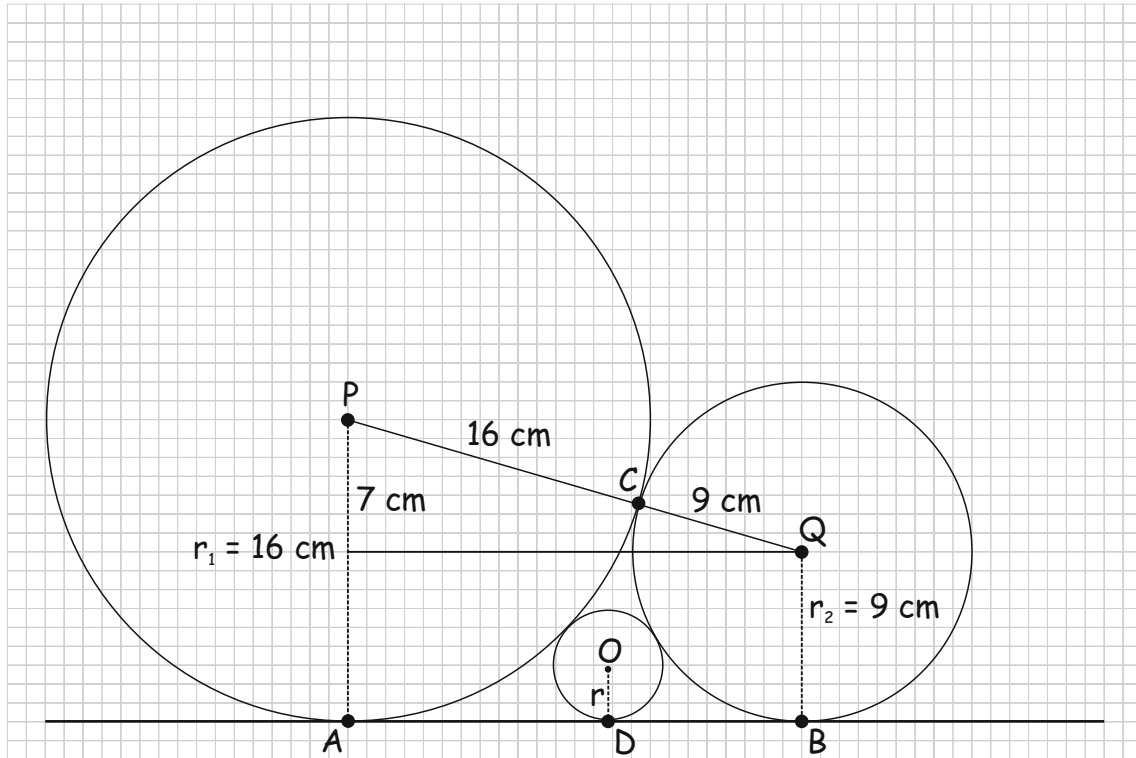
The 11 year plot is misleading as the 64 year plot shows little correlation between the unemployment rate and investment to GDP ratio. It is a prime example of someone using statistics to support their own position.

**Question 7 (f) (ii)**

Yes



**QUESTION 8 (75 MARKS)**

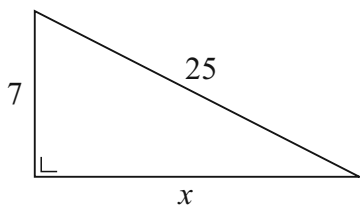


**Question 8 (a) (i)**

$$r_1 = 16 \text{ cm}$$

$$r_2 = 9 \text{ cm}$$

**Question 8 (b) (i)**



$$x^2 + 7^2 = 25^2$$

$$x = \sqrt{25^2 - 7^2} = 24 \text{ cm}$$

**Question 8 (a) (ii)**

$$|PQ| = |PC| + |CQ| = r_1 + r_2$$

$$= 16 \text{ cm} + 9 \text{ cm} = 25 \text{ cm}$$

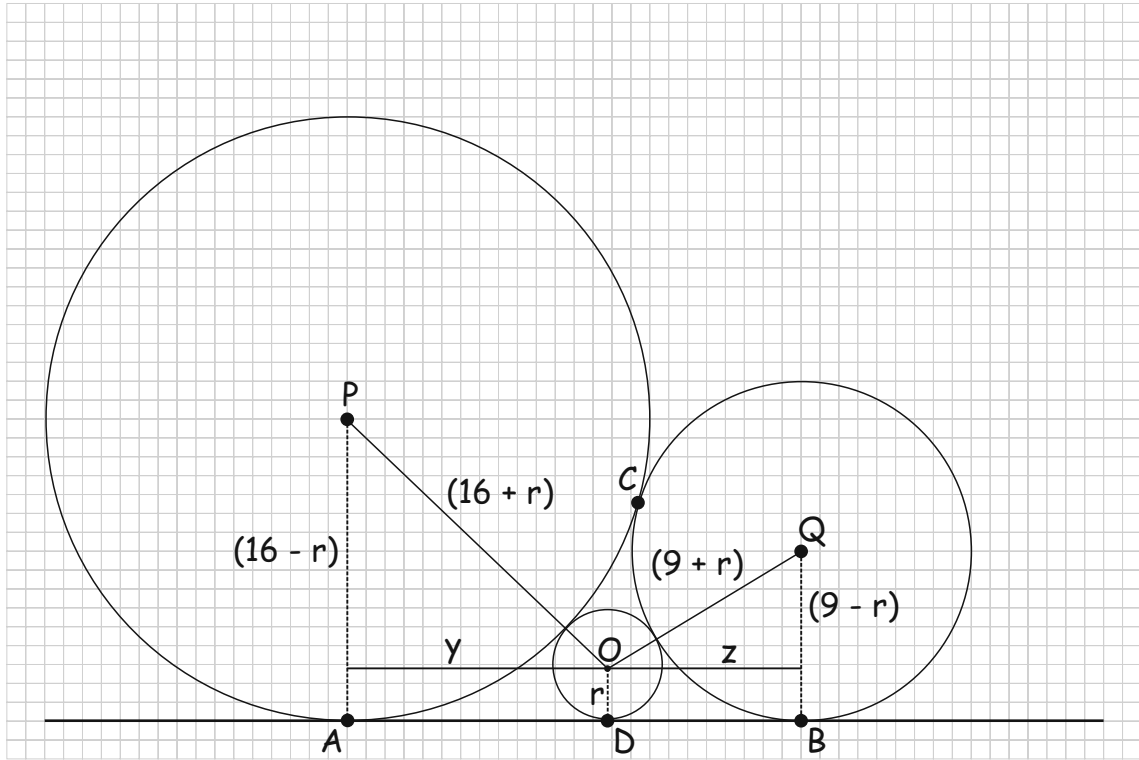
**Question 8 (b) (ii)**

$$\tan(|\angle APQ|) = \frac{24}{7}$$

$$\therefore |\angle APQ| = \tan^{-1}\left(\frac{24}{7}\right)$$

$$= 73.74^\circ = 1.287 \text{ rads}$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}(16)^2(1.287) = 164.7 \text{ cm}^2$$



**Question 8 (c) (i)**

$$y^2 + (16-r)^2 = (16+r)^2$$

$$y^2 = (16+r)^2 - (16-r)^2$$

$$y^2 = (16+r+16-r)(16+r-16+r)$$

$$y^2 = (32)(2r) = 64r$$

$$y = 8\sqrt{r}$$

**Question 8 (c) (ii)**

$$z^2 + (9-r)^2 = (9+r)^2$$

$$z^2 = (9+r)^2 - (9-r)^2$$

$$z^2 = (9+r+9-r)(9+r-9+r)$$

$$z^2 = (18)(2r) = 36r$$

$$z = 6\sqrt{r}$$

**Question 8 (d)**

$$y + z = 24$$

$$8\sqrt{r} + 6\sqrt{r} = 24$$

$$14\sqrt{r} = 24$$

$$\sqrt{r} = \frac{24}{14} = \frac{12}{7}$$

$$\therefore r = \frac{144}{49}$$

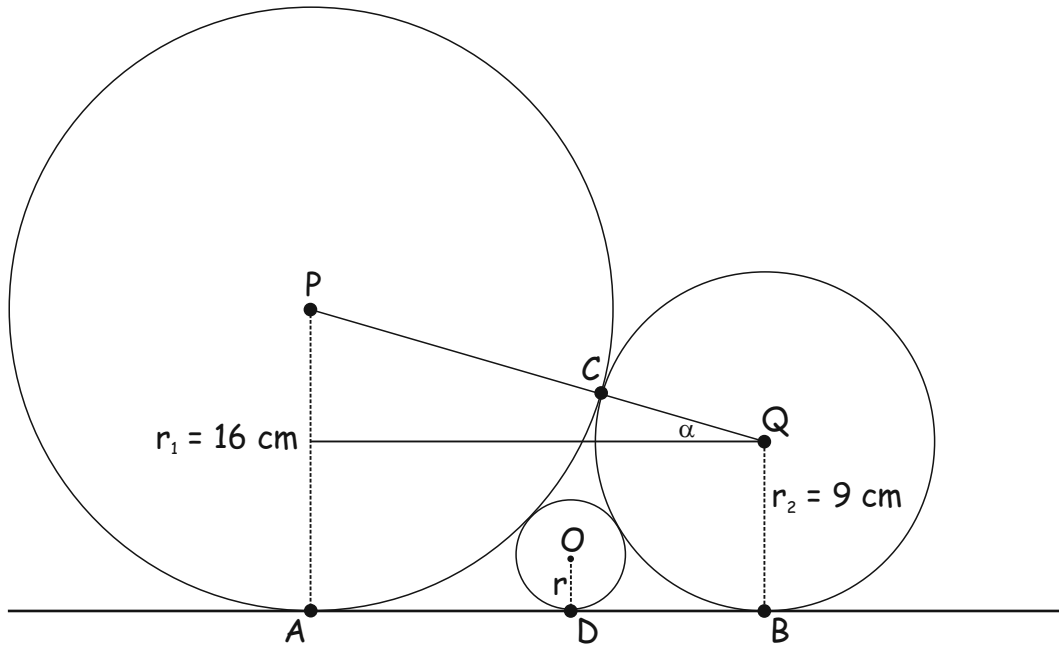
**Question 8 (e)**

Area of trapezium  $ABQP$ :  $A = \frac{1}{2}(16+9)(24) = 25(12) = 300 \text{ cm}^2$

**Question 8 (f)**

Area of space between wheels

= Area of trapezium – Area of sector  $APC$  – Area of sector  $CQB$  – Area of smallest wheel



Area of sector  $CQB$ :

$$\alpha = 90^\circ - 73.74^\circ = 16.26^\circ$$

$$|\angle CQB| = 16.26^\circ + 90^\circ = 106.26^\circ = 1.855 \text{ rads}$$

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} (9)^2 (1.855) = 75.13 \text{ cm}^2$$

$$\text{Area of space between wheels} = 300 - 164.7 - 75.13 - \pi \left( \frac{144}{49} \right)^2 \approx 33 \text{ cm}^2$$

$$\% \text{ of air space to area of three wheels} = \frac{33}{\pi \left[ 16^2 + 9^2 + \left( \frac{144}{49} \right)^2 \right]} \times 100\% \approx 3\%$$

**SAMPLE PAPER 3: PAPER 1**

**QUESTION 1 (25 MARKS)**

**Question 1 (a)**

$$\left(1 - \frac{1}{x}\right)^2 = 2$$

$$1 - \frac{1}{x} = \pm\sqrt{2}$$

$$1 \pm \sqrt{2} = \frac{1}{x}$$

$$x = \frac{1}{1 + \sqrt{2}}, \frac{1}{1 - \sqrt{2}}$$

Rationalise the denominators by multiplying above and below by the conjugate of the denominator.

$$\frac{1}{(1 + \sqrt{2})} \times \frac{(1 - \sqrt{2})}{(1 - \sqrt{2})} = \frac{1 - \sqrt{2}}{1 - 2} = \sqrt{2} - 1$$

$$\frac{1}{(1 - \sqrt{2})} \times \frac{(1 + \sqrt{2})}{(1 + \sqrt{2})} = \frac{1 + \sqrt{2}}{1 - 2} = -\sqrt{2} - 1$$

**Question 1 (b)**

$$x^3 + 3x^2 + x - 2 = 0$$

$$x = -2: (-2)^3 + 3(-2)^2 - 2 - 2 = 0$$

$\therefore (x + 2)$  is a factor

$$\therefore x^3 + 3x^2 + x - 2 = (x + 2)(x^2 + kx - 1)$$

$$= x^3 + (k + 2)x^2 + (2k - 1)x - 2 \quad [\text{Line up the coefficients.}]$$

$$\therefore 3 = k + 2 \Rightarrow k = 1$$

$$\therefore x^3 + 3x^2 + x - 2 = (x + 2)(x^2 + x - 1) = 0$$

$$x^2 + x - 1 = 0 \quad [\text{Solve the quadratic equation.}]$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\therefore x = -2, \frac{-1 \pm \sqrt{5}}{2}$$

**QUESTION 2 (25 MARKS)****Question 2 (a)**

$$\log_4(6x+1) - 2 = 2 \log_4 x$$

$$\log_4(6x+1) - 2 \log_4 x = 2$$

$$\log_a \left( \frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_4 \left( \frac{6x+1}{x^2} \right) = 2$$

$$a^x = y \Leftrightarrow \log_a y = x$$

$$\frac{6x+1}{x^2} = 4^2$$

$$6x+1 = 16x^2$$

$$16x^2 - 6x - 1 = 0$$

$$(8x+1)(2x-1) = 0 \quad [x = -\frac{1}{8} \text{ is not allowed as } \log_4(-\frac{1}{8}) \text{ is not defined.}]$$

$$x = \frac{1}{2}$$

**Question 2 (b) (i)**

$$F\left(\frac{a}{b}\right) = F(a) - F(b)$$

$$F(1) \Rightarrow a = b$$

$$F(1) = F(a) - F(a) = 0$$

**Question 2 (b) (ii)**

$$F\left(\frac{1}{x}\right) = F(1) - F(x)$$

$$= 0 - F(x)$$

$$= -F(x)$$

**Question 2 (b) (iii)**

$$F(y^2) = F\left(\frac{y}{\frac{1}{y}}\right) = F(y) - F\left(\frac{1}{y}\right)$$

$$= F(y) - [F(1) - F(y)]$$

$$= F(y) - 0 + F(y)$$

$$= 2F(y)$$

### QUESTION 3 (25 MARKS)

#### Question 3 (a)

Three consecutive numbers:  $n - 1, n, n + 1$

$$\begin{aligned} S &= (n-1)^2 + n^2 + (n+1)^2 \\ &= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1 \\ &= 3n^2 + 2 \end{aligned}$$

$$\frac{3n^2 + 2}{3} = n^2 + \frac{2}{3}$$

#### Question 3 (b)

##### STEPS FOR PROOF BY INDUCTION

1. Prove result is true for some starting value of  $n \in \mathbb{N}$ .
2. Assume result is true for  $n = k$ .
3. Prove result is true for  $n = (k + 1)$ .

1. Prove true for  $n = 1$ :  $n = 1: 7^1 + 3(1) + 8 = 18$  [Therefore, true for  $n = 1$ .]

2. Assume true for  $n = k$ : Assume  $n = k: 7^k + 3k + 8 = 9a, a \in \mathbb{N}$

3. Prove true for  $n = k + 1$ : Prove  $7^{k+1} + 3(k+1) + 8 = 9b, b \in \mathbb{N}$

##### Proof:

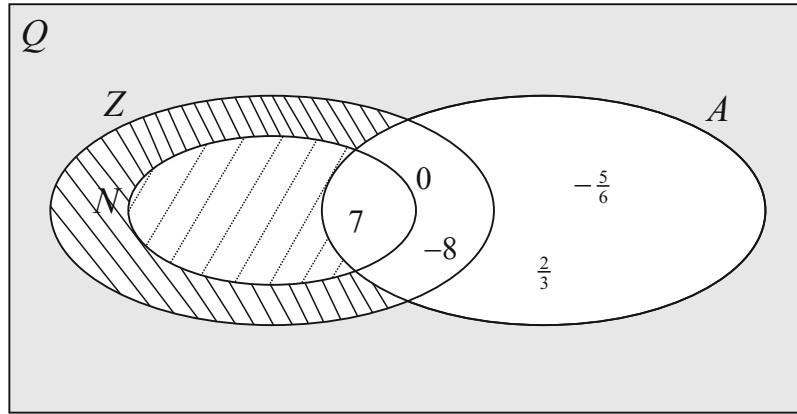
$$\begin{aligned} &7^{k+1} + 3(k+1) + 8 \\ &= 7(7^k) + 3k + 3 + 8 \\ &= 7(9a - 3k - 8) + 3k + 11 \\ &= 63a - 21k - 56 + 3k + 11 \\ &= 63a - 18k - 45 \\ &= 9[7a - 2k - 5] \\ &= 9b, b \in \mathbb{N} \end{aligned}$$

Therefore, assuming true for  $n = k$  means it is true for  $n = k + 1$ . So true for  $n = 1$  and true for  $n = k$  means it is true for  $n = k + 1$ . This implies it is true for all  $n \in \mathbb{N}$ .

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**QUESTION 4 (25 MARKS)**

**Question 4 (a) (i) & (ii)**



**Question 4 (b) (i)**

$$S = \{6, 12, 18, 24, \dots\}$$

$$S_1 = \{3, 6, 9, 12, \dots\}$$

No. 9 is an element of  $S_1$  but not an element of  $S$ .

**Question 4 (b) (iii)**

$$S = \{6, 12, 18, 24, \dots\}$$

$$S_3 = \{12, 24, 36, 48, \dots\}$$

Yes, all elements of  $S_3$  are elements of  $S$ .

**Question 4 (b) (ii)**

$$S = \{6, 12, 18, 24, \dots\}$$

$$S_2 = \{9, 18, 27, 36, \dots\}$$

No. 9 is an element of  $S_2$  but not an element of  $S$ .

**Question 4 (c)**

$$z = -4\sqrt{3} + 4i$$

$$|z| = \sqrt{(-4\sqrt{3})^2 + 4^2} = \sqrt{48 + 16} = \sqrt{64} = 8 \quad |z| = r, \arg z = \theta$$

$$|\tan \theta| = \left| \frac{4}{-4\sqrt{3}} \right| = \frac{1}{\sqrt{3}} = \tan \alpha, \text{ where } \alpha \text{ is the related angle in the first quadrant.}$$

$$\therefore \alpha = \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) = 30^\circ = \frac{\pi}{6}$$

Second quadrant:  $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$   $z = r(\cos \theta + i \sin \theta)$

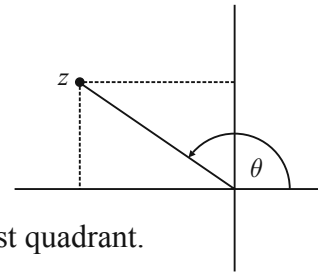
$$\therefore z = 8 \left\{ \cos \left( \frac{5\pi}{6} + 2n\pi \right) + i \sin \left( \frac{5\pi}{6} + 2n\pi \right) \right\} = 8 \left\{ \cos \left( \frac{5\pi + 12n\pi}{6} \right) + i \sin \left( \frac{5\pi + 12n\pi}{6} \right) \right\}$$

$$z^{\frac{1}{3}} = 8^{\frac{1}{3}} \left\{ \cos \left( \frac{5\pi + 12n\pi}{6} \right) + i \sin \left( \frac{5\pi + 12n\pi}{6} \right) \right\}^{\frac{1}{3}} = 2 \left\{ \cos \left( \frac{5\pi + 12n\pi}{18} \right) + i \sin \left( \frac{5\pi + 12n\pi}{18} \right) \right\}$$

$$n = 0: z_1 = 2 \left\{ \cos \left( \frac{5\pi}{18} \right) + i \sin \left( \frac{5\pi}{18} \right) \right\}$$

$$n = 1: z_2 = 2 \left\{ \cos \left( \frac{17\pi}{18} \right) + i \sin \left( \frac{17\pi}{18} \right) \right\}$$

$$n = 2: z_3 = 2 \left\{ \cos \left( \frac{29\pi}{18} \right) + i \sin \left( \frac{29\pi}{18} \right) \right\}$$

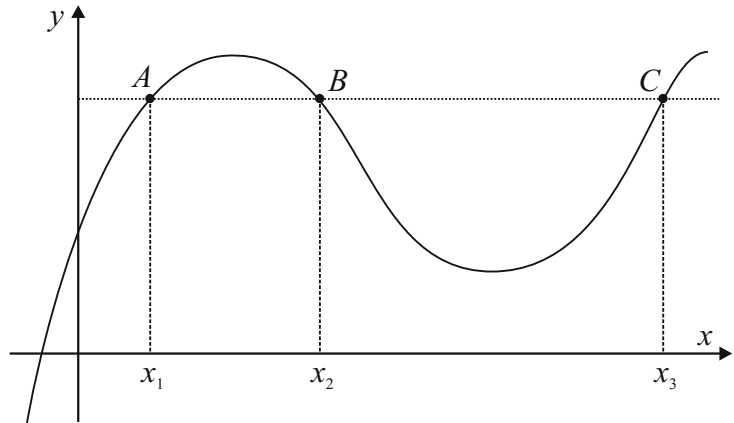


**QUESTION 5 (25 MARKS)**

**Question 5 (a)**

It is not injective because there is more than one  $x$  value mapping on to the same  $y$  value.

It is surjective because every  $y$  value has a corresponding  $x$  value.



**Question 5 (b)**

$$f(x) = y = \frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{x(\frac{1}{x}) - (\ln x)(1)}{x^2} = \frac{1 - \ln x}{x^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow \frac{1 - \ln x}{x^2} = 0$$

$$1 - \ln x = 0$$

$$1 = \ln x \Rightarrow x = e$$

$$f(e) = \frac{\ln e}{e} = \frac{1}{e}$$

$$\text{TP} : \left( e, \frac{1}{e} \right)$$

**Question 5 (c)**

$$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{x^2(-\frac{1}{x}) - (1 - \ln x)(2x)}{(x^2)^2}$$

$$= \frac{-x - 2x + 2x \ln x}{x^4}$$

$$= \frac{-3x + 2x \ln x}{x^4}$$

$$\left( \frac{d^2y}{dx^2} \right)_{x=e} = \frac{-3e + 2e \ln e}{e^4} = \frac{-3e + 2e}{e^4} = \frac{-e}{e^4} = -\frac{1}{e^3} < 0$$

$\therefore \left( e, \frac{1}{e} \right)$  is a local maximum

**Question 5 (d)**

$$a^b = b^a$$

$$\ln a^b = \ln b^a$$

$$b \ln a = a \ln b$$

$$\therefore \frac{\ln a}{a} = \frac{\ln b}{b}$$

**Question 5 (e) (i)**

$$2^4 = 4^2 \Rightarrow \frac{\ln 2}{2} = \frac{\ln 4}{4}$$

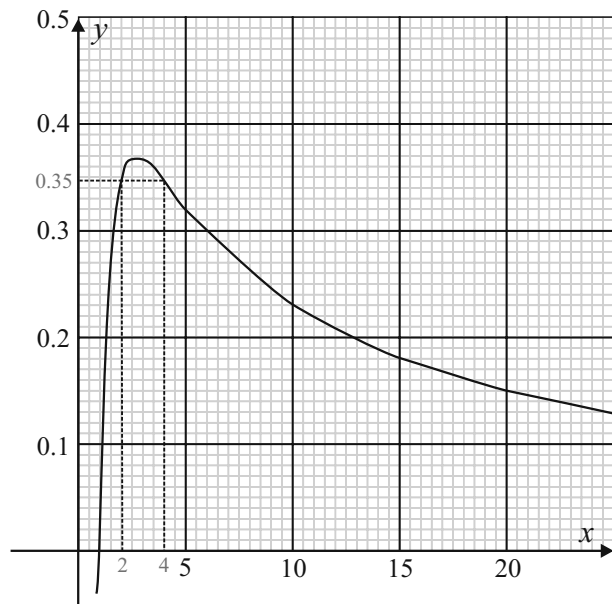
$$f(x) = \frac{\ln x}{x}$$

$$f(2) = \frac{\ln 2}{2}$$

$$f(4) = \frac{\ln 4}{4}$$

$$\therefore f(2) = f(4) = 0.35$$

$$\text{Calculator: } \frac{\ln 2}{2} = \frac{\ln 4}{4} = 0.3466$$



**Question 5 (e) (ii)**

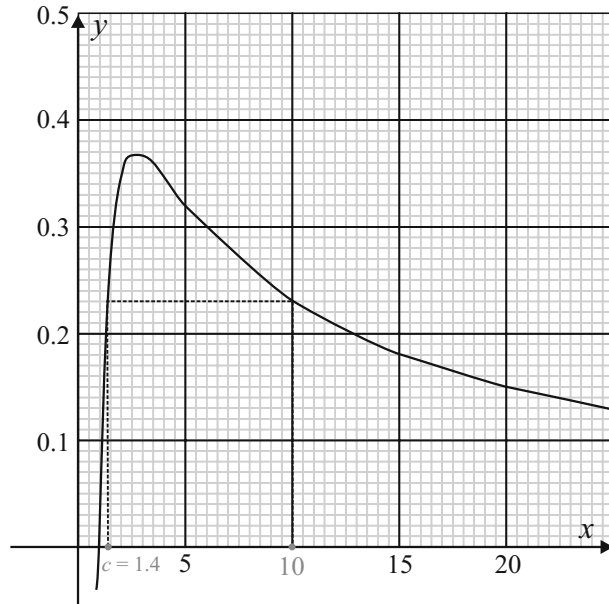
$$10^c = c^{10} \Rightarrow \frac{\ln 10}{10} = \frac{\ln c}{c}$$

$$f(x) = \frac{\ln x}{x}$$

$$f(10) = \frac{\ln 10}{10}$$

$$f(c) = \frac{\ln c}{c}$$

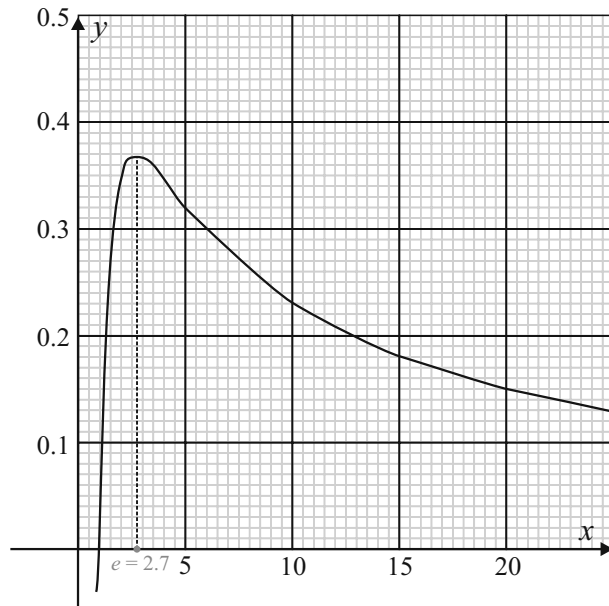
$$\therefore f(10) = f(c) \Rightarrow c \approx 1.4$$



**Question 5 (e) (iii)**

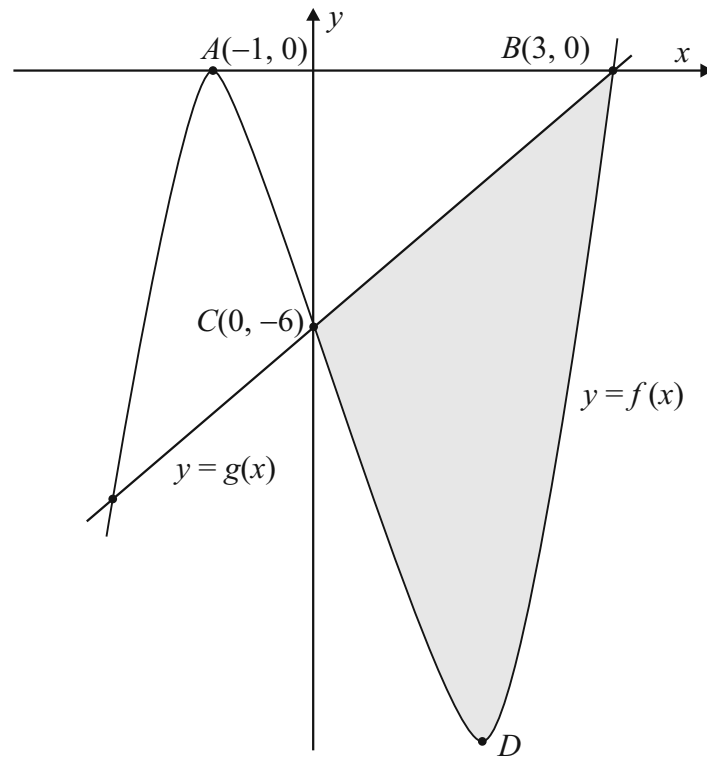
The maximum value of  $f(x)$  as calculated in part (b) is  $e$ .

$$\therefore e \approx 2.7$$



**QUESTION 6 (25 MARKS)**

**Question 6 (a)**



$$f(x) = ax^3 + bx^2 + cx + d$$

$$A(-1, 0) \in f(x) : f(-1) = a(-1)^3 + b(-1)^2 + c(-1) + d = 0$$

$$\therefore -a + b - c + d = 0 \dots (1)$$

$$B(3, 0) \in f(x) : f(3) = a(3)^3 + b(3)^2 + c(3) + d = 0$$

$$\therefore 27a + 9b + 3c + d = 0 \dots (2)$$

$$C(0, -6) \in f(x) : f(0) = a(0)^3 + b(0)^2 + c(0) + d = -6$$

$$\therefore d = -6$$

$$\therefore -a + b - c = 6 \dots (1)$$

$$\therefore 27a + 9b + 3c = 6 \Rightarrow 9a + 3b + c = 2 \dots (2)$$

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$A(-1, 0) \text{ is a turning point} \Rightarrow f'(-1) = 3a(-1)^2 + 2b(-1) + c = 0$$

$$\therefore 3a - 2b + c = 0 \dots (3)$$

Solve equations (1), (2) and (3) simultaneously.

$$-a + b - c = 6 \dots (1)$$

$$9a + 3b + c = 2 \dots (2)$$

$$3a - 2b + c = 0 \dots (3)$$

$$(1) + (2): 8a + 4b = 8 \Rightarrow 2a + b = 2 \dots (4)$$

$$(1) + (3): 2a - b = 6 \dots (5)$$

$$\therefore 4a = 8 \dots (4) + (5)$$

$$\therefore a = 2$$

Substitute into equation (4):  $2(2) + b = 2 \Rightarrow b = -2$

Substitute into equation (1):  $-2 - 2 - c = 6 \Rightarrow c = -10$

$$\therefore f(x) = 2x^3 - 2x^2 - 10x - 6$$

#### Question 6 (b)

$$f'(x) = 6x^2 - 4x - 10$$

$D$  is a turning point  $\Rightarrow f'(x) = 0 \Rightarrow 6x^2 - 4x - 10 = 0$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0$$

$$\therefore x = -1, \frac{5}{3}$$

$x$ -coordinate of  $D = \frac{5}{3}$

#### Question 6 (d)

$$A = \int_0^3 (g(x) - f(x)) dx$$

$$= \int_0^3 (2x - 6 - 2x^3 + 2x^2 + 10x + 6) dx$$

$$= \int_0^3 (-2x^3 + 2x^2 + 12x) dx$$

$$= \left[ -\frac{2x^4}{4} + \frac{2x^3}{3} + \frac{12x^2}{2} \right]_0^3$$

$$= \left[ -\frac{1}{2}x^4 + \frac{2}{3}x^3 + 6x^2 \right]_0^3$$

$$= \left\{ -\frac{1}{2}(3)^4 + \frac{2}{3}(3)^3 + 6(3)^2 \right\} - 0$$

$$= \frac{63}{2}$$

#### Question 6 (c)

$B(3, 0)$  and  $C(0, -6) \in g(x)$

$$\text{Slope } m = \frac{0 + 6}{3 - 0} = 2$$

Equation of  $g(x)$ :  $2x - y + k = 0$

$B(3, 0) \in g(x)$ :  $6 - 0 + k = 0 \Rightarrow k = -6$

$$2x - y - 6 = 0$$

Equation of  $g(x)$ :  $g(x) = y = 2x - 6$

**QUESTION 7 (50 MARKS)**

**Question 7 (a)**

$$P = \frac{F}{\left(1 + \frac{r}{100}\right)^n}$$

**Question 7 (c) (i)**

$$P = \frac{200}{(1.06)^7} = \text{€}133 \text{ million}$$

The exploration company **should sell in 2020** because you would need €133 million now to make this money. However, the multinational company is only offering you €120 million now.

**Question 7 (c) (iii)**

$$120 = \frac{200}{(1+i)^7}$$

$$(1+i)^7 = \frac{200}{120} = \frac{5}{3}$$

$$1+i = \left(\frac{5}{3}\right)^{\frac{1}{7}}$$

$$\therefore i = \left(\frac{5}{3}\right)^{\frac{1}{7}} - 1 = 0.076 = 7.6\%$$

**Question 7 (b)**

$$P = \frac{50\,000}{(1.03)^6} = \text{€}41\,874$$

**Question 7 (c) (ii)**

$$P = \frac{200}{(1.08)^7} = \text{€}116.7 \text{ million}$$

The exploration company should **take the offer now** because you would need €116.7 million now to make this money. However, the multinational company is offering you €120 million now.

**Question 7 (d) (i)**

$$P = \frac{3}{1.06} = \text{€}2.83 \text{ billion}$$

**Question 7 (d) (ii)**

Year	Reduction in Billions
1	3
2	3
3	3
4	3
5	3
6	2
7	2
8	2
9	2
10	2

$$\begin{aligned}
 P &= \frac{3}{1.06} + \frac{3}{1.06^2} + \frac{3}{1.06^3} + \frac{3}{1.06^4} + \frac{3}{1.06^5} + \frac{2}{1.06^6} + \frac{2}{1.06^7} + \frac{2}{1.06^8} + \frac{2}{1.06^9} + \frac{2}{1.06^{10}} \\
 &= \frac{3}{1.06} \left[ 1 + \frac{1}{1.06} + \frac{1}{1.06^2} + \frac{1}{1.06^3} + \frac{1}{1.06^4} \right] + \frac{2}{1.06^6} \left[ 1 + \frac{1}{1.06} + \frac{1}{1.06^2} + \frac{1}{1.06^3} + \frac{1}{1.06^4} \right] \\
 &= \left( \frac{3}{1.06} + \frac{2}{1.06^6} \right) \left[ 1 + \frac{1}{1.06} + \frac{1}{1.06^2} + \frac{1}{1.06^3} + \frac{1}{1.06^4} \right] \\
 &= \left( \frac{3}{1.06} + \frac{2}{1.06^6} \right) \left[ \frac{1 \left( 1 - \left( \frac{1}{1.06} \right)^5 \right)}{1 - \left( \frac{1}{1.06} \right)} \right] \\
 &= \text{€}18.9 \text{ billion}
 \end{aligned}$$

**QUESTION 8 (50 MARKS)**

Pure silver is, of course, 100% pure silver.  
 Britannia silver contains 95.8% pure silver.  
 Sterling silver contains 92.5% pure silver.

**Question 8 (a)**

Pure silver: 14 g  
 Mass of coin = 15 g  

$$\text{Purity} = \frac{14 \text{ g}}{15 \text{ g}} = 93.3\%$$

**Question 8 (b)**

Let  $x$  = Mass of Sterling silver  
 Let  $y$  = Mass of Britannia silver  
 Mass of ring = 28 g  
 $x + y = 28$  ( $\times 0.925$ )  
 $0.925x + 0.958y = 26.3224$

$$\begin{array}{r} 0.925x + 0.925y = 25.9 \\ 0.925x + 0.958y = 26.3224 \\ \hline 0.033y = 0.4224 \Rightarrow y = 12.8 \text{ g} \\ \therefore x = 28 - 12.8 = 15.2 \text{ g} \end{array}$$

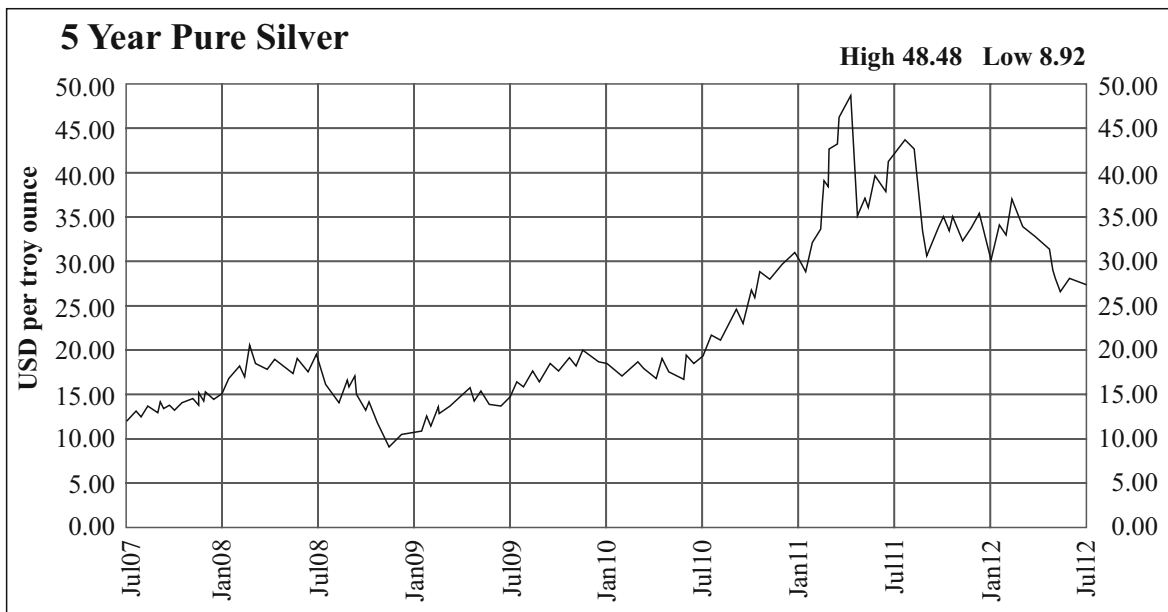
**Question 8 (c)**

1 T = 31.1034768 g  
 $\frac{1}{31.1034768} \text{ T} = 1 \text{ g}$   
 $\frac{40}{31.1034768} \text{ T} = 40 \text{ g}$   
 $\therefore 40 \text{ g} = 1.286 \text{ T}$

**Question 8 (d)**

$$\begin{aligned} S &= C \times T \times p \\ C &= 0.94 \\ T &= \frac{28}{31.1034768} = 0.9 \\ p &= \$26 \\ S &= 0.94 \times 0.9 \times 26 = \$22 \end{aligned}$$

**Question 8 (e)**



**Question 8 (e) (i)**

$$S = C \times T \times p$$

$$C = 0.96$$

$$T = \frac{60}{31.1034768} = 1.93$$

$$p = \$12.50$$

$$S = 0.96 \times 1.93 \times 12.5 = \$23.16$$

**Question 8 (e) (ii)**

$$S = C \times T \times p$$

$$C = 0.96$$

$$T = 1.93$$

$$p = \$27.50$$

$$S = 0.96 \times 1.93 \times 27.5 = \$50.95$$

**Question 8 (f)**

$$\text{Minimum value} = \$8.92$$

$$\text{Maximum value} = \$48.48$$

$$\% \text{ change} = \frac{(48.48 - 8.92)}{8.92} \times 100\% = 443.5\%$$

**Question 8 (g)**

$$S = C \times T \times p$$

$$C = ?$$

$$T = 1$$

$$p = \$30$$

$$S = \$25$$

$$25 = C \times 1 \times 30$$

$$\therefore C = \frac{25}{30} = 83.33\%$$

$$\frac{90}{25} = 3.6 \quad T = 3.6 \times 31.1034768 = 111.97 \text{ g}$$

$$\frac{93.2}{111.97} = 0.83 = 83.3\%$$

You thought you were not getting the full market value for pure silver. The scrap dealer was right.

**QUESTION 9 (50 MARKS)****Question 9 (a) (i)**

$$V = 16\pi = \pi r^2 h \Rightarrow h = \frac{16}{r^2}$$

**Question 9 (a) (ii)**

$$\begin{aligned} A &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r \left( \frac{16}{r^2} \right) \\ &= 2\pi r^2 + \frac{32\pi}{r} \end{aligned}$$

**Question 9 (a) (iii)**

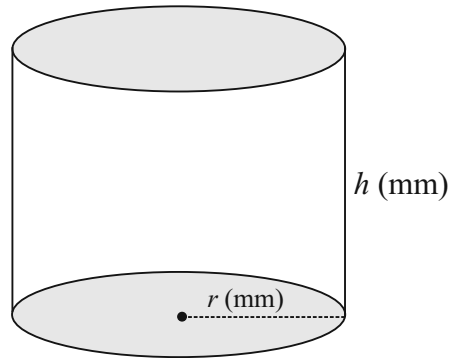
$$\begin{aligned} A &= 2\pi r^2 + 32\pi r^{-1} \\ \frac{dA}{dr} &= 0 \Rightarrow 4\pi r - 32\pi r^{-2} = 0 \\ 4\pi r - \frac{32\pi}{r^2} &= 0 \Rightarrow r^3 = 8 \\ \therefore r &= 2 \text{ mm} \end{aligned}$$

**Question 9 (b) (i)**

$$\begin{aligned} \frac{dm}{dt} &= -e^{-\frac{1}{10}t} \\ \int dm &= -\int e^{-\frac{1}{10}t} dt \\ m &= 10e^{-\frac{1}{10}t} + c \\ t = 0, m = 10 : 10 &= 10e^0 + c \Rightarrow c = 0 \\ \therefore m &= 10e^{-\frac{1}{10}t} \text{ mg} \end{aligned}$$

**Question 9 (b) (iv)**

$$\begin{aligned} 2 &= 10e^{-\frac{1}{10}T} + 10e^{-\frac{1}{10}(T+5)} \\ 0.2 &= e^{-\frac{1}{10}T} (1 + e^{-\frac{1}{2}}) \\ e^{-\frac{1}{10}T} &= \frac{0.2}{1 + \frac{1}{\sqrt{e}}} = 0.12449 \\ -\frac{1}{10}T &= \ln(0.12449) \\ \therefore T &= -10\ln(0.12449) = 20.835 \text{ hours} \end{aligned}$$

**Question 9 (a) (iv)**

$$\begin{aligned} A &= 2\pi r^2 + \frac{32\pi}{r} \\ A_{\text{Min.}} &= 2\pi(2)^2 + \frac{32\pi}{2} = 8\pi + 16\pi = 24\pi \text{ mm}^2 \end{aligned}$$

**Question 9 (b) (ii)**

$$t = 5 : m = 10e^{-\frac{1}{10}(5)} = 10e^{-0.5} = 6.065 \text{ mg}$$

**Question 9 (b) (iii)**

$$m = 10e^{-\frac{1}{10}} + 10e^{-\frac{6}{10}} = 14.536 \text{ mg}$$

**SAMPLE PAPER 3: PAPER 2**

**QUESTION 1 (25 MARKS)**

**Question 1 (a)**

$$s : (x-2)^2 + (y+3)^2 = 25$$

Centre  $O(2, -3)$ ,  $r = \sqrt{25} = 5$

Centre  $(h, k)$ , Radius  $r$

$$(x-h)^2 + (y-k)^2 = r^2$$

Substitute  $A$  into  $s$  to see if it satisfies the equation of the circle.

$$A(6, -6) \in s?$$

$$(6-2)^2 + (-6+3)^2$$

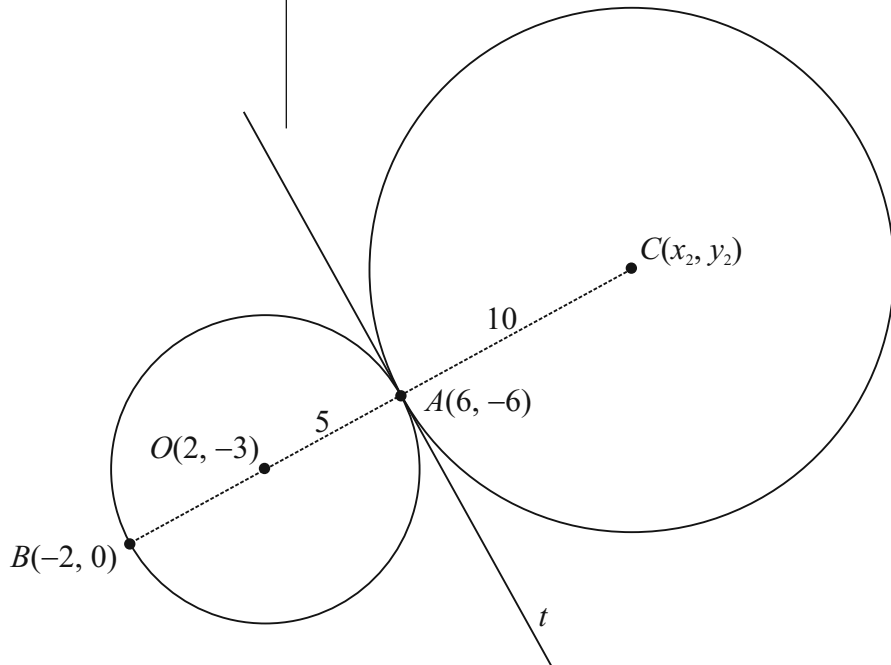
$$= 16 + 9$$

$$= 25$$

Pass  $A$  through  $O$  by a central symmetry to find  $B$ .

$$A(6, -6) \rightarrow O(2, -3) \rightarrow B(-2, 0)$$

**Question 1 (b)**



Slope of  $OA$ :  $m_1 = \frac{-6 - (-3)}{6 - 2} = \frac{-3}{4} = -\frac{3}{4}$

Slope of  $t$ :  $m_2 = \frac{4}{3}$  [Perpendicular slope]

Equation of  $t$ :

$$m = \frac{4}{3}, (x_1, y_1) = A(6, -6)$$

$$y + 6 = \frac{4}{3}(x - 6)$$

$$3y + 18 = 4x - 24$$

$$t : 4x - 3y - 42 = 0$$

**Question 1 (c)**

$OC$  is divided in the ratio  $a:b = 1:2$ .

$$O(2, -3) = (x_1, y_1), C(x_2, y_2), A(6, -6)$$

Ratio:  $a : b = 1 : 2$

$$\therefore \frac{2(2) + 1x_2}{2+1} = 6 \Rightarrow 4 + x_2 = 18 \Rightarrow x_2 = 14$$

$$\therefore \frac{2(-3) + 1y_2}{2+1} = -6 \Rightarrow -6 + y_2 = -18 \Rightarrow y_2 = -12$$

$$\therefore C(14, -12)$$

Point dividing  $[PQ]$  in the ratio  $a:b$   $\left( \frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a} \right)$

Equation of second circle: Centre  $C(14, -12)$ ,  $r = 10$

$$(x-14)^2 + (y+12)^2 = 100$$

Centre  $(h, k)$ , Radius  $r$

$$(x-h)^2 + (y-k)^2 = r^2$$

**QUESTION 2 (25 MARKS)**

**Question 2 (a)**

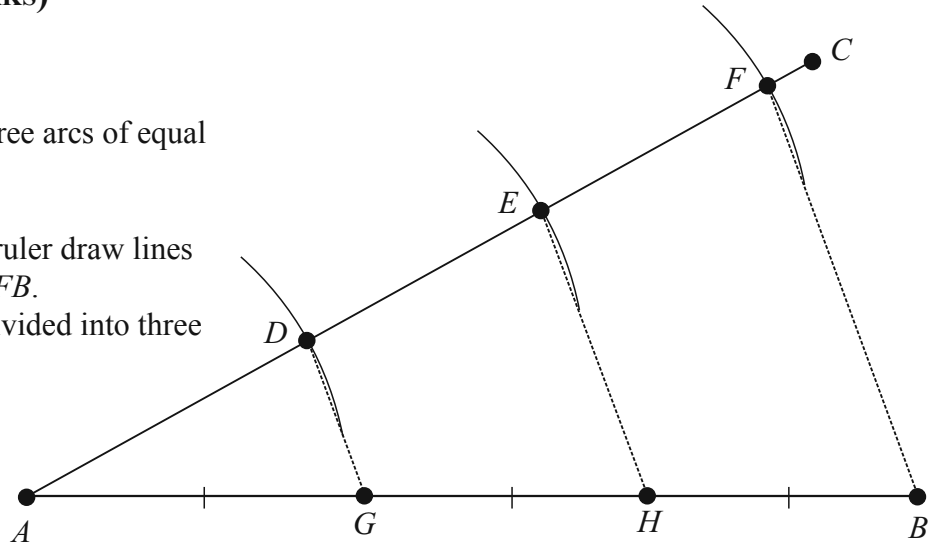
Draw a line  $AC$ .

On line  $AC$  mark out three arcs of equal radius with a compass.

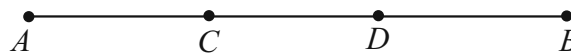
Draw the line  $FB$ .

Using a set square and ruler draw lines  $EH$  and  $DG$  parallel to  $FB$ .

The line  $AB$  has been divided into three equal parts.



**Question 2 (b)**



$A(-1, 2) = (x_1, y_1), B(5, -4) = (x_2, y_2), C = ?$

Ratio:  $a : b = 1 : 2$

$$\therefore \frac{2(-1) + 1(5)}{2+1} = \frac{-2+5}{3} = 1$$

$$\therefore \frac{2(2) + 1(-4)}{2+1} = \frac{4-4}{3} = 0$$

$\therefore C(1, 0)$

$D$  is the midpoint of  $BC$ :

$B(5, -4), C(1, 0)$

$$D = \left( \frac{5+1}{2}, \frac{-4+0}{2} \right) = (3, -2)$$

**Question 2 (c)**

$O(2, 0)$

$c : y^2 = 3x$

$P(a, b) \in c \Rightarrow b^2 = 3a$

$|PO| = \sqrt{10}$

$$\therefore \sqrt{(a-2)^2 + (b-0)^2} = \sqrt{10}$$

$(a-2)^2 + b^2 = 10$

$(a-2)^2 + 3a = 10$

$a^2 - 4a + 4 + 3a - 10 = 0$

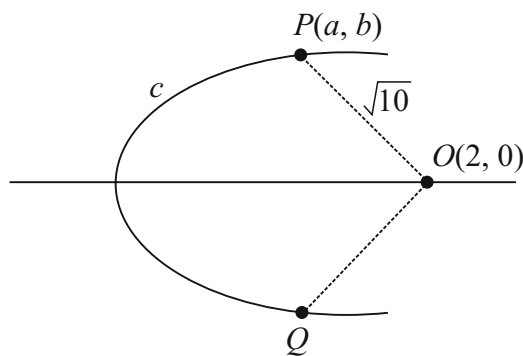
$a^2 - a - 6 = 0$

$(a+2)(a-3) = 0$

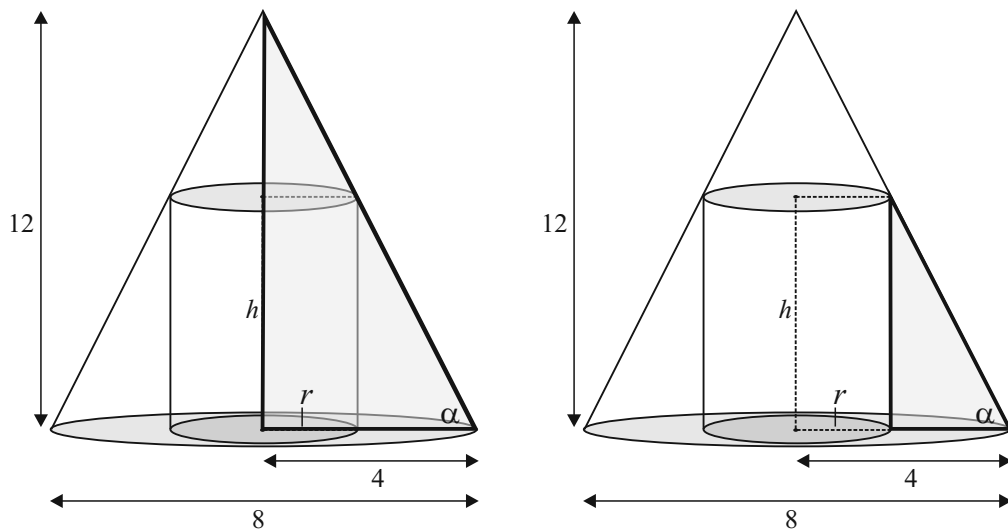
$\therefore a = \cancel{-2}, 3$

$b^2 = 3(3) = 9 \Rightarrow b = \pm 3$

$P(3, 3), Q(3, -3)$



**QUESTION 3 (25 MARKS)**



**Question 3 (a)**

The two highlighted triangles are similar triangles.

$$\frac{12}{4} = \frac{h}{4-r}$$

$$3 = \frac{h}{4-r}$$

$$\therefore 12 - 3r = h$$

**Question 3 (b)**

$$V = \pi r^2 h = \pi r^2 (12 - 3r) = 24\pi$$

$$12r^2 - 3r^3 = 24$$

$$r^3 - 4r^2 + 8 = 0$$

**Question 3 (c)**

$$f(r) = r^3 - 4r^2 + 8$$

$$f(1) = 1 - 4 + 8 \neq 0$$

$$f(2) = 8 - 16 + 8 = 0$$

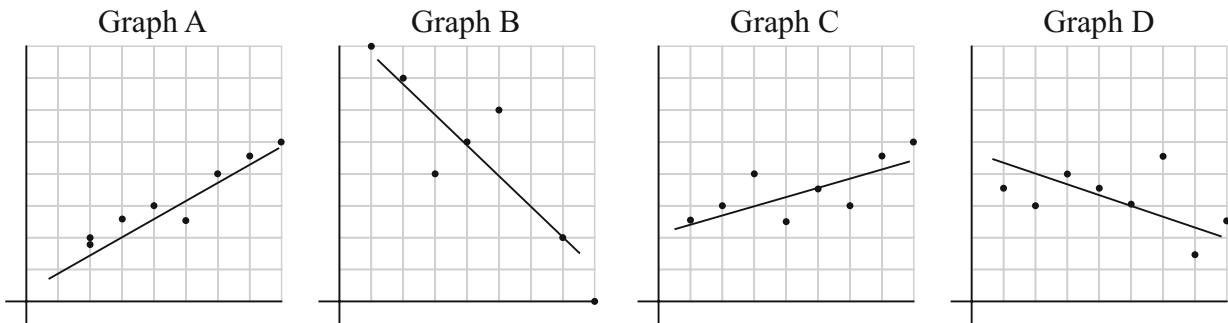
$$\therefore r = 2$$

$$h = 12 - 3r$$

$$\therefore h = 12 - 6 = 6$$

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**



Correlation	0.72	-0.90	0.96	-0.42
Graph	C	B	A	D

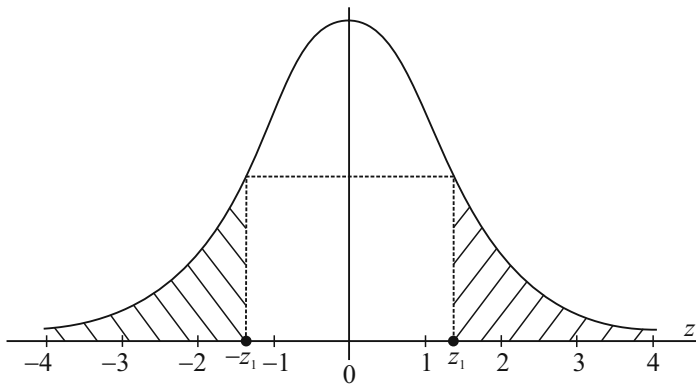
**GRAPH A:** Very tight clustering indicating  $r$  is close to 1 and a positive slope.

**GRAPH B:** Mostly tight clustering indicating  $r$  is close enough to 1 and a negative slope.

**GRAPH C:** Clustering is moderate indicating  $r$  is somewhere around 0.75 and a positive slope.

**GRAPH D:** Weaker clustering indicating  $r$  is less than 0.5 and a negative slope.

**Question 4 (b) (i)**



**Question 4 (b) (ii)**

Is  $P(z > z_1) = P(z < -z_1)$ ? Yes

**Question 4 (b) (iii)**

$$P(0) = 0$$

$$P(-\infty < z < +\infty) = 1$$

**Question 4 (b) (iv)**

$$P(z > z_1) = 1 - P(z \leq z_1)$$

**Question 4 (b) (v)**

$$P(z > 0) = 0.5 \quad P(z < 0) = 0.5$$

$$P(z \geq 0) = 0.5 \quad P(z \leq 0) = 0.5$$

**QUESTION 5 (25 MARKS)**

$G$ : Germinate,  $NG$ : Not germinate

$$P(G) = 0.6, P(NG) = 0.4$$

**Question 5 (a) (i)**

$$P(G, G, G, G) = (0.6)(0.6)(0.6)(0.6) = (0.6)^4 = \frac{81}{625}$$

**Question 5 (a) (ii)**

$$P(G, NG, NG, NG) = (0.6)(0.4)(0.4)(0.4) \times \frac{4!}{3!} = \frac{96}{625}$$

**Question 5 (a) (iii)**

$$P(NG, NG, NG, NG) = (0.4)(0.4)(0.4)(0.4) = (0.4)^4 = \frac{16}{625}$$

**Question 5 (b)**

Independence is necessary to do the calculation in (a) as the probability would change depending on whether or not the other seeds germinate. Independence may not be valid as the growing conditions (temperature, moisture and soil type) may vary from seed to seed.

**Question 5 (c)**

$$\text{Expected value} = xP(x) = 4(0.6) = 2.4$$

**Question 5 (d)**

Null hypothesis  $H_0: P = 0.9$

Alternative hypothesis  $H_1: P < 0.9$

$$\text{Sample proportion} = p = \frac{97}{120}$$

True population proportion = Sample proportion  $\pm$  1.96(Standard error of the proportion)

$$\text{Standard error of the proportion} = \sqrt{\frac{p(1-p)}{n}}$$

$$\text{True population proportion} = \frac{97}{120} \pm 1.96 \sqrt{\frac{\frac{97}{120}(1-\frac{97}{120})}{120}} = 0.738, 0.879$$

Confidence interval: 0.738  $\leftrightarrow$  0.879

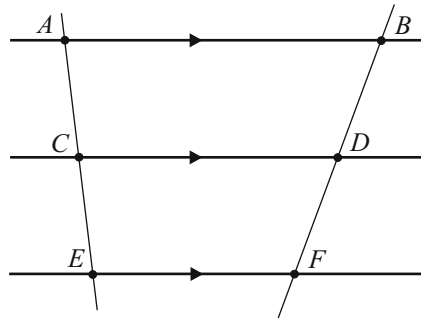
There is evidence to support the gardener's claim that less than 90% of the seeds germinate because, based on the sample data, any values in the range 73.8% – 87.9% are possible values for the proportion of seeds in the sample that germinate.

$P = 0.9$  is not in the confidence interval. At the 5% significance level, we accept the alternative hypothesis and agree that the gardener's suspicions are well-founded.

**QUESTION 6 (25 MARKS)**

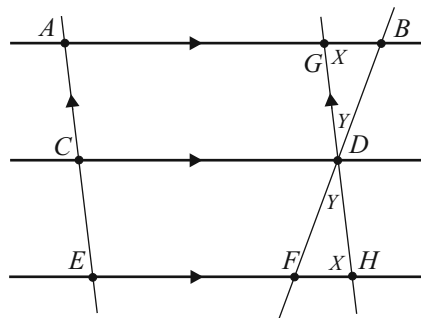
**Question 6 (a)**

**GIVEN:** Three parallel lines  $AB$ ,  $CD$  and  $EF$  such that  $C$  is on  $AE$  and  $D$  is on  $BF$  with  $|AC| = |CE|$ .



**TO PROVE:**  $|BD| = |DF|$ .

**CONSTRUCTION:** Draw a line  $GH$  through  $D$  parallel to  $AE$  such that  $G$  is on  $AB$  and  $H$  is on  $EF$ .



**PROOF:**  $ACDG$  is a parallelogram  $\Rightarrow |AC| = |GD| = |CE|$

$CEHD$  is a parallelogram  $\Rightarrow |CE| = |DH|$

$\therefore |GD| = |DH|$

Now triangle  $GDB$  and triangle  $FDH$  are congruent (ASA) because:

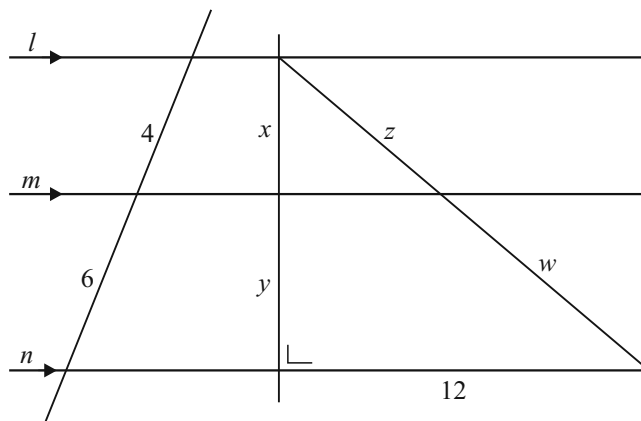
$|\angle BGD| = |\angle FHD| = X$  [Alternate angles]

$|\angle GDB| = |\angle FDH| = Y$  [Vertically opposite angles]

$|GD| = |DH|$  [Already proved]

$\therefore |BD| = |DF|$

Question 6 (b)



$$x + y = 5 \Rightarrow y = 5 - x$$

$$\frac{x}{4} = \frac{y}{6} \Rightarrow x = \frac{4y}{6} = \frac{2y}{3} = \frac{2(5-x)}{3}$$

$$\therefore 3x = 10 - 2x$$

$$5x = 10$$

$$\therefore x = 2, y = 3$$

$$z + w = 13 \text{ (Pythagoras)} \Rightarrow z = 13 - w$$

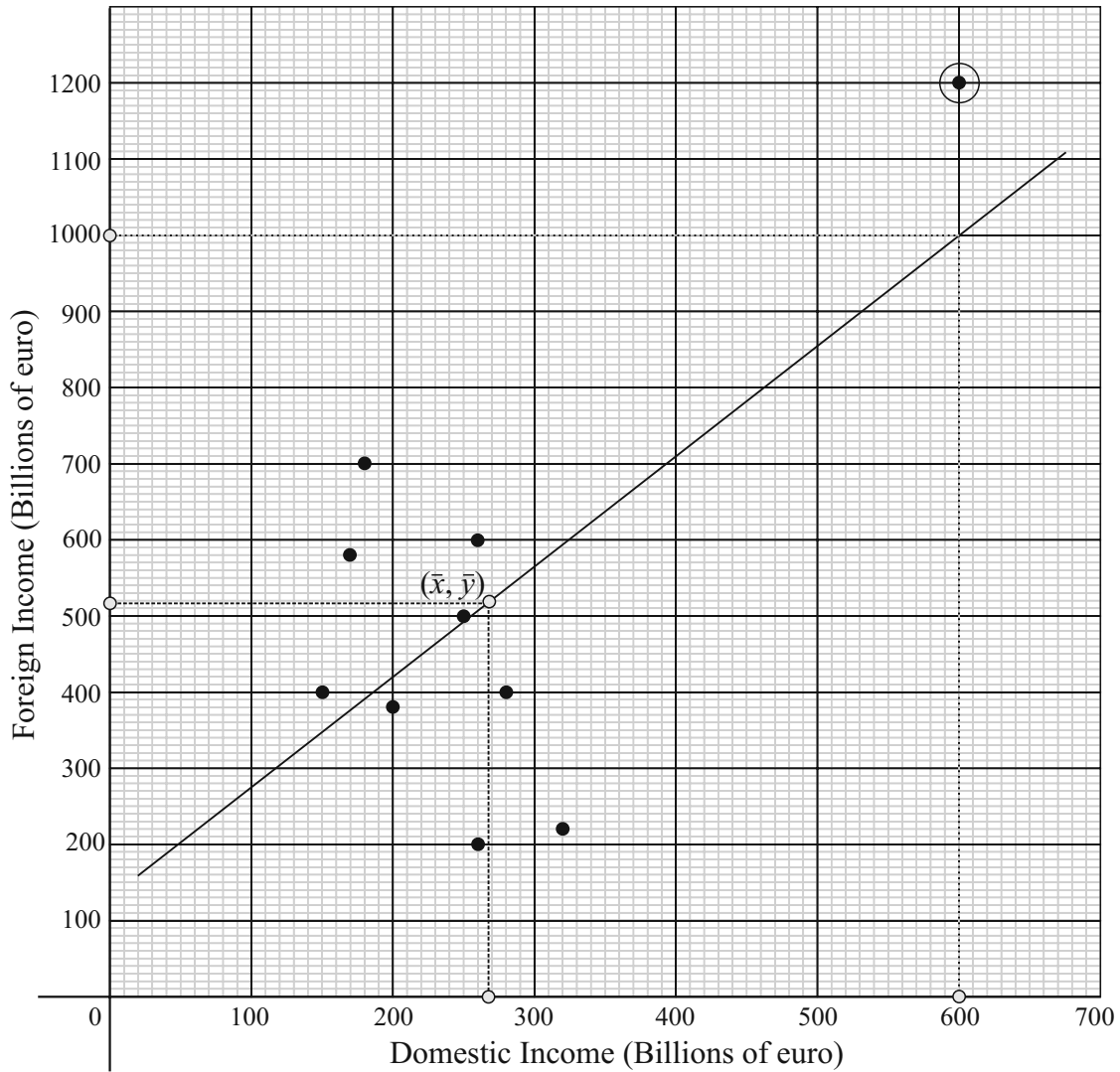
$$\frac{x}{y} = \frac{z}{w}$$

$$\frac{2}{3} = \frac{13-w}{w} \Rightarrow 2w = 39 - 3w \Rightarrow 5w = 39$$

$$\therefore w = \frac{39}{5}, z = \frac{26}{5}$$

**QUESTION 7 (75 MARKS)**

**Question 7 (a)**



**Question 7 (b)**

CASIO CALCULATOR (*fx-85GT PLUS*)  
 Steps to find  $r$ :  
 Press Mode.  
 Press 2: Stat  
 Press 2:  $A + Bx$   
 Input your  $x$  and  $y$  values  
 Press AC Button  
 Press Shift followed by the Number 1  
 Press 5: Reg  
 Press 3:  $r$   
 Press =

$r = 0.6378$

**Question 7 (c)**

$$\bar{x} = \frac{150+170+180+200+250+260+280+320+600+260}{10} = 267$$

$$\bar{y} = \frac{400+580+700+380+500+200+400+220+1200+600}{10} = 518$$

$$(\bar{x}, \bar{y}) = (267, 518)$$

$$(\bar{x}, \bar{y}) = (267, 518), (x_2, y_2) = (600, 1000)$$

$$m = \frac{1000 - 518}{600 - 267} = 1.4$$

As the yearly domestic income *increases* the foreign yearly income *increases*.

**Question 7 (d)**

Outlier (600, 1200);

Possible answer: A boom for the economy due to a huge tourist influx and a huge increase in computer/pharmaceutical exports due to the exchange rate of the euro.

**Question 7 (e)**

Remove (600, 1200) from the table and input the results again into your calculator. After pressing the AC button do the following:

**Question 7 (e) (i)**

CASIO CALCULATOR (*fx-85GT PLUS*)  
 Press Shift followed by the Number 1  
 Press 4: Var  
 Press 2:  $\bar{x}$   
 Press = [*Write down the answer*]  
 Press Shift followed by the Number 1  
 Press 4: Var  
 Press 3:  $\sigma_x$   
 Press = [*Write down the answer*]

$$\bar{x} = 230$$

$$\sigma_x = 54$$

**Question 7 (e) (iii)**

CASIO CALCULATOR (*fx-85GT PLUS*)  
 Press Shift followed by the Number 1  
 Press 5: Reg  
 Press 3:  $r$   
 Press = [*Write down the answer*]

$$r = -0.5085$$

**Question 7 (e) (ii)**

CASIO CALCULATOR (*fx-85GT PLUS*)  
 Press Shift followed by the Number 1  
 Press 4: Var  
 Press 3:  $\bar{y}$   
 Press = [*Write down the answer*]  
 Press Shift followed by the Number 1  
 Press 4: Var  
 Press 3:  $\sigma_y$   
 Press = [*Write down the answer*]

$$\bar{y} = 442$$

$$\sigma_y = 160$$

**Question 7 (f) (i)**

$$y = \bar{y} + \frac{r\sigma_y}{\sigma_x}(x - \bar{x})$$

$$y = 442 + \left[ \frac{(-0.5085)160}{54} \right] (x - 230)$$

$$y = 442 - 1.51(x - 230)$$

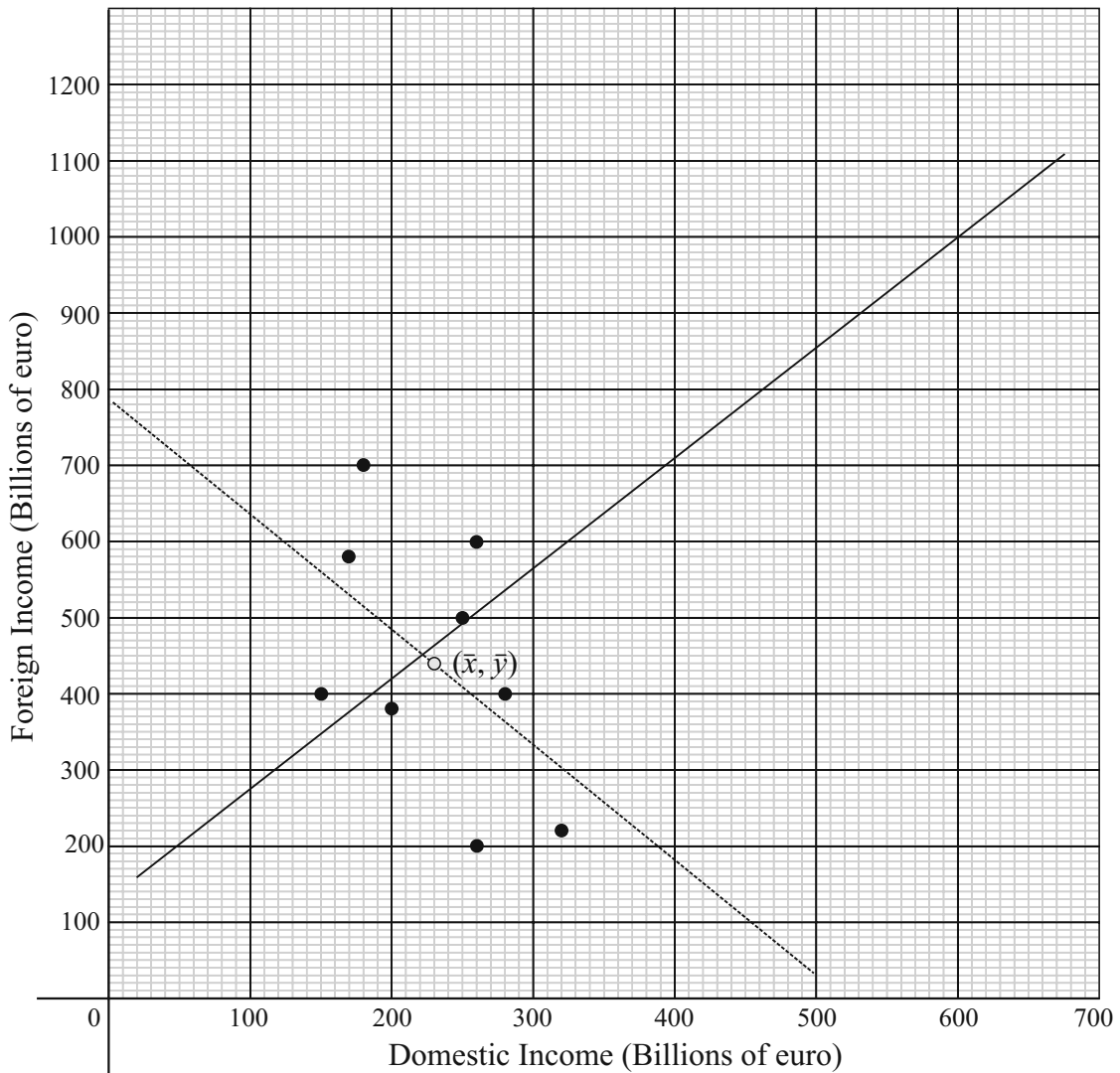
$$y = 442 - 1.51x + 347.3$$

$$y = -1.51x + 789.3$$

**Question 7 (f) (ii)**

$$\text{Slope } m = -1.5$$

**Question 7 (f) (iii)**



**Question 7 (f) (iv)**

The slope is now negative, there is a negative moderate correlation. The difference is that as domestic yearly income increases the foreign yearly income decreases.

**QUESTION 8 (50 MARKS)**

**Question 8 (a)**

$$P = 100 + 20 \cos 6t$$

Maximum value :  $\cos 6t = 1$

$$\therefore P = 100 + 20(1) = 120 \text{ mm Hg (SP)}$$

$$P = 100 + 20 \cos 6t$$

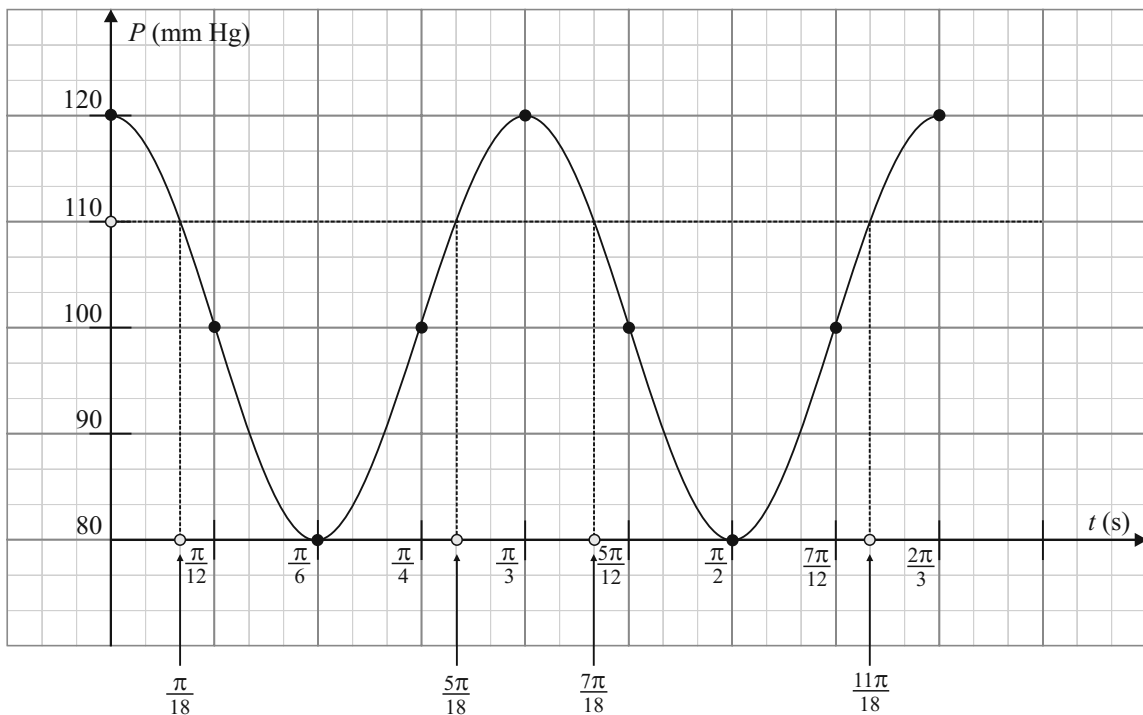
Minimum value :  $\cos 6t = -1$

$$\therefore P = 100 + 20(-1) = 80 \text{ mm Hg (DP)}$$

**Question 8 (b) (i)**

$t$ (s)	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$
$P$ (mm Hg)	120	100	80	100	120	100	80	100	120

**Question 8 (b) (ii)**



**Question 8 (b) (iii)**

Range =  $[80, 120]$

$$\text{Period} = \frac{\pi}{3}$$

**Question 8 (b) (iv)**

$$\text{Duration} = \frac{\pi}{3} \text{ s}$$

$$\text{Number of beats per minute} = \frac{60}{\frac{1}{3}\pi} = \frac{180}{\pi} = 57$$

**Question 8 (c) (i)**

Draw a line through  $P = 100$  mm Hg and read off the times as shown above.

$$P = 100 \text{ mm Hg} \Rightarrow t = \frac{\pi}{18} \text{ s}, \frac{5\pi}{18} \text{ s}, \frac{7\pi}{18} \text{ s}, \frac{11\pi}{18} \text{ s}$$

**Question 8 (c) (ii)**

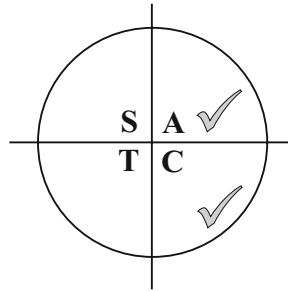
$$P = 100 + 20 \cos 6t = 110$$

$$20 \cos 6t = 10$$

$$\cos 6t = \frac{1}{2} \Rightarrow 6t = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}$$

$$6t = \frac{\pi}{3}, \frac{7\pi}{3} \text{ [First quadrant]}$$

$$= \frac{5\pi}{3}, \frac{11\pi}{3} \text{ [Fourth quadrant]}$$



$$t = \frac{\pi}{18}, \frac{7\pi}{18}$$

$$= \frac{5\pi}{18}, \frac{11\pi}{18}$$

**Question 8 (d)**

$$P = 100 + 20 \cos 6t$$

$$\frac{dP}{dt} = -20 \sin 6t \times 6 = -120 \sin 6t$$

$$\left(\frac{dP}{dt}\right)_{t=0} = -120 \sin 6(0) = 0$$

$$\left(\frac{dP}{dt}\right)_{t=\frac{\pi}{15}} = -120 \sin 6\left(\frac{\pi}{15}\right) = -120 \sin\left(\frac{2\pi}{5}\right) = -114 \text{ mm Hg}$$

$$\left(\frac{dP}{dt}\right)_{t=\frac{\pi}{5}} = -120 \sin 6\left(\frac{\pi}{5}\right) = -120 \sin\left(\frac{6\pi}{5}\right) = 70.5 \text{ mm Hg}$$

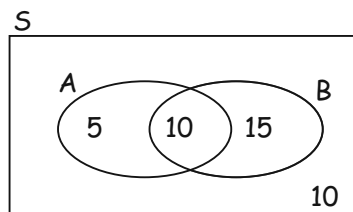
$$\left(\frac{dP}{dt}\right)_{t=\frac{\pi}{3}} = -120 \sin 6\left(\frac{\pi}{3}\right) = -120 \sin 2\pi = 0 \text{ mm Hg}$$

**QUESTION 9 (25 MARKS)**

A: People with high blood pressure

B: High level of cholesterol

- (a) People with high blood pressure and high level of cholesterol = 10



(b)  $P(\text{High BP}) = \frac{15}{40} = \frac{3}{8}$

(c)  $P(\text{High level of cholesterol}) = \frac{25}{40} = \frac{5}{8}$

(d)  $P(\text{High BP and high level of cholesterol}) = \frac{10}{40} = \frac{1}{4}$

(e)  $P(\text{High BP or high level of cholesterol}) = \frac{30}{40} = \frac{3}{4}$

(f)  $P(A \text{ or } B) = \frac{3}{4}$

$$P(A) + P(B) - P(A \text{ and } B) = \frac{3}{8} + \frac{5}{8} - \frac{1}{4} = \frac{3}{4}$$

**SAMPLE PAPER 4: PAPER 1**

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**QUESTION 1 (25 MARKS)**

**Question 1 (a) (i)**

$$z = -3 - i \Rightarrow \bar{z} = -3 + i \quad \boxed{z = a + bi \Rightarrow \bar{z} = a - bi}$$

**Question 1 (a) (ii)**

$$z = -3 - i \Rightarrow (z + 3 + i) \text{ is a factor}$$

$$\bar{z} = -3 + i \Rightarrow (z + 3 - i) \text{ is a factor}$$

$$\therefore (z + 3 + i)(z + 3 - i) = 0$$

$$z^2 + 3z - iz + 3z + 9 - 3i + iz + 3i - i^2 = 0$$

$$z^2 + 6z + 9 + 1 = 0$$

$$z^2 + 6z + 10 = 0$$

OR

$$\text{Roots: } -3 - i, -3 + i$$

$$\text{Sum } S = -6$$

$$\text{Product } P = 10$$

$$z^2 - Sz + P = 0$$

$$\therefore z^2 + 6z + 10 = 0$$

**Question 1 (b)**

If  $z$  is a root of the cubic equation, its conjugate is also a root. This is because the coefficients in the cubic are all real. Therefore,  $z^2 + 6z + 10 = 0$  is a factor of the cubic equation.

$$az^3 + 22z^2 + bz + 40 = (z^2 + 6z + 10)(az + 4)$$

$$az^3 + 22z^2 + bz + 40 = az^3 + (6a + 4)z^2 + (10a + 24)z + 40$$

$$\therefore 22 = 6a + 4 \Rightarrow a = 3$$

$$10a + 24 = b \Rightarrow 54 = b$$

$$\therefore 3z^3 + 22z^2 + 54z + 40 = (z^2 + 6z + 10)(3z + 4) = 0$$

$$z = -3 - i, -3 + i, -\frac{4}{3}$$


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**QUESTION 2 (25 MARKS)****Question 2 (a)**

$$\text{Let } \frac{ax}{b-c} = \frac{by}{c-a} = \frac{cz}{a-b} = k$$

$$ax = k(b-c)$$

$$by = k(c-a)$$

$$cz = k(a-b)$$

$$ax + by + cz = kb - kc + kc - ka + ka - kb = 0$$

**Question 2 (b) (i)**

$$x + \sqrt{5x+19} = -1$$

$$\sqrt{5x+19} = -(x+1)$$

$$5x+19 = x^2 + 2x + 1$$

$$x^2 - 3x - 18 = 0$$

$$(x+3)(x-6) = 0$$

$$x = -3, 6$$

Check solutions:

$$x = -3: -3 + \sqrt{4} = -3 + 2 = -1$$

$$x = 6: 6 + \sqrt{49} = 6 + 7 = 13$$

ANS:  $x = -3$

**Question 2 (b) (ii)**

$$\log_2 x + \frac{12}{\log_2 x} = 7$$

$$(\log_2 x)^2 - 7\log_2 x + 12 = 0$$

$$(\log_2 x - 3)(\log_2 x - 4) = 0$$

$$\therefore \log_2 x = 3 \Rightarrow x = 2^3 = 8$$

$$\therefore \log_2 x = 4 \Rightarrow x = 2^4 = 16$$

**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$2x - y + 3z = 20 \dots (1)$$

$$7x + y + z = 23 \dots (2)$$

$$3x + y - z = 3 \dots (3)$$

$$(1) + (2): 9x + 4z = 43 \dots (4)$$

$$(2) - (3): 4x + 2z = 20 \dots (5) (\times -2)$$

$$9x + 4z = 43$$

$$\frac{-8x - 4z = -40}{x = 3}$$

$$27 + 4z = 43$$

$$4z = 16 \Rightarrow z = 4$$

$$6 - y + 12 = 20$$

$$\therefore y = -2$$

ANSWER: (3, -2, 4)

**Question 3 (b)**

$$(2-3k)x^2 + (4-k)x + 2 = 0$$

$$\text{No real roots: } b^2 - 4ac < 0$$

$$(4-k)^2 - 4(2-3k)(2) < 0$$

$$16 - 8k + k^2 - 16 + 24k < 0$$

$$k^2 + 16k < 0$$

$$k(k+16) < 0$$

$$k(k+16) = 0 \Rightarrow k = -16, 0$$

ANS:  $-16 < k < 0$

**QUESTION 4 (25 MARKS)**

**Question 4 (a)**

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$$

$$a = 1, r = \frac{1}{3}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

**Question 4 (b)**

$$a - d, a, a + d$$

$$a - d + a + a + d = 3a = 27 \Rightarrow a = 9$$

$$9 - d, 9, 9 + d$$

$$(9 - d)^2 + 9^2 + (9 + d)^2 = 293$$

$$81 - 18d + d^2 + 81 + 81 + 18d + d^2 = 293$$

$$2d^2 = 293 - 243$$

$$2d^2 = 50$$

$$d^2 = 25$$

$$d = \pm 5 \text{ (Use either)}$$

Numbers: 4, 9, 14

**QUESTION 5 (25 MARKS)**

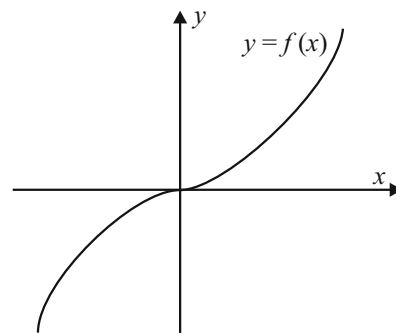
**Question 5 (a)**

This is a bijective function as there is a perfect one to one correspondence between the  $x$  and  $y$  values. A bijective function means it is both an injective and surjective function.

INJECTIVE FUNCTION

SURJECTIVE FUNCTION

BIJECTIVE FUNCTION



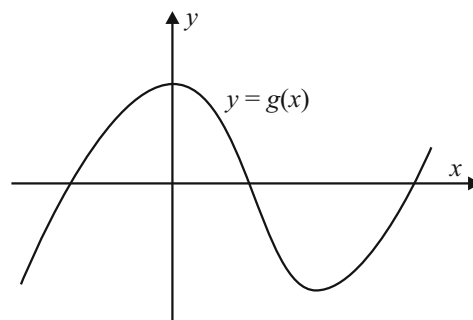
**Question 5 (b)**

This is a surjective function which means that every  $y$  value has at least one matching  $x$  value. It is not injective as many  $y$  values have more than one corresponding  $x$  value.

INJECTIVE FUNCTION

SURJECTIVE FUNCTION

BIJECTIVE FUNCTION



**Question 5 (c)**

This is an injective function which means that every  $y$  value has its own unique matching  $x$  value.

INJECTIVE FUNCTION

Domain =  $\{0, 1, 2, 3, 4, 5, \dots\}$

Range =  $\{0, 1, 4, 9, 16, \dots\}$

SURJECTIVE FUNCTION

Every element in the domain matches to a unique element in the range. There are elements in the range that do not have a matching element from the domain.

BIJECTIVE FUNCTION

**Question 5 (d)**

This is a bijective function as there is a perfect one to one correspondence between the  $x$  and  $y$  values. A bijective function means it is both an injective and surjective function.

INJECTIVE FUNCTION

Domain = {Positive real numbers}

Range = {Positive real numbers}

SURJECTIVE FUNCTION

Every element in the domain matches to a unique element in the range. Every element in the range has a unique matching element from the domain.

BIJECTIVE FUNCTION

**Question 5 (e)**

This is a surjective function which means that every  $y$  value has at least one matching  $x$  value. It is not injective as many  $y$  values have more than one corresponding  $x$  value.

INJECTIVE FUNCTION

Domain = {All real numbers}

Range = {Positive real numbers}

SURJECTIVE FUNCTION

Every element in the domain matches to at least one element in the range. Some values in the range match to two elements in the range. For example,  $2^2$  and  $(-2)^2$  both map on to 4.

BIJECTIVE FUNCTION

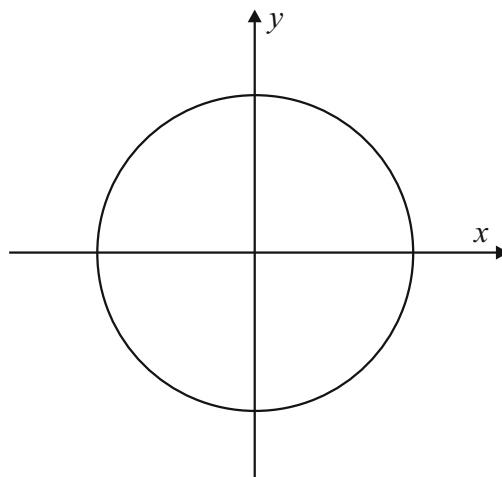
**Question 5 (f)**

This is not a function because  $x$  values have two  $y$  values.

INJECTIVE FUNCTION

SURJECTIVE FUNCTION

BIJECTIVE FUNCTION



### QUESTION 6 (25 MARKS)

#### Question 6 (a)

$f(x)$  is continuous as there are no gaps.

$g(x)$  is not continuous at there is a gap at  $x = 0$ .

#### Question 6 (b)

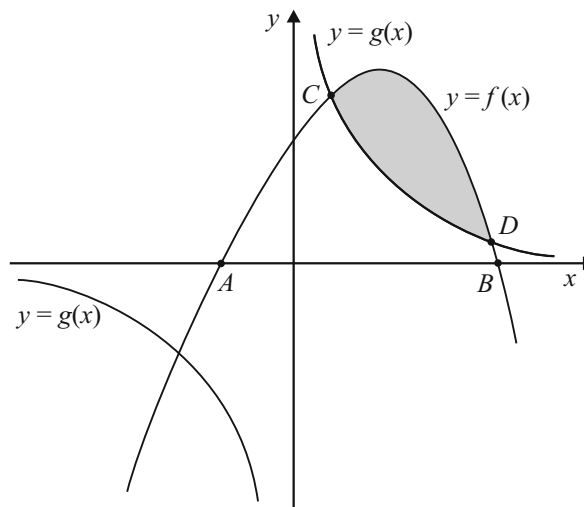
$$y = f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$$

$$f'(x) = -\frac{7}{3}x + \frac{17}{6}$$

$$f'(x) = 0 \Rightarrow -\frac{7}{3}x + \frac{17}{6} = 0$$

$$-14x + 17 = 0$$

$$\therefore x = \frac{17}{14}$$



#### Question 6 (c)

$$y = f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$$

$$y = 0 \Rightarrow -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3} = 0$$

$$7x^2 - 17x - 62 = 0$$

$$(7x - 31)(x + 2) = 0$$

$$x = -2, \frac{31}{7}$$

$$\therefore A(-2, 0), B(\frac{31}{7}, 0)$$

#### Question 6 (d)

$$(1, 12) \in f(x)?$$

$$f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$$

$$f(1) = -\frac{7}{6}(1)^2 + \frac{17}{6}(1) + \frac{31}{3}$$

$$= -\frac{7}{6} + \frac{17}{6} + \frac{31}{3} = \frac{36}{3} = 12 \text{ (True)}$$

$$(1, 12) \in g(x)?$$

$$g(x) = \frac{12}{x}$$

$$g(1) = \frac{12}{1} = 12 \text{ (True)}$$

$$(4, 3) \in f(x)?$$

$$f(x) = -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3}$$

$$f(4) = -\frac{7}{6}(4)^2 + \frac{17}{6}(4) + \frac{31}{3}$$

$$= -\frac{56}{3} + \frac{34}{3} + \frac{31}{3} = \frac{9}{3} = 3 \text{ (True)}$$

$$(4, 3) \in g(x)?$$

$$g(x) = \frac{12}{x}$$

$$g(4) = \frac{12}{4} = 3 \text{ (True)}$$

#### Question 6 (e)

$$A = \int_1^4 (f(x) - g(x)) dx$$

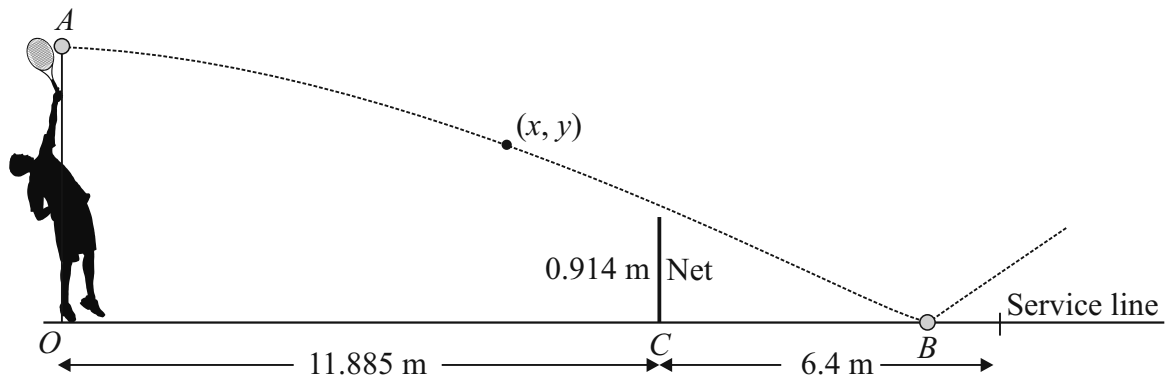
$$= \int_1^4 \left( -\frac{7}{6}x^2 + \frac{17}{6}x + \frac{31}{3} - \frac{12}{x} \right) dx$$

$$= \left[ -\frac{7}{18}x^3 + \frac{17}{12}x^2 + \frac{31}{3}x - 12 \ln x \right]_1^4$$

$$= \left\{ -\frac{7}{18}(4)^3 + \frac{17}{12}(4)^2 + \frac{31}{3}(4) - 12 \ln(4) \right\} - \left\{ -\frac{7}{18}(1)^3 + \frac{17}{12}(1)^2 + \frac{31}{3}(1) - 12 \ln(1) \right\}$$

$$= \frac{111}{4} - 12 \ln 4 \approx 11.1$$

**QUESTION 7 (50 MARKS)**



**Question 7 (a)**

Height of player + arm span above head = 1.85 m + 0.8 m = 2.65 m

$$y = 3.3 - 13t - 4.9t^2$$

$$t = 0: y = 3.3 \text{ m}$$

Height above base of racket = 3.3 - 2.65 = 0.65 m

**Question 7 (b)**

$$y = 3.3 - 13t - 4.9t^2$$

$$y = 0: 4.9t^2 + 13t - 3.3 = 0$$

$$t = \frac{-13 \pm \sqrt{13^2 - 4(4.9)(-3.3)}}{2(4.9)} = 0.23 \text{ s}$$

**Question 7 (c)**

$$x = 70t$$

$$t = 0.23: x = 70(0.23) = 16.1 \text{ m}$$

Distance from baseline to far serve line = 11.885 m + 6.4 m = 18.285 m

Ball is in as 16.1 m < 18.285 m

**Question 7 (d)**

$$x = 70t$$

$$x = 11.885: 11.885 = 70t \Rightarrow t = 0.17 \text{ s}$$

**Question 7 (e)**

$$y = 3.3 - 13t - 4.9t^2$$

$$t = 0.17 \text{ s}: y = 3.3 - 13(0.17) - 4.9(0.17)^2 = 0.948 \text{ m}$$

Yes, it will clear the net.

**Question 7 (f) (i)**

$$y = 3.3 - 13t - 4.9t^2$$

$$y = 0.914: 0.914 = 3.3 - 13t - 4.9t^2$$

$$4.9t^2 + 13t - 2.386 = 0$$

$$t = \frac{-13 \pm \sqrt{13^2 - 4(4.9)(2.386)}}{2(4.9)} = 0.172 \text{ s}$$

**Question 7 (f) (ii)**

$$x = bt$$

$$t = 0.172: 11.885 = b(0.172) \Rightarrow b = 69.1$$

### QUESTION 8 (50 MARKS)

#### Question 8 (a)

$$y = ab^t$$

$$\log_{10} y = \log_{10}(ab^t)$$

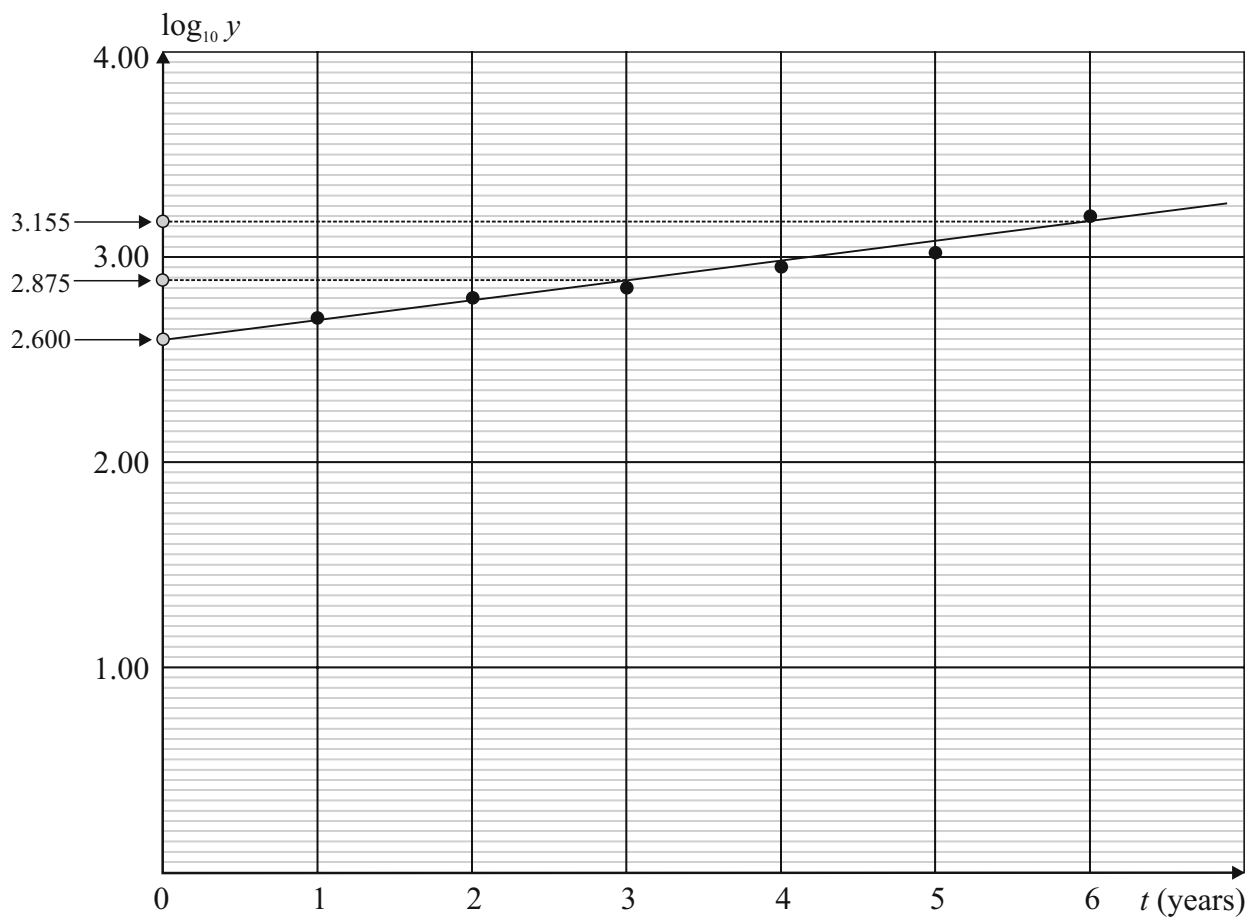
$$\log_{10} y = \log_{10} a + \log_{10} b^t$$

$$\log_{10} y = \log_{10} a + t \log_{10} b$$

#### Question 8 (b) (i)

$t$ (Number of years after 2006)	1	2	3	4	5	6
$y$ (Cost €)	500	610	720	890	1050	1540
$\log_{10} y$	2.70	2.79	2.86	2.95	3.02	3.19

#### Question 8 (b) (ii)



**Question 8 (c)**

$$y = mx + c$$

$$\log_{10} y = t \log_{10} b + \log_{10} a$$

$$c = \log_{10} a$$

$$2.6 = \log_{10} a \Rightarrow a = 10^{2.6} = 398$$

$$m = \log_{10} b$$

Points on graph: (3, 2.875), (6, 3.155)

$$m = \frac{3.155 - 2.875}{6 - 3} = 0.093$$

$$0.093 = \log_{10} b \Rightarrow b = 10^{0.093} = 1.24$$

**Question 8 (d)**

€398

**Question 8 (e)**

$$y = ab^t$$

$$2200 = 398 \times 1.24^t$$

$$\frac{2200}{398} = 1.24^t$$

$$\log_{10} \left( \frac{2200}{398} \right) = t \log_{10} 1.24$$

$$\therefore t = \frac{\log_{10} \left( \frac{2200}{398} \right)}{\log_{10} 1.24} = 7.9 \approx 8 \text{ years}$$

---

**QUESTION 9 (50 MARKS)**

**Question 9 (a) (i)**

$$h = 1 + x$$

**Question 9 (a) (ii)**

$$x^2 + r^2 = 1^2$$

$$r = \sqrt{1 - x^2}$$

**Question 9 (b)**

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (1 - x^2)(1 + x)$$

$$= \frac{1}{3} \pi (1 + x - x^2 + x^3)$$

**Question 9 (c)**

$$V = \frac{1}{3} \pi (1 + x - x^2 - x^3)$$

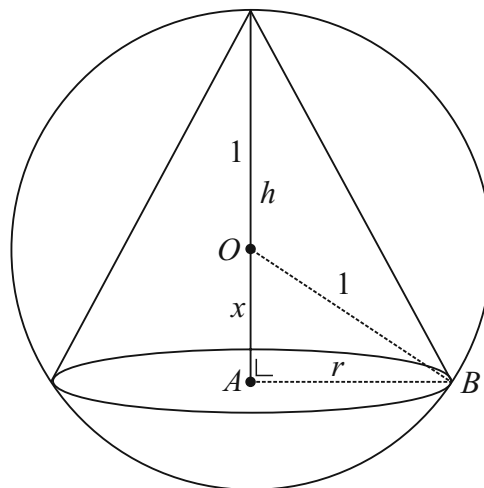
$$\frac{dV}{dx} = \frac{1}{3} \pi (1 - 2x - 3x^2) = 0$$

$$3x^2 + 2x - 1 = 0$$

$$(3x - 1)(x + 1) = 0$$

$$x = \cancel{-1}, \frac{1}{3}$$

$$\therefore V_{\text{Max.}} = \frac{1}{3} \pi \left( 1 + \left(\frac{1}{3}\right) - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^3 \right) = \frac{32}{81} \pi$$



**Question 9 (d)**

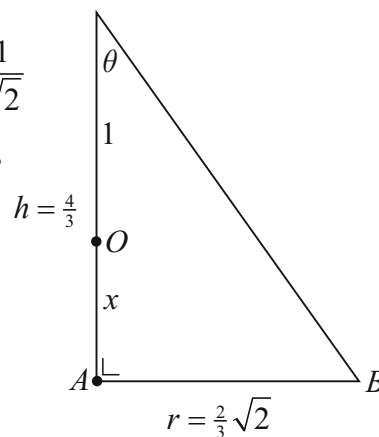
$$x = \frac{1}{3}$$

$$h = 1 + x = \frac{4}{3}$$

$$r = \sqrt{1 - x^2} = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}} = \frac{2}{3} \sqrt{2}$$

$$\tan \theta = \frac{r}{h} = \frac{\frac{2}{3} \sqrt{2}}{\frac{4}{3}} = \frac{1}{\sqrt{2}}$$

$$\theta = \tan^{-1} \left( \frac{1}{\sqrt{2}} \right) \approx 35^\circ$$



**Question 9 (e)**

Cone:  $\frac{dV}{dt} = -0.2 \text{ cm}^3 \text{ s}^{-1}$

Sphere:  $\frac{dV}{dt} = +0.2 \text{ cm}^3 \text{ s}^{-1}$

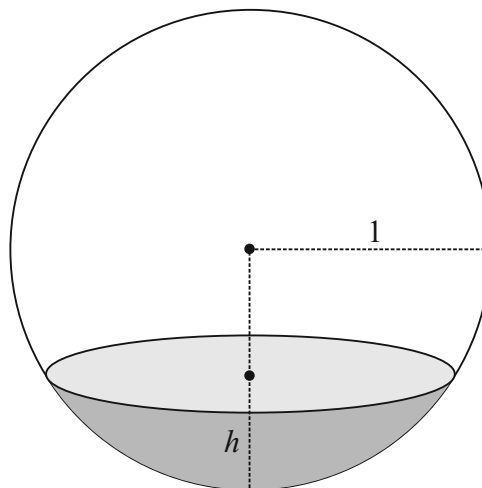
$$V = \pi h^2 - \frac{1}{3} \pi h^3$$

$$\frac{dV}{dh} = 2\pi h - \pi h^2$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} = (2\pi h - \pi h^2) \frac{dh}{dt}$$

$$\left( \frac{dV}{dt} \right)_{h=0.3} = 0.2 = [2\pi(0.3) - \pi(0.3)^2] \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{0.2}{[2\pi(0.3) - \pi(0.3)^2]} = 0.125 \text{ cm s}^{-1}$$



**SAMPLE PAPER 4: PAPER 2**

**QUESTION 1 (25 MARKS)**

**Question 1 (a)**

$$x + y - 3 = 0 \Rightarrow m_1 = -1$$

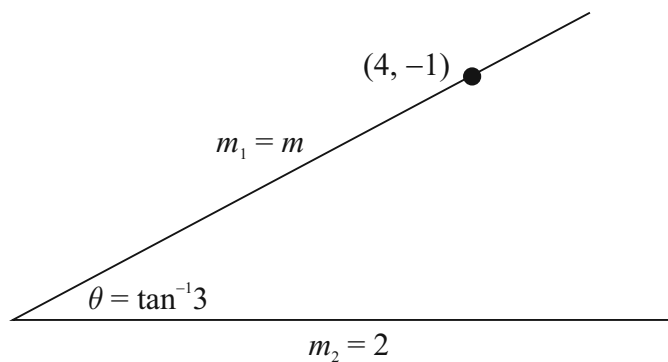
$$2x + y - 1 = 0 \Rightarrow m_2 = -2$$

$$\tan \theta = + \left( \frac{-1 - (-2)}{1 + (-1)(-2)} \right) = \left( \frac{-1 + 2}{1 + 2} \right) = \frac{1}{3}$$

$$\tan \theta = \pm \left( \frac{m_1 - m_2}{1 + m_1 m_2} \right)$$

$$\therefore \theta = \tan^{-1} \left( \frac{1}{3} \right) = 18.4^\circ$$

**Question 1 (b)**



$$\theta = \tan^{-1} 3 \Rightarrow \tan \theta = 3$$

$$2x - y - 8 = 0 \Rightarrow m_2 = 2$$

$$3 = \pm \left( \frac{m - 2}{1 + 2m} \right)$$

$$\therefore 3(1 + 2m) = (m - 2)$$

$$3 + 6m = m - 2$$

$$5m = -5$$

$$m = -1$$

$$\therefore 3(1 + 2m) = -(m - 2)$$

$$3 + 6m = -m + 2$$

$$7m = -1$$

$$m = -\frac{1}{7}$$

Equations of line:

$$l_1 : m = -1 \Rightarrow x + y + k = 0$$

$$(4, -1) \in l_1 \Rightarrow 4 - 1 + k = 0 \Rightarrow k = -3$$

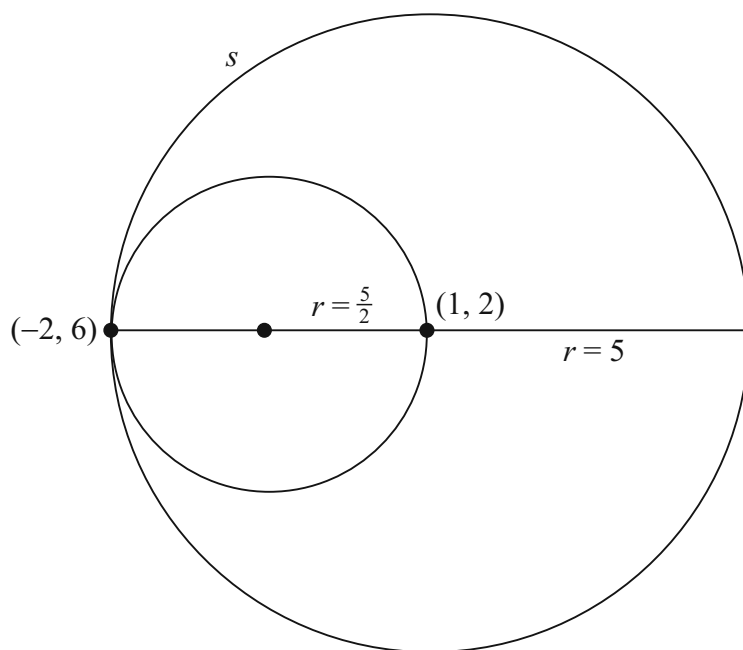
$$l_1 : x + y - 3 = 0$$

$$l_2 : m = -\frac{1}{7} \Rightarrow x + 7y + k = 0$$

$$(4, -1) \in l_2 \Rightarrow 4 - 7 + k = 0 \Rightarrow k = 3$$

$$l_2 : x + 7y + 3 = 0$$

**QUESTION 2 (25 MARKS)**



$$s: x^2 + y^2 - 2x - 4y - 20 = 0$$

$$\text{Centre } (1, 2), r = \sqrt{1^2 + 2^2 + 20} = \sqrt{25} = 5$$

Centre  $(h, k)$ , Radius  $r$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\text{Radius of new circle: } r = \frac{5}{2}$$

Centre of new circle is midpoint of  $(1, 2)$  and  $(-2, 6)$ .

$$\text{Centre} = \left( \frac{-2+1}{2}, \frac{6+2}{2} \right) = \left( -\frac{1}{2}, 4 \right)$$

$$\text{Equation of new circle: } (x + \frac{1}{2})^2 + (y - 4)^2 = (\frac{5}{2})^2$$

$$x^2 + x + \frac{1}{4} + y^2 - 8y + 16 = \frac{25}{4}$$

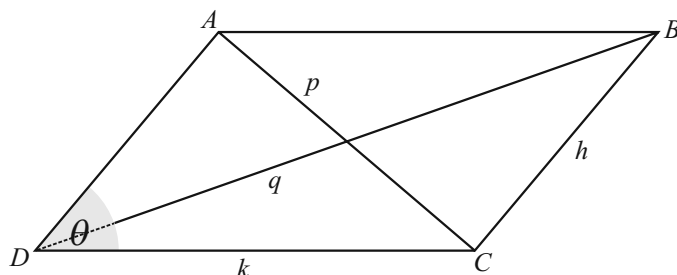
$$x^2 + y^2 + x - 8y + 10 = 0$$

### QUESTION 3 (25 MARKS)

#### Question 3 (a)

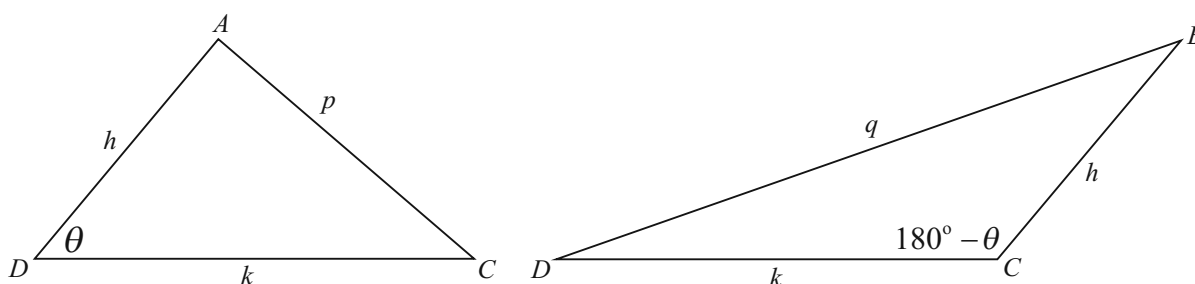
The adjacent interior angles in a parallelogram add up to  $180^\circ$ .

#### Question 3 (b)



$$\begin{aligned}\cos(180^\circ - \theta) &= \cos 180^\circ \cos \theta - \sin 180^\circ \sin \theta \\ &= (-1) \cos \theta - (0) \sin \theta \\ &= -\cos \theta\end{aligned}$$

#### Question 3 (c)



$$p^2 = h^2 + k^2 - 2kh \cos \theta \dots (1)$$

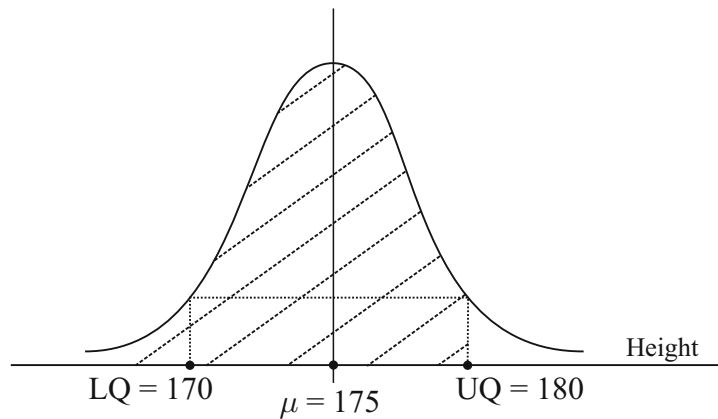
$$q^2 = h^2 + k^2 - 2kh \cos(180^\circ - \theta)$$

$$q^2 = h^2 + k^2 + 2kh \cos \theta \dots (2)$$

Add equations (1) and (2):

$$\therefore p^2 + q^2 = 2h^2 + 2k^2$$

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**QUESTION 4 (25 MARKS)****Question 4 (a)**

- (i) Median  $M = 175$  cm
- (ii) Lower quartile (LQ) = 170 cm
- (iii) Interquartile range =  $180 - 170 = 10$  cm

**Question 4 (b)**

75% of students have a height less than 180 cm.

**Question 4 (c)**

$$P(x \leq 180) = 0.75$$

$$P(z \leq Z) = 0.75 \Rightarrow z = 0.675$$

$$x = 180 : z = \frac{x - \mu}{\sigma} \Rightarrow 0.675 = \frac{180 - 175}{\sigma}$$

$$\therefore \sigma = \frac{180 - 175}{0.675} = 7.4 \text{ cm}$$

**Question 4 (d)**

$$x = 170 : z = \frac{170 - 175}{7.4} = -0.675$$

$$x = 180 : z = \frac{180 - 175}{7.4} = 0.675$$

$$\begin{aligned} &P(170 \leq x \leq 180) \\ &= P(-0.675 \leq z \leq 0.675) \\ &= P(z \leq 0.675) - P(z \leq -0.675) \\ &= P(z \leq 0.675) - \{1 - P(z \leq 0.675)\} \\ &= 2P(z \leq 0.675) - 1 \\ &= 2(0.75) - 1 \\ &= 1.5 - 1 \\ &= 0.5 \end{aligned}$$

**QUESTION 5 (25 MARKS)**

$x$	1	2	3	4	5
$P(x)$	0.1	0.2	0.3	0.3	0.1

**Question 5 (a) (i)**

$$P(\text{At least 3}) = 1 - [P(1) + P(2)]$$

$$= 1 - 0.1 - 0.2 = 0.7$$

*Or*

$$P(3) + P(4) + P(5) = 0.3 + 0.3 + 0.1 = 0.7$$

**Question 5 (a) (ii)**

$$\bar{x} = \frac{\sum xP(x)}{\sum P} = \frac{9(0.1) + 2(0.2) + 3(0.3) + 4(0.3) + 5(0.1)}{0.1 + 0.2 + 0.3 + 0.3 + 0.1} = 3.1$$

Gender	For	Against	Total
Male	58	85	143
Female	84	73	157
Total	142	158	300

**Question 5 (b) (i)**

$$P(\text{Male and Against treaty}) = \frac{85}{300} = \frac{17}{60}$$

**Question 5 (b) (ii)**

$$P(\text{Female or For treaty}) = \frac{58 + 157}{300} = \frac{43}{60}$$

**Question 5 (b) (iii)**

$$P = \frac{58}{142} = \frac{29}{71}$$


---

**QUESTION 6 (25 MARKS)**

**Question 6 (a)**

$A(0, 0), B(1, 4), C(4, 1)$

$$\begin{aligned} A &= \frac{1}{2}|x_1y_2 - x_2y_1| \\ &= \frac{1}{2}|(1)(1) - (4)(4)| \\ &= \frac{1}{2}|-15| \\ &= \frac{15}{2} \end{aligned}$$

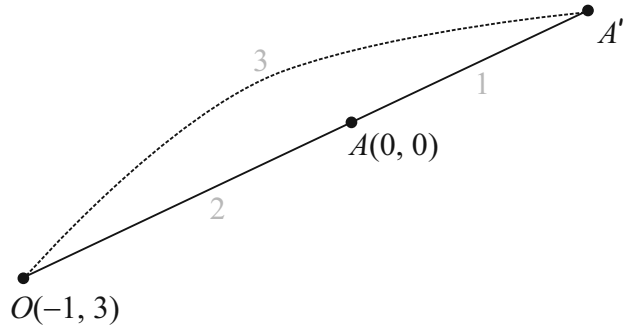
**Question 6 (b)**

$$k = \frac{|\text{Image Length}|}{|\text{Object Length}|} = \frac{|OA'|}{|OA|} = \frac{3}{2}$$

$a = 3, b = 1$

$O(-1, 3) = (x_1, y_1); A(0, 0) = (x_2, y_2)$

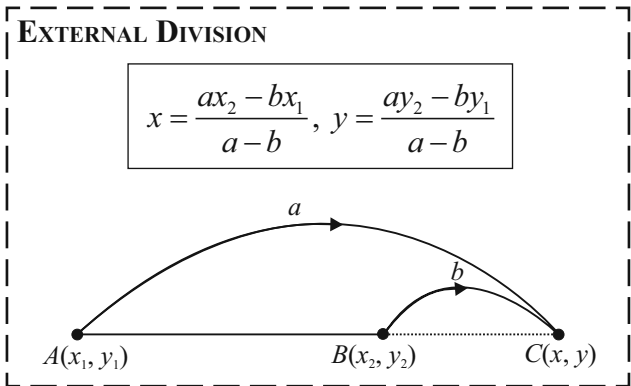
$$A' = \left( \frac{3(0) - 1(-1)}{3 - 1}, \frac{3(0) - 1(3)}{3 - 1} \right) = \left( \frac{1}{2}, -\frac{3}{2} \right)$$



$a = 3, b = 1$

$O(-1, 3) = (x_1, y_1); B(1, 4) = (x_2, y_2)$

$$B' = \left( \frac{3(1) - 1(-1)}{3 - 1}, \frac{3(4) - 1(3)}{3 - 1} \right) = \left( 2, \frac{9}{2} \right)$$



$a = 3, b = 1$

$O(-1, 3) = (x_1, y_1); C(4, 1) = (x_2, y_2)$

$$C' = \left( \frac{3(4) - 1(-1)}{3 - 1}, \frac{3(1) - 1(3)}{3 - 1} \right) = \left( \frac{13}{2}, 0 \right)$$

**Question 6 (c)**

$$A' = \left( \frac{1}{2}, -\frac{3}{2} \right) \rightarrow \left( -6, -\frac{3}{2} \right)$$

$$B' = \left( 2, \frac{9}{2} \right) \rightarrow \left( -\frac{9}{2}, \frac{9}{2} \right)$$

$$C' = \left( \frac{13}{2}, 0 \right) \rightarrow (0, 0)$$

$$A = \frac{1}{2} \left| (-6) \left( \frac{9}{2} \right) - \left( -\frac{3}{2} \right) \left( -\frac{9}{2} \right) \right| = \frac{1}{2} \left| -27 - \frac{27}{4} \right| = \frac{1}{2} \left| -\frac{135}{4} \right| = \frac{135}{8}$$

$$\frac{\text{Area } A'B'C'}{\text{Area } ABC} = \frac{\left( \frac{135}{8} \right)}{\left( \frac{15}{2} \right)} = \frac{9}{4} = \left( \frac{3}{2} \right)^2 = k^2$$

## QUESTION 7 (50 MARKS)

### Question 7 (a)

- (i) A random variable is a function that associates a unique numerical value with every outcome of an experiment. It may vary from trial to trial as the experiment is repeated.
- (ii) DISCRETE: **Ex.** A coin is tossed five times. The random variable is the number of heads. Its values can be 0, 1, 2, 3, 4, 5.  
CONTINUOUS: **Ex.** The temperature in a house during the day can take any positive or negative value within a certain range.
- (iii) Expected value  $\mu = \sum xP(x)$  = Mean of a probability distribution.

### Question 7 (b)

- (i) A prime number is a whole positive integer (excluding 1) divisible by itself and 1 only.

(ii)

		Die A					
		1	2	3	4	5	6
Die B	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- (iii) Primes in table: 2, 3, 5, 7, 11  
Number of primes = 15

$$P(\text{Sum that is a prime}) = \frac{\text{Number of primes}}{\text{Number of numbers}} = \frac{15}{36} = \frac{5}{12}$$

- (iv)  $P(\text{Sum that is not a prime}) = \frac{\text{Number of non-primes}}{\text{Number of numbers}} = \frac{21}{36} = \frac{7}{12}$

### Question 7 (c)

Outcome	Not a Prime Sum	Prime Sum
$P$	$\frac{7}{12}$	$\frac{5}{12}$
Net income to Bob	-3	3
$xP(x)$	$-\frac{7}{4}$	$\frac{5}{4}$

### Question 7 (d)

- (i)  $E = \sum xP(x) = -\frac{7}{4} + \frac{5}{4} = -\frac{1}{2}$

$E = -50$  c, on average Bob loses 50 c per game.

- (ii) Expected losses:  $30 \times (-\frac{1}{2}) = -\text{€}15$

**Question 7 (e)**

$$-3 \times \frac{7}{12} + \frac{5}{12}x = \frac{1}{2}$$

$$-21 + 5x = 6$$

$$5x = 27$$

$$x = \frac{27}{5} = \text{€}5.40$$

**QUESTION 8 (25 MARKS)****Question 8 (a)**

$$P(4) = \frac{1}{6}$$

$$P(\text{Not } 4) = \frac{5}{6}$$

$$\begin{aligned}
 P &= 1 - \left\{ {}^{20}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{20} + {}^{20}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{19} + {}^{20}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{18} + {}^{20}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{17} + {}^{20}C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^{16} + {}^{20}C_5 \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^{15} \right\} \\
 &= 1 - 0.898 \\
 &= 0.102 \\
 &= 10.2\%
 \end{aligned}$$

**Question 8 (b)**

A probability of 10.2% means that a fair die will give six or more 4s 10.2% of the time in 20 throws of a die. So there is nothing unusual about the die used in the casino. If the probability of at least six 4s when a fair die was tossed 20 times was less than 5% then this probability is so small that it hasn't happened by chance and so the die is biased.

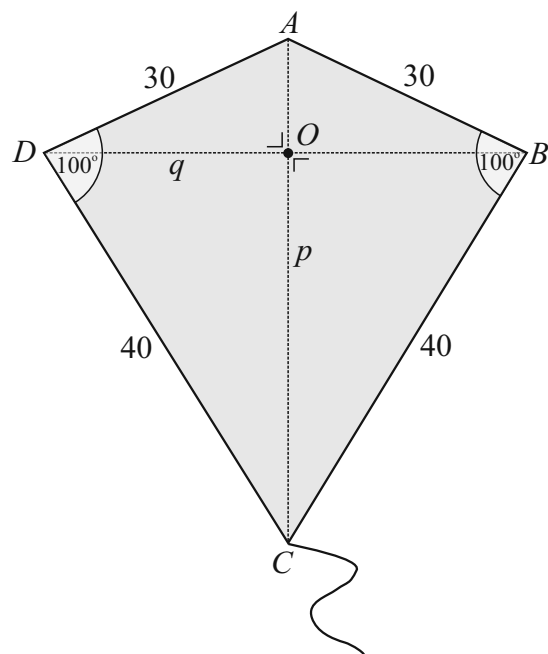
**QUESTION 9 (75 MARKS)****Question 9 (a)**

- (i) A **rhombus** is a simple quadrilateral whose sides are equal.  
 (ii) The angles in a quadrilateral add up to  $360^\circ$ . If each angle is equal, each angle will be  $90^\circ$  and hence a **square** is the shape formed.

**Question 9 (b)**

(i) Perimeter  $P = 2a + 2b$

- (ii) Area of  $\triangle ADC = \frac{1}{2}(p)\left(\frac{1}{2}q\right) = \frac{1}{4}pq$   
 Area of  $\triangle ABC = \frac{1}{2}(p)\left(\frac{1}{2}q\right) = \frac{1}{4}pq$   
 Area of  $ABCD = \frac{1}{4}pq + \frac{1}{4}pq = \frac{1}{2}pq$   
  
 Area of  $\triangle ADC = \frac{1}{2}ab \sin \theta$   
 Area of  $\triangle ABC = \frac{1}{2}ab \sin \theta$   
 Area of  $ABCD = ab \sin \theta$



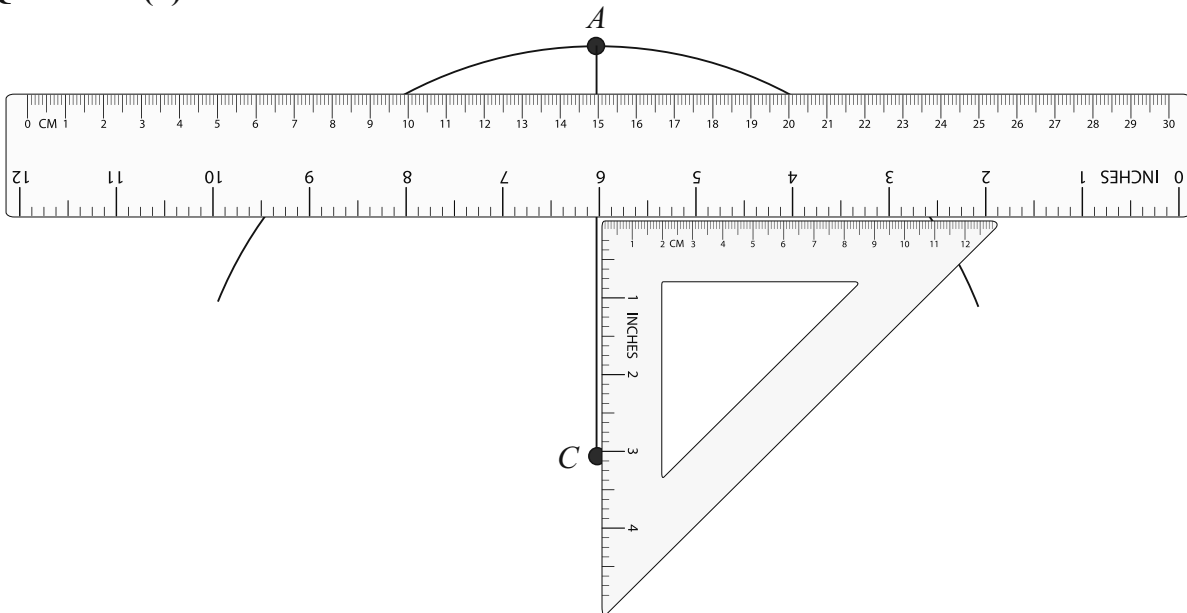
**Question 9 (c)**

(i)  $|AC|^2 = 30^2 + 40^2 - 2(30)(40)\cos 100^\circ$   
 $\therefore |AC| = 54 \text{ cm}$

(ii)  $\frac{\sin(|\angle CAD|)}{40} = \frac{\sin 100^\circ}{54}$   
 $\sin(|\angle CAD|) = \frac{40 \sin 100^\circ}{54}$   
 $|\angle CAD| = \sin^{-1}\left(\frac{40 \sin 100^\circ}{54}\right) = 46.8^\circ$

(iii)  $\sin(|\angle CAD|) = \frac{\frac{1}{2}|BD|}{30}$   
 $\therefore \sin 46.8^\circ = \frac{|BD|}{60}$   
 $|BD| = 60 \sin 46.8^\circ = 43.7 \text{ m}$

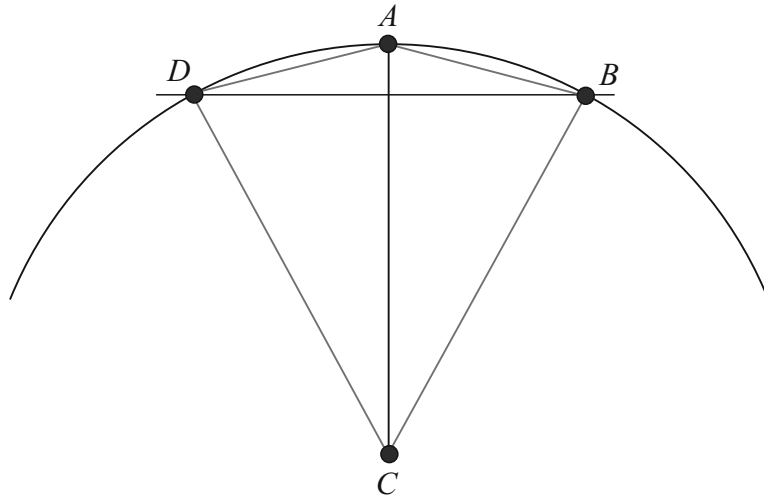
(iv) Area =  $(30)(40)\sin 100^\circ = 1182 \text{ cm}^2$

**Question 9 (d)**

Draw a line segment  $[AC]$  of length 10 cm.

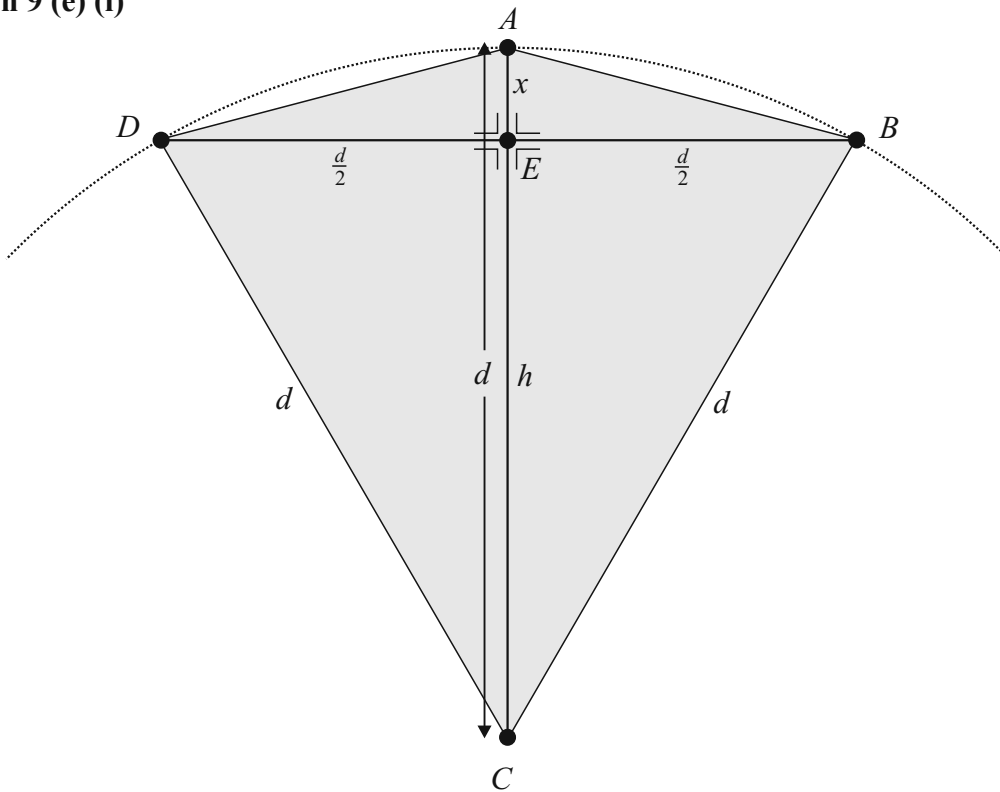
Using a compass draw an arc of radius 10 cm with  $C$  as centre.

Using a set square place a rule perpendicular to  $AC$  and move the ruler into position such that it measures a chord of distance of 10 cm. Draw a line along the ruler in this position.



Draw the kite  $ABCD$ .

**Question 9 (e) (i)**



Consider the right-angled triangle  $BEC$ :

$$d^2 = h^2 + \left(\frac{d}{2}\right)^2$$

$$d^2 = h^2 + \frac{d^2}{4}$$

$$\therefore h^2 = d^2 - \frac{d^2}{4} = \frac{3d^2}{4}$$

$$h = \sqrt{\frac{3d^2}{4}} = \frac{\sqrt{3}}{2}d$$

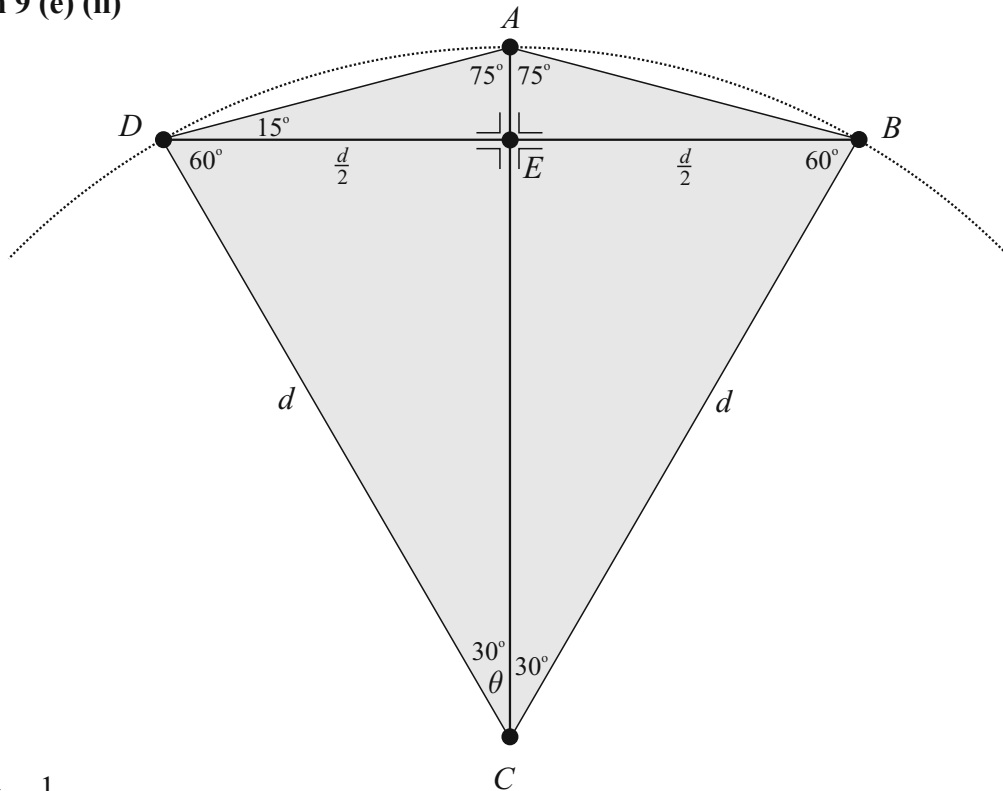
Consider the right-angled triangle  $BEA$ :

$$x = d - h = d - \frac{\sqrt{3}}{2}d = d\left(1 - \frac{\sqrt{3}}{2}\right) = d\left(\frac{2 - \sqrt{3}}{2}\right)$$

$$\tan(|\angle ABD|) = \frac{d\left(\frac{2 - \sqrt{3}}{2}\right)}{\frac{d}{2}} = (2 - \sqrt{3})$$

$$\therefore |\angle ABD| = \tan^{-1}(2 - \sqrt{3}) = 15^\circ$$

**Question 9 (e) (ii)**



$$\sin \theta = \frac{\frac{d}{2}}{d} = \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

$$|\angle ADC| = 75^\circ, |\angle ABC| = 75^\circ, |\angle BAD| = 150^\circ, |\angle BCD| = 60^\circ$$

**Question 9 (f)**

$$\tan 15^\circ = \tan(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\left(1 - \frac{1}{\sqrt{3}}\right)}{\left(1 + \frac{1}{\sqrt{3}}\right)} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

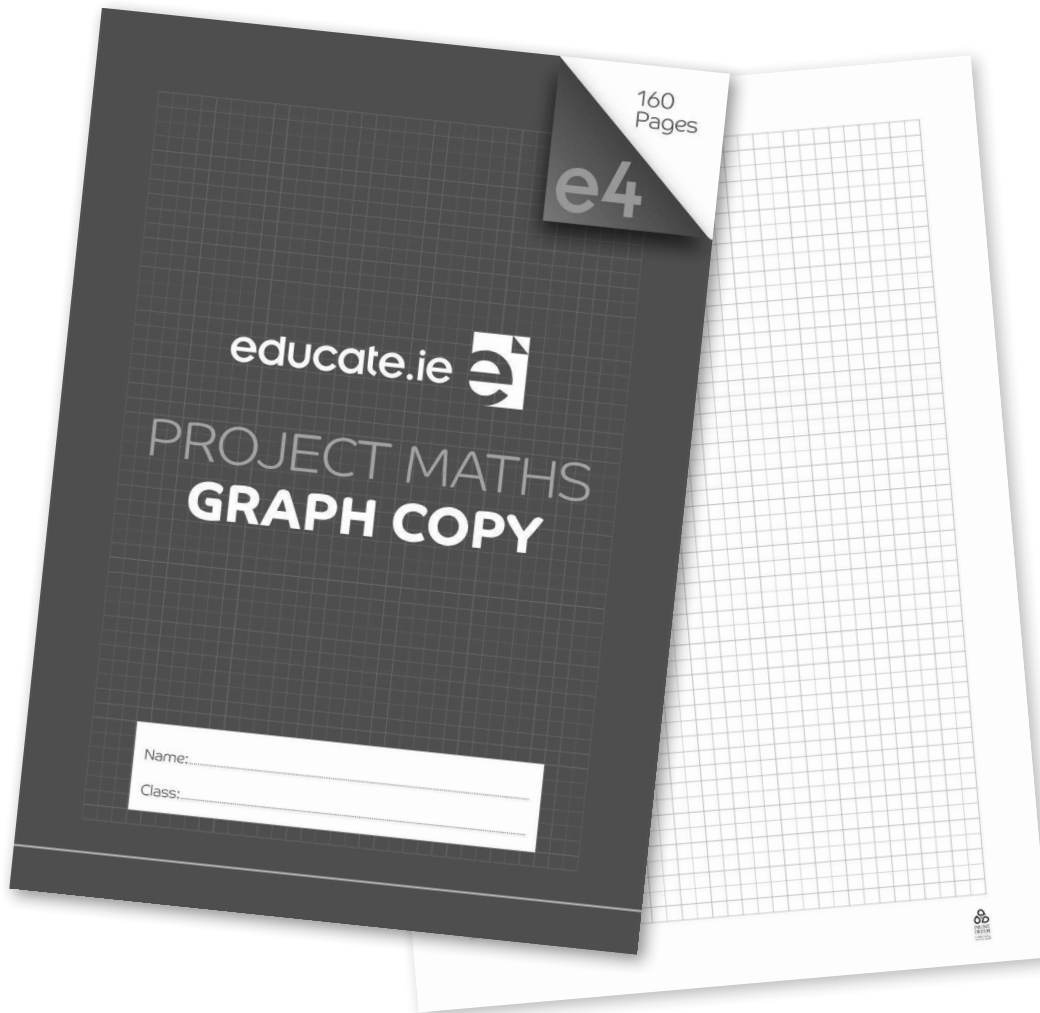
$$= \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

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