This book contains the solutions to a large number of questions that provide a comprehensive workout for students preparing for the Leaving Certificate Maths exam 2019.

It includes the solutions to the SEC Papers from 2018 to 2012 (Sets A to G) in addition to the official sample paper produced in 2014 (Set H). It also includes solutions to our own sample papers (Sample Papers 1 to 4) which follow the style of questioning required to prepare you for the Leaving Certificate exam in 2019. Sample Papers 5, 6 & 7 and accompanying solutions are available online at **www.educateplus.ie**.

This book of the solutions to SEC and sample papers provides an excellent guide for students where every aspect of the course is covered. So, enjoy and good luck with your studies.

Tony Kelly & Kieran Mills

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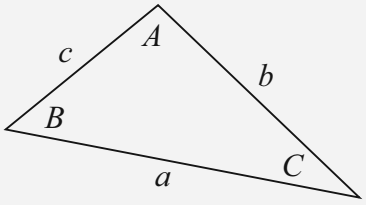
How will this book help you?

1. Formulae

The *Formulae and Tables* Book available to you in the exam is an invaluable resource. It is wonderful that students no longer have to learn formulae off by heart.

There is a lot of information in your *Formulae and Tables* Book so you need to be completely familiar with its layout. Our solutions book makes constant reference to where formulae are placed in the book with their page numbers as shown on the right. These references are placed strategically throughout these solutions.

Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Area: $\frac{1}{2}ab\sin C$

Sine Rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

Cosine rule: $a^2=b^2+c^2-2bc\cos A$

EPL 104

2. Calculator

Your calculator is your friend. You should be comfortable carrying out all the operations on your calculator from simple arithmetic calculations to being able to change modes to carrying out more complicated statistical operations. Once again, this book contains constant references to how you use your calculator as shown on the right.

The calculator used in this book is a CASIO (*fx-83GT PLUS*). This calculator is by far the most popular calculator used in schools.

Using your calculator
Input the number
Press **Shift** followed by **Mode**
Press **7**: Sci
Press **2** (two significant figures)
Press =

3. How the marking scheme works

The marking scheme used to allocate marks to your paper is an extremely precise method to make sure you get exactly the marks you deserve for the type of answer you have written in the exam. Read the official explanation of this scheme presented below before we give you a user-friendly guide as to how it operates.

Example of the structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No. of categories	2	3	4	5	6
5-mark scale		0, 3, 5	0, 3, 4, 5		
10-mark scale		0, 5, 10	0, 4, 8, 10		
15-mark scale					
20-mark scale			0, 7, 18, 20	0, 7, 10, 18, 20	
25-mark scale				0, 15, 20, 22, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

• incorrect response (no credit)
• correct response (full credit)

B-scales (three categories)

• response of no substantial merit (no credit)
• partially correct response (partial credit)
• correct response (full credit)

C-scales (four categories)

• response of no substantial merit (no credit)
• response with some merit (low partial credit)
• almost correct response (high partial credit)
• correct response (full credit)

D-scales (five categories)

• response of no substantial merit (no credit)
• response with some merit (low partial credit)
• response about half-right (middle partial credit)
• almost correct response (high partial credit)
• correct response (full credit)

E-scales (six categories)

• response of no substantial merit (no credit)
• response with some merit (low partial credit)
• response almost half-right (lower middle partial credit)
• response more than half-right (upper middle partial credit)
• almost correct response (high partial credit)
• correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

USER-FRIENDLY GUIDE TO THE MARKING SCHEME USING AN EXAMPLE:

MARKING SCHEME NOTES

Question 7 (b) (iv) [Scale 10C* (0, 4, 8, 10)]

4: • Calculates or gives one correct distance.

8: • Calculates the total distance.
• Sets up correct fraction to calculate time without complete and correct calculation.
• Sets up incorrect fraction to calculate time with correct calculation.
* Penalise one mark for incorrect or omitted units, provided full marks otherwise.

The first note of importance is that you have no idea of the marking scheme when you are sitting the paper. The original marking scheme may undergo a number of changes over the Summer before the final marking scheme is released. This final marking scheme is constantly referenced throughout the solutions.
Let us examine how it works using the example above for a particular question (7 (b) (iv)):
[Scale 10C* (0, 4, 8, 10)] 10 means **10 marks are allocated to this question**.

[Scale 10C* (0, 4, 8, 10)] C means there are **4 possible marks you can receive for this question**.

[Scale 10C* (0, 4, 8, 10)] * means **1 mark can be taken off for leaving out units**. So 4, 8 or 10 marks could become 3, 7 or 9 marks if you forgot to put in your correct units.

[Scale 10C* (0, 4, 8, 10)] 0 means **no credit**. This is what you get when the grid is left blank or you write something totally irrelevant in it. No matter how much pressure you feel, always write something. Even though you may feel overwhelmed by the paper you still have access to formulae, your calculator and your head is full of opinions.

Do not leave any grids blank. Always write something of relevance to the question.

[Scale 10C* (0, 4, 8, 10)] 4 means **low partial credit**. You will get this for a minimal amount of effort - any effort in the right direction. This gives you 40% of the marks. However, other marking schemes may even give you 7 for a low partial credit - 70% of the marks. You just never know. Once again, leave no blanks. Every grid should have something written on it. Correctors want to give you marks. Make sure you give them that opportunity.

[Scale 10C* (0, 4, 8, 10)] 8 means **high partial credit**.

[Scale 10C* (0, 4, 8, 10)] 10 means **full credit**.

4. Our Sample Papers get you ready for the real thing

Our sample papers were written and researched over a number of years to try and capture the essence and style of the new method of questioning brought in by the new Project Maths course.

Initially when students start using our sample papers, they think they are difficult. But when they persevere they actually start enjoying them because they feel they are cracking the course. This is what learning is all about: being challenged, struggling and fighting with a problem and then it all becomes clear - that lovely *ah ha* moment. Students who complete our sample papers feel totally prepared for the real thing.

One thing you will notice is the SEC Leaving Cert questions bear a remarkable similarity to a number of our sample questions. What a compliment!

LC 2018 (Set A): Paper 1

Question 1 (25 marks)

	Salary	End of year commission on total sales
Contract A	€35 000	2%
Contract B	€30 000	3%

Question 1 (a)

Contract A

€35 000 + €400 000 × 0·02 = €43 000

Contract B

€30 000 + €400 000 × 0·03 = €42 000

Question 1 (b)

Commision earned = €50 000 − €35 000
= €15 000

€15 000 = 2% of sales

€15 000 × 50 = 100% of sales

Total sales = €750 000

Question 1 (c)

$35\,000 + 0\cdot02x = 30\,000 + 0\cdot03x$

$35\,000 - 30\,000 = 0\cdot03x - 0\cdot02x$

$5000 = 0\cdot01x$

$x = \dfrac{5000}{0\cdot01} = €500\,000$

Marking Scheme Notes

Question 1 (a) [Scale 10D (0, 2, 6, 8, 10)]

2: • Use of 2% or 3%

6: • 8000 or 12000 or both

8: • One contract correct

Note: Accept correct answer without work

Question 1 (b) [Scale 5C (0, 3, 4, 5)]

3: • 50000 − 35000 formulated or 15000

4: • 15000 × 0·02 or equivalent = 300

Note: Accept correct answer without work

Question 1 (c) [Scale 10C (0, 3, 8, 10)]

3: • 35000 − 30000 or 5000 written
• correct answer without work

8: • $35000 + 0\cdot02x = 30000 + 0\cdot03x$
• $\dfrac{x}{100} = 5000$

Question 2 (25 marks)

Question 2 (a)

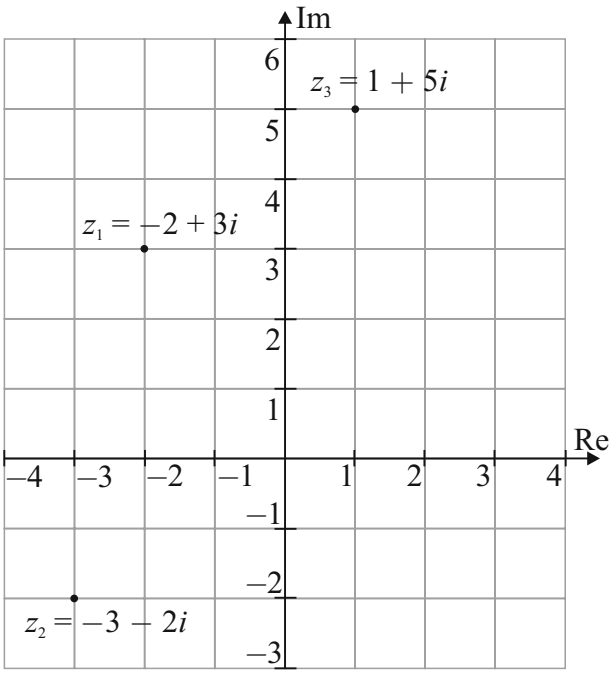
$z_1 = -2 + 3i$

$z_2 = -3 - 2i$

$z_3 = z_1 - z_2 = -2 + 3i - (-3 - 2i)$

$= -2 + 3i + 3 + 2i$

$= 1 + 5i$



Marking Scheme Notes

Question 2 (a) [Scale 15D (0, 3, 7, 13, 15)]

3: • $z_3 = z_1 - z_2$ with some substitution
• 1 or 2 correct plots (z_1 and/or z_2) without labels

7: • z_3 correct, with or without plotting
• 2 correct plots (z_1 and z_2) with labels

13: • Plot of incorrect z_3 and 2 correct plots labelled
• Mixes up real and imaginary axes but plots and labels 3 points 'correctly'

Question 2 (b)

$z_1 = -2 + 3i \Rightarrow |z_1| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

$z_2 = -3 - 2i \Rightarrow |z_2| = \sqrt{(-3)^2 + (-2)^2} = \sqrt{13}$

$z_3 = 1 + 5i \Rightarrow |z_3| = \sqrt{(1)^2 + 5^2} = \sqrt{26}$

$|z_1| + |z_2| = 2\sqrt{13} = 7 \cdot 2$

$|z_3| = \sqrt{26} = 5 \cdot 1$

$\therefore |z_1| + |z_2| \neq |z_3|$

Marking Scheme Notes

Question 2 (b) [Scale 5D (0, 2, 3, 4, 5)]

2: • $|z_1|$ or $|z_2|$ or $|z_3|$ formulated with some substitution
• Correct formula

3: • $|z_1|$ or $|z_2|$ or $|z_3|$ correct

4: • $|z_1|$ and $|z_2|$ and $|z_3|$ correct but no conclusion or incorrect conclusion

Question 2 (c) [Scale 5C (0, 3, 4, 5)]

3: • Some correct substitution
• Correct formula
• Some multiplication above and below by same number, even if incorrect conjugate
• correct answer without work

4: • $\dfrac{(-2+3i)(-3+2i)}{(-3-2i)(-3+2i)}$

Question 2 (c)

$z_4 = \dfrac{z_1}{z_2} = \dfrac{(-2+3i)}{(-3-2i)} \times \dfrac{(-3+2i)}{(-3+2i)} = \dfrac{6 - 4i - 9i + 6i^2}{9 - 6i + 6i - 4i^2}$

$= \dfrac{6 - 13i - 6}{9 + 4} = \dfrac{-13i}{13} = 0 - 1i$

Question 3 (25 marks)

Question 3 (a)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$2x^2 - 7x - 3 = 0$

$a = 2,\ b = -7,\ c = -3$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)} = \frac{7 \pm \sqrt{49 + 24}}{4}$$

$$= \frac{7 \pm \sqrt{73}}{4} = -0 \cdot 39,\ 3 \cdot 89$$

Question 3 (b)

$2a + 3b = 15 \ldots (\mathbf{1})$

$5a + b = -8 \ldots . (\mathbf{2})(\times -3)$

$$\begin{array}{r} 2a + 3b = 15 \\ -15a - 3b = 24 \\ \hline -13a \qquad = 39 \end{array} \Rightarrow a = -3$$

Into (**2**): $5(-3) + b = -8 \Rightarrow -15 + b = -8$

$\therefore b = 7$

Answer: $(-3,\ 7)$

Marking Scheme Notes

Question 3 (a) [Scale 10D (0, 2, 6, 8, 10)]

2: • correct roots formula
• a or b or c identified
• attempt at factorising
• correct answer without work

6: • formula fully substituted

8: • one root calculated
• $x = \frac{7 \pm \sqrt{73}}{4}$ and stops

Question 3 (b) [Scale 15D (0, 3, 7, 13, 15)]

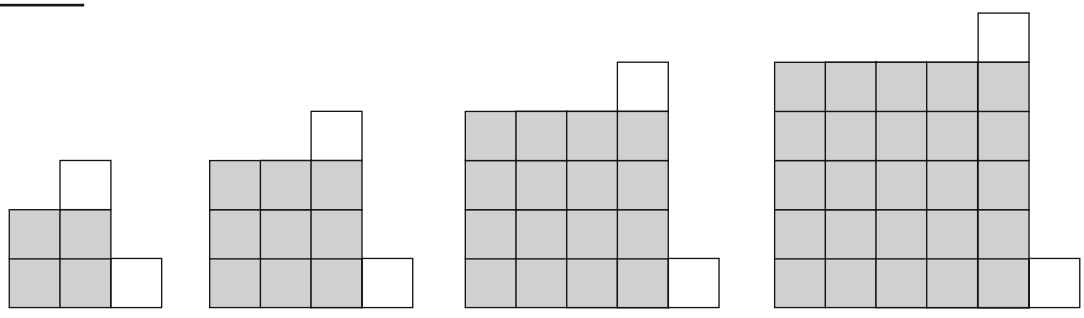
3: • effort to equate a or b coefficient in both equations
• correct answer without work

7: • eliminates one unknown

13: • 1 unknown found

Question 4 (25 marks)

Question 4 (a)



Stage 1 **Stage 2** **Stage 3** **Stage 4**

Question 4 (b)

Stage (n)	Number of Grey Tiles	Number of White Tiles	Total
1	4	2	6
2	9	2	11
3	16	2	18
4	25	2	27
5	36	2	38

Marking Scheme Notes
Question 4 (a) [Scale 5B (0, 3, 5)]
3: • correct diagram, no shading
• 5^2 written

Question 4 (b) [Scale 5C (0, 3, 4, 5)]
3: • At least 1 correct new entry
4: • At least 6 correct or consistent new entries

Question 4 (c)

$T_n = n^2 + bn + c$

$T_1 = 6 = (1)^2 + b(1) + c \Rightarrow b + c = 5 ... (\mathbf{1})$

$T_2 = 11 = (2)^2 + b(2) + c \Rightarrow 2b + c = 7 ... (\mathbf{2})$

$(\mathbf{2}) - (\mathbf{1}) : b = 2$

Into (**1**): $(2) + c = 5 \Rightarrow c = 3$

$\therefore T_n = n^2 + 2n + 3$

Question 4 (d)

$n^2 + 2n + 3 = 443$

$n^2 + 2n - 440 = 0$

$(n - 20)(n + 22) = 0$

$\therefore n = 20$

Marking Scheme Notes
Question 4 (c) [Scale 5C (0, 3, 4, 5)]
3: • $(n)^2 + 2$ written and stops or continues
• Effort at substitution to derive one equation in a and b
• correct answer without work
4: • $(n + 1)^2 + 2$
• 2 relevant equations derived

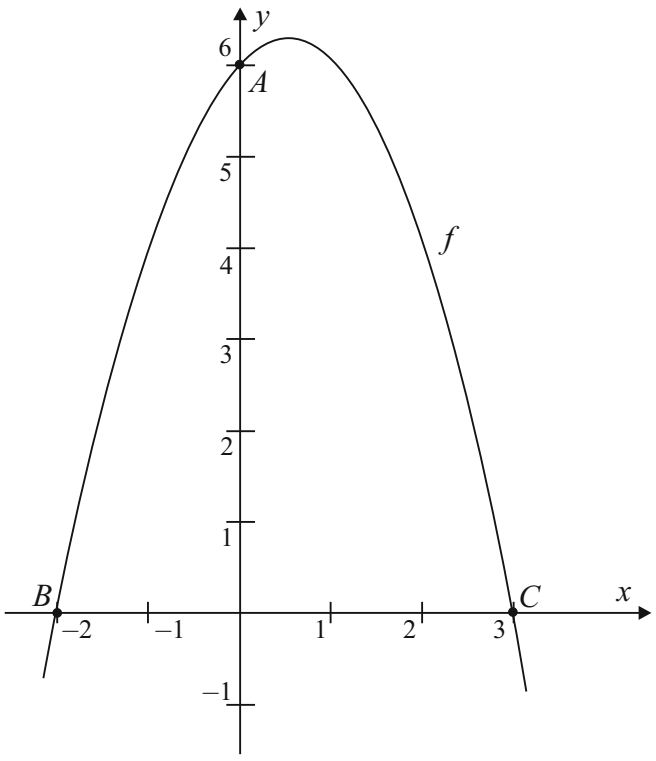
Question 4 (d) [Scale 10C (0, 3, 8, 10)]
3: • $n^2 + 2n + 3 = 443$
• effort at trial and error involving any term beyond T_5
8: • $n^2 + 2n - 440 = 0$
• $(21)^2 + 2 = 443$
Note: Accept $n = 20$ without work for full marks

Question 5 (25 marks)

Question 5 (a)

$A = (0, 6)$
$B = (-2, 0)$
$C = (3, 0)$

Marking Scheme Notes
Question 5 (a) [Scale 5C (0, 3, 4, 5)]
3: • 1 point correctly identified
4: • 2 points correctly identified
• mixes up axes consistently for the 3 points



Question 5 (b)

As it is a quadratic, the three points where the quadratic cuts the axes should satisfy its equation.

$f(x) = -x^2 + x + 6$

$f(0) = -(0)^2 + (0) + 6 = 6 \Rightarrow A(0,\ 6) \in f(x)$

$f(-2) = -(-2)^2 + (-2) + 6 = 0 \Rightarrow B(-2,\ 0) \in f(x)$

$f(3) = -(3)^2 + (3) + 6 = 0 \Rightarrow C(3,\ 0) \in f(x)$

Or

$x = -2 \Rightarrow (x+2)$ is a factor

$x = 3 \Rightarrow (x-3)$ is a factor

$f(x) = k(x+2)(x-3) = k(x^2 - x - 6)$

$f(0) = 6 \Rightarrow k(-6) = 6$

$\therefore k = -1$

$f(x) = -1(x^2 - x - 6) = -x^2 + x + 6$

Marking Scheme Notes

Question 5 (b) [Scale 10C (0, 3, 8, 10)]

3: • $c = 6$
• substitution of A or B or C into given equation or $ax^2 + bx + c$
• 1 factor written

8: • substitution of any 2 of A, B, or C into given equation worked out
• substitution of A, B, and C into $ax^2 + bx + c$
• $\pm(x + 2)(x - 3)$

Note: A, B and C substituted into $-x^2 + x + 6$ and fully verified merits full marks

Question 5 (c) [Scale 10C (0, 3, 8, 10)]

3: • any effort at differentiation
• states $f'(x) = 0$

8: • $x = \frac{1}{2}$

Question 5 (c)

$f(x) = -x^2 + x + 6$

$f'(x) = -2x + 1$

$f'(x) = 0 \Rightarrow -2x + 1 = 0$

$\therefore x = 0 \cdot 5$

$f''(x) = -2 < 0$

$\therefore$ Maximum at $x = 0 \cdot 5 : f(0 \cdot 5) = -(0 \cdot 5)^2 + (0 \cdot 5) + 6 = 6 \cdot 25$

Maximum point: $(0 \cdot 5,\ 6 \cdot 25)$

Question 6 (25 marks)

Question 6 (a)

$(x+5)(3x-4) - 3(x^2+2) + 4 = 0$

$3x^2 - 4x + 15x - 20 - 3x^2 - 6 + 4 = 0$

$11x - 22 = 0$

$11x = 22$

$\therefore x = 2$

Question 6 (b)

$\frac{5}{x+3} - \frac{1}{x} = \frac{1}{2}$ [Multiply across by $2x(x+3)$]

$\frac{5 \times 2x(x+3)}{(x+3)} - \frac{2x(x+3)}{x} = \frac{2x(x+3)}{2}$

$10x - 2(x+3) = x(x+3) \Rightarrow 10x - 2x - 6 = x^2 + 3x$

$x^2 + 5x + 6 = 0 \Rightarrow (x-3)(x-2) = 0$

$\therefore x = 2,\ 3$

Marking Scheme Notes

Question 6 (a) [Scale 10C (0, 3, 8, 10)]

3: • any correct multiplication
• correct answer without work

8: • multiplication fully correct
• error in expanding brackets but finishes correctly

Question 6 (b) [Scale 15D (0, 3, 7, 13, 15)]

3: • common denominator identified
• $5x - (x + 3)$ or similar
• correct answer without work, 2 and/or 3
• writes $\frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$

7: • $10x - 2(x + 3) = x(x + 3)$ or similar

13: • $x^2 - 5x + 6 = 0$ or equivalent

Question 7 (55 marks)

Question 7 (a)

$$\boxed{T_n = a + (n-1)d}$$

Number of seats in each row: 28, 29, 30,...

$a = 28,\ d = 1,\ n = 10$

$T_{10} = 28 + (10-1)(1) = 28 + 9 = 37$

Question 7 (b)

Number of seats in each row: 28, 29, 30,...

$a = 28,\ d = 1,\ n = ?,\ T_n = 50$

$T_n = 28 + (n-1)(1) = 50$

$28 + n - 1 = 50$

$\therefore n = 23$

Question 7 (c)

$$\boxed{S_n = \tfrac{n}{2}[2a + (n-1)d]}$$

Number of seats in each row: 28, 29, 30,...

$a = 28,\ d = 1,\ n = 23$

$S_{23} = \frac{23}{2}[2(28) + (23-1)(1)] = \frac{23}{2}[56 + 22]$

$= \frac{23}{2}[78] = 897$

Question 7 (d)

Number of seats in each row: 28, 29, 30,...

$a = 28,\ d = 1,\ S_n = 600$

$S_n = \frac{n}{2}[2(28) + (n-1)(1)] = 600$

$n[56 + n - 1] = 1200$

$n[n + 55] = 1200$

$n^2 + 55n - 1200 = 0$

$$n = \frac{-55 \pm \sqrt{55^2 - 4(1)(-1200)}}{2} = 16 \cdot 7$$

$\therefore n = 16$

16 full rows: $S_{16} = \frac{16}{2}[2(28) + (16-1)(1)]$

$= 8[56 + 15] = 568$

Number of people in next row $= 600 - 568$

$= 32$

Question 7 (e)

Total income $= 276 \times €25 + 212 \times €12$

$= €9444$

Marking Scheme Notes

Question 7 (a) [Scale 5B (0, 3, 5)]

3: • correct number of seats for any row between row 4 and row 9 (inclusive)

Note: Accept 37 without work

Question 7 (b) [Scale 10C (0, 3, 8, 10)]

3: • any evidence of counting (e.g. listing 28, 29,…)
• 13 or 22 without work
• writes $T_n = a + (n - 1)d$
• a or d identified

8: • 50 – 28 or 50 – 37
• $28 + (n - 1)1 = 50$

Note: Accept 23 without work

Question 7 (c) [Scale 10C (0, 3, 8, 10)]

3: • S_n formula written
• any evidence of counting (e.g. listing 28 + 29 + 30 +…)
• a or d identified

8: • full listing with addition indicated but incorrect solution
• S_n formula fully substituted

Note: Accept 897 without work

Question 7 (d) [Scale 10C (0, 3, 8, 10)]

3: • S_n formula written
• a or d identified
• any evidence of counting (e.g. listing 28 + 29 + 30 +…)

8: • $n^2 + 55n - 1200 = 0$
• $\frac{n}{2}(56 + n - 1) = 600$
• full listing to 16 or 17 terms with addition indicated but incorrect solution

Note: Accept (16, 32) without work

Question 7 (e) [Scale 5C (0, 3, 4, 5)]

3: • 276(25)
• 212(12)

4: • 76(25) + 212(12) formulated

Note: Accept correct answer without work

Question 7 (f)

(i) Adults : Children = 3 : 1

Number of adult tickets $= \frac{3}{4} \times 752 = 564$

Number of child tickets $= \frac{1}{4} \times 752 = 188$

(ii) Let x = Cost of child ticket

$\therefore 2 \cdot 5x$ = Cost of adult ticket

$188x + 564 \times 2 \cdot 5x = 17\ 578$

$1598x = 17\ 578$

$\therefore x = €11$

Marking Scheme Notes

Question 7 (f) (i) [Scale 5C (0, 3, 4, 5)]

3: • 4

• $\frac{752}{x}$, $x \neq 4, 1$

4: • 188 found or 564

Note: Accept correct answer without work

Question 7 (f) (ii) [Scale 10C (0, 3, 8, 10)]

3: • x, $2 \cdot 5x$

• correct answer without work

8: • $188x + 564(2 \cdot 5x) = 17\ 578$, or consistent with f (i)

Question 8 (65 marks)

$C(t) = -t^3 + 4 \cdot 5t + 54t,\ 0 \leq t \leq 9,\ t \in \mathbb{R}$

Question 8 (a)

$t = 4 : C(4) = -(4)^3 + 4 \cdot 5(4) + 54(4) = 224$ units

Question 8 (b)

t (Hours)	0	1	2	3	4	5	6	7	8	9
$C(t)$ (units)	0	57·5	118	175·5	224	257·5	270	255·5	208	121·5

Marking Scheme Notes

Question 8 (a) [Scale 10C (0, 3, 8, 10)]

3: • Function with some substitution of 4

8: • Function fully substituted

Question 8 (b) [Scale 10C (0, 3, 8, 10)]

3: • 1, 2 or 3 correct new entries

8: • 4, 5 or 6 correct new entries

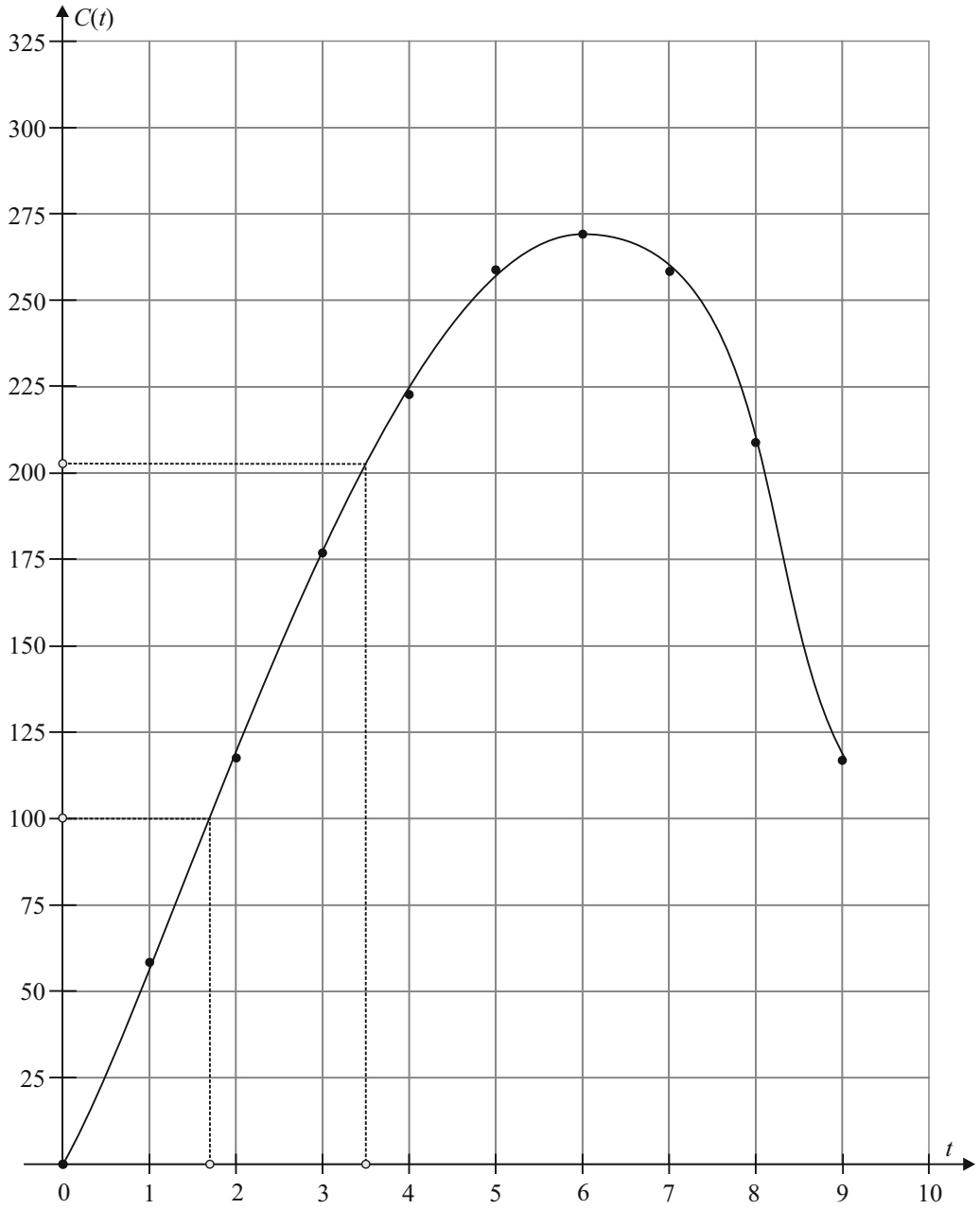
Question 8 (c) [Scale 5C (0, 3, 4, 5)]

3: • 1 to 4 correct plots

4: • 5 to 9 correct plots

• all points correctly plotted but incorrect or no joining

Question 8 (c)



Question 8 (d)

(i) $t = 3{\cdot}5$ hours: $C(t) \approx 201$ units

(ii) $C(t) = 100$ units: $t \approx 1{\cdot}7$ hours

Question 8 (d) (i) [Scale 5C (0, 3, 4, 5)]

3: • 3·5 shown on horizontal axis

4: • correct construction shown but no indication of 200

Question 8 (d) (ii) [Scale 10C (0, 3, 8, 10)]

3: • 100 clearly indicated on vertical axis

8: • correct construction shown but no indication of 1 hour 45 minutes

Question 8 (e)

(i) $C(t) = -t^3 + 4 \cdot 5t^2 + 54t$

$$\frac{dC}{dt} = -3t^2 + 9t + 54$$

(ii) $\left(\frac{dC}{dt}\right)_{t=4} = -3(4)^2 + 9(4) + 54 = 42$ units/hour

(iii) $\frac{dC}{dt} = 0 \Rightarrow -3t^2 + 9t + 54 = 0$

$t^2 - 3t - 18 = 0$

$(t-6)(t+3) = 0$

$\therefore t = 6$ hours

$C(6) = -(6)^3 + 4 \cdot 5(6)^2 + 54(6) = 270$ units

(iv) $\left(\frac{dC}{dt}\right)_{t=7} = -3(7)^2 + 9(7) + 54$

$= -30$ units/hour < 0

Since $\frac{dC}{dt}$ is negative at a time of 7 hours, the concentration is decreasing.

Marking Scheme Notes

Question 8 (e) (i) [Scale 10C (0, 3, 8, 10)]

3: • any correct differentiation or indication of differentiation

8: • 2 terms correctly differentiated

Question 8 (e) (ii) [Scale 5C (0, 3, 4, 5)]

3: • some substitution of 4 into C' or writes $C'(4)$

4: • $C'(4) = -3(4)^2 + 9(4) + 54$

Note: Accept correct or consistent answer without work

Question 8 (e) (iii) [Scale 5C (0, 3, 4, 5)]

3: • $C'(t) = 0$

• correct answer without work

4: • $t = 6$ from calculus

Question 8 (e) (iv) [Scale 5D (0, 2, 3, 4, 5)]

2: • some substitution of 7 into C' or writes $C'(7)$

3: • $C'(7) = -3(7)^2 + 9(7) + 54$

4: • -30 units / hour

Question 9 (30 marks)

$s = \sqrt{g \times d}$

Question 9 (a)

(i) $d = 2000 : s = \sqrt{9 \cdot 8 \times 2000} = 140$ m/s

(ii) Speed $= \frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

$$\text{Time} = \frac{400 \times 1000}{140} = 2857 \cdot 14 \text{ s} \approx 48 \text{ minutes}$$

Question 9 (b)

(i) $s = \sqrt{g \times d}$

$s^2 = g \times d$

$d = \frac{s^2}{g}$

(ii) $s = 55$ m/s: $d = \frac{55^2}{9 \cdot 8} \approx 309$ m

Marking Scheme Notes

Question 9 (a) (i) [Scale 5B (0, 3, 5)]

3: • Any relevant substitution

Note: Accept correct answer without work

Question 9 (a) (ii) [Scale 5C (0, 3, 4, 5)]

3: • use of answer to Part (a)(i)

• correct answer without work

• 400000 written

4: • $\frac{400\,000}{140}$

Question 9 (b) (i) [Scale 5B (0, 3, 5)]

3: • effort to square both sides

Question 9 (b) (ii) [Scale 15C (0, 4, 12, 15)]

4: • some substitution into either formula

• correct answer without work

12: • $d = \frac{55^2}{9 \cdot 8}$

LC 2018 (Set A): Paper 2

Question 1 (25 marks)

Question 1 (a)

(i) Table

Marking Scheme Notes
Question 1 (a) (i) [Scale 5C (0, 2, 3, 5)]
0: • Correct answers outside box with no work
2: • One correct entry
• Work of merit
3: • Four correct entries
5: • Correct answer without work

Die 1

Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	**7**	**8**	9	10
5	6	**7**	8	9	**10**	11
6	**7**	8	9	10	11	**12**

(ii) $P(7 \text{ or } 11) = \dfrac{\text{Number of 7's or 11's}}{\text{Number of numbers}}$

$= \dfrac{8}{36} = \dfrac{2}{9}$

Marking Scheme Notes
Question 1 (a) (ii) [Scale 5B (0, 2, 5)]
0: • Probability > 1, without work
2: • Work of merit
5: • Correct answer without work

Die 1

Die 2	1	2	3	4	5	6
1	2	3	4	5	6	**7**
2	3	4	5	6	**7**	8
3	4	5	6	**7**	8	9
4	5	6	**7**	8	9	10
5	6	**7**	8	9	10	**11**
6	**7**	8	9	10	**11**	12

(iii) $P(\text{Prime}) = \dfrac{\text{Number of primes}}{\text{Number of numbers}}$

$= \dfrac{15}{36} = \dfrac{5}{12}$

Marking Scheme Notes
Question 1 (a) (iii) [Scale 5B (0, 2, 5)]
0: • Probability > 1, without work
2: • Prime Number identified
• Work of merit
5: • Correct answer without work

Die 1

Die 2	1	2	3	4	5	6
1	**2**	**3**	4	**5**	6	**7**
2	**3**	4	**5**	6	**7**	8
3	4	**5**	6	**7**	8	9
4	**5**	6	**7**	8	9	10
5	6	**7**	8	9	10	**11**
6	**7**	8	9	10	**11**	12

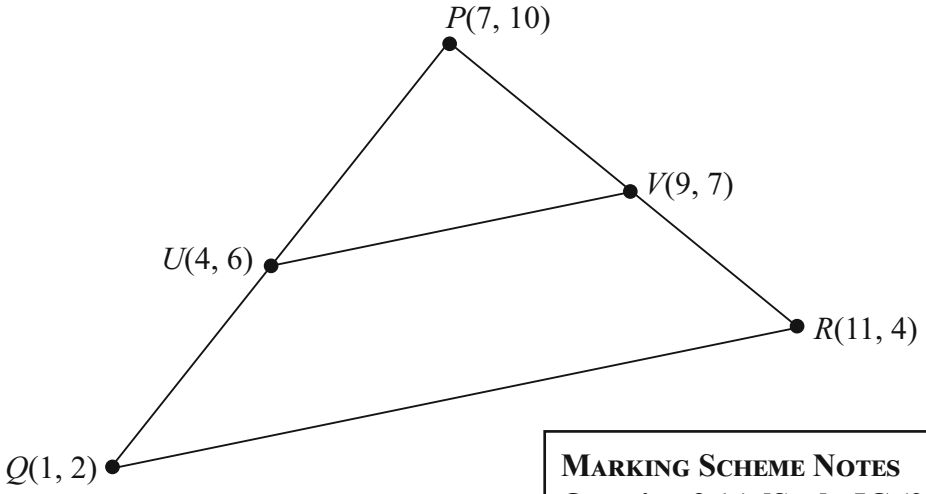
Question 1 (b)

$P(\text{Ford or Not red}) = \dfrac{26}{30} = \dfrac{13}{15}$

	Ford	Renault
Red	**7**	4
Not red	**11**	**8**

Marking Scheme Notes
Question 1 (b) [Scale 10C (0, 3, 7, 10)]
0: • Probability > 1, without work
3: • Any relevant work towards creating at least one correct probability
• (18 + 8, 30 – 4, 18 + 19 –11)
7: • Significant work towards answer
10: • Correct answer without work

QUESTION 2 (25 MARKS)



Question 2 (a)

$P(7, 10) = (x_1, y_1),\ R(11, 4) = (x_2, y_2)$

$$V\left(\frac{7+11}{2}, \frac{10+4}{2}\right) = V(9, 7)$$

Question 2 (b)

$U(4, 6) = (x_1, y_1),\ V(9, 7) = (x_2, y_2)$

$$m_1 = \frac{7-6}{9-4} = \frac{1}{5}$$

$Q(1, 2) = (x_1, y_1),\ R(11, 4) = (x_2, y_2)$

$$m_2 = \frac{4-2}{11-1} = \frac{2}{10} = \frac{1}{5}$$

$m_1 = m_2 \Rightarrow UV \parallel QR$

Question 2 (c)

$P(7, 10) \rightarrow (6, 8) = (x_1, y_1)$

$Q(1, 2) \rightarrow (0, 0)$

$R(11, 4) \rightarrow (10, 2) = (x_2, y_2)$

$$\text{Area} = \tfrac{1}{2}\left|6(2) - 8(10)\right| = \tfrac{1}{2}\left|12 - 80\right| = 34$$

Question 2 (d)

$U(4, 6) \rightarrow V(9, 7)$ [Add 5, Add 1]

$Q(1, 2) \rightarrow S(6, 3)$

MARKING SCHEME NOTES

Question 2 (a) [Scale 5C (0, 2, 3, 5)]

2: • Correct relevant formula with some correct substitution
• Correct translation identified

3: • Formula fully substituted

5: • Correct answer without work

Question 2 (b) [Scale 5D (0, 2, 3, 4, 5)]

2: • Correct relevant formula with some correct or consistent substitution

3: • One or both formulae fully substituted

4: • Both slopes calculated and no conclusion

Question 2 (c) [Scale 10D (0, 2, 5, 8, 10)]

2: • Area of triangle formula with some relevant substitution
• Translation of any relevant point to (0, 0)

5: • Area of triangle formula substituted correctly with TWO from P, Q and R

8: • Formula fully substituted with translated points
• Area of triangle found from formula fully substituted with TWO from P, Q and R

Question 2 (d) [Scale 5C (0, 2, 3, 5)]

2: • Work with a translation which indicates an understanding of the concept
• Mid-Point Formula with some relevant substitution

3: • An incorrect translation applied correctly
• Correct translation applied to an incorrect point
• Correct formula fully substituted

5: • Correct answer without work

Question 3 (25 marks)

Question 3 (a)

(i) $7! = 5040$

(ii) ${}^{7}P_{3} = 7 \times 6 \times 5 = 210$

Marking Scheme Notes
Question 3 (a) (i) + (ii) [Scale 10C (0, 3, 7, 10)]
3: • Any work of merit towards creating at least one correct solution
7: • One correct answer
• Answers given as correct list with multiplication clearly indicated but answers not worked out
10: • Correct answers without work

Question 3 (b)

(i) Probability table:

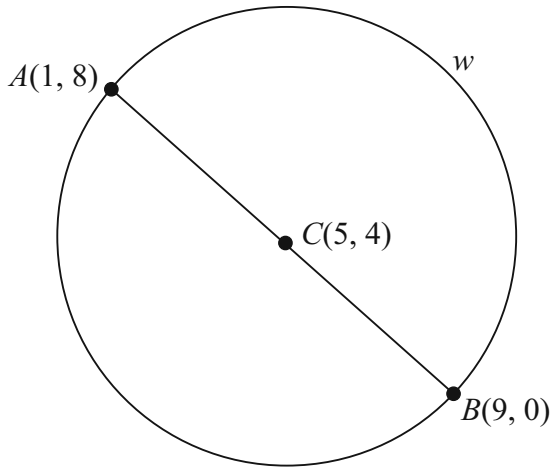
Colour	Angle	Probability	Prize
Red	72^{o}	$\frac{72}{360} = \frac{1}{5}$	€20
Orange	30^{o}	$\frac{30}{360} = \frac{1}{12}$	€60
Yellow	45^{o}	$\frac{1}{8}$	€24
Green	90^{o}	$\frac{90}{360} = \frac{1}{4}$	€8
Blue	60^{o}	$\frac{60}{360} = \frac{1}{6}$	€42
Indigo	18^{o}	$\frac{18}{360} = \frac{1}{20}$	€90
Violet	45^{o}	$\frac{45}{360} = \frac{1}{8}$	€48

(ii) Expected value:

$$E(x) = \tfrac{1}{5} \times 20 + \tfrac{1}{12} \times 60 + \tfrac{1}{8} \times 24 + \tfrac{1}{4} \times 8 + \tfrac{1}{6} \times 42 + \tfrac{1}{20} \times 90 + \tfrac{1}{8} \times 48$$
$$= 4 + 5 + 3 + 2 + 7 + 4 \cdot 5 + 4$$
$$= €31 \cdot 50$$

Marking Scheme Notes
Question 3 (b) (i) + (ii) [Scale 15D (0, 4, 7, 11, 15)]
4: • Any work of merit e.g. One correct entry into table
7: • Four correct entries into table
• All incorrect consistent probabilities found
• $E(x)$ correct with no work and no table
11: • Table correct or $E(x)$ correct
• Error/s in completed table and significant work in formulation of $E(x)$
Note: If expected values are correct but not added then award *F**

QUESTION 4 (25 MARKS)



Question 4 (a)

$A(1, 8) = (x_1, y_1),\ B(9, 0) = (x_2, y_2)$

$C\left(\frac{1+9}{2}, \frac{8+0}{2}\right) = C(5, 4)$

Question 4 (b)

$A(1, 8) = (x_1, y_1),\ C(5, 4) = (x_2, y_2)$

$r = |AC| = \sqrt{(5-1)^2 + (4-8)^2} = \sqrt{16+16} = 4\sqrt{2}$

Question 4 (c)

$C(5, 4) = (h, k),\ r = 4\sqrt{2}$

$w: (x-5)^2 + (y-4)^2 = (4\sqrt{2})^2$

$w: (x-5)^2 + (y-4)^2 = 32$

Question 4 (d)

$A(1, 8) = (x_1, y_1),\ B(9, 0) = (x_2, y_2)$

Slope of $AB = m = \frac{8-0}{1-9} = \frac{8}{-8} = -1$

Slope of $t = m_{\perp} = 1$

$y - y_1 = m(x - x_1)$

$t: y - 8 = 1(x - 1)$

$t: y - 8 = x - 1$

$t: x - y + 7 = 0$

MARKING SCHEME NOTES

Question 4 (a) [Scale 5B (0, 2, 5)]

2: • Mid-point formula with some correct substitution
• Leaves answer in fraction form
• Indicates centre
• Correct translation identified

5: • Correct answer without work

Question 4 (b) [Scale 5C (0, 2, 3, 5)]

2: • Correct formula with some correct or consistent substitution

3: • Formula fully substituted
• Correct answer without work

Question 4 (c) [Scale 10C (0, 3, 7, 10)]

3: • Correct formula with some correct or consistent substitution

7: • Answer as $(x-5)^2 + (y-4)^2 = (\sqrt{32})^2$

10: • Correct or consistent answer without work

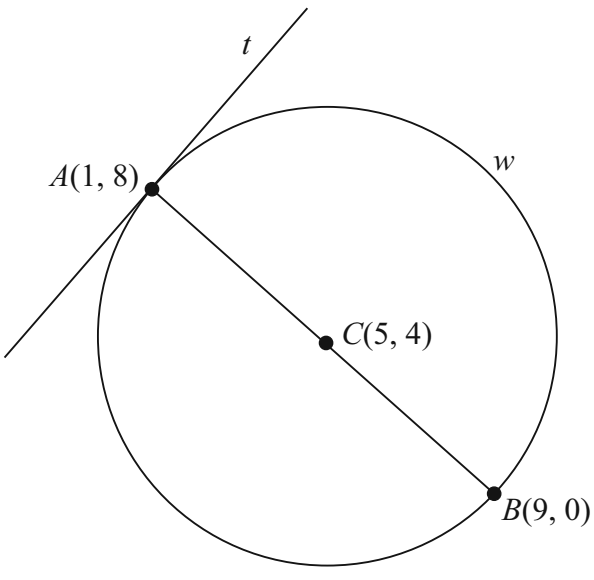
Question 4 (d) [Scale 5C (0, 2, 3, 5)]

2: • Correct relevant formula with some correct substitution
• Finds slope of radius and stops

3: • Eq. of line formula fully substituted

5: • Answer as $(-x + y - 7 = 0)$

Misreading: Equation of tangent through B $(x - y - 9 = 0)$



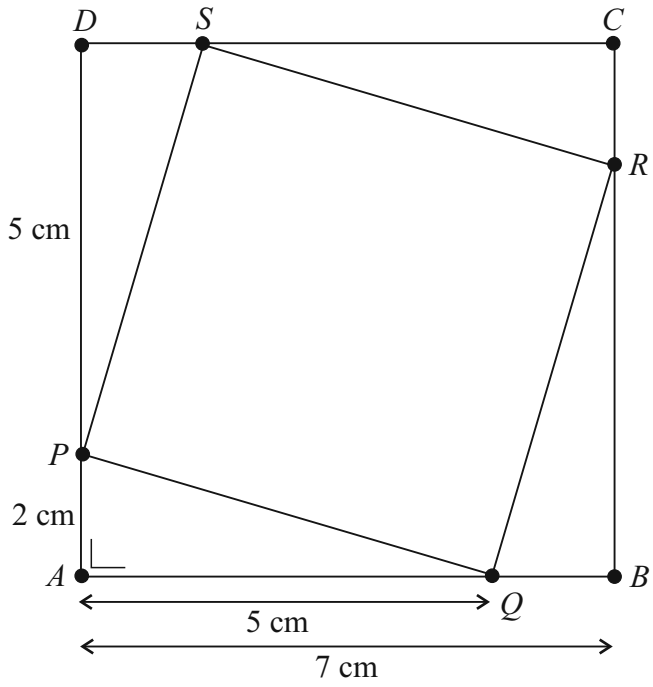
EPL 104

Question 5 (25 marks)

Question 5 (a)

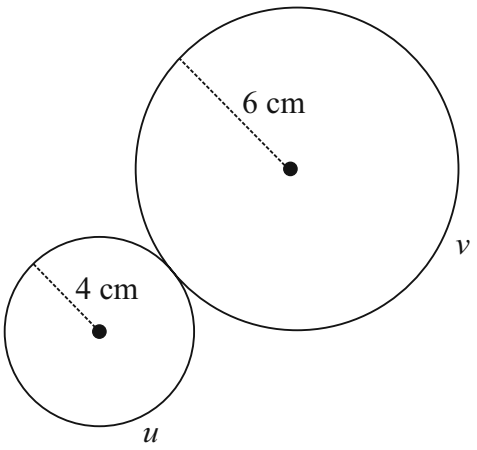
$|PQ| = \sqrt{5^2 + 2^2} = \sqrt{29}$ cm

Area $= \sqrt{29} \times \sqrt{29} = 29$ cm^2



Marking Scheme Notes
Question 5 (a) [Scale 10C (0, 3, 7, 10)]
3: • Any work of merit e.g. Identifies $|QB| = 2$
7: • Correct relevant expression for Area of $PQRS$ fully substituted
10: • Correct answer without work

Question 5 (b)



Marking Scheme Notes
Question 5 (b) (i) + (ii) [Scale 15D (0, 4, 7, 11, 15)]
4: • Circumference formula with some substitution
• Any work of merit e.g. ratio of two circles
7: • One circumference found (in terms of π)
• Both substitutions fully correct
11: • Both circumferences found (in terms of π)
• Finds the number of rotations wheel u makes

(i) $u: L_1 = 2\pi(4) = 8\pi$ cm

$v: L_2 = 2\pi(6) = 12\pi$ cm

(ii) Total distance travelled by $v = 12\pi \times 1000 = 12\,000\pi$ cm

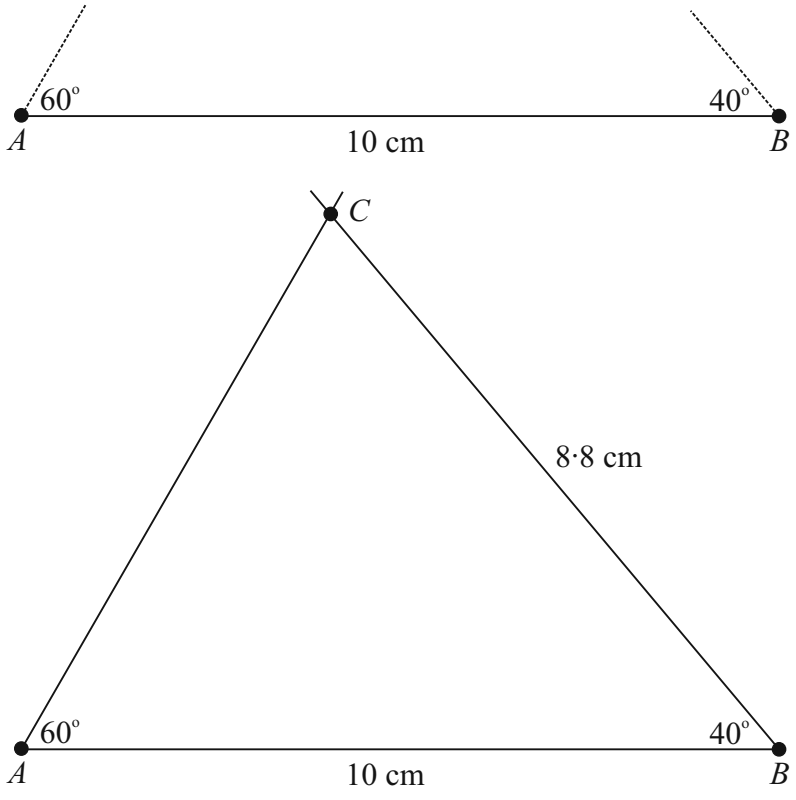
Total distance travelled by $u = 8\pi \times$ Number of rotations $= 12\,000\pi$ cm

$\therefore$ Number of rotations $= \dfrac{12\,000\pi}{8\pi} = 150$

Question 6 (25 marks)

Question 6 (a)

(i) Construction:

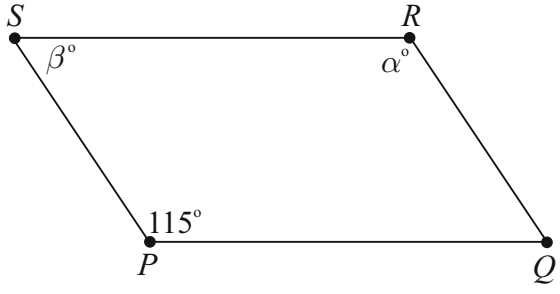


(ii) $|BC| = 8{\cdot}8$ cm

Marking Scheme Notes
Question 6 (a) (i) + (ii) [Scale 10D (0, 2, 5, 8, 10)]
2: • A labelled pilot diagram drawn
• One correct length or angle drawn
5: • Any two components correct and stops
• Both substitutions fully correct
8: • Construction correct but vertices incorrectly named or omitted and stops
Misreading: Works with $|AC|$
Note: Tolerance ± 0·2 cm
$|BC|$ must be ONE from the list: [(8·6),(8·7),(8·8),(8·9),(9·0)]

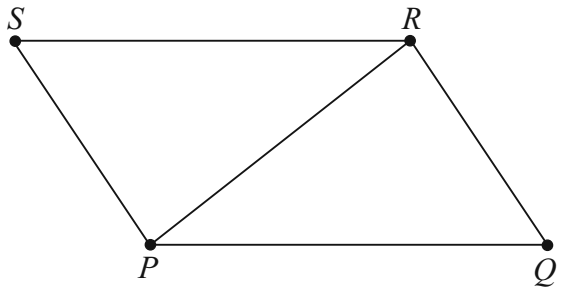
Question 6 (b)

(i) $\alpha = 115^\circ$ [Opposite angles in a parallelogram are equal]
$\beta = 65^\circ$ [Adjacent angles in a parallelogram add up to 180°]



Marking Scheme Notes
Question 6 (b) (i) [Scale 5B (0, 2, 5)]
2: • One correct angle
5: • Correct answers without work

(ii) $|PR| = |PR|$ [Common side]
$|SR| = |PQ|$ [Opposite sides of a parallelogram are equal in length]
$|SP| = |RQ|$ [Opposite sides of a parallelogram are equal in length]
$\therefore \Delta RSP \equiv \Delta PQR$ **[SSS]**



Marking Scheme Notes
Question 6 (b) (ii) [Scale 10C (0, 3, 7, 10)]
3: • Any work of merit e.g. One correct statement
7: • Two statements & Two reasons correct
10: • Three statements & Three reasons correct

Question 7 (60 marks)

Question 7 (a)

Total rainfall and total sunshine at Valentia in June										
Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Total Rainfall (mm)	72	133	155	101	94	47	149	134	94	84
Total Sunshine (hours)	169	124	180	173	173	239	159	168	228	205

(i) Range = 155 mm − 47 mm = 108 mm
(ii) Year with highest June rainfall is 2003 [155 mm]
(iii) Year with least June sunshine is 2002 [124 hours]

Marking Scheme Notes
Question 7 (a) [Scale 10C (0, 3, 7, 10)]
3: • Any work of merit e.g. (a) (i): [155 and/or 47]; (a) (ii): [155]; (a) (iii): [124]
7: • Two correct answers
• Significant work in all answers

Question 7 (b)

Year with best June weather is 2006 as this month had the least rainfall (47 mm) and the highest number of sunshine hours (239 hours).

Marking Scheme Notes
Question 7 (b) [Scale 5B (0, 2, 5)]
0: • A reason with no year identified
2: • Mention of year without reference to rainfall or sunshine
5: • Mention of year with reference to rainfall and/or sunshine

Question 7 (c)

47, 72, 84, 94, **94**, **101**, 133, 134, 149, 155

Median is the average of the two middle values: $\text{Median} = \frac{94+101}{2} = 97 \cdot 5$ mm

Marking Scheme Notes
Question 7 (c) [Scale 10C (0, 3, 7, 10)]
3: • Effort at ordering data
• Median calculated as $\left(\frac{94+47}{2}\right)=70\cdot5$
• Consistent Median from an incorrect list (without work)
7: • All Data ordered
• Correct Median without list
• Correct Median with Sunshine data [173]
10: • Correct list and correct Median without work
Misreading: • Works with a decreasing list

Question 7 (d)

(i) Mean $=\frac{169+124+180+173+173+239+159+168+228+205}{10}=181\cdot8$ hours

(ii) 5% of $181\cdot8=9\cdot1$ hours
$181\cdot8\pm9\cdot1=[172\cdot7, 190\cdot9]$
Years: 2003, 2004, 2005

Marking Scheme Notes

Question 7 (d) (i) [Scale 10C (0, 3, 7, 10)]
3: • Effort at adding data
• Division by 10
7: • Formula fully substituted without calculation
• Gets mean of the rainfall [= 106·3]
10: • Correct answer without work

Question 7 (d) (ii) [Scale 5C (0, 2, 3, 5)]
2: • Any work of merit e.g. Identifies One year (without work)
• Finds 5% of mean [Answer (d) (i)]
3: • Two of the years identified (with work)
• Establishes [172·71, 190·89] or consistent
• Correct answers without work

Question 7 (e)

Standard deviation σ = 33·5 mm

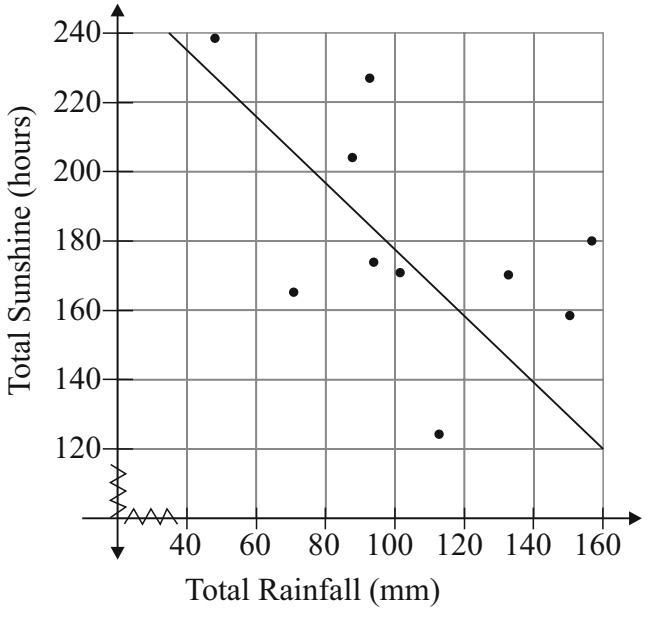
Question 7 (f)

(i) Scatterplot

(ii)

	Tick one box
0·6	
0·1	
−0·1	
−0·6	✓

The line of best fit has a moderate, negative slope which looks to be about −0·6.



Marking Scheme Notes

Question 7 (e) [Scale 5B (0, 2, 5)]

2: • Mean calculated as 106·3
• Calculates σ for sunshine data [32·1]

Note: • Answer as 33, or 33·46 merits *F**

Question 7 (f) (i) [Scale 10C (0, 3, 7, 10)]

3: • One correct plot

7: • Four correct plots

Tolerance: Within the correct box

Question 7 (f) (ii) [Scale 5A (0, 5)]

5: • Correct box ticked and explanation is either:
• Correlation is negative and strong *or* Correlation is negative and moderate

0: • All other answers

Question 8 (50 marks)

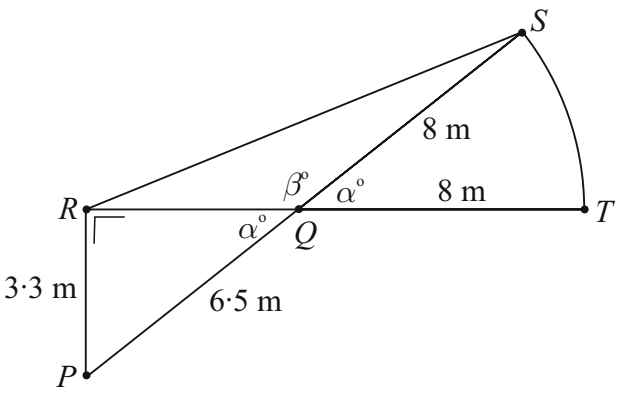
Question 8 (a)

$|RQ|^2 + 3\cdot3^2 = 6\cdot5^2$

$|RQ| = \sqrt{6\cdot5^2 - 3\cdot3^2} = 5\cdot6$ m

Question 8 (b)

$\sin\alpha = \dfrac{3\cdot3}{6\cdot5} \Rightarrow \alpha = \sin^{-1}\left(\dfrac{3\cdot3}{6\cdot5}\right) \approx 31^\circ$



Question 8 (c)

$\beta = 180^\circ - 31^\circ = 149^\circ$ [Straight angle]

Marking Scheme Notes

Question 8 (a) [Scale 10C (0, 3, 7, 10)]

3: • Correct relevant formula with some substitution

7: • Correct formula fully substituted
• Answer as $\sqrt{31\cdot36}$
• Correct answer without work

Question 8 (b) [Scale 10C (0, 3, 7, 10)]

3: • Correct trigonometric ratio with some correct or consistent substitution

7: • $\sin\alpha = \frac{3\cdot3}{6\cdot5}$ (or similar).
• Finds $|\angle QPR| = 59°$ and stops
• Incorrect Calculator Mode (once only) [Rad = 1, Grad = 34]

Question 8 (c) [Scale 5B (0, 2, 5)]

2: • Any work of merit

5: • Correct answer without work

Question 8 (d)

$|RS|^2 = 8^2 + 5\cdot 6^2 - 2(8)(5\cdot 6)\cos 149^\circ$

$|RS| = \sqrt{8^2 + 5\cdot 6^2 - 2(8)(5\cdot 6)\cos 149^\circ} \approx 13 \text{ m}$

Question 8 (e)

$|TS| = \frac{31^\circ}{360^\circ} \times 2\pi(8) \approx 4\cdot 3 \text{ m}$

Question 8 (f)

Area of sector $SQT = \frac{31^\circ}{360^\circ} \times \pi(8)^2 \approx 17\cdot 3 \text{ m}^2$

Marking Scheme Notes

Question 8 (d) [Scale 15D (0, 4, 7, 11, 15)]

0: • Treats triangle as right angled/Sine Rule

4: • Any work of merit to identify the correct side required (Pilot Diagram)
• Correct relevant formula with some correct or consistent substitution

7: • Two relevant variables correctly substituted into correct relevant formula and stops

11: • Fully correct substitution
• Correct answer without work (13)
• Incorrect Calculator Mode (once only) [Rad = 115, Grad = 158]

Question 8 (e) [Scale 5C (0, 2, 3, 5)]

2: • Correct relevant formula with some correct or consistent substitution

3: • Correct formula fully substituted $2\pi(8) \times \frac{31}{360}$

Question 8 (f) [Scale 5C (0, 2, 3, 5)]

2: • Correct relevant formula with some substitution

3: • Correct formula fully substituted $\pi(8)^2 \times \frac{31}{360}$

Question 9 (40 marks)

Question 9 (a)

(i) Sphere: $r = 0\cdot 3$ cm

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0\cdot 3)^3 = 0\cdot 113 \text{ cm}^3$

Marking Scheme Notes

Question 9 (a) (i) [Scale 10B (0, 5, 10)]

5: • Volume formula with radius substituted

10: • Correct answer without work

Note 1: • Answer: 0·113097 (without work) Merits *F**

Note 2: $\pi = 3\cdot 14$, Volume = 0·11304; $\pi = \frac{22}{7}$, $V = 0\cdot 1131428571$

(ii) Number of grams of gold $= 0\cdot 113 \times 19\cdot 3 \text{ g} = 2\cdot 18 \text{ g}$

Marking Scheme Notes

Question 9 (a) (ii) [Scale 5B (0, 2, 5)]

2: • Any work of merit e.g. Indicates multiplication by 19·3

5: • Correct answer without work

(iii) $197\text{ g} \rightarrow 6\cdot02\times10^{23}$ atoms

$1\text{ g} \rightarrow \dfrac{6\cdot02\times10^{23}}{197}$ atoms

$2\cdot18\text{ g} \rightarrow \dfrac{6\cdot02\times10^{23}\times2\cdot18}{197}$ atoms $= 6\cdot7\times10^{21}$ atoms

Marking Scheme Notes
Question 9 (a) (iii) [Scale 5C (0, 2, 3, 5)]
2: • Finds expression for atoms in 1 g
• Two incorrect operations and finishes
3: • Finds expression for atoms in 2·18 g
• One incorrect operation and finishes

Question 9 (b)

(i) $n = 1560$

$$\text{Margin of error} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1560}}$$
$$= 0\cdot025 = 2\cdot5\%$$

(ii) $\hat{p} = \dfrac{546}{1560} = 0\cdot35$

Confidence interval: $\hat{p} - \dfrac{1}{\sqrt{n}} \leftrightarrow \hat{p} + \dfrac{1}{\sqrt{n}}$

$0\cdot35 - 0\cdot025 \leftrightarrow 0\cdot35 + 0\cdot025$

$0\cdot325 \leftrightarrow 0\cdot375$

$32\cdot5\% \leftrightarrow 37\cdot5\%$

(iii) Null hypothesis H_0: 40% of viewers liked the show

$p = 0\cdot4$ (population proportion)

Alternative hypothesis H_1: $p \neq 0\cdot4$

p is not in the confidence interval.

Therefore, do not accept the null hypothesis.

Marking Scheme Notes
Question 9 (b) (i) [Scale 5C (0, 2, 3, 5)]
2: • Writes $\dfrac{1}{\sqrt{n}}$
• Sets up work as $\sqrt{1560}$
3: • Correct formula fully substituted
i.e. $\left(\dfrac{1}{\sqrt{1560}}\right)$
• MOE formula substituted and calculated as 0·025318 without work
5: • Correct answer without work

Question 9 (b) (ii) [Scale 5C (0, 2, 3, 5)]
2: • Work of merit e.g. writes answer from (b) (i) in this part
• Answer as $\dfrac{546}{1560}$
• $\hat{p} \pm \dfrac{1}{\sqrt{n}}$ or $\hat{p} \pm 1\cdot96 \times \dfrac{1}{\sqrt{n}}$
3: • One boundary formed
i.e. (35 + 2·5 or 35 – 2·5)
• Correct answer without work

Question 9 (b) (iii) [Scale 10C (0, 3, 7, 10)]
3: • **Any** of: H_0, H_1, or C.I. correct
7: • **All** of: H_0, H_1, and C.I. corrrect
10: • Result of Hypothesis Test plus conclusion in context e.g. Reject the Null Hypothesis and hence reject the executive's claim

LC 2017 (Set B): Paper 1

Question 1 (25 marks)

Question 1 (a)

$$\boxed{F = P(1-i)^t}$$

$P = €30\,000,\ i = 0{\cdot}15,\ t = 5$ years

$$F = P(1-i)^t = 30\,000(1-0{\cdot}15)^5$$
$$= 30\,000(0{\cdot}85)^5$$
$$= €13\,311{\cdot}16$$

Question 1 (b)

$$\boxed{F = P(1+i)^t}$$

$F = €30\,000,\ i = 0{\cdot}03,\ t = 2$ years, $P = ?$

$$F = P(1+i)^t \Rightarrow 30\,000 = P(1+0{\cdot}03)^2$$
$$30\,000 = P \times 1{\cdot}03^2$$
$$P = \frac{30\,000}{1{\cdot}03^2} = €28\,277{\cdot}88$$

Question 1 (c)

$P = €25\,000,\ t = 3$ years,

$F = €26\,530{\cdot}20,\ i = ?$

$$F = P(1+i)^t \Rightarrow 26\,530{\cdot}20$$
$$= 25\,000(1+i)^3$$
$$\frac{26\,530{\cdot}20}{25\,000} = (1+i)^3$$
$$1{\cdot}061208 = (1+i)^3$$
$$1+i = \sqrt[3]{1{\cdot}061208} = 1{\cdot}02$$
$$i = 0{\cdot}02$$
$$r\% = 2\%$$

Marking Scheme Notes

Question 1 (a) [Scale 10C (0, 4, 6, 10)]

4:
- $30\,000 \times 15\%$ or equivalent
- $0{\cdot}85$
- at least one correct substitution
- correct formula written

6:
- $(0{\cdot}85)^5$
- 3 or 4 years correct by list method
- fully correct substitution into formula but fails to finish
- correct answer without work (13 311)

Question 1 (b) [Scale 10C (0, 4, 6, 10)]

4:
- $1{\cdot}03$
- at least 1 correct substitution
- correct formula written
- unsuccessful efforts at trial and improvement

6:
- $\frac{30\,000}{1{\cdot}03^2}$
- correct answer without work, 28 277 or 28 278

Question 1 (c) [Scale 5C (0, 2, 3, 5)]

2:
- $\frac{26\,530{\cdot}20}{25\,000}$
- $1530{\cdot}20$
- $\sqrt[3]{\ }$
- at least 1 correct substitution or correct formula written
- correct answer without work

3:
- $1{\cdot}02$

Question 2 (25 marks)

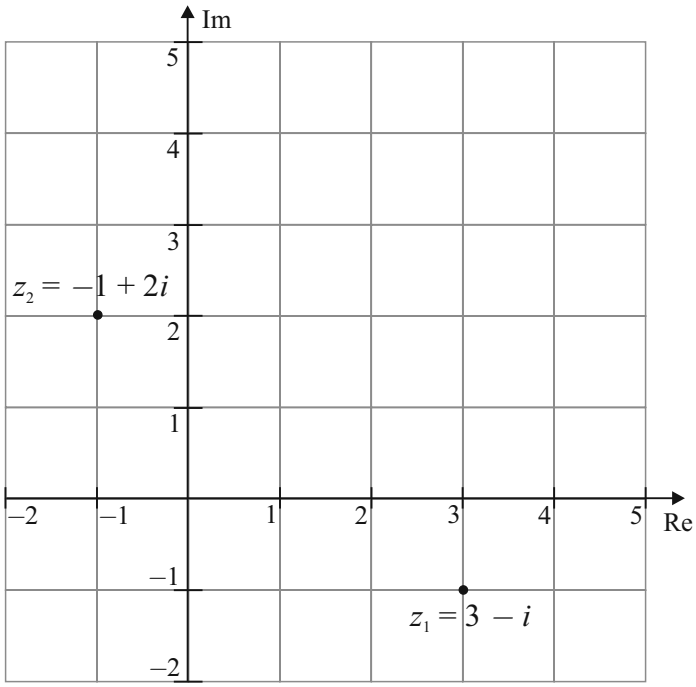
Question 2 (a) (i)

$z_1 = 3 - i$

$a = 3,\ b = -1$

Question 2 (a) (ii)

$z_2 = -1 + 2i$



Question 2 (a) (iii)

$$\frac{z_1}{z_2} = \frac{3-i}{-1+2i} = \frac{(3-i)}{(-1+2i)} \times \frac{(-1-2i)}{(-1-2i)}$$

$$= \frac{-3-6i+i+2i^2}{1+2i-2i-4i^2} = \frac{-3-6i+i-2}{1+2i-2i+4}$$

$$= \frac{-5-5i}{5}$$

$$= -1-i$$

Question 2 (b)

$2z - 6(4-6i) = (-1+i)(4-2i)$

$2z - 24 + 36i = -4 + 2i + 4i - 2i^2$

$2z - 24 + 36i = -4 + 6i + 2$

$2z = -2 + 24 + 6i - 36i$

$2z = 22 - 30i$

$z = 11 - 15i$

Marking Scheme Notes

Question 2 (a) (i) [Scale 5B (0, 2, 5)]

2: • $a = -1,\ b = 3$
• one correct value only

Question 2 (a) (ii) [Scale 5A (0, 5)]

Question 2 (a) (iii) [Scale 10D (0, 4, 6, 8, 10)]

4: • at least 1 correct substitution

6: • conjugate identified
• some multiplication above and below by the same number, even if incorrect conjugate

8: • correct multiplication using correct conjugate
• answer not in correct form

Question 2 (b) [Scale 5C (0, 2, 3, 5)]

2: • any correct multiplication

3: • multiplication fully correct

Question 3 (25 marks)

Question 3 (a)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$3x^2 - 6x - 8 = 0$

$a = 3,\ b = -6,\ c = -8$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-8)}}{2(3)} = \frac{6 \pm \sqrt{36 + 96}}{6}$$

$$= \frac{6 \pm \sqrt{132}}{6} = -0{\cdot}9,\ 2{\cdot}9$$

Question 3 (b)

$f(x) = 3x^2 - 6x - 8$

$f'(x) = 6x - 6$

Minimum: $f'(x) = 0 \Rightarrow 6x - 6 = 0$

$6x = 6$

$x = 1$

$f(1) = 3(1)^2 - 6(1) - 8 = 3 - 6 - 8 = -11$

Minimum point $= (1, -11)$

Marking Scheme Notes

Question 3 (a) [Scale 10D (0, 4, 6, 8, 10)]

4: • correct roots formula
• a, b, and c explicitly identified
• attempt at factorising

6: • formula fully substituted
• $x = \frac{6 \pm \sqrt{-60}}{6}$ and stops or continues

8: • one correct root
• $x = \frac{6 \pm \sqrt{132}}{6}$ and stops

Question 3 (b) [Scale 15D (0, 5, 7, 10, 15)]

5: • $\frac{dy}{dx}$ or $f'(x)$ *or* $f'(x) = 0$
• any correct differentiation
• some work towards a graphical solution
• correct answer without work

7: • $6x - 6$ or $6x - 6 = 0$

10: • finds x value correctly

Note: Accept correct turning point found graphically for full marks

Question 4 (25 marks)

Question 4 (a)

$11x-5(2x-1)=3(6-x)+3$

$11x-10x+5=18-3x+3$

$11x-10x+3x=18+3-5$

$4x=16$

$\therefore x=\frac{16}{4}=4$

Question 4 (b)

$y+5=2x \Rightarrow y=2x-5...(\mathbf{1})$

$x^2+y^2=25...(\mathbf{2})$

$x^2+(2x-5)^2=25$

$x^2+4x^2-20x+25=25$

$x^2+4x^2-20x+25-25=0$

$5x^2-20x=0$

$x^2-4x=0$

$x(x-4)=0 \Rightarrow x=0,\ 4$

Into (**1**):

$x=0: y=2(0)-5=0-5=-5$

$x=4: y=2(4)-5=8-5=3$

Solutions:

$(x,\ y)=(0,\ -5)$

$(x,\ y)=(4,\ 3)$

Marking Scheme Notes

Question 4 (a) [Scale 10C (0, 4, 6, 10)]

4: • any correct multiplication
• correct answer without work

6: • multiplication fully correct
• error in expanding brackets but finishes correctly

Question 4 (b) [Scale 15D (0, 5, 7, 10, 15)]

5: • isolates one variable

7: • substitution into quadratic

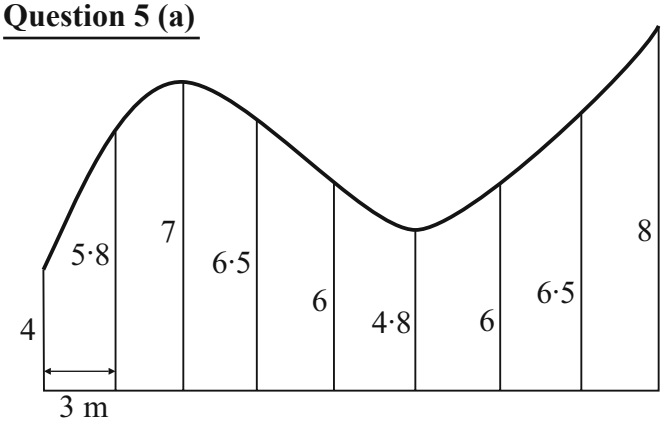
10: • solve (2 values) for 1 variable
• finds one correct couple from a quadratic

Note: $(2x-5)^2 = 4x^2 + 25$ and continues correctly merits MPC

QUESTION 5 (25 MARKS)

Trapezoidal rule: Area $\approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + ... + y_{n-1})]$

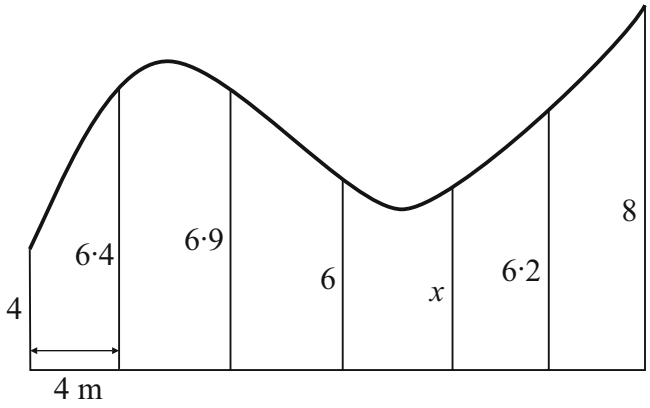
Question 5 (a)



MARKING SCHEME NOTES
Question 5 (a) [Scale 10C (0, 4, 6, 10)]
4: • correct relevant area formula with some substitution
6: • formula fully substituted
• uses Simpsons Rule and finishes correctly

Area $A \approx \frac{3}{2}[4 + 8 + 2(5 \cdot 8 + 7 + 6 \cdot 5 + 6 + 4 \cdot 8 + 6 + 6 \cdot 5)] = 145 \cdot 8 \text{ m}^2$

Question 5 (b)



MARKING SCHEME NOTES
Question 5 (b) [Scale 15D (0, 5, 7, 10, 15)]
5: • correct relevant area formula with some substitution
7: • Formula fully substituted
10: • x isolated from formula
• uses Simpsons Rule and finishes correctly

Area $A \approx \frac{4}{2}[4 + 8 + 2(6 \cdot 4 + 6 \cdot 9 + 6 + x + 6 \cdot 2)] = 145 \cdot 6 \text{ m}^2$

$2[12 + 2(x + 25 \cdot 5)] = 145 \cdot 6$

$12 + 2(x + 25 \cdot 5) = 72 \cdot 8$

$2(x + 25 \cdot 5) = 72 \cdot 8 - 12$

$2(x + 25 \cdot 5) = 60 \cdot 8$

$x + 25 \cdot 5 = 30 \cdot 4$

$x = 30 \cdot 4 - 25 \cdot 5 = 4 \cdot 9 \text{ m}$

EPL 104

Question 6 (25 marks)

Question 6 (a)

Basic weekly salary = €150
Total sales = €3000
Commission: €1000 × 0·2 + €2000 × 0·3
= €200 + €600 = €800
Total income = €150 + €800 = €950

Question 6 (b)

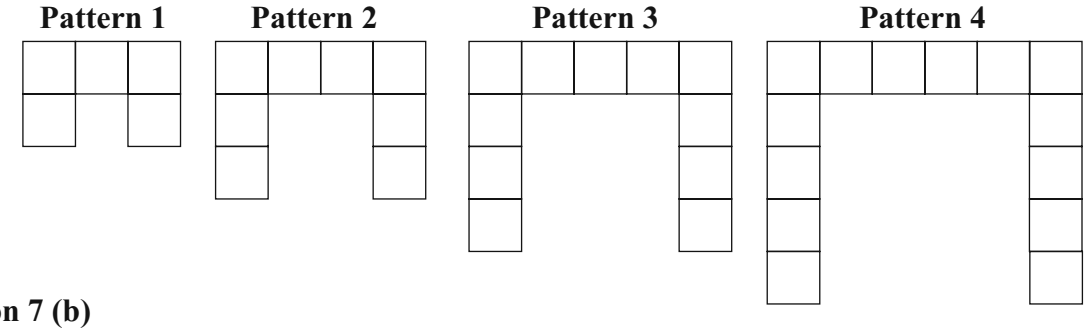
Basic weekly salary = €150
Total income = €1160
Commission = €1160 − €150 = €1010
€1000 × 0·2 + €x × 0·3 = €1010
€200 + €x × 0·3 = €1010
€x × 0·3 = €1010 − €200 = €810
x = €2700
Total sales = €2700 + €1000 = €3700

Marking Scheme Notes
Question 5 (a) [Scale 10C (0, 4, 6, 10)]
4: • 1000 × 20% or equivalent
• 2000
6: • both commissions found, 200 and 600

Question 5 (b) [Scale 15D (0, 5, 7, 10, 15)]
5: • 1160 – 150
• 1160 – 200
7: • 810
• $\frac{x}{0\cdot3}$
10: • 2700

Question 7 (50 marks)

Question 7 (a)



Question 7 (b)

Pattern number (n)	Number of Tiles
1	5
2	8
3	11
4	14
5	17

Marking Scheme Notes
Question 7 (a) [Scale 5A (0, 5)]

Question 7 (b) [Scale 5C (0, 2, 3, 5)]
2: • 1 or 2 correct new entries
3: • 3 correct or consistent new entries

Question 7 (c) (i)

Method 1:

$$\boxed{T_n = a + (n-1)d}$$

$a = 5,\ d = 8 - 5 = 3$

$T_n = a + (n-1)d = 5 + (n-1)3$

$= 5 + 3n - 3$

$= 3n + 2$

$\therefore p = 3,\ q = 2$

Method 2:

$T_n = pn + q$

$T_1 = p + q = 5 \ldots\ldots (\mathbf{1})$

$T_2 = 2p + q = 8 \ldots (\mathbf{2})$

$(\mathbf{2}) - (\mathbf{1}): \quad p \qquad = 3$

Into (**1**): $3 + q = 5 \Rightarrow q = 2$

Marking Scheme Notes

Question 7 (c) (i) [Scale 10C (0, 4, 6, 10)]

4: • formula written with some substitution
• 1 or 2 relevant equations in p & q
• $d = 3$ or $a = 5$ written

6: • T_n formula fully substituted
• value of p or value of q found

Note: Accept $3n + 2$ without work for full marks

Question 7 (c) (ii)

$T_n = 3n + 2$

$T_{20} = 3(20) + 2 = 60 + 2 = 62$

Question 7 (c) (iii)

$T_n = 3n + 2 = 290$

$3n = 288 \Rightarrow n = \dfrac{288}{3} = 96$

The 96th pattern has 290 tiles.

Marking Scheme Notes

Question 7 (c) (ii) [Scale 5B (0, 2, 5)]

2: • formula with some substitution
• listing of terms (> 10) but T_{20} not identified or incorrectly identified

Note: Accept correct answer without work

Question 7 (c) (iii) [Scale 5B (0, 2, 5)]

2: • correct answer without work
• equation (with formula) written
• $3n + 2$ on its own
• finds $T_{290} = 872$, but 872 without work is zero marks

Question 7 (d) (i)

$$\boxed{S_n = \tfrac{n}{2}[2a + (n-1)d]}$$

$a = 5,\ d = 3$

$S_n = \tfrac{n}{2}[2(5) + (n-1)3] = \tfrac{n}{2}[10 + 3n - 3]$

$= \tfrac{n}{2}[3n + 7] = \dfrac{3n^2 + 7n}{2}$

Question 7 (d) (ii)

$S_n = \dfrac{3n^2 + 7n}{2}$

$S_{30} = \dfrac{3(30)^2 + 7(30)}{2} = 1455$

Marking Scheme Notes

Question 7 (d) (i) [Scale 15C (0, 6, 10, 15)]

6: • correct formula with some correct substitution
• $a = 5$ and/or $d = 3$

10: • formula fully substituted

Question 7 (d) (ii) [Scale 5C (0, 2, 3, 5)]

2: • formula = 1455
• finds $S_n = 30$
• $a = 5$ and/or $d = 3$
• evidence of addition of terms in trial and improvement method

3: • quadratic equation formed
• substitutes correctly into either S_n formula

Question 8 (45 marks)

$P(x) = 275x - x^2 - 2000,\ x \leq 200$

Question 8 (a)

$x = 0 : P(0) = 275(0) - (0)^2 - 2000$

$= -2000$ [Loss of €2000]

Question 8 (b)

$P(x) = 8350 \Rightarrow P(x) = 275x - x^2 - 2000 = 8350$

$x^2 - 275x + 10\,350 = 0$

$a = 1,\ b = -275,\ c = 10\,350$

$x = \dfrac{-(-275) \pm \sqrt{(-275)^2 - 4(1)(10\,350)}}{2}$

$= 45,\ 230$

$x = 45$ km as $x \leq 200$ km

Marking Scheme Notes

Question 8 (a) [Scale 5B (0, 2, 5)]

2: • $x = 0$

Question 8 (b) [Scale 10C (0, 4, 6, 10)]

4: • quadratic equation written
• 45 without work

6: • roots of quadratic

Note: 45 subbed in to get 8350 is full Marks

Question 8 (c) (i)

$P(50) = 275(50) - (50)^2 - 2000 = €9250$

$P(60) = 275(60) - (60)^2 - 2000 = €10\ 900$

$P(70) = 275(70) - (70)^2 - 2000 = €12\ 350$

$P(80) = 275(80) - (80)^2 - 2000 = €13\ 600$

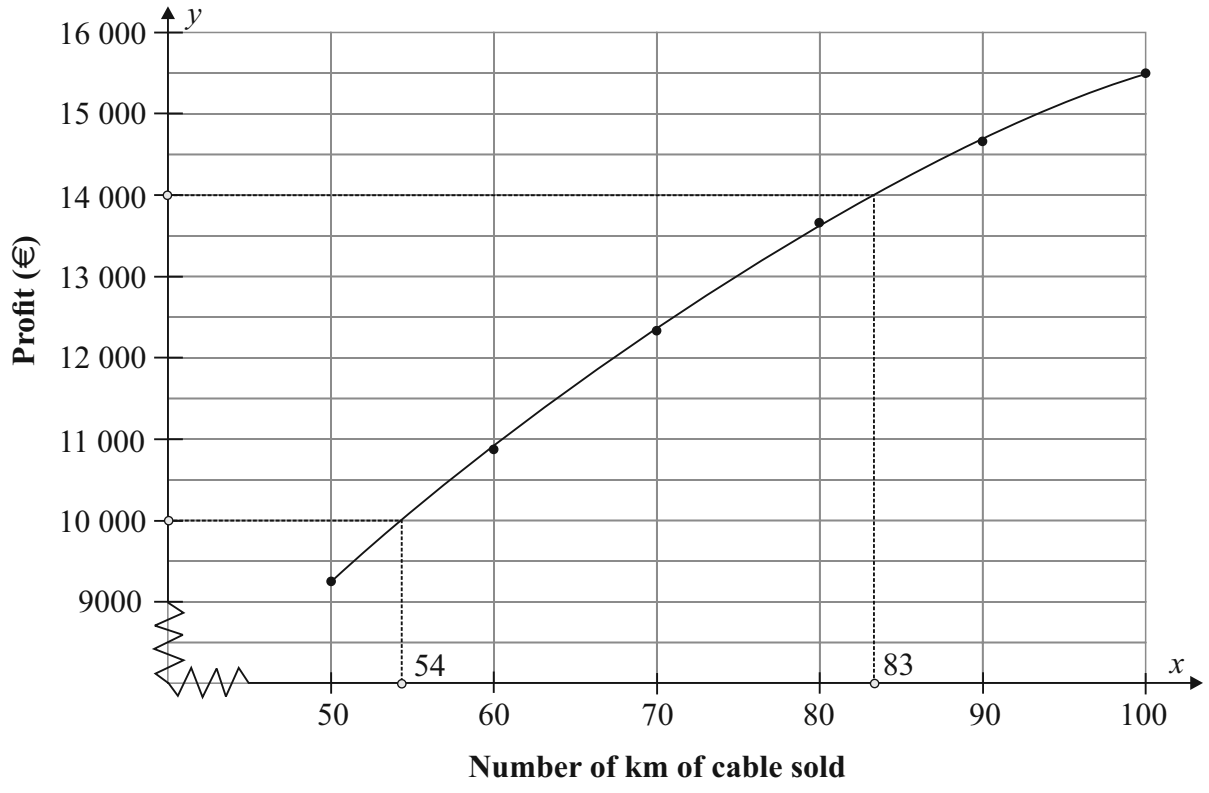
$P(90) = 275(90) - (90)^2 - 2000 = €14\ 650$

$P(100) = 275(100) - (100)^2 - 2000 = €15\ 500$

Number of km of cable sold (x)	50	60	70	80	90	100
Profit (€)	9250	10 900	12 350	13 600	14 650	15 500

Marking Scheme Notes
Question 8 (c) (i) [Scale 10C (0, 4, 6, 10)]
4: • 1 or 2 correct new entries
6: • 3 or 4 correct new entries

Question 8 (c) (ii)



Marking Scheme Notes
Question 8 (c) (ii) [Scale 5C (0, 2, 3, 5)]
2: • 1, 2 or 3 correct plots
• bar chart
3: • 4 or 5 correct plots
• 6 correct plots but not joined

Question 8 (c) (iii)

Lower range = 54 km
Upper range = 83 km

Question 8 (d)

$P(x) = 275x - x^2 - 2000$

$\frac{dP}{dx} = 105 \Rightarrow 275 - 2x = 105$

$275 - 105 = 2x$

$170 = 2x$

$\therefore x = \frac{170}{2} = 85$ km

Marking Scheme Notes

Question 8 (c) (iii) [Scale 5B (0, 2, 5)]

2: • evidence of use of graph
• one correct range found
• correct answers without work

Question 8 (d) [Scale 10C (0, 4, 6, 10)]

4: • effort at calculus
6: • correct differentiation

Question 9 (55 marks)

Male: $m(x) = 2 \cdot 3x + 65 \cdot 53$

Female: $f(x) = 2 \cdot 5x + 54 \cdot 13$

Question 9 (a)

$x = 47 \cdot 54$:

$$m(47 \cdot 54) = 2 \cdot 3(47 \cdot 54) + 65 \cdot 53$$
$$= 174 \cdot 87 \text{ cm}$$

$x = 47 \cdot 54$:

$$f(47 \cdot 54) = 2 \cdot 5(47 \cdot 54) + 54 \cdot 13$$
$$= 172 \cdot 98 \text{ cm}$$

Question 9 (b)

$$m(x) = 184 \Rightarrow 2 \cdot 3x + 65 \cdot 53 = 184$$
$$2 \cdot 3x = 184 - 65 \cdot 53$$
$$2 \cdot 3x = 118 \cdot 47$$
$$x = \frac{118 \cdot 47}{2 \cdot 3} = 51 \cdot 51 \text{ cm}$$

Question 9 (c)

True height $= 171$ cm

$$m(44 \cdot 2) = 2 \cdot 3(44 \cdot 2) + 65 \cdot 53 = 167 \cdot 19 \text{ cm}$$
$$\% \text{ error} = \frac{|171 - 167 \cdot 19|}{171} \times 100\% = 2 \cdot 23\%$$

Question 9 (d)

$$m(x) = f(x) \Rightarrow 2 \cdot 3x + 65 \cdot 53 = 2 \cdot 5x + 54 \cdot 13$$
$$65 \cdot 53 - 54 \cdot 13 = 2 \cdot 5x - 2 \cdot 3x$$
$$11 \cdot 4 = 0 \cdot 2x$$
$$x = \frac{11 \cdot 4}{0 \cdot 2} = 57 \text{ cm (Length of femur)}$$
$$m(57) = 2 \cdot 3(57) + 65 \cdot 53 = 196 \cdot 63 \text{ cm (Height)}$$

Question 9 (e)

$$P = \frac{M}{h^3}$$

(i) $h = 1 \cdot 60$ m, $M = 72 \cdot 5$ kg: $P = \dfrac{72 \cdot 5 \text{ kg}}{(1 \cdot 60 \text{ m})^3} = 17 \cdot 7 \text{ kg/m}^3$

(ii) $P = \dfrac{M}{h^3} \Rightarrow h^3 = \dfrac{M}{P} \Rightarrow h = \sqrt[3]{\dfrac{M}{P}}$

(iii) $P = 13$, $M = 67 \cdot 5$ kg: $h = \sqrt[3]{\dfrac{67 \cdot 5}{13}} = 1 \cdot 73$ m

Marking Scheme Notes

Question 9 (a) [Scale 10C (0, 4, 6, 10)]
4: • substitution into either formula
6: • one relevant height found
Note: Correct answer without work, award full marks

Question 9 (b) [Scale 5B (0, 2, 5)]
2: • equation formed

Question 9 (c) [Scale 10D (0, 4, 6, 8, 10)]
4: • formula = 167·19
• correct substitution into formula
• correct answer no work
6: • error found, 3·81
8: • % error formulated
• 97·77% found

Question 9 (d) [Scale 10C (0, 4, 6, 10)]
4: • Formulas equated
6: • transpose completed
• length of femur found (x) and stops

Question 9 (e) (i) [Scale 5C (0, 2, 3, 5)]
2: • some relevant substitution into formula
3: • formula fully substituted

Question 9 (e) (ii) [Scale 10B (0, 5, 10)]
5: • some relevant transpose

Question 9 (e) (iii) [Scale 5C (0, 2, 3, 5)]
2: • some relevant substitution into formula
• correct answer without work
3: • formula fully substituted

LC 2017 (Set B): Paper 2

Question 1 (25 marks)
Number of babies born in 2015 = 65 909
Number of girls born in 2015 = 32 290

Question 1 (a)

(i) Number of boys born in 2015
$= 65\,909 - 32\,290 = 33\,619$

(ii) $P(\text{Boy}) = \dfrac{\text{Number of boys}}{\text{Number of babies}}$
$= \dfrac{33\,619}{65\,909} = 0{\cdot}51$

Marking Scheme Notes
Question 1 (a) (i) [Scale 5B (0, 3, 5)]
3: • Indicates relevant subtraction
5: • Correct answer without work

Question 1 (a) (ii) [Scale 5C (0, 2, 3, 5)]
2: • Either #(*E*) or #(*S*) correct
(Answer must be shown in fraction format)
3: • $P(B) = \dfrac{33\,619}{65\,909}$
5: • Correct answer without work

Special Case	
Without Work	**Award**
51%	F* *Full Credit* −1

Question 1 (b)

(i) $P(\text{Boy, Boy, Boy}) = 0{\cdot}51 \times 0{\cdot}51 \times 0{\cdot}51 = 0{\cdot}1327$

(ii) $P(\text{Girl}) = 1 - 0{\cdot}51 = \mathbf{0{\cdot}49}$

$P(\text{Boy, Boy, Girl}) = 0{\cdot}51 \times 0{\cdot}51 \times 0{\cdot}49 = 0{\cdot}1274$

Marking Scheme Notes
Question 1 (b) (i) & (ii) [Scale 5C (0, 2, 3, 5)]
2: • $(0{\cdot}51)^x$
• 0·49
3: • One correct answer
• (0·51)(0·51)(0·51) & (0·51)(0·51)(0·49)
• Substantial work in both parts
5: • Correct answers without work

Question 1 (c)

(i) Table:

Day	Mon	Tues	Wed	Thu	Fri	Sat	Sun
Probability	0·14	**0·16**	0·15	0·18	0·15	0·12	0·1

(ii) Expected number of babies born on Tuesday $= 1300 \times 0{\cdot}16 = 208$

Marking Scheme Notes
Question 1 (c) (i) [Scale 5B (0, 3, 5)]
3: • Formulates addition of some probabilities
• 1 – (incorrect) Sum
5: • Correct answer without work

Question 1 (c) (ii) [Scale 5B (0, 3, 5)]
3: • Probability of (Tuesday) used
• Work of merit with 1300
5: • Correct answer without work

Question 2 (25 marks)

Question 2 (a)

$c : x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (0, 0) and radius r

$$x^2 + y^2 = r^2$$

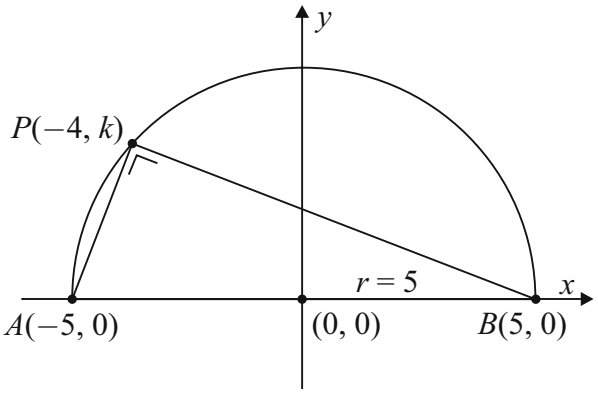
Marking Scheme Notes
Question 2 (a) [Scale 5C (0, 2, 3, 5)]
2: • Equation of circle formula with some substitution. e.g. $x^2 + y^2 =$
• 25 or 5^2
3: • Equation of circle formula fully substituted. i.e. $x^2 + y^2 = 5^2$

Question 2 (b)

(i) $P(-4, k) \in c : (-4)^2 + (k)^2 = 25$

$16 + k^2 = 25 \Rightarrow k^2 = 25 - 16 = 9$

$\therefore k = \sqrt{9} = 3,\ k > 0$



(ii) $A(-5, 0),\ P(-4, 3),\ B(5, 0)$

Slope of AP: $m_1 = \dfrac{3-0}{-4+5} = \dfrac{3}{1} = 3$

Slope of BP: $m_2 = \dfrac{0-3}{5+4} = \dfrac{-3}{9} = -\dfrac{1}{3}$

$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1 \Rightarrow AP \perp BP$

$\therefore \Delta ABP$ is right-angled at P.

Marking Scheme Notes
Question 2 (b) (i) [Scale 5C (0, 2, 3, 5)]
2: • (–4) or k substituted into equation of circle
3: • $(-4)^2 + k^2 = 25$
• Leaves as $k^2 = 9$
• Correct answer without work

Question 2 (b) (ii) [Scale 5C (0, 2, 3, 5)]
2: • Correct relevant formula with some substitution
• AB is the diameter (but fails to draw conclusion)
• Demonstrates understanding of a requirement to use slopes
e.g. $[m_1 \times m_2 = -1]$
• One correct slope, or length
3: • Both slopes correct but no conclusion
• Triangle ABP is right angled
0: • Triangle drawn with P identified as right angled

Question 2 (c)

Area of ΔAPB:

$A(-5, 0) \rightarrow (0, 0)$

$P(-4, 3) \rightarrow (1, 3)$

$B(5, 0) \rightarrow (10, 0)$

Area $= \frac{1}{2}\left|1(0) - 3(10)\right| = 15$

Area of semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi$

Area of region = Area of semi-circle – Area of ΔAPB

$= \frac{25}{2}\pi - 15 = 24 \cdot 27$ units squared

Marking Scheme Notes
Question 2 (c) [Scale 10C (0, 3, 7, 10)]

3: • Area of circle or semi-circle formula with some substitution
• Area of triangle formula with some substitution
• Area = Area of semicircle – Area of triangle (or equivalent)

7: • Both areas found
• Answer as $A = \frac{1}{2}\pi 5^2 - \frac{1}{2}10(3)$

Note:

π (Calculator) : Answer $= 24 \cdot 27$

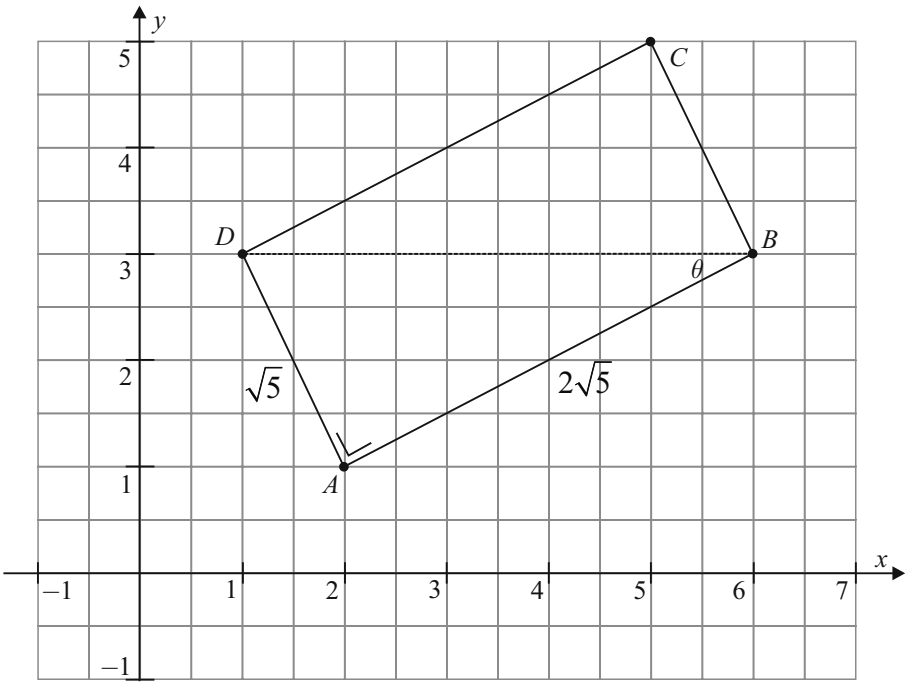
π $(3 \cdot 14)$: Answer $= 24 \cdot 25$

π $(\frac{22}{7})$: Answer $= 24 \cdot 29$

Question 3 (25 marks)

Question 3 (a)

$A(2, 1)$, $B(6, 3)$, $C(5, 5)$, $D(1, 3)$



(i) $|AD| = \sqrt{(1-2)^2 + (3-1)^2} = \sqrt{(-1)^2 + 2^2}$

$= \sqrt{1+4} = \sqrt{5}$ units

(ii) $|AB| = \sqrt{(6-2)^2 + (3-1)^2} = \sqrt{4^2 + 2^2}$

$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ units

Area $ABCD = |AB| \times |AD| = \sqrt{5} \times 2\sqrt{5} = 10$ square units

Marking Scheme Notes

Question 3 (a) (i) [Scale 5C (0, 2, 3, 5)]

2: • Correct relevant formula with some substitution
• Work with Pythagoras (check candidates diagram)

3: • Distance formula fully substituted
• Pythagoras's fully substituted

0: • Measurement from diagram (2·2)

Question 3 (a) (ii) [Scale 5C (0, 2, 3, 5)]

2: • Area = $|AD| \times |CD|$
• Area = $A_{\text{Rectangle}} - A_{\text{4 Triangles}}$
• Correct relevant formula with some substitution

3: • Area of triangles calculated
• Area $= \sqrt{20} \times \sqrt{5}$
• Area = 20 – (4 + 4 + 1 + 1)

0: • Measurement from diagram (2·2, 4·5)

5: • Correct answer without work

Question 3 (b)

$B(6, 3), C(5, 5) = (x_1, y_1)$

Slope of BC: $m = \dfrac{5-3}{5-6} = \dfrac{2}{-1} = -2$

Equation of BC : $y - y_1 = m(x - x_1)$

$y - 5 = -2(x - 5)$

$y - 5 = -2x + 10$

$2x + y - 15 = 0$

Question 3 (c)

$\tan\theta = \dfrac{\sqrt{5}}{2\sqrt{5}} = \dfrac{1}{2} \Rightarrow \theta = \tan^{-1}(\frac{1}{2}) \approx 27^\circ$

Marking Scheme Notes

Question 3 (b) [Scale 10C (0, 3, 7, 10)]

3: • Correct relevant formula with some substitution

7: • Eq. of line formula fully substituted
• One incorrect substitution followed by correct solution

Note: Accept $(-2x - y + 15 = 0)$

Question 3 (c) [Scale 5C (0, 2, 3, 5)]

2: • Any work of merit e.g. identifies $\angle ABD$
• Work of merit with correct trigonometric ratio
• Correct relevant formula with some substitution

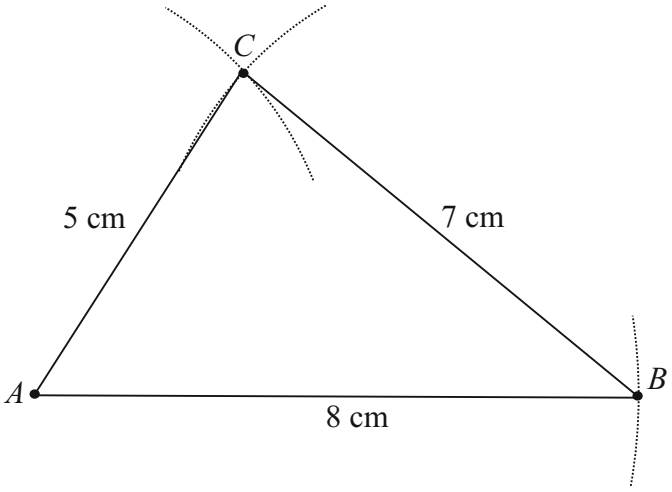
3: • Sets up a correct trigonometric ratio
• Correct relevant formula fully substituted

Misread (–1):
• Finds $|\angle ABD|$ correctly

Note:
Rad = 0·4636
Grad = 29·5167

Question 4 (25 marks)

Question 4 (a)



Using a compass, measure out a length of 8 cm.

With the point of the compass on A, draw an arc $|AB|$ of length 8 cm.

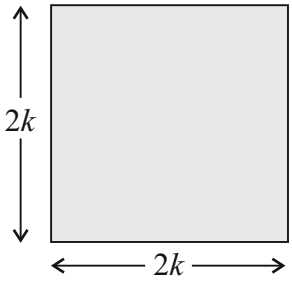
Draw a radius of 5 cm with the point of the compass placed at A. Draw a radius of 7 cm with the point of the compass placed at B. Where the arcs intersect is point C.

Join A to C and B to C to form triangle ABC.

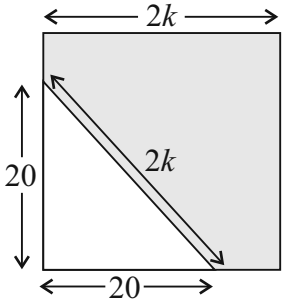
Marking Scheme Notes
Question 4 (a) [Scale 10C (0, 3, 7, 10)]
3: • Pilot diagram drawn
• One correct length drawn
7: • TWO correct lengths drawn.
• Construction correct without arcs (having checked length of sides)
• Construction correct but vertices incorrectly named or omitted
10: • Correct construction with sides or vertices labelled correctly
Note: Tolerance ± 0·25 cm

Question 4 (b)

(i) Area $= (2k)(2k)$
$= 4k^2$ cm^2



(ii) $20^2 + 20^2 = (2k)^2$
$400 + 400 = 4k^2$
$800 = 4k^2$
$200 = k^2$
$\therefore k = \sqrt{200}$
$= 14{\cdot}14$ cm



Shaded area
= Area of square – Area of triangle
$= 4k^2 - \frac{1}{2}(20)(20)$
$= 4(200) - 200$
$= 800 - 200 = 600$ cm^2

Marking Scheme Notes
Question 4 (b) (i) [Scale 5C (0, 2, 3, 5)]
2: • Area of square written as L × W (or similar)
3: • Correct relevant formula fully substituted
5: • Correct answer without work

Question 4 (b) (ii) [Scale 10D (0, 4, 5, 7, 10)]
4: • Any use of Pythagoras's Theorem
• Area of Triangle Formula with some substitution
• Formulates for area using subtraction
5: • Value of k found
• Area of Triangle = 200
7: • Area of square and area of triangle found
Note: Accept correct rounding of answers

QUESTION 5 (25 MARKS)

Question 5 (a)

(i) Mean $\mu = 100$, standard deviation $\sigma = 15$

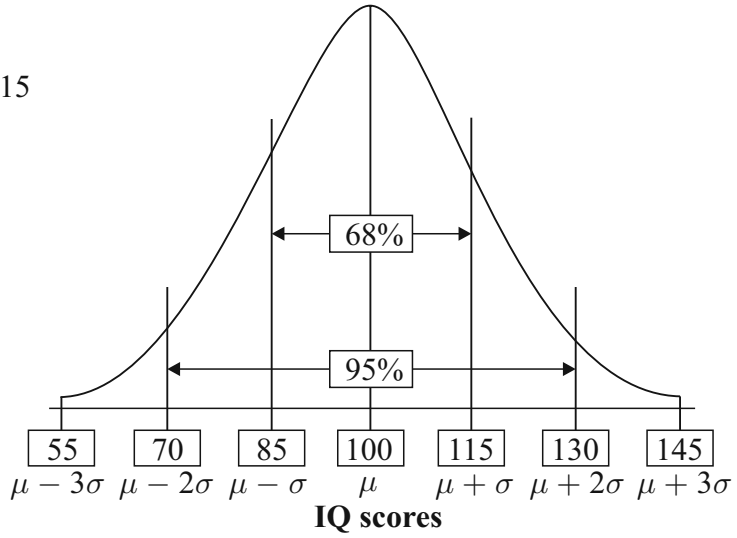
(ii) 95% of those surveyed lie within 2σ of the mean [70 to 130 points]

$P(\text{IQ score is 70 to 130}) = 0{\cdot}95$

(iii) 68% of those surveyed lie within 1σ of the mean [85 to 115 points]

Number of people $= 1200 \times 0{\cdot}68$

$= 816$



MARKING SCHEME NOTES

Question 5 (a) (i) [Scale 15C (0, 7, 10, 15)]

7: • One correct new entry.
• Writes $\bar{x} = 100$ or $\sigma = 15$
• Writes $\bar{x}$, σ, 2σ or 3σ on axis correctly

10: • Four correct new entries

Question 5 (a) (ii) & (iii) [Scale 5C (0, 2, 3, 5)]

2: • Shows some understanding of the Empirical Rule
• Work of merit with 1200

3: • One correct answer [e.g. $p = 0{\cdot}95$ (95%)]
• Error in the use of the Empirical Rule
[e.g. (a) (ii) 68% and (a) (iii) 95% of 1200 = 1140]
• Substantial work in both parts

Question 5 (b)

	Studies Chemistry	**Does not study Chemistry**
Boys	**6**	**4**
Girls	9	**5**

$P(\text{Boy or a pupil not studying Chemistry}) = \frac{15}{24} = \frac{5}{8}$

MARKING SCHEME NOTES

Question 5 (b) [Scale 5B (0, 3, 5)]

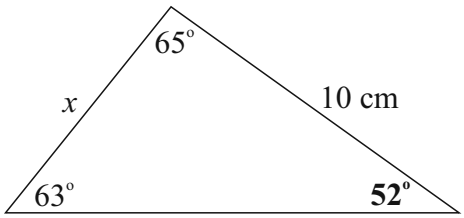
3: • Any relevant work towards creating at least one correct probability
[e.g. $P = \frac{x}{24}$, $x \neq 15$]
• Answer as 10 + 5

0: • Answer as 15

5: • Correct answer without work

Question 6 (25 marks)

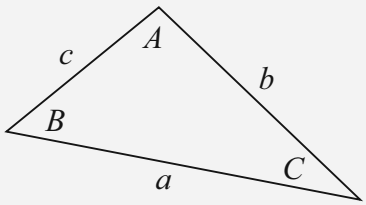
Question 6 (a)



Third angle $= 180^\circ - 65^\circ - 63^\circ = 52^\circ$

$$\frac{x}{\sin 52^\circ} = \frac{10}{\sin 63^\circ} \Rightarrow x = \frac{10 \sin 52^\circ}{\sin 63^\circ} = 8 \cdot 84 \text{ cm}$$

Formulae and Tables Book
Trigonometry of the triangle:

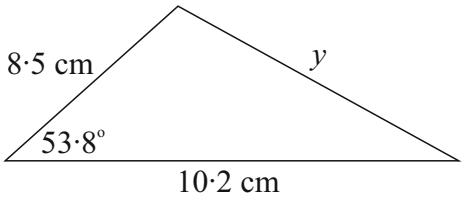


Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Marking Scheme Notes
Question 6 (a) [Scale 10D (0, 4, 5, 7, 10)]
4: • Any work of merit [e.g. Third Angle = 52°]
• Sine rule with some substitution
5: • Finds the length of the base side (10·17)
7: • Full correct substitution without calculation
• Incorrect calculator mode but otherwise correct (once only)
• Correct answer without work
Rad = 58·95392809; Grad = 8·721730163

Question 6 (b)



$y^2 = 8 \cdot 5^2 + 10 \cdot 2^2 - 2(8 \cdot 5)(10 \cdot 2) \cos 53 \cdot 8^\circ$

$y = \sqrt{8 \cdot 5^2 + 10 \cdot 2^2 - 2(8 \cdot 5)(10 \cdot 2) \cos 53 \cdot 8^\circ} = 8 \cdot 60 \text{ cm}$

Marking Scheme Notes
Question 6 (b) [Scale 15D (0, 5, 7, 9, 15)]
5: • Writes Cosine Rule correctly with some substitution
7: • Two variables correctly substituted into correct relevant formula
• $y^2 = 73 \cdot 878977$ without work and stops
9: • Fully correct substitution
• One incorrect substitution followed by correct calculation
• Incorrect calculator mode but otherwise correct (once only)
• Correct answer without work
0: • Treating triangle as right-angled
Note:
Rad = 18·34; Grad = 7·82

Question 7 (55 marks)

Question 7 (a)

(i) $r = 2 \cdot 25$ m

$h = 2r = 2(2 \cdot 25)$

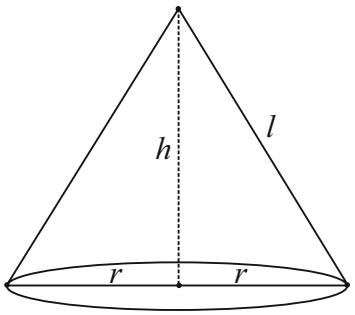
$= 4 \cdot 5$ m

(ii) $l^2 = h^2 + r^2$

$= 4 \cdot 5^2 + 2 \cdot 25^2$

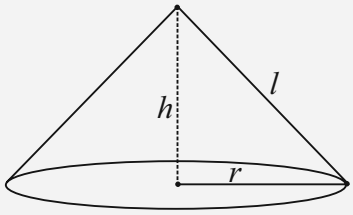
$l = \sqrt{4 \cdot 5^2 + 2 \cdot 25^2}$

$= 5 \cdot 03$ m



Formulae and Tables Book

Cone: [page 10]



Surface Area $A = \pi rl$

Volume $V = \frac{1}{3}\pi r^2 h$

Marking Scheme Notes

Question 7 (a) (i) [Scale 10B (0, 5, 10)]

5: • Correct relationship identified
• 2·25 ÷ 2
• $h = d$ drawn on a diagram

10: • Correct answer without work

Question 7 (a) (ii) [Scale 5D (0, 2, 3, 4, 5)]

2: • Some use of Pythagoras.
• Work of merit [e.g. Work with 2·25/4·5]

3: • Uses (5·03) to give: $(5 \cdot 03)^2 = (4 \cdot 5)^2 + (2 \cdot 25)^2$

4: • Pythagoras fully substituted
• Finishes with only one error
• Stops at $\sqrt{25 \cdot 3125}$

Question 7 (b)

(i) Curved surface area $= 14 \times \pi rl$

$= 14 \times \pi(2 \cdot 25)(5 \cdot 03) = 497 \cdot 77$ m^2

(ii) Number of litres of paint $= \dfrac{497 \cdot 77}{12 \cdot 25}$

≈ 41 litres

(iii) Container $= 5$ litres

Number of containers $= \dfrac{41}{5} = 8 \cdot 2$

$\therefore 9$ containers are needed.

Cost $= 9 \times$ A\$110 $=$ A\$990

$= 990 \times €0 \cdot 68 \approx €673$

Marking Scheme Notes

Question 7 (b) (i) [Scale 10C (0, 3, 7, 10)]

3: • CSA formula written with some correct substitution
• Work of merit [e.g. Use of 2·25/5·03]
• Indicates multiplication by 14

7: • Fully correct substitution
• Finishes with only one error
• CSA of one cone calculated (35·55)

Question 7 (b) (ii) [Scale 10B (0, 5, 10)]

5: • Works with answer to (b) (i)
• Works with 12·25

Question 7 (b) (iii) [Scale 10D (0, 4, 5, 7, 10)]

4: • Indicates answer as $\dfrac{\text{b (ii)}}{5}$ and stops

5: • Finds correct number of containers (without work)

7: • Finds *A*\$990 or consistent equivalent
• Finishes with only one error

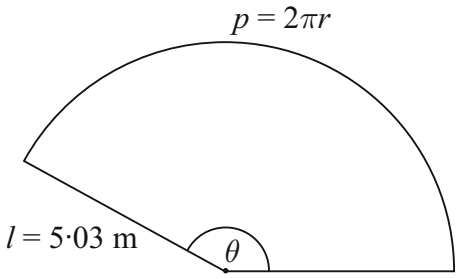
Question 7 (c)

(i) Length of sector $p = 2\pi r$

$= 2\pi(2 \cdot 25) = 14 \cdot 14$ m

(ii) $\dfrac{\theta}{360^{\circ}} \times 2\pi(5 \cdot 03) = 14 \cdot 14$

$\theta = \dfrac{14 \cdot 14 \times 360}{2\pi(5 \cdot 03)} \approx 161^{\circ}$



Marking Scheme Notes

Question 7 (c) (i) & (ii) [Scale 10C (0, 3, 7, 10)]

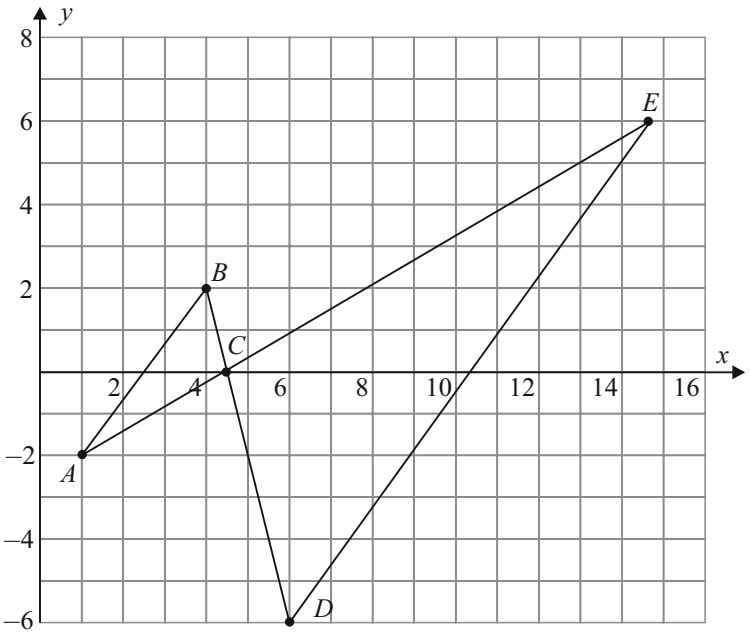
3: • Correct relevant formula written with some correct substitution
• Identifies $r = 2 \cdot 25$ or CSA = (35·55)
• Correct use of 360°

7: • One correct
• Uses $l = r\theta$ as degrees ($\theta = 2 \cdot 8$) (1 rad ≈ 57·296°)
• Substantial work in both parts

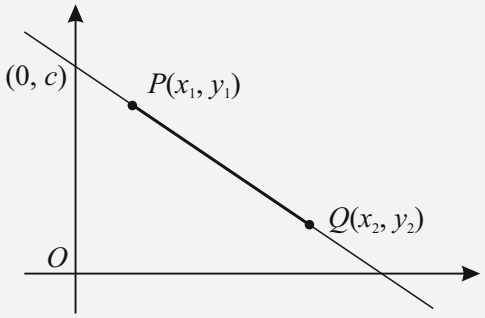
Question 8 (40 marks)

Question 8 (a)

Points: $A(1, -2)$, $B(4, 2)$, $D(6, -6)$, $E(15, 6)$



Formulae and Tables Book
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Marking Scheme Notes
Question 8 (a) [Scale 10C (0, 3, 7, 10)]
3: • ONE correct point
7: • THREE correct points
• All answers written consistently as (y, x)

Question 8 (b)

$A(1, -2)$, $B(4, 2)$

Slope of AB: $m_1 = \frac{2+2}{4-1} = \frac{4}{3}$

$D(6, -6)$, $E(15, 6)$

Slope of DE: $m_2 = \frac{6+6}{15-6} = \frac{12}{9} = \frac{4}{3}$

$m_1 = m_2 \Rightarrow [AB] \| [DE]$

Marking Scheme Notes
Question 8 (b) [Scale 10D (0, 4, 5, 7, 10)]
4: • Any mention of same slopes or similar
• Slope formula written with some relevant substitution
• Rise over Run (Written or indicated)
5: • Slope formula fully substituted for one slope
• One correct slope
7: • Both slopes but no conclusion
• One error in substitution followed by correct calculation and consistent conclusion
10: • Equal slopes and conclusion

Question 8 (c)

(i) $A(1, -2)$, $B(4, 2)$, $C(4{\cdot}5, 0)$

$A(1, -2) \rightarrow (0, 0)$

$B(4, 2) \rightarrow (3, 4)$

$C(4{\cdot}5, 0) \rightarrow (3{\cdot}5, 2)$

Area $\Delta CBA = \frac{1}{2}|3(2) - 4(3{\cdot}5)| = \frac{1}{2}|6 - 14|$

$= \frac{1}{2}|-8| = 4$ square units

(ii) $|AB| = \sqrt{(4-1)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

(iii) $|DE| = k|AB| \Rightarrow 15 = k \times 5$

$\therefore k = 3$

(iv) Area $\Delta CDE = k^2 \times$ (Area ΔCBA)

$= 3^2 \times 4 = 9 \times 4 = 36$ square units

Marking Scheme Notes

Question 8 (c) (i) [Scale 5C (0, 2, 3, 5)]

2: • Correct relevant formula with some correct/consistent substitution
• One point translated to (0, 0)

3: • Area $= \frac{1}{2}|x_1y_2 - x_2y_1|$ fully substituted with translated points
• Error in translation and area calculated correctly
• No translation but uses TWO of the points A, B, C and area calculated correctly
• One error in substitution followed by correct calculation

Question 8 (c) (ii) [Scale 5C (0, 2, 3, 5)]

2: • Correct relevant formula with some substitution
• Any work of merit indicated in this part

3: • Correct substitution without calculation
• One error in substitution followed by correct calculation
• Correct answer without work

Question 8 (c) (iii) [Scale 5B (0, 3, 5)]

3: • Use of answer to part (c) (ii)
• Scale factor formula written or indicated
• Attempts to calculate $|AB|$ in this part

5: • Correct answer without work

Question 8 (c) (iv) [Scale 5C (0, 2, 3, 5)]

2: • Work of merit with answer from (c) (iii)
• Work of merit with answer from (c) (i)

3: • Area as $3 \times 4 = 12$ or equivalent

5: • Correct answer without work

Question 9 (55 marks)

Table 1	
United Kingdom EU Membership Referendum 2016	
	Votes
Leave the EU	17 410 742
Remain a member of the EU	16 141 241
Valid votes	33 551 983
Invalid or blank votes	**25 359**
Total votes	33 577 342

Question 9 (a)

(i) Number of invalid or blank votes = Total votes − Number of valid votes
= 33 577 342 − 33 551 983 = 25 359

(ii) % who voted to leave EU $= \frac{17\ 410\ 742}{33\ 551\ 983} \times 100\% \approx 52\%$

Marking Scheme Notes
Question 9 (a) (i) [Scale 5B (0, 3, 5)]
3: • Relevant addition or subtraction indicated
5: • Correct answer without work
Note: If Correct answer not written in Table 1 Award F*

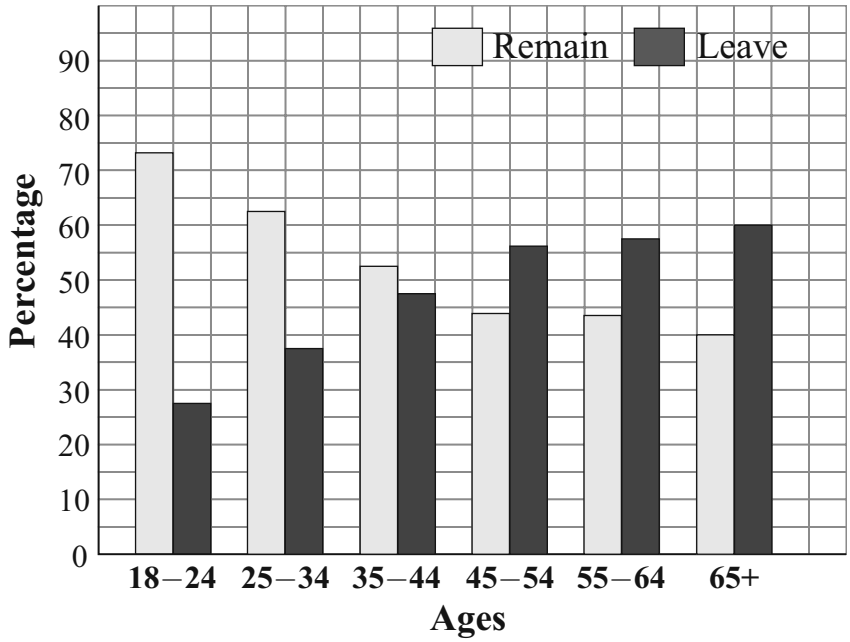
Question 9 (a) (ii) [Scale 5C (0, 2, 3, 5)]
2: • Any correct element in relevant fraction
3: • Ignores or mishandles % calculation [e.g. = 0·52% or 0·5189%]
5: • Correct answer without work

Question 9 (b)

Table 2		
Age Group	Remain (%)	Leave (%)
18 – 24	73	27
25 – 34	62	38
35 – 44	52	48
45 – 54	44	**56**
55 – 64	43	57
65+	40	60

Marking Scheme Notes
Question 9 (b) (i) [Scale 15C (0, 7, 10, 15)]
7: • Effort to draw a chart [e.g. One data point displayed correctly]
10: • One fully correct chart.
• Two substantial charts shown [e.g. Three or more data elements illustrated with scales and axes]

(i) Chart:



(ii) Remain: Mean $= \dfrac{73+62+52+44+43+40}{6}$

$= 52{\cdot}33\%$

Leave: Mean $= \dfrac{27+38+48+56+57+60}{6}$

$= 47{\cdot}67\%$

Formulae and Tables Book
Statistics and probability:
Mean from a list of n numbers
[page 33]

$$\mu = \frac{\text{Sum of all the numbers}}{\text{Number of numbers}} = \frac{\sum x}{n}$$

(iii) The true percentage of those who voted to leave is 52%.
The mean of the 'Leave' values from Table 2 is 47·67%.
These values are different because the numbers of people in each category differ.
For example, far fewer young people voted than older people.

Marking Scheme Notes
Question 9 (b) (ii) [Scale 10C (0, 3, 7, 10)]
3: • Indicates some relevant addition.
• Relevant formula with some correct substitution
7: • One mean correct
10: • Correct answer without work

Special Case	
Without Work	**Award**
52·3 and 47·6	F* *Full Credit* −1
52·3 or 47·6	*High Partial*

Question 9 (b) (iii) [Scale 5B (0, 3, 5)]
3: • The number of voters in each age group is not used
• The mean values calculated in (b) (ii) are based on percentage data

Question 9 (c)

(i) $n = 1200$

$$\text{Margin of error} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1200}} = 0\cdot029 \approx 3\%$$

(ii) Confidence interval: $\frac{578}{1200} - 0\cdot03 \leftrightarrow \frac{578}{1200} + 0\cdot03 = 0\cdot45 \leftrightarrow 0\cdot51$

(iii) Null hypothesis H_0 : Population proportion $p = 0\cdot53$

Alternative hypothesis H_1 : Population proportion $p \neq 0\cdot53$

0·53 does not lie within the confidence interval. Therefore, reject the null hypothesis and accept the alternative hypothesis.

Marking Scheme Notes

Question 9 (c) (i) [Scale 5C (0, 2, 3, 5)]

2: • Writes $\frac{1}{\sqrt{n}}$

• Sets up work as $\sqrt{1200}$

3: • Correct formula fully substituted i.e. $\left(\frac{1}{\sqrt{1200}}\right)$

• Answer as 0·0288675 or $\left(\frac{\sqrt{3}}{60}\right)$ without work

5: • Correct answer without work

Special Case	
Without Work	**Award**
2·88675%	F* *Full Credit* −1

Question 9 (c) (ii) [Scale 5C (0, 2, 3, 5)]

2: • Work of merit with margin of error from (c) (i)

• Answer as $\frac{578}{1200}$

• $\hat{p} \pm \frac{1}{\sqrt{n}}$

3: • One boundary formed (48 + 3 or 48 – 3)
• Correct answer without work
• Use of version of formula from HL

Question 9 (c) (iii) [Scale 5C (0, 2, 3, 5)]

2: • Null hypothesis stated
• C. I. written

3: • Conclusion OK but not given in the context of the question
(Reject null hypothesis but no mention of party claim)

LC 2016 (Set C): Paper 1

Question 1 (25 marks)

Question 1 (a)

Total pay = €28
Deliveries: 5 × €2 = €10
Payment for 2 hours' work = €28 − €10 = €18
∴ Payment per hour = €9

Question 1 (b)

She works for h hours at €9 per hour.
She makes d deliveries at €2 per delivery.
Wages $w = 9h + 2d$

Question 1 (c)

She works for 6 hours ($h = 6$) and makes 12 deliveries ($d = 12$):
$w = 9h + 2d = 9(6) + 2(12) = 54 + 24 = €78$
Sunday: Hourly rate = 9 × 1·5 = €13·50
She makes d deliveries on the Sunday.
Sunday wages = €13·50 × 5 + $2d$ = €67·50 + $2d$
Total wages = €161·50
∴ 78 + 67·50 + $2d$ = 161·50
$2d = 16$
$d = 8$
She makes 8 deliveries on Sunday.

Marking Scheme Notes

Question 1 (a) [Scale 10C (0, 2, 5, 10)]

2: • Any use of 2, 5 or 28
5: • 18 calculated

Question 1 (b) [Scale 5A (0, 5)]

Question 1 (c) [Scale 10C (0, 2, 5, 10)]

2: • 9(6) or 12(2) or 78 formulated calculated or used
• 13·5 written
5: • 83·5 and/or 67·5

QUESTION 2 (25 MARKS)

Question 2 (a)

$z_1 = 1 + 3i, z_2 = 2 - i$

$$\begin{aligned} z_3 &= z_1 + 2z_2 \\ &= 1 + 3i + 2(2 - i) \\ &= 1 + 3i + 4 - 2i \\ &= 5 + i \end{aligned}$$

MARKING SCHEME NOTES

Question 2 (a) [Scale 5C (0, 2, 4, 5)]

2: • Any correct substitution

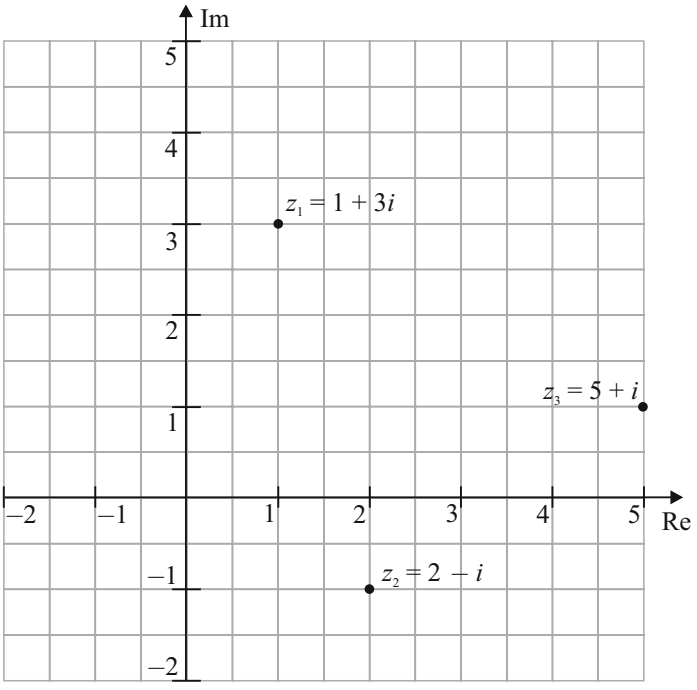
4: • Correct multiplication

Question 2 (b) [Scale 5C (0, 2, 4, 5)]

2: • One Point correctly plotted
• 2 points correctly plotted but without labels

4: • 2 points correctly plotted and labelled
• 3 points correctly plotted without labels
• Mixes up real and imaginary axes with label of 3 correct points

Question 2 (b)



Question 2 (c)

$$\begin{aligned} |z_2 - z_3| &= |2 - i - (5 + i)| \\ &= |2 - i - 5 - i| \\ &= |-3 - 2i| \\ &= \sqrt{(-3)^2 + (-2)^2} \\ &= \sqrt{9 + 4} = \sqrt{13} \end{aligned}$$

$$\begin{aligned} |z_1 + z_2| &= |1 + 3i + 2 - i| \\ &= |3 + 2i| \\ &= \sqrt{(3)^2 + (2)^2} \\ &= \sqrt{9 + 4} = \sqrt{13} \end{aligned}$$

$$\therefore |z_2 - z_3| = |z_1 + z_2|$$

Question 2 (d)

$$\begin{aligned} w = \frac{z_1}{z_2} &= \frac{1 + 3i}{2 - i} \\ &= \frac{(1 + 3i)}{(2 - i)} \times \frac{(2 + i)}{(2 + i)} \quad \leftarrow \text{Multiply above and below by the conjugate of } (2 - i). \\ &= \frac{2 + i + 6i + 3i^2}{4 + 2i - 2i - i^2} \\ &= \frac{2 + i + 6i - 3}{4 + 2i - 2i + 1} = \frac{-1 + 7i}{5} \\ &= -\tfrac{1}{5} + \tfrac{7}{5}i \end{aligned}$$

Marking Scheme Notes
Question 2 (c) [Scale 5A (0, 5)]
Question 2 (d) [Scale 10D (0, 2, 4, 6, 10)]
2: • Any correct substitution
4: • Conjugate identified
• Some multiplication above and below, even if by wrong conjugate
6: • multiply out correctly

Question 3 (25 marks)

Question 3 (a)

$3(x-7)+5(x-4)=15$

$3x-21+5x-20=15$

$3x+5x=15+21+20$

$8x=56$

$x=\frac{56}{8}=7$

Question 3 (b)

$4a+3b=-3$

$5a=25+2b$

$4a+3b=-3...(\mathbf{1})(\times 2)$

$5a-2b=25....(\mathbf{2})(\times 3)$

$8a+6b=-6$

$15a-6b=75$

$23a \quad =69 \Rightarrow a=3$

Into (**1**): $4(3)+3b=-3$

$12+3b=-3$

$3b=-15 \Rightarrow b=-5$

$\therefore (a, b)=(3, -5)$

Question 3 (c)

$2(2x-3)+6x<25$

$4x-6+6x<25$

$4x+6x<25+6$

$10x<31$

$x<\frac{31}{10}$

$x<3\cdot 1,\ x\in\mathbb{N}$

$\therefore x=1, 2, 3$

Marking Scheme Notes
Question 3 (a) [Scale 5C (0, 2, 4, 5)]
2: • Any correct multiplication
• Correct answer no work
4: • Both multiplications done correctly and fails to finish
• Error in expanding brackets and finishes correctly

Question 3 (b) [Scale 10B (0, 3, 10)]
3: • Some correct work in solving

Question 3 (c) [Scale 10C (0, 2, 5, 10)]
2: • Any correct step
• Correct answer without work
5: • Solves for x
• Error in multiplication but finishes correctly

Question 4 (25 marks)

$f(x) = x^3 + x^2 - 2x + 7,\ x \in \mathbb{R}$

Question 4 (a)

$f(x) = y = x^3 + x^2 - 2x + 7$

$x = 0 : f(0) = 0 + 0 - 0 + 7 = 7$

(0, 7) is the point it cuts the y-axis.

Question 4 (b)

$A(1, 7) \in f(x) = x^3 + x^2 - 2x + 7$

$x = 1 : f(1) = (1)^3 + (1)^2 - 2(1) + 7$

$= 1 + 1 - 2 + 7 = 7$

$\therefore A(1, 7) \in f(x).$

Question 4 (c) (i)

$f(x) = x^3 + x^2 - 2x + 7$

$f'(x) = 3x^2 + 2x - 2$

$x = 1 : f'(1) = 3(1)^2 + 2(1) - 2$

$= 3(1) + 2(1) - 2$

$= 3 + 2 - 2 = 3$

$\therefore$ Slope $= 3$

Question 4 (c) (ii)

Point $(x_1, y_1) = A(1, 7)$, slope $m = 3$

$y - y_1 = m(x - x_1)$

$y - 7 = 3(x - 1)$

$y - 7 = 3x - 3$

$3x - y + 4 = 0$

Marking Scheme Notes

Question 4 (a) [Scale 5B (0, 2, 5)]

2: • $x = 0$, written
• Zero correctly substituted into equation

Question 4 (b) [Scale 5B (0, 2, 5)]

2: • Any correct substitution

Question 4 (c) (i) [Scale 10D (0, 2, 4, 6, 10)]

2: • Any correct differentiation

4: • Fully correct differentiation

6: • Correct substitution for slope

Question 4 (c) (ii) [Scale 5B (0, 2, 5)]

2: • Shows $m = 3$
• Correct equation of a line formula
• Any correct substitution

Question 5 (25 marks)

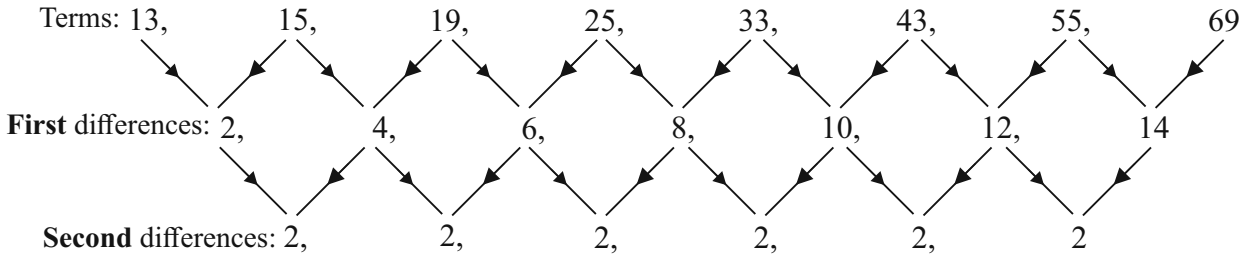
Question 5 (a) (i)

Sequence: 13, 15, 19, 25, 33, ...
[13 + **2** = 15, 15 + **4** = 19, 19 + **6** = 25, ...]

Sequence: 13, 15, 19, 25, 33, 43, 55, 69

Term	Number
U_1	13
U_2	15
U_3	19
U_4	25
U_5	33
U_6	43
U_7	55
U_8	69

Question 5 (a) (ii)



The sequence of second differences is a constant showing the pattern is quadratic.

Marking Scheme Notes
Question 5 (a) [Scale 5C (0, 2, 4, 5)]
2: • One or two correct terms
4: • 3 terms correct but fails to show quadratic

Question 5 (b)

$U_n = n^2 + bn + c$

$U_1 = (1)^2 + b(1) + c = 13$

$1 + b + c = 13$

$\therefore b + c = 12 ...(\mathbf{1})$

$U_2 = (2)^2 + b(2) + c = 15$

$4 + 2b + c = 15$

$\therefore 2b + c = 11 ...(\mathbf{2})$

$(\mathbf{2}) - (\mathbf{1}) : b = -1$

Into (**1**): $(-1) + c = 12 \Rightarrow c = 13$

Marking Scheme Notes
Question 5 (b) [Scale 15C (0, 3, 7, 15)]
3: • Some relevant substitution into U_n
• Derives one correct equation
7: • Derives two correct equations

Question 5 (c)

$a = T_1 = 12,\ d = 14 - 12 = 2$

$T_n = a + (n-1)d$

$= 12 + (n-1)2$

$= 12 + 2n - 2$

$= 2n + 10$

$T_n = 2n + 10$

$T_{30} = 2(30) + 10 = 60 + 10 = 70$

Term	Number
T_1	12
T_2	14
T_3	16
T_4	18
T_5	20

Marking Scheme Notes
Question 5 (c) [Scale 5C (0, 2, 4, 5)]
2: • a or d identified
• Writes $T_n = a + (n - 1)d$
4: • T_n correct and stops
• Finds T_{30} correctly using list method, without finding T_n

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Question 6 (25 marks)
Gross fortnightly wage = €1550

Question 6 (a)
Income tax
20% of €1300 = €1300 × 0·2 = €260
40% of €(1550 − 1300) = €250 × 0·4 = €100
Gross income tax = €260 + €100 = €360
Tax credits = €126
Net income tax = €360 − €126 = €234

Question 6 (b)
USC
1% of €462 = €462 × 0·01 = €4·62
3% of €214 = €214 × 0·03 = €6·42
5·5% of €(1550 − 462 − 214) = €874 × 0·055 = €48·07
Total USC = €4·62 + €6·42 + €48·07 = €59·11

Question 6 (c) (i)
PRSI = €18
Total deductions = €234 + €59·11 + €18 = €311·11

Question 6 (c) (ii)

$$\frac{311{\cdot}11}{1550} \times 100\% = 20{\cdot}1\%$$

Marking Scheme Notes
Question 6 (a) [Scale 5C (0, 2, 4, 5)]
2: • Sets up 20% of 1300
• calculates €250
• Finds €260 or €100
4: • Finds €260 and €100, or finds €360

Question 6 (b) [Scale 10C (0, 2, 5, 10)]
2: • Finds one correct amount
• Formulates a % correctly
• Finds €874 or €676
5: • Finds two correct amounts

Question 6 (c) (i) & (ii) [Scale 10C (0, 2, 5, 10)]
2: • €234 or €59·11 written in this section
• Some correct addition for **(c)(i)**
5: • Correct deductions found, €311·11
• Formulates % correctly for **(c)(ii)**

Question 7 (55 marks)

Paul's money to invest: €8000

Option 1	Option 2
2% interest in Year 1	3·7% compound interest per year, for 3 years
3% interest in Year 2	
5% interest in Year 3	
Money can be taken out at the end of Year 1 or Year 2 without penalty.	Money may not be taken out until the end of Year 3

Question 7 (a)

End of Year 1: €8000 × 1·02 = €8160
End of Year 2: €8160 × 1·03 = €8404·80
End of Year 3: €8404·80 × 1·05 = €8825·04

Value of the investment using **Option 1** = €8825·04

Marking Scheme Notes
Question 7 (a) [Scale 10D (0, 2, 4, 6, 10)]
2: • Any effort at calculation of interest on €8000 at 2% or 3% or 5%
4: • One correct calculation, €160 or €8160
• Uses simple interest leading to €8800 or €800
6: • Two correct calculations, €244·80 or €8404·80
• Finds correct interest for each year but fails to finish
• Fully correct formulation of amount

Question 7 (b)

Value of the investment using **Option 2** = €8000 × $(1{\cdot}037)^3$ = €8921·26

Marking Scheme Notes
Question 7 (b) [Scale 10D (0, 2, 4, 6, 10)]

2: • Relevant use of 3·7%

4: • One correct calculation, €296 or €8296
• Use of simple interest leading to €8888, or €888

6: • Two correct calculations, €306·95 or €8602·95
• Finds correct interest for each year but fails to finish
• Fully correct formulation of amount

Question 7 (c)

Access to his money. An emergency may arise during the 3 years where Paul may need some money. Option 1 allows him access to his money after Year 1 or Year 2.

Marking Scheme Notes
Question 7 (c) [Scale 5A (0, 5)]

Question 7 (d)

P = €8000
F = €9000
t = 3 years
r = ?

$$F = P(1+i)^t$$
$$9000 = 8000(1+i)^3$$
$$\frac{9}{8} = (1+i)^3$$
$$1+i = \sqrt[3]{\frac{9}{8}} = 1{\cdot}04$$
$$\therefore i = 0{\cdot}04 \Rightarrow r = 4\%$$

Marking Scheme Notes
Question 7 (d) [Scale 10C (0, 2, 5, 10)]

2: • At least one correct substitution
• Correct formula
• Correct answer no work
• 9000 – 8000 = 1000

5: • Fully correct substitution into formula but fails to finish

Question 7 (e)

(i) $v = 8000 + 36t - 1{\cdot}2t^2$

$t = 12$: $v = 8000 + 36(12) - 1{\cdot}2(12)^2 = €8259{\cdot}20$

(ii) Interest earned = €8259·20 − €8000 = €259·20

Mary's interest rate $= \frac{259{\cdot}20}{8000} \times 100\% = 3{\cdot}24\%$

Marking Scheme Notes
Question 7 (e) (i) [Scale 10C (0, 2, 5, 10)]
2: • Any correct substitution into formula
5: • Fully correct substitution into formula but fails to finish
• Subs $t = 1$ giving answer of 8034·80

Question 7 (e) (ii) [Scale 10C (0, 2, 5, 10)]
2: • Brings down interest or amount from **e (i)**
• Relevant work
5: • Formulates correctly

Question 8 (50 marks)

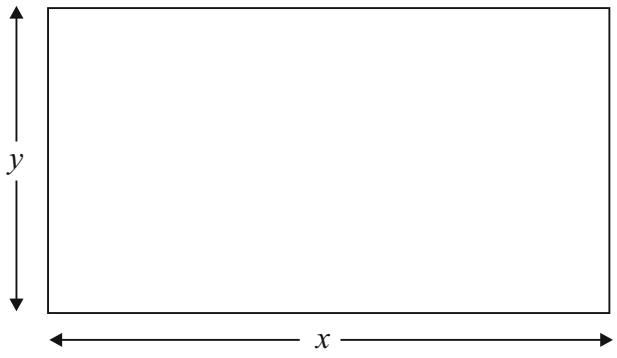
Question 8 (a)

Perimeter $P = 2x + 2y$

$2x + 2y = 21$

$x + y = 10 \cdot 5$

$y = 10 \cdot 5 - x$



Marking Scheme Notes
Question 8 (a) [Scale 5B (0, 2, 5)]
2: • Some correct work on perimeter
• Relevant work on diagram

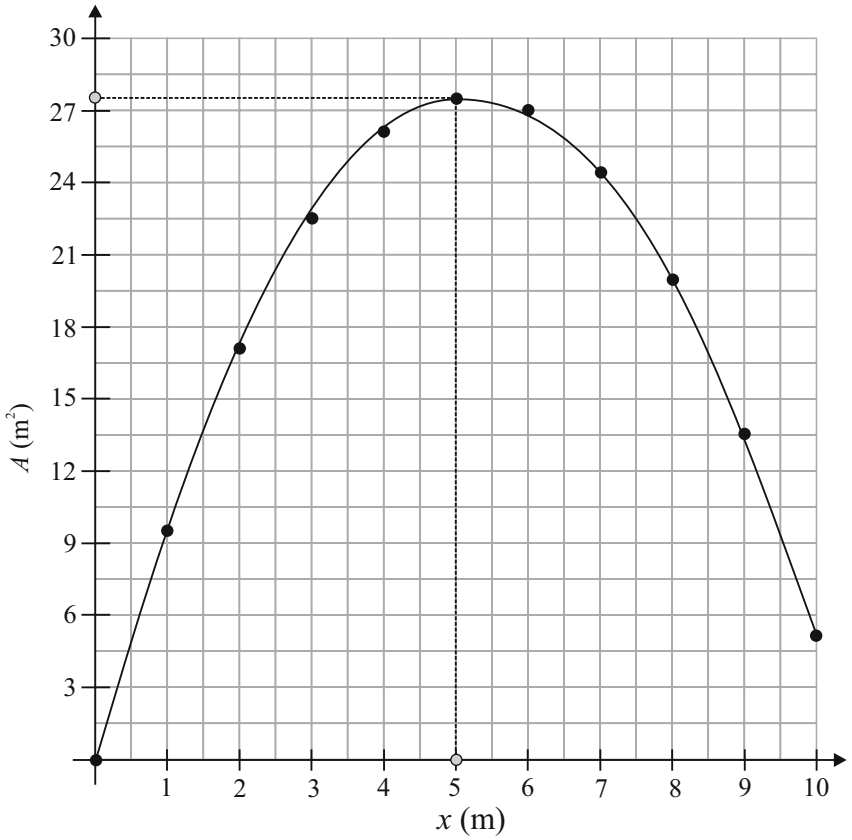
Question 8 (b) (i)

$x = 0$: $y = 10{\cdot}5 - (0) = 10{\cdot}5$ m $\Rightarrow A = xy = (0)(10{\cdot}5) = 0$ m^2
$x = 1$: $y = 10{\cdot}5 - (1) = 9{\cdot}5$ m $\Rightarrow A = xy = (1)(9{\cdot}5) = 9{\cdot}5$ m^2
$x = 2$: $y = 10{\cdot}5 - (2) = 8{\cdot}5$ m $\Rightarrow A = xy = (2)(8{\cdot}5) = 17{\cdot}0$ m^2
$x = 3$: $y = 10{\cdot}5 - (3) = 7{\cdot}5$ m $\Rightarrow A = xy = (3)(7{\cdot}5) = 22{\cdot}5$ m^2
$x = 4$: $y = 10{\cdot}5 - (4) = 6{\cdot}5$ m $\Rightarrow A = xy = (4)(6{\cdot}5) = 26{\cdot}0$ m^2
$x = 5$: $y = 10{\cdot}5 - (5) = 5{\cdot}5$ m $\Rightarrow A = xy = (5)(5{\cdot}5) = 27{\cdot}5$ m^2
$x = 6$: $y = 10{\cdot}5 - (6) = 4{\cdot}5$ m $\Rightarrow A = xy = (6)(4{\cdot}5) = 27{\cdot}0$ m^2
$x = 7$: $y = 10{\cdot}5 - (7) = 3{\cdot}5$ m $\Rightarrow A = xy = (7)(3{\cdot}5) = 24{\cdot}5$ m^2
$x = 8$: $y = 10{\cdot}5 - (8) = 2{\cdot}5$ m $\Rightarrow A = xy = (8)(2{\cdot}5) = 20{\cdot}0$ m^2
$x = 9$: $y = 10{\cdot}5 - (9) = 1{\cdot}5$ m $\Rightarrow A = xy = (9)(1{\cdot}5) = 13{\cdot}5$ m^2
$x = 10$: $y = 10{\cdot}5 - (10) = 0{\cdot}5$ m $\Rightarrow A = xy = (10)(0{\cdot}5) = 5{\cdot}0$ m^2

x (m)	0	1	2	3	4	5	6	7	8	9	10
y (m)	10·5	9·5	8·5	7·5	6·5	5·5	4·5	3·5	2·5	1·5	0·5
A (m^2)	0	9·5	17	22·5	26	27·5	27	24·5	20	13·5	5

Marking Scheme Notes
Question 8 (b) (i) [Scale 5D (0, 2, 3, 4, 5)]
2: • relevant calculations without any correct entry
• 1, 2, 3 or 4 correct entries to table (y or A)
3: • 5 or 6 correct pairs of entries (y or A) or full y line correct
4: • 7, 8 or 9 correct pairs of entries (y or A)

Question 8 (b) (ii)



Marking Scheme Notes
Question 8 (b) (ii) [Scale 5D (0, 2, 3, 4, 5)]
2: • 1, 2, 3 or 4 correct plots
• Graphs y only
3: • 5, 6, 7 or 8 correct plots
4: • 9, 10 or 11 correct plots but no joining or incorrect joining

Question 8 (c)

A: Maximum area (m^2)	27·5
x (m)	5
y (m)	5·5

Question 8 (d)

(i) $y = 10 \cdot 5 - x$

Area $A = xy = x(10 \cdot 5 - x)$

$= 10 \cdot 5x - x^2$

(ii) $A = 10 \cdot 5x - x^2$

$\frac{dA}{dx} = 10 \cdot 5 - 2x$

(iii) $\frac{dA}{dx} = 0 \Rightarrow 10 \cdot 5 - 2x = 0$

$2x = 10 \cdot 5$

$x = \frac{10 \cdot 5}{2} = 5 \cdot 25$ m

(iv) $A_{\text{Max}} = 10 \cdot 5(5 \cdot 25) - (5 \cdot 25)^2$

$= 27 \cdot 5625$ m^2

Marking Scheme Notes

Question 8 (c) [Scale 5C (0, 2, 4, 5)]

2: • Maximum area identified from graph

4: • Area and one corresponding value

Question 8 (d) (i) [Scale 15C (0, 3, 7, 15)]

3: • Relevant area formula

7: • Correct substitution into area formula

Question 8 (d) (ii) [Scale 5B(0, 2, 5)]

2: • Any correct differentiation

Question 8 (d) (iii) [Scale 5C (0, 2, 4, 5)]

Note: Must have calculus in **(d) (ii)** to get any marks in this section

2: • Any use of answer to **(d) (ii)**

4: • Lets their derivative = 0

Question 8 (d) (iv) [Scale 5B (0, 2, 5)]

2: • Any substitution into area formula or equation given in **(d) (i)**

• Writes area formula

QUESTION 9 (45 MARKS)

Question 9 (a) (i)

$A(h) = 30 + 9 \cdot 5h$

$t = 0: A(0) = 30 + 9 \cdot 5(0) = €30 \cdot 00$

$t = 1: A(1) = 30 + 9 \cdot 5(1) = €39 \cdot 50$

$t = 2: A(2) = 30 + 9 \cdot 5(2) = €49 \cdot 00$

$t = 3: A(3) = 30 + 9 \cdot 5(3) = €58 \cdot 50$

$t = 4: A(4) = 30 + 9 \cdot 5(4) = €68 \cdot 00$

$t = 5: A(5) = 30 + 9 \cdot 5(5) = €77 \cdot 50$

$B(h) = 10(1 \cdot 74)^h$

$t = 0: B(0) = 10(1 \cdot 74)^0 = €10 \cdot 00$

$t = 1: B(1) = 10(1 \cdot 74)^1 = €17 \cdot 40$

$t = 2: B(2) = 10(1 \cdot 74)^2 = €30 \cdot 28$

$t = 3: B(3) = 10(1 \cdot 74)^3 = €52 \cdot 68$

$t = 4: B(4) = 10(1 \cdot 74)^4 = €91 \cdot 66$

$t = 5: B(5) = 10(1 \cdot 74)^5 = €159 \cdot 49$

Time (hours)	0	1	2	3	4	5
$A(h)$ (€)	30·00	39·50	49·00	58·50	68·00	77·50
$B(h)$ (€)	10·00	17·40	30·28	52·68	91·66	159·49

MARKING SCHEME NOTES

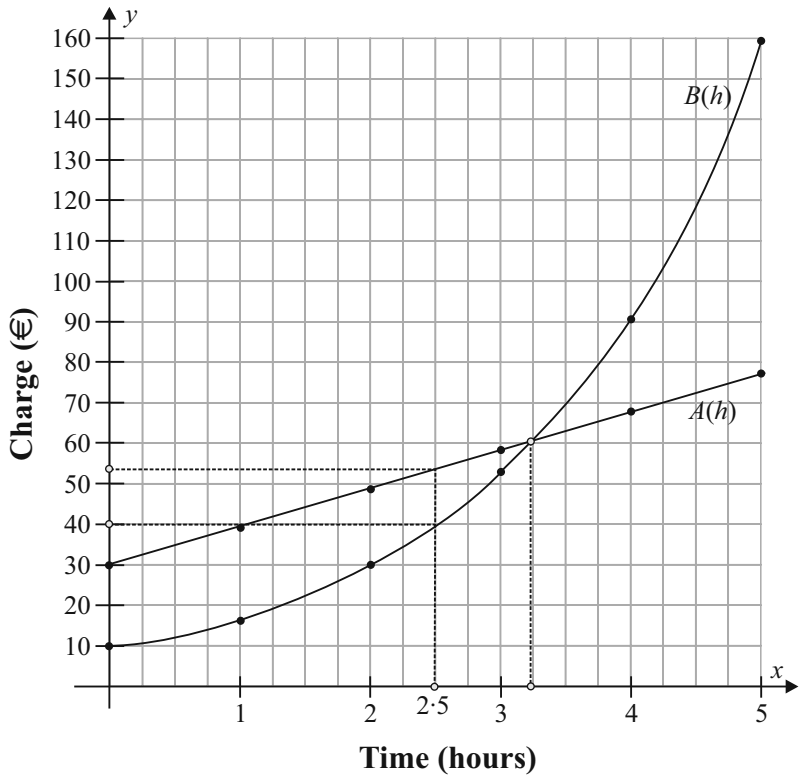
Question 9 (a) (i) [Scale 10D (0, 2, 4, 6, 10)]

2: • relevant calculations
• 1, 2, 3 or 4 correct entries into table

4: • 5 or 6 correct entries

6: • 7, 8 or 9 correct entries

Question 9 (a) (ii)



EPL 104

Marking Scheme Notes
Question 9 (a) (ii) [Scale 15D (0, 4, 7, 11, 15)]
4: • 1, 2, 3 or 4 correct plots
7: • 5 to 11 correct plots
11: • All plots correct but no joining or incorrect joining
• 9, 10 or 11 points plotted correct and joined correctly

(b) B charges least for a job taking 2·5 hours as B charges about €40 whereas A charges over €50.

(c) $h \approx 3{\cdot}2$ hours

(d) $A(h) = 30 + 9{\cdot}5h$

$A(6) = 30 + 9{\cdot}5(6) = €87$

$B(h) = 10(1{\cdot}74)^h$

$B(6) = 10(1{\cdot}74)^6 = €277{\cdot}52$

$B(6) - A(6) = €277{\cdot}52 - €87 = €190{\cdot}52$

Marking Scheme Notes
Question 9 (b) [Scale 5B (0, 2, 5)]
2: • A or B clearly identified on graph
• B given as answer but no reason

Question 9 (c) [Scale 5B (0, 2, 5)]
2: • Point clearly indicated on graph

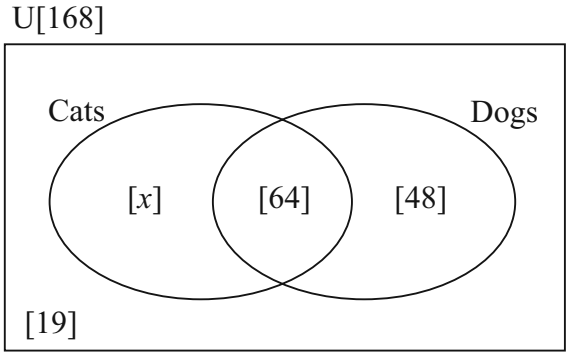
Question 9 (d) [Scale 10D (0, 2, 4, 6, 10)]
2: • $A(6)$ or $B(6)$ substituted correctly
4: • $A(6)$ or $B(6)$ evaluated correctly
6: • $A(6)$ and $B(6)$ evaluated correctly

LC 2016 (Set C): Paper 2

Question 1 (25 marks)

Question 1 (a) (i)

$x + 64 + 48 + 19 = 168$
$x + 131 = 168$
$x = 168 - 131 = 37$



Question 1 (a) (ii)

Number of people who owned a cat and dog = 64
Total number of people = 168

$$P(\text{Owning a cat and a dog}) = \frac{64}{168} = \frac{8}{21}$$

$$P(\text{Event}) = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

Question 1 (a) (iii)

Number of people who just own a dog = 37
Number of people who just own a cat = 48
Number of people who own one animal only = 37 + 48 = 85

$$\%\text{ owning one animal only} = \frac{85}{168} \times 100\% = 50 \cdot 6\%$$

Marking Scheme Notes

Question 1 (a) (i) [Scale 5C (0, 1, 2, 5)]

1: • Any work of merit
2: • One correct/consistent entry calculated or inserted in diagram
5: • Correctly filled in diagram without work

Question 1 (a) (ii) [Scale 5C (0, 1, 2, 5)]

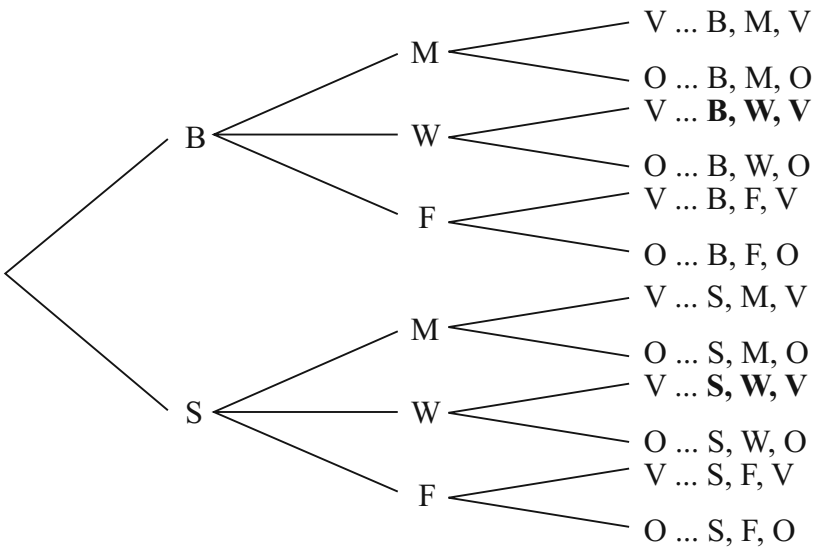
1: • Works with 64 or $168 \neq \#(E), \#(S)$
2: • $\frac{x}{168}$, $(x \neq 64,\ x < 168)$
• $\frac{64}{x}$, $(x \neq 168,\ x > 64)$

Question 1 (a) (iii) [Scale 5C (0, 1, 2, 5)]

1: • Works with 37, 48, 168 (or consistent)
• $\frac{*}{\#} \times 100$
2: • Works to 85 (or consistent)
• Correct answer without work

EPL 104

Question 1 (b)

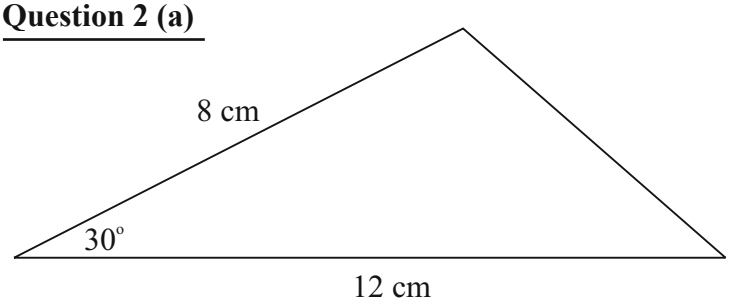


Total number of outcomes = 12
Total number of outcomes on Wednesday (W) with a VIP ticket (V) = 2

$$P(\text{Mandy chooses a VIP ticket on a Wednesday}) = \frac{2}{12} = \frac{1}{6}$$

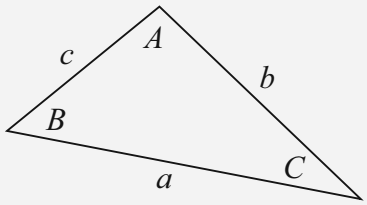
Marking Scheme Notes
Question 1 (b) [Scale 10C (0, 2, 4, 10)]
2: • Any relevant continuation of the tree diagram.
• One probability formulated correctly, e.g. $\frac{1}{2}$
4: • Tree diagram fully correct.
• Probability worked to $\frac{2}{12}$ correct but tree diagram incomplete.

Question 2 (25 marks)
Question 2 (a)



$A = \frac{1}{2}(8)(12)\sin 30^\circ = 24 \text{ cm}^2$

Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Area: $\frac{1}{2}ab\sin C$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Marking Scheme Notes

Question 2 (a) [Scale 5D (0, 2, 3, 4, 5)]

0: • Answer as $\frac{1}{2}(8)(12)$ *or* $\frac{1}{2}\sin(30^\circ)$ without formula

2: • Writes correct relevant formula for area of triangle. e.g. $\frac{1}{2}ab\sin C$

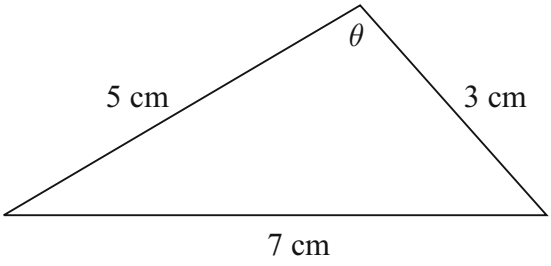
3: • One/Two variables correctly substituted into sine formula
• $h_{\perp}$ calculated

4: • Full correct substitution without calculation
• One incorrect or omitted length substitution with area calculated
• Incorrect calculator mode but otherwise correct (once only)
• Radian = −47·42 , Gradian = 21·79
• Correct answer without work

Question 2 (b)

The largest angle is opposite the longest side.

$$7^2 = 5^2 + 3^2 - 2(5)(3)\cos\theta$$
$$30\cos\theta = 25 + 9 - 49$$
$$30\cos\theta = -15$$
$$\cos\theta = -\frac{15}{30} = -\frac{1}{2}$$
$$\therefore \theta = \cos^{-1}(-\tfrac{1}{2}) = 120^\circ$$



Marking Scheme Notes

Question 2 (b) [Scale 20D (0, 5, 10, 15, 20)]

0: • Treats triangle as right angled

5: • Any work of merit to identify the correct angle required.(Pilot Diagram)
• Writes Cosine Rule correctly

10: • Two variables correctly substituted into correct relevant formula

15: • Fully correct substitution
• One incorrect substitution followed by correct calculation
• Calculates a different angle correctly
• (21·79°, 38·21°)
• A correct or consistent answer without work
• Incorrect calculator mode but otherwise correct (only once)
• Radian = 2·09, Gradian = 133·33
• Correct answer without work

Question 3 (25 marks)

Question 3 (a)

$\text{Centre} = (h, k) = (2, -3),\ r = 4$

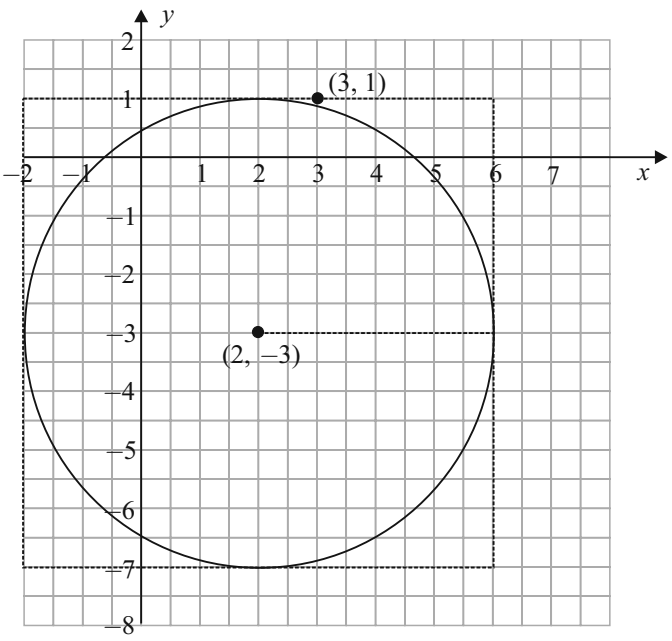
$c : (x-2)^2 + (y-(-3))^2 = 4^2$

$(x-2)^2 + (y+3)^2 = 16$

Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r
$(x-h)^2 + (y-k)^2 = r^2$

Marking Scheme Notes
Question 3 (a) [Scale 5C (0, 1, 2, 5)]
1: • Identifies correct formula
• Labels any one of (2, –3) or 4 correctly
• Works to $r^2 = 4^2$
2: • Answer as $(x - 2)^2 + (y - 3)^2 = 4^2$
• Substitutes centre or radius correctly into formula

Question 3 (b)



Marking Scheme Notes
Question 3 (b) [Scale 5C (0, 1, 2, 5)]
1: • Any work of merit e.g. Marks centre on grid
2: • Any TWO of the three components, centre, radius or drawing correct

Question 3 (c)

$(3, 1) : ((3) - 2)^2 + ((1) + 3)^2$

$= 1^2 + 4^2 = 1 + 16$

$= 17 > 16$

Therefore, (3, 1) is outside the circle.

Marking Scheme Notes
Question 3 (c) [Scale 10C (0, 2, 4, 10)]
2: • Plots (3,1)
• Identifies correct relevant formula
• Substitutes (3, 1) into correct/consistent formula
4: • (1 + 16) or 17 but no conclusion. (> 16)
• $\sqrt{17}$ but no conclusion. (> 4)

Question 3 (d)

The smallest four-sided figure that fits around the circle is a square of side 8 cm
Area = 8 cm × 8 cm = 64 cm^2

Marking Scheme Notes
Question 3 (d) [Scale 5C (0, 1, 2, 5)]
1: • Drawing on grid. (Any one dimension)
2: • Identifies correct/consistent square
• Scale error in dimensions calculated correctly

Question 4 (25 marks)
Question 4 (a)

$A(4, 5) = (x_1, y_1)$, $B(2, 0) = (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-5}{2-4} = \frac{-5}{-2} = \frac{5}{2}$$

$(x_1, y_1) = (2, 0)$, $m = \frac{5}{2}$

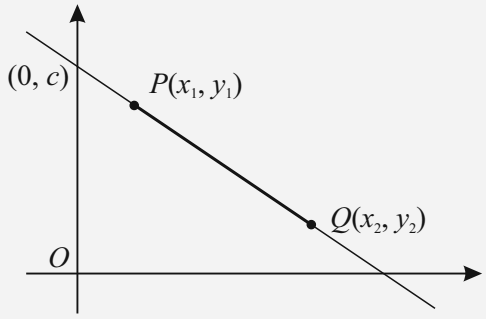
$y - 0 = \frac{5}{2}(x - 2)$

$2y = 5x - 10$

$l : 5x - 2y - 10 = 0$

Marking Scheme Notes
Question 4 (a) [Scale 15D (0, 2, 4, 8, 15)]
2: • Identifies a correct relevant formula
4: • Slope formula with some substitution
• Equation of line formula with some substitution
• Slope correct and stops
8: • Equation of line formula fully substituted correctly (consistently)
Note: Accept ($-5x + 2y + 10 = 0$, *in any order*)

Formulae and Tables Book
Co-ordinate geometry: Line



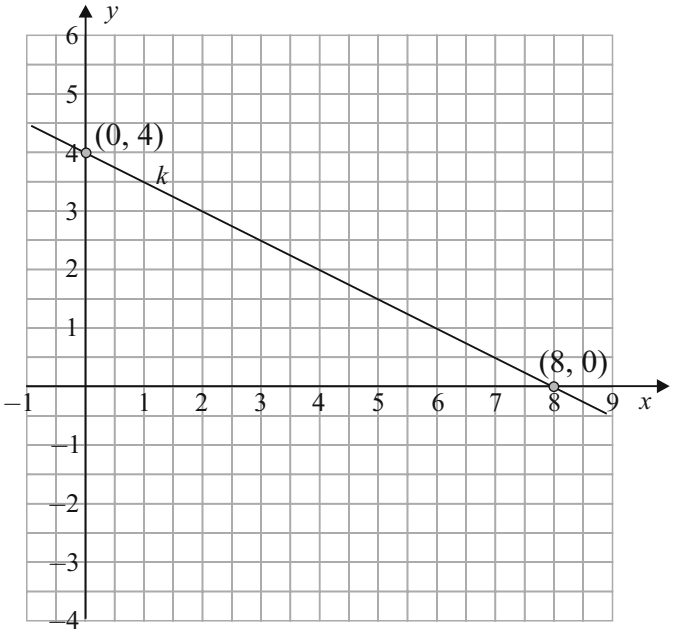
Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$
$$y = mx + c$$

Question 4 (b)



$k : x + 2y = 8$

$x = 0 : (0) + 2y = 8 \Rightarrow y = 4$

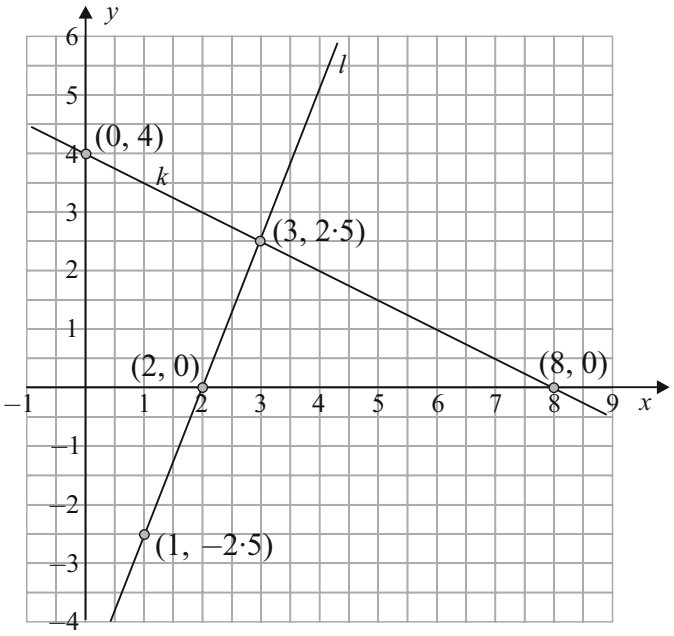
$y = 0 : x + 2(0) = 8 \Rightarrow x = 8$

(8, 0), (0, 4) are points on k.

Marking Scheme Notes
Question 4 (b) [Scale 5C (0, 1, 2, 5)]
1: • $x = 0$ or $y = 0$ written
• Any work of merit working with line k
2: • Finds two correct points but no plots or line on graph

Question 4 (c)

Graphical



$l : 5x - 2y - 10 = 0$

$x = 1 : 5(1) - 2y - 10 = 0$

$-2y = 5 \Rightarrow y = -\frac{5}{2}$

$y = 0 : 5x - 2(0) - 10 = 0 \Rightarrow x = 2$

$(1, -\frac{5}{2})$, $(2, 0)$ are points on l.

From graph: $l \cap k = (3, 2\frac{1}{2})$

Marking Scheme Notes
Question 4 (c) [Scale 5C (0, 1, 2, 5)]

1: • Effort to draw line l

2: • l drawn correctly on grid (in this part) and point of intersection marked but not written

5: • $l \cap k = (3, 2\frac{1}{2})$

Algebraic

$l : 5x - 2y = 10 ...(\mathbf{1})$

$k : x + 2y = 8 .....(\mathbf{2})$

$6x \quad = 18 \Rightarrow x = 3$

Into $(\mathbf{2}) : (3) + 2y = 8 \Rightarrow 2y = 5 \Rightarrow y = \frac{5}{2}$

$l \cap k = (3, \frac{5}{2})$

Marking Scheme Notes
Question 4 (c) [Scale 5C (0, 1, 2, 5)]

1: • Effort to solve using simultaneous equations

2: • x and/or y calculated correctly
• Writes $l \cap k$ as (y, x)

5: • $l \cap k = (3, 2\frac{1}{2})$

Question 5 (25 marks)

Question 5 (a)

Week 2		Week 1
	0	5 8
	1	**2 2 2** 3
	2	0 **4 5** 7
	3	0 1 2 4 4
	4	4

Key: 1|3 = 13 minutes

Mode = 12 minutes (this data value occurs three times)

$$\text{Median} = \frac{24+25}{2} = 24 \cdot 5 \text{ minutes}$$

Marking Scheme Notes
Question 5 (a) [Scale 15C (0, 3, 5, 15)]
3: • Any work of merit in finding Mode or Median
5: • Mode or Median correct
15: • Both answers correct without work

Question 5 (b)

Sum of the data values in week 1
= 5 + 8 + 12 + 12 + 12 + 13 + 20 + 24 + 25 + 27 + 30 + 31 + 32 + 34 + 34 + 44 = 363

$$\text{Mean} = \frac{\text{Sum of data values}}{\text{Number of patients}} = \frac{363}{16} = 22 \cdot 7 \text{ minutes}$$

Marking Scheme Notes
Question 5 (b) [Scale 5C (0, 1, 2, 5)]
1: • Indicates addition of values.(5 + 8 ….
2: • Sum correct (53) or (63) or $\frac{x}{16}, \frac{x}{21}$
• Count correct (16), (21)
5: • Correct answer without work

Formulae and Tables Book
Statistics and probability:
Mean from a list of n numbers
[page 33]

$$\mu = \frac{\text{Sum of all the numbers}}{\text{Number of numbers}} = \frac{\sum x}{n}$$

Question 5 (c)

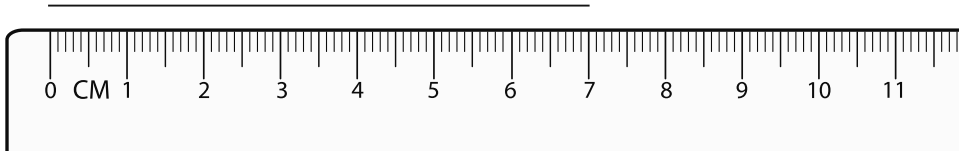
Week 2									Week 1					
						9	6	0	5	8				
8	8	7	6	6	5	5	2	1	**2**	**2**	**2**	3		
				9	8	7	3	2	0	**4**	**5**	7		
							2	3	0	1	2	4	4	
							0	4	4					

Marking Scheme Notes
Question 5 (c) [Scale 5C (0, 1, 2, 5)]
1: • Any effort at ordering results
• Any correct entry
2: • Nine correct entries

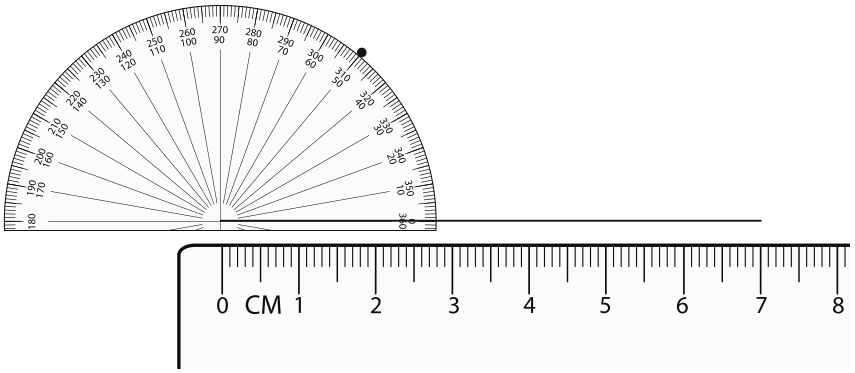
Question 6 (25 marks)

Question 6 (a) (i)

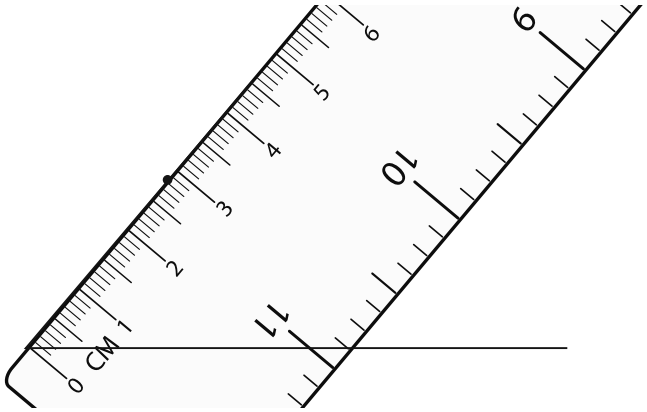
Step 1: Using a ruler draw a line [*AB*] of length 7 cm.



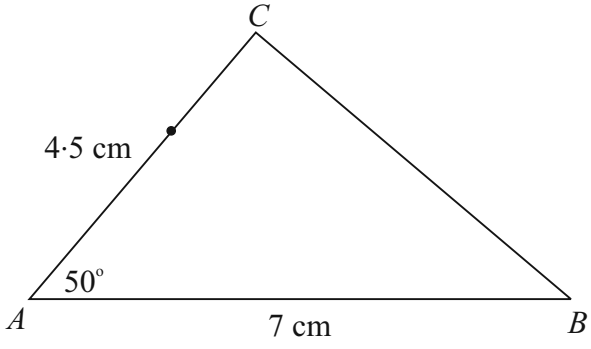
Step 2: Place a protractor at one end of the line segment and mark a 50° angle.



Step 3: Place the ruler along the line joining the end of the line and the point at the 50° angle and draw a line segment [*AC*] of length 4·5 cm.

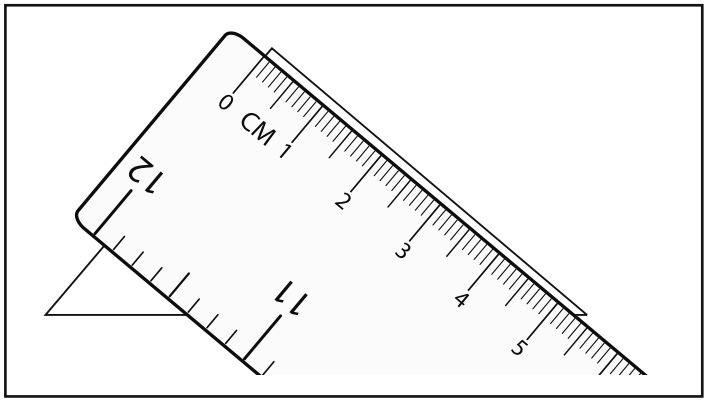


Step 4: Use your ruler to draw a line joining *C* to *B*.



Question 6 (a) (ii)

$|AC| = 4{\cdot}5$ cm
$|BC| = 5{\cdot}4$ cm
Sum $= |AC| + |BC|$
$= 4{\cdot}5$ cm $+ 5{\cdot}4$ cm
$= 9{\cdot}9$ cm



Marking Scheme Notes
Question 6 (a) (i) & (ii) [Scale 5D (0, 2, 3, 4, 5)]
2: • One correct line or angle drawn
• Any triangle drawn (Pilot Diagram)
3: • Two correct elements drawn & stops
4: • Construction correct
• Two correct elements and finished correctly/consistently
• Tolerance : ±0·25 cm, ± 2·5°

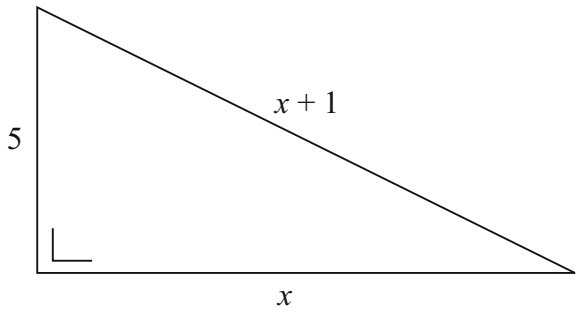
Question 6 (b)

Triangle 1	Triangle 2
Sides of length (cm) 3·2, 2·9, 5·4	Sides of length (cm) 6, 7, 15

Triangle 2 cannot be constructed as $6 + 7 = 13 < 15$

Marking Scheme Notes
Question 6 (b) [Scale 5B (0, 1, 5)]
0: • Nominates Triangle 1
1: • Triangle correct but no reason
• Reason correct but no Triangle
• Any work of merit leading towards correct triangle or reason

Question 6 (c)



Marking Scheme Notes
Question 6 (c) [Scale 15C (0, 3, 5, 15)]
3: • Pythagoras with some substitution
• Trial and error to try and find $x = 12$
5: • Pythagoras fully substituted correctly
15: • $13^2 = 12^2 + 5^2$
• (5, 12, 13) identified

$$(x+1)^2 = x^2 + 5^2$$
$$x^2 + 2x + 1 = x^2 + 25$$
$$2x = 25 - 1 = 24$$
$$\therefore x = 12$$

Question 7 (55 marks)

Question 7 (a)

Number in Scientific notation:
$a \times 10^n,\ n \in \mathbb{Z},\ 1 \leq a < 10$

165 billion
= 165 000 000 000
$= 1{\cdot}65 \times 10^{11}$

Question 7 (b)

(i) Diameter (d) = 2 × Radius (r)
$d = 18$ m $\Rightarrow r = 9$ m

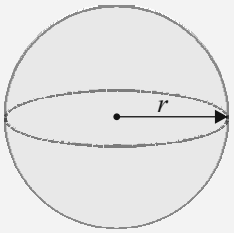
(ii) Volume $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3$
$= 972\pi = 3053{\cdot}63$ m^3

Question 7 (c)

Combined surface area $= 9 \times 4\pi(9)^2$
$= 2916\pi$
$= 9161$ m^2

Marking Scheme Notes
Question 7 (a) [Scale 15B (0, 4, 15)]
4: Accept any one of the following:
- $16{\cdot}5 \times 10^{10}$
- 165×10^9
- 10^9
- 10^6
- 10^3
- 165 000 000 000
- 1·65

Formulae and Tables Book
Surface area and volume:
Sphere [page 8]



$A = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Marking Scheme Notes
Question 7 (b) (i) [Scale 5B (0, 1, 5)]
1: • Writes answer as $\frac{18}{2}$ or similar

Question 7 (b) (ii) [Scale 5C (0, 1, 2, 5)]
1: • Identifies correct volume formula
• Writes answer from part **(b) (i)** in this section
2: • Formula fully substituted correctly (consistently)
• One error in substitution followed by correct calculation
• Answer as 972π
Note: π = 3·14, (3052·08 m^3); π = $\frac{22}{7}$, (3054·86 m^3)

Question 7 (c) [Scale 5C (0, 1, 2, 5)]
1: • Identifies correct formula
• Identifies radius correctly or consistently (in this part)
• Indicates multiplication by 9
2: • Expression fully substituted
• S.A. of one sphere correctly calculated.(1018 m^2)
Note: π = 3·14, (9156 m^2); π = $\frac{22}{7}$, (9165 m^2)

Question 7 (d)

(i) $r = 1{\cdot}65$ m, $h = 23$ m

Combined surface areas $= 8 \times 2\pi(1{\cdot}65)(23)$

$= 607{\cdot}2\pi$

$= 1908$ m^2

(ii) $r = 1{\cdot}45$ m, $h = ?$

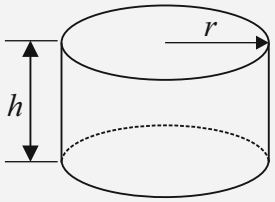
Combined surface areas $= 3170$ m^2

$12 \times 2\pi(1{\cdot}45)h = 3170$

$34{\cdot}8\pi h = 3170$

$\therefore h = \dfrac{3170}{34{\cdot}8\pi} = 29$ m

Formulae and Tables Book
Surface area and volume:
Cylinder [page 10]



$A = 2\pi rh$

$V = \pi r^2 h$

(iii) Total surface area:
9 spheres + 8 shorter cylindrical pipes + 12 longer cylindrical pipes
= 9161 + 1908 + 3170
= 14 239 m^2
Cost of painting the Atomium at €70 per square metre = 14 239 × 70 = €996 730

Marking Scheme Notes
Question 7 (d) (i) [Scale 5C (0, 1, 2, 5)]
1: • Identifies correct formula
• Identifies $r = 1{\cdot}65$ *or* $h = 23$
• Indicates multiplication by 8
2: • Expression fully substituted
• Area of one pipe correct
Note: $\pi = 3{\cdot}14$, (1907 m^2); $\pi = \frac{22}{7}$, (1908 m^2)

Question 7 (d) (ii) [Scale 15C (0, 3, 5, 15)]
3: • ÷ 12 indicated.
• Identifies correct formula.
• Indicates $r = 1{\cdot}45$.
5: • Equation fully substituted.

Question 7 (d) (ii) [Scale 5A (0, 5)]

EPL 104

Question 8 (50 marks)

Activity	Percentage used	Angle (Degrees)
WC flushing	30%	
Personal Washing – Baths and Taps	21%	
Personal Washing – Showers	12%	
Clothes Washing	13%	$46{\cdot}8^{o}$
Other		

Question 8 (a)

Other Activity = 100% – 30% – 21% – 12% – 13% = 24%

Marking Scheme Notes
Question 8 (a) [Scale 5B (0, 1, 5)]
1: • 100% − (any relevant percentage)
• Indicates addition of given percentages

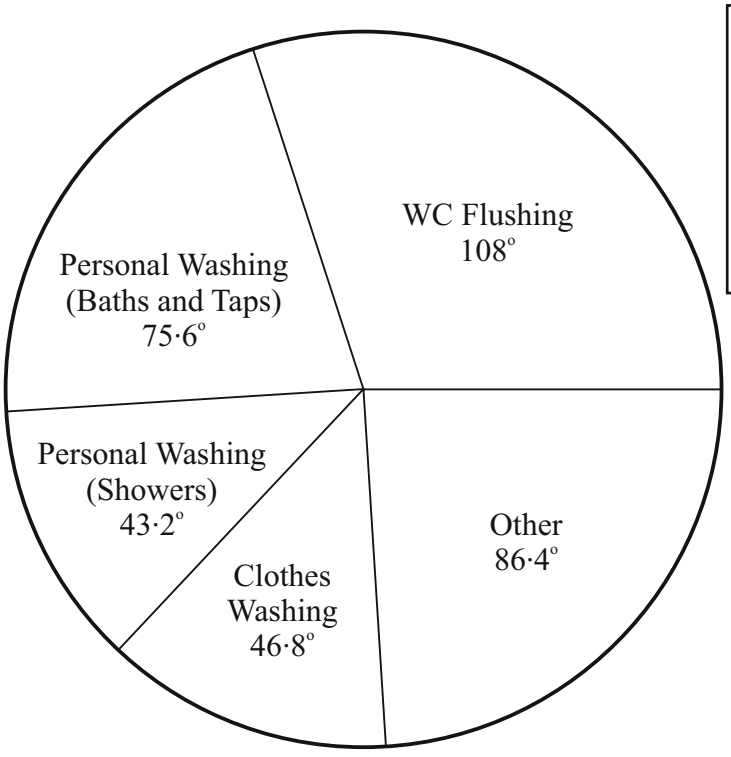
Question 8 (b) (i)

100% → 360^{o} ⇒ 1% = $3{\cdot}6^{o}$
WC flushing: 30% → $3{\cdot}6^{o} \times 30 = 108^{o}$
Personal Washing – Baths and Taps: 21% → $3{\cdot}6^{o} \times 21 = 75{\cdot}6^{o}$
Personal Washing – Showers: 12% → $3{\cdot}6^{o} \times 12 = 43{\cdot}2^{o}$
Other: 24% → $3{\cdot}6^{o} \times 24 = 86{\cdot}4^{o}$

Activity	Percentage used	Angle (Degrees)
WC flushing	30%	108^{o}
Personal Washing – Baths and Taps	21%	$75{\cdot}6^{o}$
Personal Washing – Showers	12%	$43{\cdot}2^{o}$
Clothes Washing	13%	$46{\cdot}8^{o}$
Other	24%	$86{\cdot}4^{o}$

Marking Scheme Notes
Question 8 (b) (i) [Scale 5C (0, 1, 2, 5)]
1: • Works with 3·6 or similar
• One correct angle
• Three incorrect but consistent angles
• Any use of 360°
2: • Three correct angles
• All incorrect consistent angles
• Correct answer without work

Question 8 (b) (ii)



Marking Scheme Notes
Question 8 (b) (ii) [Scale 5C (0, 1, 2, 5)]
1: • Circle with a sector(s) drawn
2: • Any 3 sectors drawn correctly, i.e. with angle and label indicated
• All sectors drawn correctly but either angle or label omitted

Question 8 (c)

		John's weekly household water usage	
Activity	**Water Required per activity**	**Frequency**	**Number of litres used**
One Bath	80 litres	3 baths	240
One Shower	125 litres	20 showers	2500
Brushing Teeth with Tap Running	6 litres per minute	32 minutes	192
One WC Flush	6 litres	60 flushes	360
One Use of Washing Machine	45 litres	8 uses	360
One Use of Dishwasher	20 litres	7 uses	140
Washing One Car with a Bucket	10 litres	1 wash	10
Hosepipe	9 litres per minute	15 minutes	135
		Total number of litres used	3937

(i) Total number of litres used = 3937

(ii) Total yearly cost = 3·937 × 1·85 × 52 = €378·74

(iii) Total yearly cost including VAT = €378·74 × 1·135 = €429·87

Marking Scheme Notes

Question 8 (c) (i) [Scale 5C (0, 1, 2, 5)]

1: • Indicates one correct water calculation

2: • Four correct water calculations

Question 8 (c) (ii) [Scale 5C (0, 1, 2, 5)]

1: • Any use of answer to **(c) (i)**
• Any ONE of tasks: × by 52, × 1·85, ÷ 1000 correct

2: • Any TWO of tasks: × by 52, × 1·85, ÷ 1000 correct

Question 8 (c) (iii) [Scale 5A (0, 5)]

Question 8 (d)

Desired yearly cost = €260 = $x \times 1{\cdot}85 \times 1{\cdot}135$

$$x = \frac{260}{1{\cdot}85 \times 1{\cdot}135} = 123{\cdot}824$$

Number of litres $= 123{\cdot}824 \times 1000 = 123\ 824$

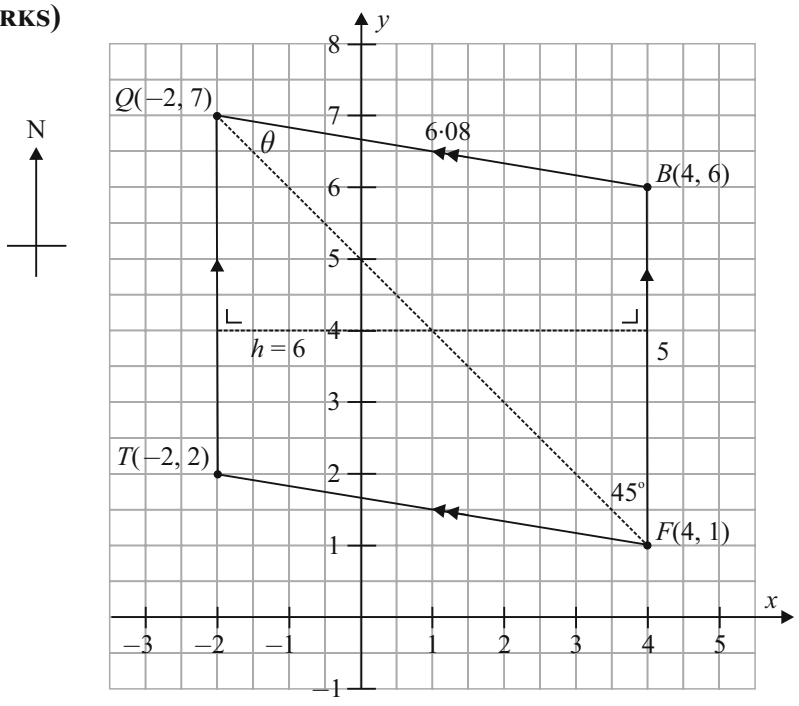
Marking Scheme Notes

Question 8 (d) [Scale 20C (0, 5, 10, 20)]

5: • Any ONE of tasks: × by 52, × 1·85, ÷ 1000 correct

10: • Any TWO of tasks: × by 52, × 1·85, ÷ 1000 correct
• Formulates correctly with both divisions (may be done separately)
• 123·824 or equivalent and stops

QUESTION 9 (45 MARKS)



Question 9 (a)

(i) $F = (4, 1)$

(ii) $B = (4, 6)$

MARKING SCHEME NOTES

Question 9 (a) (i) [Scale 5B (0, 1, 5)]

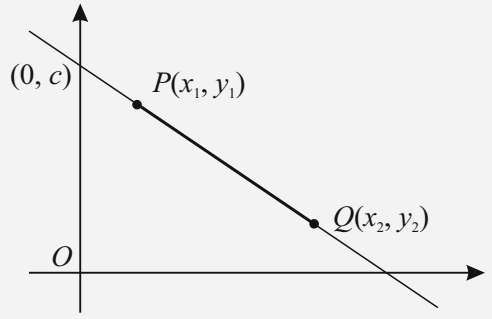
1: • One ordinate correct
• Answer as (1, 4)

Question 9 (a) (ii) [Scale 5B (0, 1, 5)]

0: • All other incorrect points with no labels

1: • Point correct (No label)
• Point plotted as (B), but not in the correct position
• Plot and label as (4, 6)
• Any plot North of F

FORMULAE AND TABLES BOOK
Co-ordinate geometry: Line



Length of PQ [page 18]

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Question 9 (b)

$B(4, 6) = (x_1, y_1)$, $Q(-2, 7) = (x_2, y_2)$

$$|BQ| = \sqrt{(-2-4)^2 + (7-6)^2}$$
$$= \sqrt{(-6)^2 + (1)^2}$$
$$= \sqrt{36+1}$$
$$= \sqrt{37} = 6 \cdot 08$$

MARKING SCHEME NOTES

Question 9 (b) [Scale 5C (0, 1, 2, 5)]

1: • Writes co-ordinates of B and/or Q
• Any work of merit

2: • Fully correct/consistent substitution without calculation
• One incorrect substitution with correct calculation
• One consistent error throughout

Question 9 (c)

$B(4, 6) \rightarrow Q(-2,\ 7)$ [Add -6, Add 1]
$F(4, 1) \rightarrow T(4 - 6, 1 + 1) = T(-2, 2)$

Marking Scheme Notes
Question 9 (c) [Scale 5B (0, 1, 5)]
1: • Any work of merit (Join F to B)
• One ordinate correct
• Answer as $(2, -2)$

Question 9 (d)

Area of $FBQT = 5 \times 6 = 30$

Marking Scheme Notes
Question 9 (d) [Scale 5D (0, 2, 3, 4, 5)]
2: • Correct relevant formula identified
• Triangle co-ordinates identified correctly
• Triangle translated correctly
• Works with 5, 6, or 6·08
3: • Triangle formula fully substituted
• One error in substitution followed by correct calculation of triangle
4: • Area of triangle calculated correctly
• One error in substitution followed by correct calculation of parallelogram
• Parallelogram formula fully substituted
• A correct answer without work

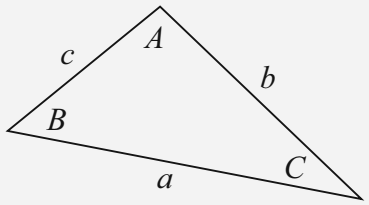
Question 9 (e)

$$\frac{\sin\theta}{5} = \frac{\sin 45^\circ}{6\cdot 08}$$

$$\sin\theta = \frac{5\sin 45^\circ}{6\cdot 08}$$

$$\theta = \sin^{-1}\left(\frac{5\sin 45^\circ}{6\cdot 08}\right) = 35\cdot 6^\circ$$

Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Marking Scheme Notes
Question 9 (e) [Scale 20C (0, 5, 10, 20)]
0: • Treats as right-angled or isosceles triangle
5: • Correct relevant formula
• Work of merit on diagram
10: • Sine rule fully substituted
• $|\angle BQF|$calculated with one incorrect substitution
• $|\angle QBF| = 99\cdot 47^\circ$
• Incorrect calculator mode (once only): Radian = 0·8, Gradian = 35·9
• Correct answer without work

LC 2015 (Set D): Paper 1

Question 1 (25 marks)

Question 1 (a)

€1 = $1·29 ← Each euro gives you $1·29.

€2000 = $1·29 × 2000 = $2580 ← €2000 gives you $2580.

Question 1 (b)

$21\,000 = €15\,000$ ← $21 000 is worth €15 000.

$\$\frac{21\,000}{15\,000} = €1$ ← Divide both sides by 15 000 to find how many dollars you get for €1.

$\therefore €1 = \$1{\cdot}40$ ← You get $1·40 for your €1.

Question 1 (c)

$€1 = \$1{\cdot}24$ ← You get $1·24 for your €1.

$€1 = £0{\cdot}83$ ← You get £0·83 for your €1.

$\$1{\cdot}24 = £0{\cdot}83$ ← Therefore, $1·24 is worth £0·83.

$\$1 = \frac{£0{\cdot}83}{1{\cdot}24} = £0{\cdot}67$ ← Divide both sides by 1·24 to find out how much sterling you get for $1.

Marking Scheme Notes

Question 1 (a) [Scale 10B (0, 3, 10)]

3: • Incorrect use of 1·29

Question 1 (b) [Scale 10B (0, 3, 10)]

3: • Equates
• Indicates division

Question 1 (c) [Scale 5C (0, 3, 4, 5)]

3: • Work on Dollar Euro rate or Sterling Euro rate

4: • Correct Dollar Euro rate or Sterling Euro rate

Question 2 (25 marks)

Question 2 (a)

John paid €25, Mary paid €15 and Eileen paid €10 for each part of the ticket.
25:15:10 is the ratio of how they will divide out the €20 000 prize money.
25:15:10 ← Divide across by 5.

5:3:2 ← Add the numbers to get 10 (5 + 3 + 2). This means John gets $\frac{5}{10}$ of the prize money, Mary gets $\frac{3}{10}$ of the prize money and Eileen gets $\frac{2}{10}$ of the money.

John: $\frac{5}{10} \times €20\,000 = €10\,000$

Mary: $\frac{3}{10} \times €20\,000 = €6\,000$

Eileen: $\frac{2}{10} \times €20\,000 = €4\,000$

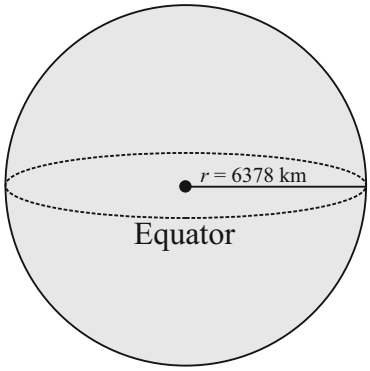
Question 2 (b) (i)

The equator is a circle that passes around the Earth which is a sphere. You need to find the length l of this circle.

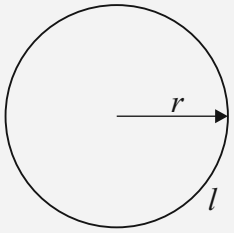
$l = 2\pi r$

$= 2\pi(6378)$

$\approx 40\ 074$ km



Formulae and Tables Book
Length and area: Circle [page 8]



$l = 2\pi r$

$A = \pi r^2$

Question 2 (b) (ii)

Scientific Notation

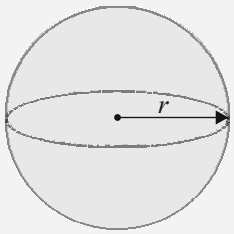
$a \times 10^n,\ 1 \le a < 10,\ n \in \mathbb{Z}$

Ex. $2 \cdot 3 \times 10^{-5}$

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6378)^3$

$= 1 \cdot 087 \times 10^{12}$ km^3 ← Your calculator turns this into scientific notation automatically.

Formulae and Tables Book
Surface area and volume: Sphere [page 8]



$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Question 2 (c)

$$\frac{\text{Mass of Sun}}{\text{Mass of Earth}} = \frac{1 \cdot 99 \times 10^{30}\ \text{kg}}{5 \cdot 97 \times 10^{24}\ \text{kg}} \approx 333\ 333$$

Using your calculator
Press the **fraction button**.
Input 1.99.
Press **×10^x** button followed by 30.
Scroll down to bottom of the fraction.
Input $5 \cdot 97 \times 10^{24}$
Press =

Marking Scheme Notes

Question 2 (a) [Scale 15B (0, 3, 15)]

3: • Any correct ratio established or one correct amount

Question 2 (b) (i) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Correct formula or work of relevance in either part
• Finds diameter (12 756 km)

4: • One part correct
• Substitutes correctly into both formulae

Question 2 (c) [Scale 5B (0, 3, 5)]

3: • Formulates division

QUESTION 3 (25 MARKS)

Question 3 (a)

$3(4-5x)-2(5-6x)$ ← Multiply out brackets.

$=12-15x-10+12x$ ← Add like terms.

$=2-3x$

Question 3 (b)

$2-3x \geq -6,\ n \in \mathbb{N}$ ← Bring numbers to the left and x's to the right.

$2+6 \geq 3x$

$8 \geq 3x$ ← Divide both sides by 3.

$\frac{8}{3} \geq x$

$\therefore x \leq \frac{8}{3}$

$\therefore x = 1,\ 2$ ← The natural numbers less than or equal to $\frac{8}{3}$ are 1 and 2.

Question 3 (c)

$(2-3x) \times g(x) = 15x^2 - 22x + 8$ ← Isolate $g(x)$ by dividing both sides by $(2 - 3x)$.

$g(x) = \dfrac{15x^2 - 22x + 8}{(2-3x)}$ ← Factorise the quadratic.

$= \dfrac{(5x-4)(3x-2)}{(2-3x)}$ ← Take a minus sign out of $(3x - 2)$ so the bracket is the same as on the bottom.

$= \dfrac{-(5x-4)\cancel{(2-3x)}}{\cancel{(2-3x)}}$ ← Cancel brackets that are the same.

$= -(5x-4)$ ← The negative sign changes the signs of each term inside the bracket.

$= 4-5x$

MARKING SCHEME NOTES

Question 3 (a) [Scale 15B (0, 3, 15)]

3: • Some correct attempt at simplifying

Question 3 (b) [Scale 5C (0, 3, 4, 5)]

Note: Accept {0, 1, 2}

3: • Any correct transposition

4: • Not more than two errors in solving

• $x \leq \frac{8}{3}$

Question 3 (c) [Scale 5C (0, 3, 4, 5)]

3: • Formulate division

• Attempt at factorising

4: • One full correct step in division

• $(3x - 2)(5x - 4)$ and stops

Question 4 (25 marks)

Question 4 (a)

$-x^2 + 6x - 4 = 0$ ← Multiply across by −1.

$\therefore x^2 - 6x + 4 = 0$ ← Write the values of a, b and c.

$a = 1,\ b = -6,\ c = 4$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{36 - 16}}{2}$$

$$= \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$$

$$= \frac{6 - 2\sqrt{5}}{2},\ \frac{6 + 2\sqrt{5}}{2}$$ ← Input each fraction into your calculator.

$\approx 0{\cdot}8,\ 5{\cdot}2$

Formulae and Tables Book
Algebra: Roots of the quadratic equation
$ax^2 + bx + c = 0$ [page 20]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Marking Scheme Notes
Question 4 (a) [Scale 15C (0, 3, 10, 15)]
3: • Correct quadratic formula written
• Attempt at factorising
10: • Correct substitution into quadratic formula
• Finds one correct solution
• Solutions found with not more than 3 errors
Note: $x = 3 + \sqrt{5},\ x = 3 - \sqrt{5}$ merits 14 marks

Question 4 (b)

$f(x) = -x^2 + 6x - 4 = 0$

$f'(x) = -2x + 6$

$f'(x) = 0 \Rightarrow -2x + 6 = 0$

$6 = 2x$

$\therefore x = 3$ ← Put this value into the original function.

$f(3) = -(3)^2 + 6(3) - 4 = -9 + 18 - 4 = 5$

Turning point (3, 5)

Find turning points (local maxiumum/minimum)
Put $\frac{dy}{dx} = 0$ and solve for x

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Marking Scheme Notes
Question 4 (b) [Scale 5C (0, 3, 4, 5)]
3: • Differentiates at least one term correctly
• States $f'(x) = 0$ or writes $f'(x)$ or writes $\frac{dy}{dx}$
• Correct answer without work
• Some work towards a graphical solution
4: • Finds correct x value
• Correct structure to $f'(x) = 0$, but with errors and finishes correctly
Note: Accept correct turning point found graphically for full marks

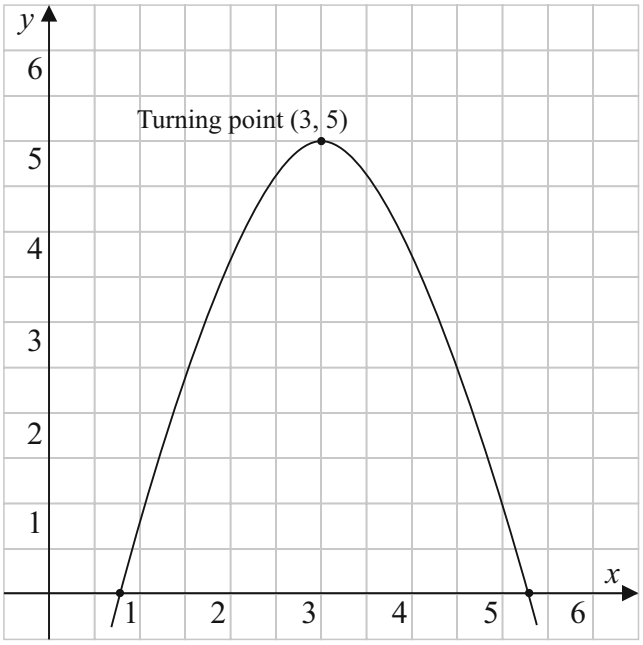
Question 4 (c)

$f(x) = -x^2 + 6x - 4$

The number in front of x^2 is negative. This means the graph is shaped as shown.

You know the 2 places where $f(x)$ cuts the x-axis: $x = 0{\cdot}8$ and $x = 5{\cdot}2$.

The turning point is (3, 5).



Marking Scheme Notes

Question 4 (c) [Scale 5B (0, 3, 5)]

3: • Any correct information shown on graph, e.g. indicates scale
• General shape of curve drawn

Question 5 (25 marks)

Question 5 (a)

$A(1, k) \in g(x) = x + 3$

$\therefore g(1) = k$

$g(1) = (1) + 3 = 4 = k$

$\therefore A(1, k) = A(1, 4)$

$f(x) = 5x - x^2$
$g(x) = x + 3$ ← Put $f(x) = g(x)$.

$f(x) = g(x) \Rightarrow 5x - x^2 = x + 3$ ← Bring all terms to the left-hand side.

$-x^2 + 4x - 3 = 0$ ← Multiply each term by -1.

$x^2 - 4x + 3 = 0$ ← Factorise the quadratic.

$(x-1)(x-3) = 0$ ← Put each bracket equal to zero and solve for x.

$\therefore x = 1, 3$

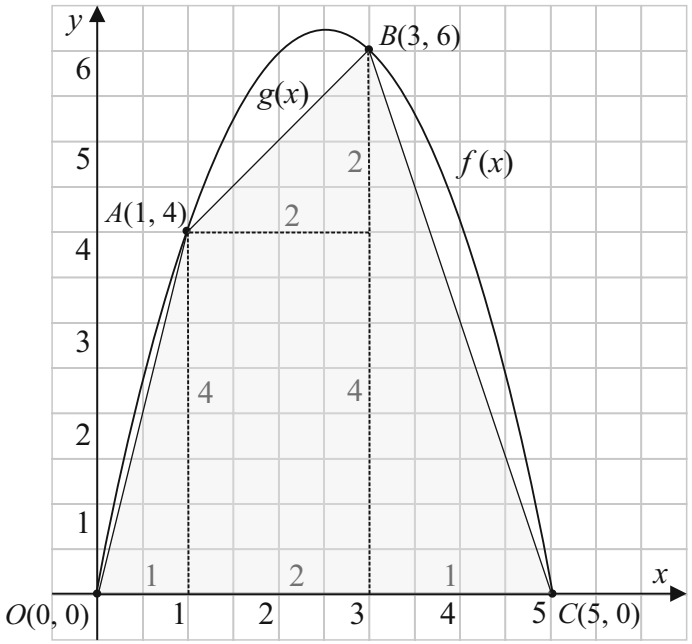
$g(1) = 1 + 3 = 4 \Rightarrow A(1, 4)$

$g(3) = 3 + 3 = 6 \Rightarrow B(3, 6)$

Marking Scheme Notes
Question 5 (a) [Scale 15C (0, 3, 10, 15)]
3: • $g(x) = f(x)$
• Any work towards a graphical solution
• Finds A and stops
• Finds A and B without work
10: • Solve for x values
• Show points on the graph
• Not more than one error working out points algebraically

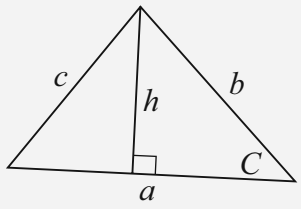
Question 5 (b) (i)

Plot the points $O(0, 0)$, $A(1, 4)$, $B(3, 6)$ and $C(5, 0)$.
Join these 4 points with straight lines to form a **quadrilateral**.



Question 5 (b) (ii)

Formulae and Tables Book: **Length and area** (page 8)
Triangle



$A = \frac{1}{2}ah$
$A = \frac{1}{2}ab\sin C$

Break up the quadrilateral into triangles and rectangles and add up their areas.

$$A = \tfrac{1}{2}(1)(4) + (2)(4) + \tfrac{1}{2}(2)(2) + \tfrac{1}{2}(2)(6)$$
$$= 2 + 8 + 2 + 6$$
$$= 18$$

Marking Scheme Notes
Question 5 (b) (i) (ii) [Scale 10D (0, 2, 5, 9, 10)]
2: • One or more points plotted correctly
• Any incorrect quadrilateral drawn
• Any relevant formula written
5: • Correct quadrilateral drawn
• Correct area of any section, without a quadrilateral established (e.g. OAC)
9: • Correct area of any section, provided quadrilateral established
• Area formula established, with correct substitutions, for total area

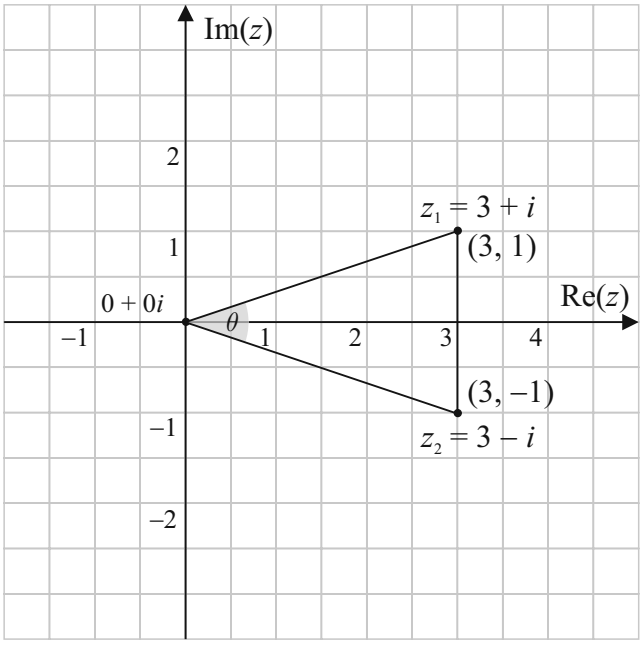
Question 6 (25 marks)

Question 6 (a)

Complex numbers
$a + bi = c + di$
Equate the real parts: $a = c$
Equate the imaginary parts: $b = d$

(i) $z_1 = 3 + i = a + bi \Rightarrow a = 3,\ b = 1$

(ii) $z_2 = 3 - i = c + di \Rightarrow c = 3,\ d = -1$



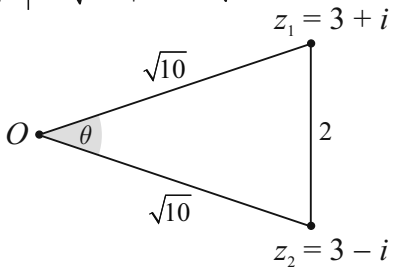
Marking Scheme Notes
Question 6 (a) [Scale 15D (0, 3, 7, 11, 15)]
3: • One correct value
• Plots z_2 correctly and stops
7: • Two correct values
11: • Three correct values

Question 6 (b) (i)

Find the modulus of each complex number. This is the distance from the origin (0, 0) to each complex number.

$|z_1| = |3 + i| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$|z_2| = |3 + i| = \sqrt{3^2 + 1^2} = \sqrt{10}$



Apply the Cosine rule as you are given 3 sides in the triangle.

$2^2 = (\sqrt{10})^2 + (\sqrt{10})^2 - 2(\sqrt{10})(\sqrt{10})\cos\theta$

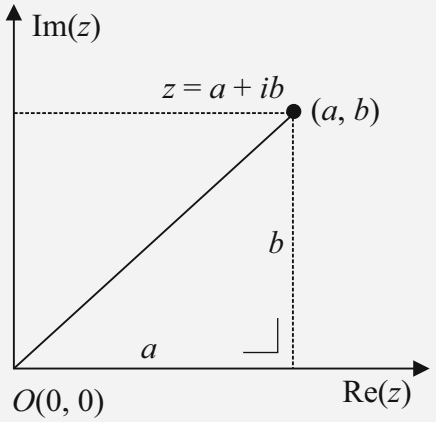
$4 = 10 + 10 - 20\cos\theta$

$4 = 20 - 20\cos\theta$

$20\cos\theta = 16$

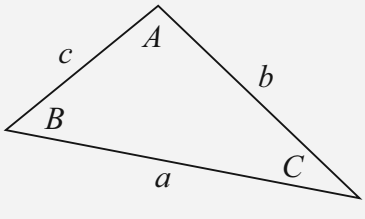
$\therefore \cos\theta = \frac{16}{20} = \frac{4}{5} = 0{\cdot}8$

Formula: Complex Numbers
Modulus $|z|$



$|z| = |a + bi| = \sqrt{a^2 + b^2}$

Formulae and Tables Book
Trigonometry of the triangle:
Cosine rule [page 16]



$a^2 = b^2 + c^2 - 2bc\cos A$

EPL 104

Marking Scheme Notes
Question 6 (b) (i) [Scale 5B (0, 3, 5)]
3: • Cosine formula written
• Finds $|z_1|$
• Any work of merit
• Joins oz_1 or oz_2 or z_1z_2

Question 6 (b) (ii)

Show the left-hand side (**LHS**) is equal to the right-hand side (**RHS**).

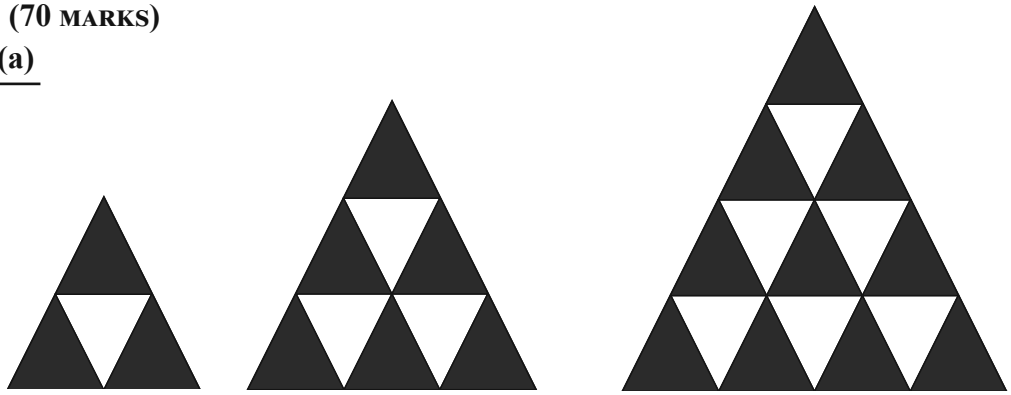
LHS	**RHS**
$\lvert z_1\rvert \times \lvert z_2\rvert \times \cos\theta$	$ac + bd$
$= \sqrt{10}\sqrt{10}(0\cdot 8)$	$= (3)(3) + (1)(-1)$
$= 8$	$= 9 - 1$
	$= 8$

$\therefore \lvert z_1\rvert \times \lvert z_2\rvert \times \cos\theta = ac + bd$

Marking Scheme Notes
Question 6 (b) (ii) [Scale 5C (0, 3, 4, 5)]
3: • $ac + bd$ correct
• Any work of merit
• Finds $|z_1|$ and/or $|z_2|$
7: • one error only in substitution, with conclusion

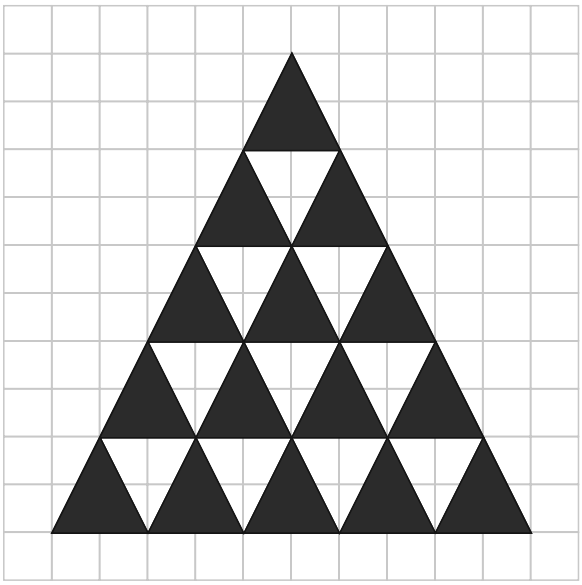
Question 7 (70 marks)

Question 7 (a)



1 2 3

This is the fourth pattern:



Marking Scheme Notes

Question 7 (a) [Scale 15B (0, 3, 15)]

3: • Outline triangle drawn

• Division consistent with pattern, but not 9 triangles in fifth row

Question 7 (b)

	Number of black triangles	Number of white triangles	Total Number of small triangles
Pattern 1	3	1	$T_1 = 4$
Pattern 2	6	3	$T_2 = 9$
Pattern 3	10	6	$T_3 = 16$
Pattern 4	15	10	$T_4 = 25$
Pattern 5	21	15	$T_5 = 36$

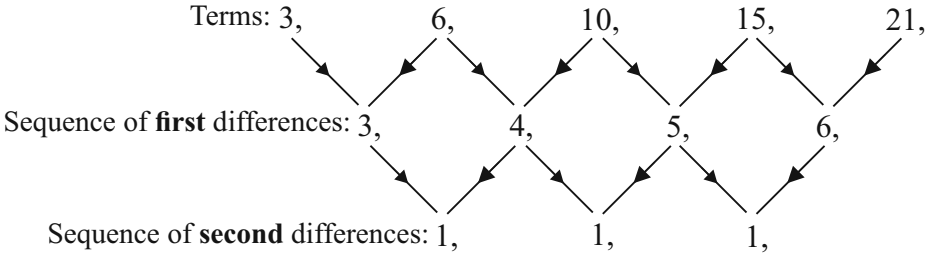
Marking Scheme Notes

Question 7 (b) [Scale 15B (0, 3, 15)]

3: • One correct entry

Question 7 (c)

How do you show that the numbers of black triangles: 3, 6, 10, 15, 21,... forms a quadratic sequence? Subtract the terms to find the sum of first sequences: 3, 4, 5, 6,... Now subtract these terms again to find the sequence of second differences. This sequence 1, 1, 1,..., where all the terms have the same value, shows that the original sequence was quadratic.



Marking Scheme Notes
Question 7 (c) [Scale 5D (0, 2, 3, 4, 5)]
2: • Correct sequence of numbers
3: • First difference established
4: • Second difference established

Question 7 (d)

	Number of black triangles	Number of white triangles	Total Number of small triangles
Pattern 1	3	1	$T_1 = 4$
Pattern 2	6	3	$T_2 = 9$
Pattern 3	10	6	$T_3 = 16$
Pattern 4	15	10	$T_4 = 25$
Pattern 5	21	15	$T_5 = 36$
Pattern 6	28	21	$T_6 = 49$
Pattern 7	36	28	$T_7 = 64$
Pattern 8	45	36	$T_8 = 81$
Pattern 9	55	45	$T_9 = 100$

(i) There are 55 **black** triangles in the 9th pattern.
(ii) There are 45 **white** triangles in the 9th pattern.
(iii) There are 100 **small** triangles, in total, in the 9th pattern.

Marking Scheme Notes
Question 7 (d) (i) (ii) (iii) [Scale 15C (0, 3, 10, 15)]
3: • One correct answer
10: • Two correct answers

Question 7 (e)

Total number of triangles: 4, 9, 16, 25, 36, 49...

$T_1 = (1 + 1)^2 = 2^2 = 4$

$T_2 = (2 + 1)^2 = 3^2 = 9$

$T_3 = (3 + 1)^2 = 4^2 = 16$

$T_4 = (4 + 1)^2 = 5^2 = 25$

$T_5 = (5 + 1)^2 = 6^2 = 36$

$T_6 = (6 + 1)^2 = 7^2 = 49$

Therefore, in general the total number of triangles $T_n = (n + 1)^2$

Marking Scheme Notes
Question 7 (e) [Scale 5B (0, 3, 5)]
3: • n^2 as answer
• Writes $an^2 + bn + c$

Question 7 (f)

B_n is the number of black triangles. The sequence of black triangles is given by: 3, 6, 10, 15,...

$B_n = \frac{1}{2}n^2 + \frac{3}{2}n + c$

$B_1 = 3 \leftarrow B_1$, the first term in the sequence, has a value of 3.

$\therefore \frac{1}{2}(1)^2 + \frac{3}{2}(1) + c = 3 \leftarrow$ Replace n by 1 in the formula to find B_1.

$\frac{1}{2} + \frac{3}{2} + c = 3 \leftarrow$ Multiply each term by 2.

$2 + c = 3$

$\therefore c = 1$

$\therefore B_n = \frac{1}{2}n^2 + \frac{3}{2}n + 1$

Marking Scheme Notes
Question 7 (f) [Scale 5B (0, 3, 5)]
3: • Any relevant substitution into B_n

Question 7 (g)

Total number of tiles T_n = Number of black tiles B_n + Number of white tiles W_n

$T_n = B_n + W_n \Rightarrow W_n = T_n - B_n$

$W_n = (n+1)^2 - (\frac{1}{2}n^2 + \frac{3}{2}n + 1)$

$= n^2 + 2n + 1 - \frac{1}{2}n^2 - \frac{3}{2}n - 1$

$= \frac{1}{2}n^2 + \frac{1}{2}n$

Marking Scheme Notes
Question 7 (g) [Scale 5C (0, 3, 4, 5)]
3: • Formula for the number of black and/or T_n from (e) written down
4: • $W_n + B_n = (n + 1)^2$

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Question 7 (h)

$T_n = (n+1)^2 = 625$ ← Total number of tiles T_n is 625. Solve for n.

$n+1 = \sqrt{625}$ ← Take the square root of both sides.

$n+1 = 25$

$\therefore n = 24$ ← This is the 24th pattern.

$B_{24} = \frac{1}{2}(24)^2 + \frac{3}{2}(24) + 1 = 325$ ← Number of black tiles in the 24th pattern B_{24}

$W_{24} = \frac{1}{2}(24)^2 + \frac{1}{2}(24) = 300$ ← Number of white tiles in the 24th pattern W_{24}

Marking Scheme Notes
Question 7 (h) [Scale 5C (0, 3, 4, 5)]
3: • Black = 625 – White or White = 625 – Black
• Pattern extended
• Correct answer without work
4: • Finds n, or number of black, or number of white

Question 8 (50 marks)

Question 8 (a)

$p = 96x - 0{\cdot}03x^2$

$x = 1000 : p = 96(1000) - 0{\cdot}03(1000)^2 = 66\ 000$

$x = 1500 : p = 96(1500) - 0{\cdot}03(1500)^2 = 76\ 500$

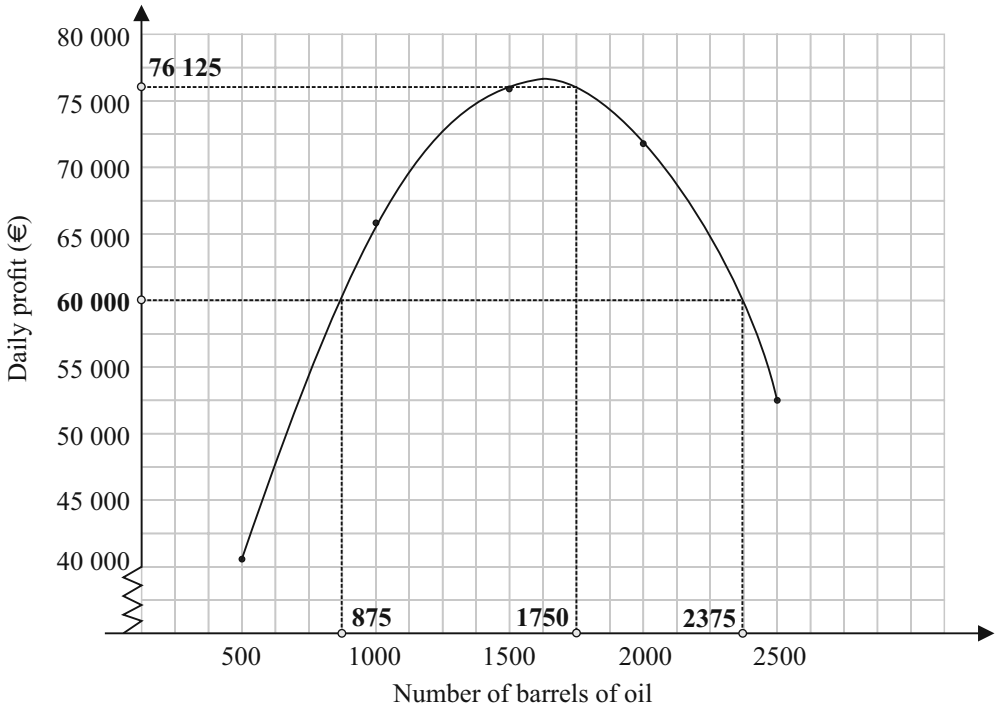
$x = 2000 : p = 96(2000) - 0{\cdot}03(2000)^2 = 72\ 000$

$x = 2500 : p = 96(2500) - 0{\cdot}03(2500)^2 = 52\ 500$

Number of barrels traded in a day	x	500	1000	1500	2000	2500
Daily profit (€)	p	40 500	66 000	76 500	72 000	52 500

Marking Scheme Notes
Question 8 (a) [Scale 15D (0, 3, 7, 11, 15)]
3: • Relevant work towards finding one p value
7: • One correct p
11: • Two correct p values

Question 8 (b)



Marking Scheme Notes
Question 8 (b) [Scale 15D (0, 3, 7, 11, 15)]
3: • One point plotted correctly
• Correct shape of graph
7: • Three or four points plotted incorrectly, with a correct shape
11: • Fails to join the five points or joins with a straight line
• One or two points plotted incorrectly, with a correct shape

Question 8 (c)

(i) The daily profit when 1750 barrels are traded is €76 125.

(ii) The number of barrels traded when the daily profit is €60 000 is 875 or 2375 barrels.

Marking Scheme Notes
Question 8 (c) (i) (ii) [Scale 5C (0, 3, 4, 5)]
3: • Some indication from graph that 1750 or 60 000 or both are used
4: • One correct answer

Question 8 (d) (i)

$p = 96x - 0{\cdot}03x^2$

$\frac{dp}{dx} = 96 - 0{\cdot}06x$

$\frac{dp}{dx} = 0 \Rightarrow 96 - 0{\cdot}06x = 0$

$96 = 0{\cdot}06x$

$\therefore x = \frac{96}{0{\cdot}06} = 1600$

1600 barrels oil traded will yield the maximum profit.

Find turning points (local maxiumum/minimum)
Put $\frac{dy}{dx} = 0$ and solve for x

Marking Scheme Notes
Question 8 (d) (i) [Scale 5C (0, 3, 4, 5)]
3: • Indicates that differentiation is being used
4: • Correct differentiation

Question 8 (d) (ii)

To find the maximum profit $p_{\text{Max.}}$ replace x by 1600 in the profit formula.

$x = 1600 : p_{\text{Max}} = 96(1600) - 0 \cdot 03(1600)^2 = €76\ 800$

Marking Scheme Notes
Question 8 (d) (ii) [Scale 5A (0, 5)]

Question 8 (e)

$p = 0,\ k = ?$ ← Find the value of k, the number of barrels produced, that will give you no profit.

$p = 96x - 0 \cdot 03x^2$ ← Replace x by k and p by 0.

$\therefore 96k - 0 \cdot 03k^2 = 0$ ← Factorise the quadratic by taking out k in common.

$k(96 - 0 \cdot 03k) = 0$ ← Put the bracket equal to 0. $k = 0$ is a trivial solution which you can ignore.

$96 - 0 \cdot 03k = 0$ ← Solve for k.

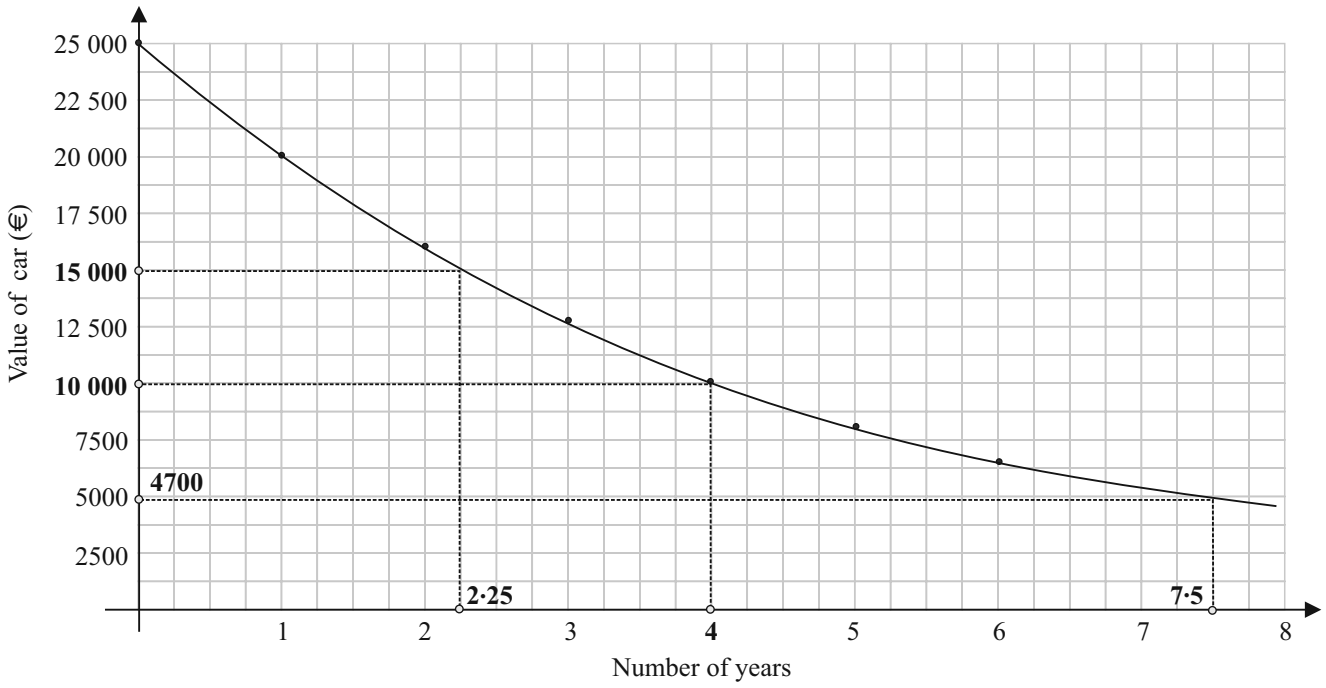
$96 = \mathbf{0} \cdot 03k$ ← Divide both sides by 0·03.

$\therefore k = \dfrac{96}{0 \cdot 03} = 3200$ ← 3200 barrels yields no profit.

Marking Scheme Notes
Question 8 (e) [Scale 5C (0, 3, 4, 5)]
3: • $p = 0$
• Evidence of more than one substitution
• Graph extended to x-axis
4: • Not more than two errors in solving

Question 9 (30 marks)

Question 9 (a)



(i) It takes 4 – 2·25 = 1·75 years for Amanda's car to depreciate from €15 000 to €10 000.

(ii) After 7·5 years, the approximate value of Amanda's car is €4700.

(iii) The graph above is an **exponential** graph.

Marking Scheme Notes
Question 9 (a) (i) [Scale 15B (0, 3, 15)]
3: • 10 000 or 15 000 indicated on graph, with no further work
• One value read correctly
• Answer between 1·55 and 2·05 (inclusive), without work

Question 9 (a) (ii) [Scale 5A (0, 5)]

Question 9 (a) (iii) [Scale 5A (0, 5)]

Question 9 (b)

Amanda

15 km → €1·50 (1 litre)

$1 \text{ km} \rightarrow \dfrac{€1 \cdot 50}{15} = €0 \cdot 10$

Amanda: Cost of car = €25 000
Cost of fuel per kilometre = €0·10
Let x = Distance travelled (km)
Total cost = $25\,000 + 0 \cdot 1d$

Cathal

10 km → €1·50 (1 litre)

$1 \text{ km} \rightarrow \dfrac{€1 \cdot 50}{10} = €0 \cdot 15$

Cathal: Cost of car = €22 500
Cost of fuel per kilometre = €0·15
Let d = Distance travelled (km)
Total cost = $22\,500 + 0 \cdot 15d$

$$25\,000 + 0 \cdot 1d = 22\,500 + 0 \cdot 15d$$
$$25\,000 - 22\,500 = 0 \cdot 15d - 0 \cdot 1d$$
$$2500 = 0 \cdot 05d$$
$$\therefore d = \frac{2500}{0 \cdot 05} = 50\,000 \text{ km}$$

Marking Scheme Notes
Question 9 (b) [Scale 5B (0, 3, 5)]
3: • 2500 written
• 1 or 2 costs per km found (Amanda and/or Cathal)

LC 2015 (Set D): Paper 2

Question 1 (25 marks)

Question 1 (a)

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6 = 1\,000\,000$$

0	**7**	**1**	**7**	**3**	**7**

There are 10 ways to fill the first box and 10 ways to fill the second box and so on.
Number of different passwords = $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6 = 1\,000\,000$

Marking Scheme Notes
Question 1 (a) [Scale 10B (0, 4, 10)]
4: • Any work of merit e.g. Mention of 10 or 6, use of !
• Works with relevant ${}^nP_r$ e.g. ${}^{10}P_6$
• Partial list of possibilities, ignoring repetition. e.g. $10 \times 9 \times 8 \times 7 \times 6$ ….
• Answer almost correct e.g. 10^6 or $10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151\,200$ or $9 \times 9 \times 9 \times 9 \times 9 \times 9 = 531\,441$
10: • Accept correct answers without work

Question 1 (b) (i)

$$9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^6 = 531\,441$$

1	**7**	**1**	**7**	**3**	**7**

There are 9 ways to fill the first box and 9 ways to fill the second box and so on.
Number of different passwords = $9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^6 = 531\,441$

Question 1 (b) (ii)

P(At least one zero) = 1 – P(No zeros)

$$= 1 - \frac{\text{Number of passwords that do } \textbf{not} \text{ contain zero}}{\text{Total number of passwords}}$$

$$= 1 - \frac{531\,441}{1\,000\,000}$$

$$= 0 \cdot 468559$$

Marking Scheme Notes
Question 1 (b) (i) (ii) [Scale 10C (0, 4, 9, 10)]
Note: Accept correct or consistent work from (a)
4: • Any work of merit e.g. Works with value from (a) reduced to exclude zero
• Works with relevant ${}^nP_r$ e.g. ${}^9P_6$
9: • Any one correct answer
• Two almost correct answers e.g. $9 \times 8 \times 7 \times 6 \times 5 \times 4 = 60\,480 \rightarrow$ P(b)(ii) $= \dfrac{60\,480}{\text{Ans}\,(a)}$
10: • Accept correct answers without work

Question 1 (c)

*		*			*
1	**7**	**1**	**7**	**3**	**7**

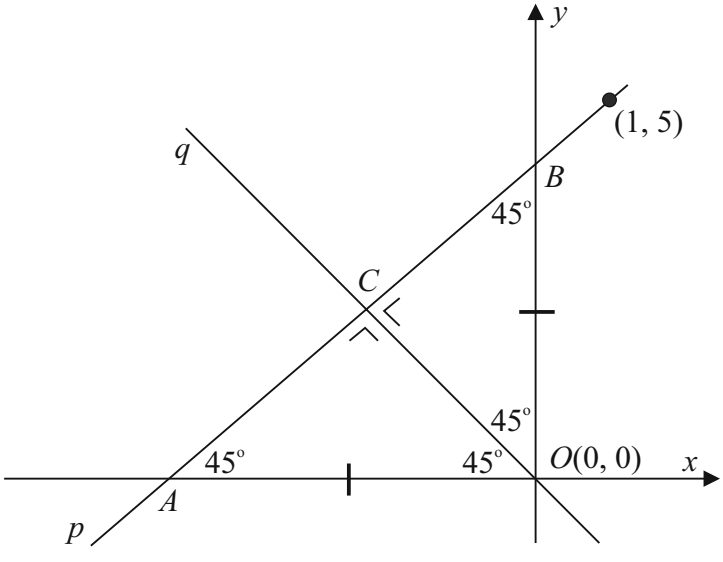
In how many ways can 3 numbers be selected from the 6 numbers in his password?

$${}^6C_3 = \frac{6\times5\times4}{3\times2\times1} = 20$$

Author's note: Selections are not on the Ordinary Level course. **Question 1 (c)** should not have been on the paper.

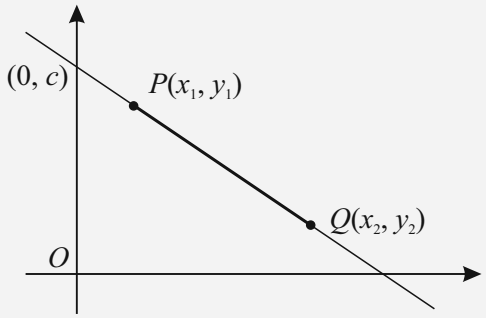
Marking Scheme Notes
Question 1 (c) [Scale 5B (0, 3, 5)]
Note: Accept correct or consistent work from (a)
3: • Any work of merit
5: • Correct answer without work

Question 2 (25 marks)
Question 2 (a)



(i) Slope of p = tan 45° = 1
(ii) $m = 1$, $(x_1, y_1) = (1, 5)$
Equation of p: $y - 5 = 1(x - 1)$
$y - 5 = x - 1$
$x - y + 4 = 0$

Formulae and Tables Book
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

Finding the perpendicular slope:

$$m = \frac{a}{b} \Rightarrow m_\perp = -\frac{b}{a}$$

Marking Scheme Notes
Question 2 (a) (i) (ii) [Scale 10C (0, 4, 9, 10)]
4: • Writes a correct relevant formula e.g. [Slope or equation of line]
• Identifies slope and stops
9: • One incorrect or omitted substitution into equation of line formula e.g. [$y - 5 = \#(x - 1)$]

Question 2 (b)

$m = -1$, $(x_1, y_1) = O(0, 0)$
Equation of q: $y - 0 = -1(x - 0)$ ← Multiply out the bracket.
$y = -x$ ← Bring all terms to the left-hand side.
$x + y = 0$

Marking Scheme Notes
Question 2 (b) [Scale 10B (0, 4, 10)]
4: • Writes equation of line formula.
• Indicates $m_1 \times m_2 = -1$

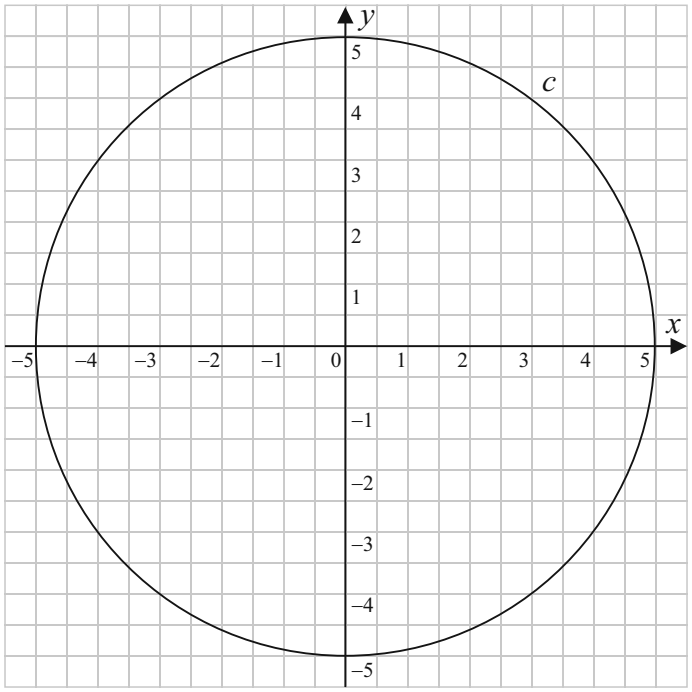
Question 2 (c)

Triangles OCA and OBC are congruent because:
$|OA| = |OB|$ [Given] (Side **S**)
$|OC| = |OC|$ [Common] (Side **S**)
$|\angle AOC| = |\angle BOC| = 45^o$ (Angle **A**)
Two sides and an included angle are equal (**SAS**).

Marking Scheme Notes
Question 2 (c) [Scale 5C (0, 3, 4, 5)]
3: • Correctly identifies any one pair of corresponding sides or angles.
• States SSS, SAS, RHS, ASA
4: • Correctly identifies either two or three of the corresponding sides or angle.
• Fails to finish
• Any 2 elements identified
• Diagram marked with 3 elements identified
5: • Accept correct explanation on basis of diagram information

Question 3 (25 marks)

Question 3 (a)



Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r
$$(x-h)^2 + (y-k)^2 = r^2$$

c is a unit circle with centre (0, 0) with radius 5 units.

$c : x^2 + y^2 = 25$
Centre $= (0, 0)$,
Radius $r = \sqrt{25} = 5$

Marking Scheme Notes
Question 3 (a) [Scale 10D (0, 2, 5, 8, 10)]

2: • Any relevant work e.g. Identifies correct centre of circle or works to $r^2 = 25$ and/or $\sqrt{25}$ and stops
• Attempt to scale an axis

5: • Any two of the four components Centre, Radius, Scale or Drawing correct

8: • Any three of the four components Centre, Radius, Scale or Drawing correct

Question 3 (b)

$A(-4, 3) \in c : x^2 + y^2 = 25$?

$(-4)^2 + (3)^2$

$= 16 + 9$

$= 25$

[Substitute the point $A(-4, 3)$ into the equation of the circle c. $A(-4, 3)$ is on c if A satisfies the equation of the circle.]

Marking Scheme Notes
Question 3 (b) [Scale 10B (0, 4, 10)]

4: • Identifies correct relevant formula, e.g. writes $[\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},\ x^2 + y^2 = r^2]$
• Any correct or consistent substitution.
• Radius = 5 (relevant work).
• No conclusion

Question 3 (c)

Circle s : Centre $= (h, k) = (-4, 3)$, radius $= r$

$s : (x - (-4))^2 + (y - 3)^2 = r^2$

$s : (x + 4)^2 + (y - 3)^2 = r^2$

$(3, 4) \in s \Rightarrow (3 + 4)^2 + (4 - 3)^2 = 49 + 1 = 50$

$\therefore s : (x + 4)^2 + (y - 3)^2 = 50$

or

The centre of the circle is $O(-4, 3)$. Call the point on the circle $B(3, 4)$. The radius r is the distance $|OB|$.

$O(-4, 3) = (x_1, y_1),\ A(3, 4) = (x_2, y_2)$

$r = |OA| = \sqrt{(3 - (-4))^2 + (4 - 3)^2}$

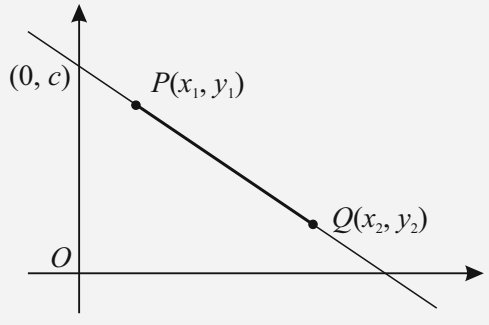
$= \sqrt{7^2 + 1^2}$

$= \sqrt{49 + 1} = \sqrt{50}$

$\therefore s : (x + 4)^2 + (y - 3)^2 = 50$

[Use the circle formula to find the equation of s. Substitute in (3, 4) to find the right-hand side of the equation.]

Formulae and Tables Book
Co-ordinate geometry: Line



Length of PQ [page 18]

$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

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Marking Scheme Notes

Question 3 (c) [Scale 5B (0, 3, 5)]

3: • Identifies a correct relevant formula

e.g. $[\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}, (x-h)^2+(y-k)^2=r^2, x^2+y^2+2gx+2fy+c=0]$

• Any correct substitution and stops

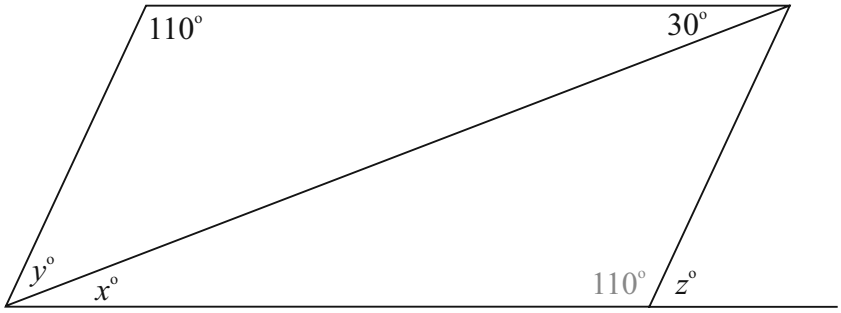
• Radius calculated

• Fully correct or consistent substitution without calculation

• Correct calculation with one incorrect substitution

Question 4 (25 marks)

Question 4 (a)

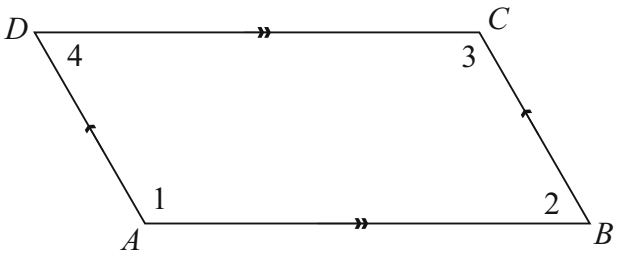


Properties of parallelograms

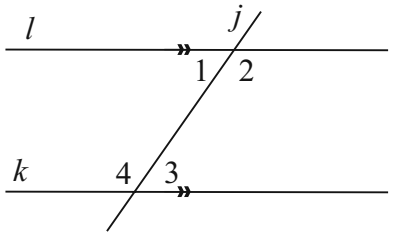
1. Opposite sides are equal in length: $|AB| = |DC|$ and $|AD| = |BC|$.

2. Opposite sides are parallel: $AB \parallel DC$ and $AD \parallel BC$.

3. Opposite angles are equal in measure: $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

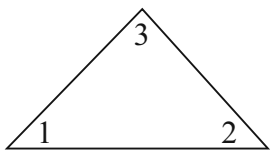


Alternate angles



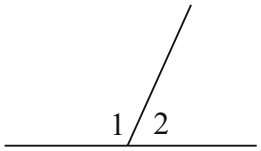
Lines k and l are parallel.
Line j is a transversal.
Alternate angles are equal.
$\therefore \angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

3 angles in a triangle



$\angle 1 + \angle 2 + \angle 3 = 180^\circ$

Straight angle



$\angle 1 + \angle 2 = 180^\circ$

$x^\circ = 30^\circ$ [Alternate angles]

$y^\circ = 180^\circ - 110^\circ - 30^\circ = 40^\circ$ [Three angles in a triangle add up to 180°]

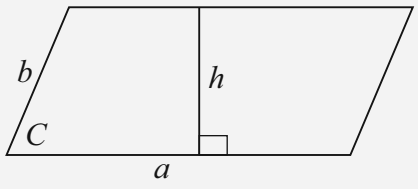
$110^\circ + z^\circ = 180^\circ \Rightarrow z^\circ = 180^\circ - 110^\circ = 70^\circ$ [A straight angle is 180°]

Marking Scheme Notes
Question 4 (a) [Scale 20D (0, 5, 10, 15, 20)]
5: • Any correct relevant work or statement for x, y, or z
• One correct value without reason
10: • One correct value and reason.
• Two correct values without reasons.
• Two correct with one reason
15: • Two correct values, both with reasons.
• All correct values with one reason.
Note: Answers may be on the diagram

Question 4 (b)

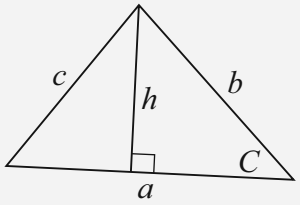
Formulae and Tables Book: **Length and area** (page 8)

Parallelogram

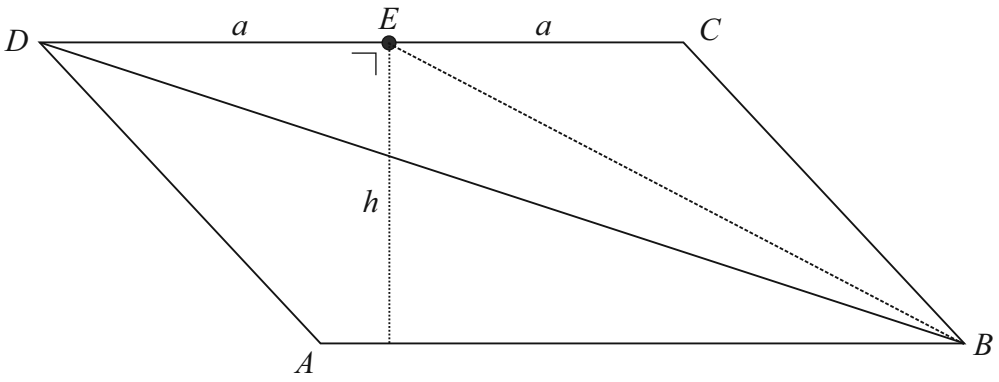


$A = ah$
$A = ab\sin C$

Triangle



$A = \frac{1}{2}ah$
$A = \frac{1}{2}ab\sin C$



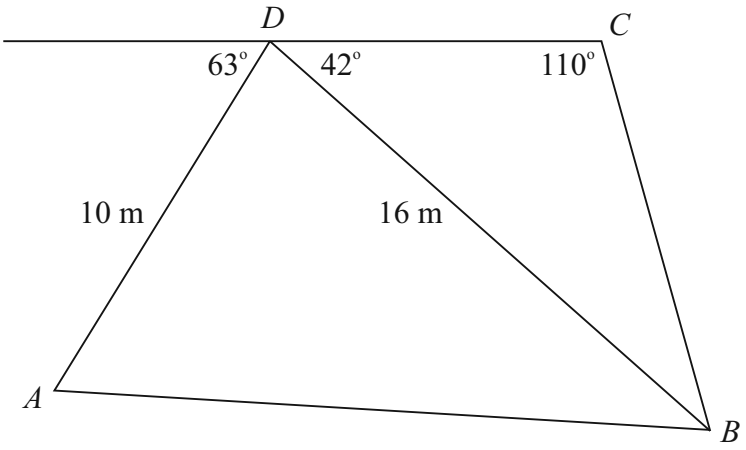
(i) Area of parallelogram $ABCD = 480$ m^2
A diagonal bisects the area of a parallelogram.
$\therefore\ |\Delta ABD| = 240$ m^2

(ii) $|\Delta BCD| = 240$ m^2
$|\Delta BCE| = |\Delta BED|$ [Area of each triangle $= \frac{1}{2}ah$]
$\therefore\ |\Delta BCE| = 120$ m^2

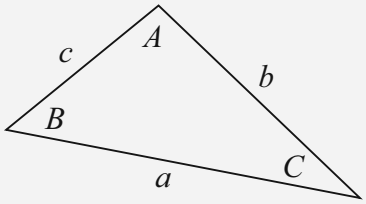
Marking Scheme Notes
Question 4 (b) (i) (ii) [Scale 5C (0, 3, 4, 5)]
3: • Any correct relevant work on the area of triangle in either part
4: • One correct area
5: • Accept correct answer without work

Question 5 (25 marks)

Question 5 (a)



Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Area: $\frac{1}{2}ab\sin C$

Sine Rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

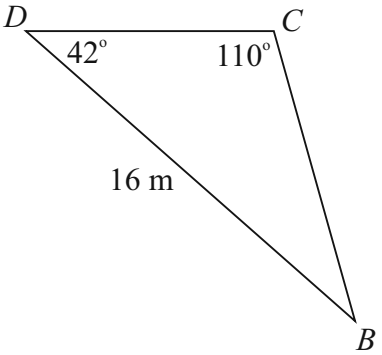
Cosine rule: $a^2=b^2+c^2-2bc\cos A$

Question 5 (a) (i)

$\frac{|BC|}{\sin 42^\circ}=\frac{16}{\sin 110^\circ}$ ← Apply the sine rule.

$|BC|=\frac{16\sin 42^\circ}{\sin 110^\circ}$ ← Multiply both sides by sin 42°.

$\therefore |BC|=11\cdot 39$ m



Marking Scheme Notes
Question 5 (a) (i) [Scale 15D (0, 4, 7, 11, 15)]

4: • Some correct substitution into a correct relevant formula $[\sin 42^\circ, |BC|=\frac{16}{\sin 110^\circ}]$

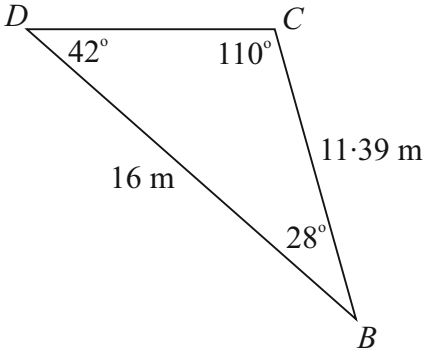
• Writes Sine Rule

7: • Fully correct substitution without calculation e.g. $\frac{|BC|}{\sin 42^\circ}=\frac{16}{\sin 110^\circ}$

11: • Calculates $|BC|$ with one incorrect substitution.
• Incorrect calculator mode. (Once only). [Radian = 331·452…, Gradian = 9·928…]
• A correct answer without work

Question 5 (a) (ii)

Area $=\frac{1}{2}(16)(11\cdot 39)\sin 28^\circ=42\cdot 78$ m^2



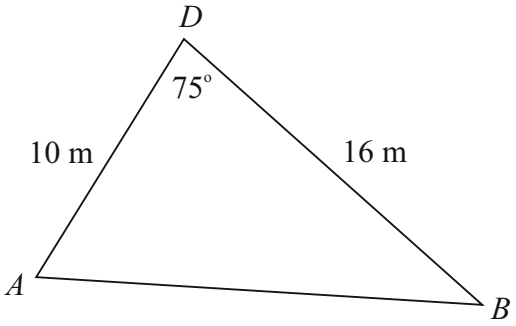
Marking Scheme Notes
Question 5 (a) (ii) [Scale 5C (0, 3, 4, 5)]
Note: Accept consistent answer from previous work
3: • Writes the correct formula for the area of a triangle
• Any correct or consistent substitution into formula
• Any work of merit e.g. Indicates total angle measure in a triangle = 180°, 28°
4: • Formula substituted correctly without calculation [Area $= \frac{1}{2}(16)(11\cdot39)\sin 28^{\circ}$]
• Calculates area of the triangle with one incorrect substitution
• Incorrect calculator mode (Once only). [Radian = 24·684…,Gradian = 38·797…]
• A correct or consistent answer without work

Question 5 (b)

Apply the Cosine rule as you have 2 sides and an included angle.

$|AB|^2 = 10^2 + 16^2 - 2(10)(16)\cos 75^{\circ}$

$\therefore |AB| = \sqrt{10^2 + 16^2 - 2(10)(16)\cos 75^{\circ}} = 16\cdot53 \text{ m}$



Marking Scheme Notes
Question 5 (b) [Scale 5D (0, 2, 3, 4, 5)]
2: • Some correct substitution into a correct relevant formula [$a = 10$, $b = 16$]
• Any work of merit such as identification of any correct information from the diagram [$C = 75°$]
• Writes Cosine Rule correctly
3: • An almost correct formula substitution and stops.
e.g. [$|AB|^2 = (10)^2 + (16)^2 - 2(10)(16)\cos\#$]
4: • Formula substituted correctly [$|AB|^2 = (10)^2 + (16)^2 - 2(10)(16)\cos 75^{\circ}$]
• Calculates $|AB|$ with one incorrect substitution
• Leaves answer as $|AB|^2$.
• Incorrect calculator mode (Once only). [Radian = 7·81…, Gradian = 15·28 …]
• A correct or consistent answer without work

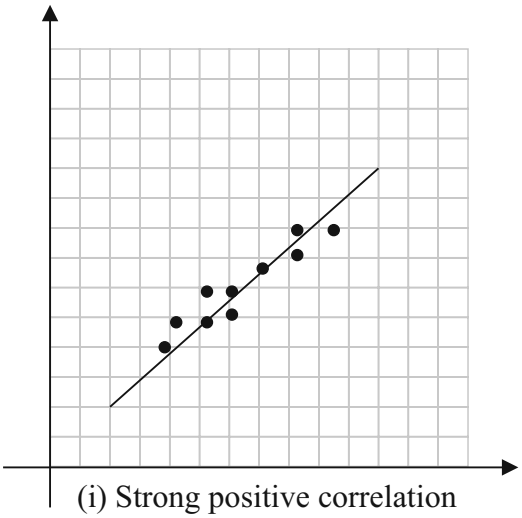
Question 6 (25 marks)

Question 6 (a)

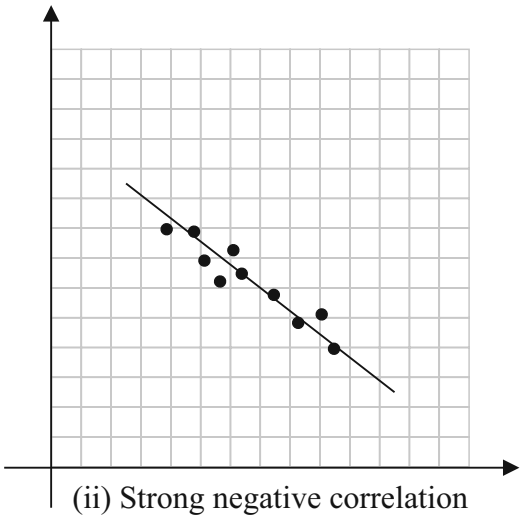
(i) **Strong positive correlation**:
The 10 points cluster closely to a straight line that has a positive slope.

(ii) **Strong negative correlation**:
The 10 points cluster closely to a straight line that has a negative slope.

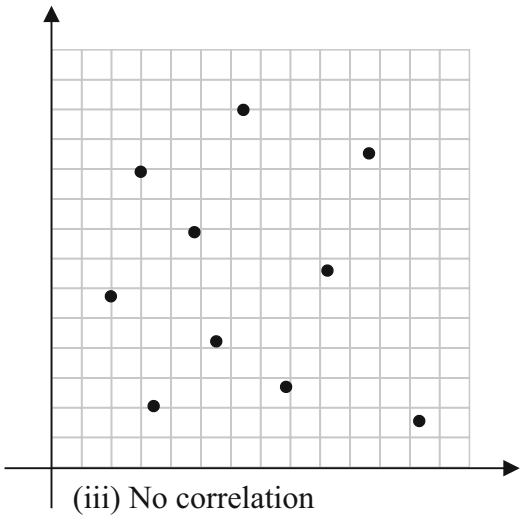
(iii) **No correlation**:
The 10 points form no overall pattern.



(i) Strong positive correlation



(ii) Strong negative correlation



(iii) No correlation

Marking Scheme Notes

Question 6 (a) (i) (ii) (iii) [Scale 20D (0, 5, 10, 15, 20)]

5: • Plots some points to illustrate a scatter graph but shows no understanding of correlation in any of the cases.
• Plots one correlation with less than ten points

10: • Plots one correct type of correlation.
• Plots two types of correlation only with less than ten points

15: • Plots two correct types of correlation
• Plots three types of correlation some with less than ten points

Question 6 (b)

Proportion p of people who support the 'No' campaign: $p = 0{\cdot}54 = 54\%$
Number of people in the sample $n = 1000$

(i) $n = 1000$

$$\text{Margin of error} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1000}} = 0{\cdot}0316 = 3{\cdot}2\%$$

(ii) $p = 54\%$

$$\text{Confidence interval} = [54\% - 3{\cdot}2\%,\ 54\% + 3{\cdot}2\%]$$
$$= [50{\cdot}8\%,\ 57{\cdot}2\%]$$

Formula: Statistics

$$\text{Margin of error} = \frac{1}{\sqrt{n}}$$

$$\text{Confidence interval} = \left[p - \frac{1}{\sqrt{n}},\ p + \frac{1}{\sqrt{n}} \right]$$

Marking Scheme Notes
Question 6 (b) (i) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Writes one/both formulae and stops e.g. $[\hat{p} - E \leq p \leq \hat{p} + E,\ \hat{p} \pm \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}]$

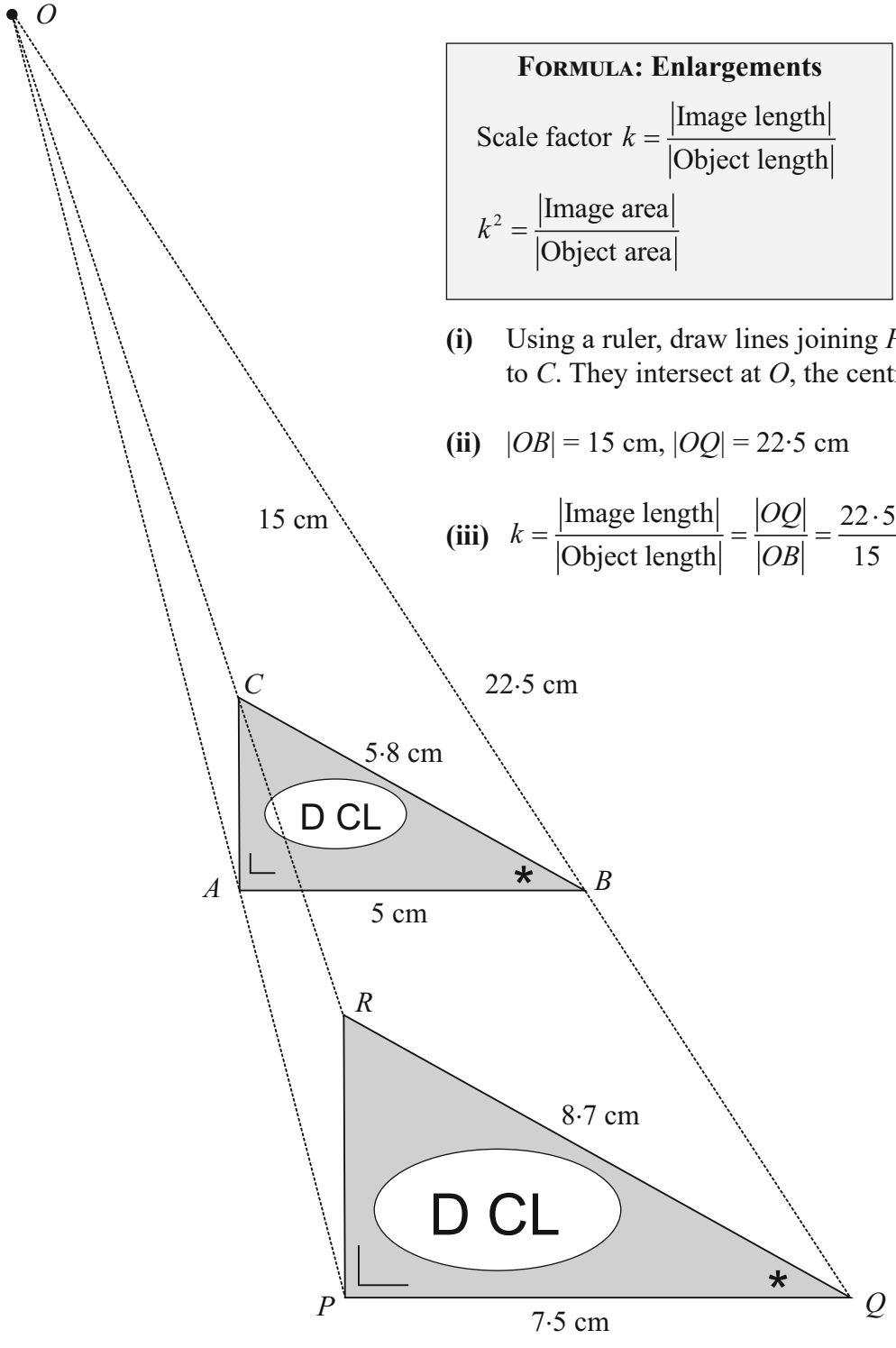
• Works with $\sqrt{1000}$

4: • One correct solution.

• Substantive work in both parts

Question 7 (35 marks)

Question 7 (a)



Formula: Enlargements

Scale factor $k = \frac{|\text{Image length}|}{|\text{Object length}|}$

$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}$

(i) Using a ruler, draw lines joining P to A, Q to B and R to C. They intersect at O, the centre of enlargement.

(ii) $|OB| = 15$ cm, $|OQ| = 22{\cdot}5$ cm

(iii) $k = \frac{|\text{Image length}|}{|\text{Object length}|} = \frac{|OQ|}{|OB|} = \frac{22{\cdot}5}{15} = \frac{3}{2} = 1{\cdot}5$

EPL 104

Marking Scheme Notes

Question 7 (a) (i) [Scale 15C (0, 4, 11, 15)]

4: • An attempt to draw any construction ray e.g. [Join B to Q]

11: • Two construction rays drawn.

• Construction rays drawn but centre not identified correctly

Question 7 (a) (ii) (iii) [Scale 10C (0, 4, 9, 10)]

Note: Accept tolerance of 5 mm Accept correct or consistent work from previous section

4: • One measurement correct.

• Works with two sides of given triangles to get k e.g. $[k = \frac{|PQ|}{|AB|} = \frac{7 \cdot 5}{5} = 1 \cdot 5]$

9: • Two measurements correct

• Error in scale factor

Question 7 (b)

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|} \Rightarrow 1 \cdot 5^2 = \frac{|\text{Image area}|}{7 \cdot 5}$$

$$\therefore |\text{Image area}| = 7 \cdot 5 \times 1 \cdot 5^2 = 16 \cdot 875 \text{ cm}^2$$

Marking Scheme Notes

Question 7 (b) [Scale 5B (0, 3, 5)]

3: • Use of an incorrect relevant formula [Use of k]

• Leaves as $7 \cdot 5 \times |1 \cdot 5|^2$

• Correct answer without use of scale factor $[|PQ| = \frac{|AB|}{k}]$

• Correct answer without work

Question 7 (c) (i)

$|\text{Object length}| = 5 \text{ cm}$

$k = 1 \cdot 5$

$|\text{Image length}| = |PQ| = ?$

$$1 \cdot 5 = \frac{|PQ|}{5} \Rightarrow |PQ| = 5 \times 1 \cdot 5 = 7 \cdot 5 \text{ cm}$$

Question 7 (c) (ii)

$|\text{Image length}| = |QR| = 8 \cdot 7 \text{ cm}$

$k = 1 \cdot 5$

$|\text{Object length}| = |BC| = ?$

$$1 \cdot 5 = \frac{|QR|}{|BC|} = \frac{8 \cdot 7}{|BC|} \Rightarrow |BC| = \frac{8 \cdot 7}{1 \cdot 5} = 5 \cdot 8 \text{ cm}$$

Question 7 (c) (iii)

$$\cos|\angle ABC| = \frac{5}{5 \cdot 8} = \frac{25}{29} \Rightarrow |\angle ABC| = \cos^{-1}\left(\frac{25}{29}\right) = 30 \cdot 45^\circ$$

$$\cos|\angle PQR| = \frac{7 \cdot 5}{8 \cdot 7} = \frac{25}{29} \Rightarrow |\angle PQR| = \cos^{-1}\left(\frac{25}{29}\right) = 30 \cdot 45^\circ$$

Formulae and Tables Book

Trigonometry: Right-angled triangle [page 16]

$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b}$$

Marking Scheme Notes
Question 7 (c) (i) (ii) (iii) [Scale 5D (0, 2, 3, 4, 5)]
Note: Accept correct or consistent from previous work
0: • Correct answer without work
2: • Any use of scale factor e.g. $|PQ| = \frac{|AB|}{k}$
3: • $|PQ|$ or $|BC|$ correct
4: • $PQ|$ and $|BC|$ correct

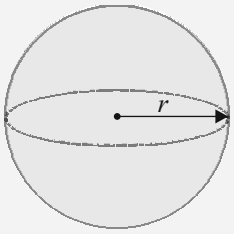
Question 8 (45 marks)

Question 8 (a)

(i) Diameter $d = 12$ m
Radius $r = 6$ cm $[d = 2r]$

(ii) $r = 6 \text{ m} : V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi \text{ m}^3 \approx 905 \text{ m}^3$

Formulae and Tables Book
Surface area and volume:
Sphere [page 8]



$A = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Marking Scheme Notes
Question 8 (a) (i) [Scale 5A (0, 5)]
5: • Accept a correct answer without work (6 m)

Question 8 (a) (ii) [Scale 10B (0, 4, 10)]
Note: Accept correct /consistent answer from (a)(i)
4: • Any work of merit e.g. writes the correct formula, identifies correct/consistent radius
• Treats as $k\pi(r)^3$ with correct/consistent radius substituted and stops
• Correct answer given in terms of π … 288π
Note: $[905 \cdot 14(\frac{22}{7}), 904 \cdot 32(3 \cdot 14)]$
10: • Accept a correct/consistent answer without work

Question 8 (b)

(i) $r = 6 \text{ m} : \text{Surface area} = 4\pi r^2 = 4\pi(6)^2$
$= 144\pi \text{ m}^2 \approx 452 \cdot 4 \text{ m}^2$

(ii) $\text{Number of litres} = \frac{452 \cdot 4 \text{ m}^2}{3 \cdot 5 \text{ m}^2} \approx 129 \text{ litres}$

(iii) $\text{Number of tins of paint} = \frac{129 \text{ litres}}{25 \text{ litres}} = 5 \cdot 17 \text{ tins}$ [You need to purchase 6 tins of paint.]
Cost: 6 × €180 = €1080

Marking Scheme Notes

Question 8 (b) (i) [Scale 5C (0, 3, 4, 5)]

Note: Accept correct/consistent answer from (a)(i)

3: • Any work of merit e.g. writes the correct formula, identifies correct/consistent radius
• Treats as $k\pi(r)^2$ with correct/consistent radius substituted and stops

4: • Correct formula with correct/consistent substitution and stops
• Correct answer given in terms of … 144π
• Finishes with only one error…e.g. $k\pi(r)^2$
Note: $[452\cdot57(\frac{22}{7}), 452\cdot57(3\cdot14)]$

5: • Accept a correct/consistent answer without work

Question 8 (b) (ii) [Scale 5A (0, 5)]

Note: Accept correct/consistent answer from (b)(i)

5: • Accept a correct/consistent answer without work

Question 8 (b) (iii) [Scale 5C (0, 3, 4, 5)]

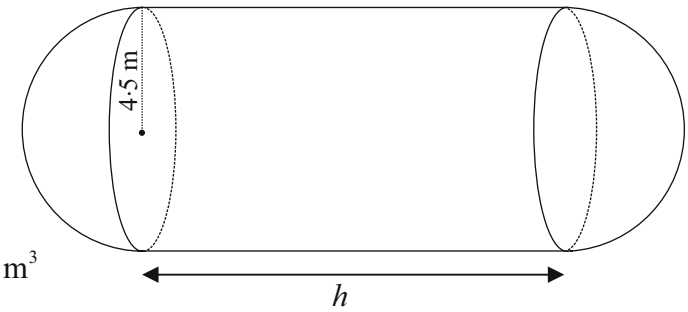
Note: Accept correct/consistent answer from (b)(ii)

3: • Any correct relevant work, e.g. Writes answer from (b) (ii), sets up division
• Calculates the required number of tins [Accept $(5\cdot17)/(5\cdot2)$]

4: • Finishes with only one error…e.g. incorrect number of tins
• Sets up correct cost multiplication and stops

Question 8 (c)

(i) The volume of a hemisphere is half the volume of a sphere of the same radius r.



$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3}\pi(4\cdot5)^3 \approx 191 \text{ m}^3$$

(ii) The tank consists of 2 **hemispheres** with a **cylinder** between them.

Volume of cylinder
= Volume of tank – 2(Volume of hemisphere)
$= 905 - 2 \times 191 = 523 \text{ m}^3$

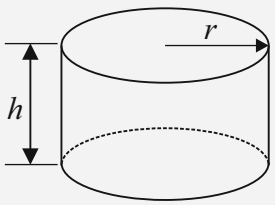
Cylinder

$V = 523 \text{ m}^3$

$r = 4\cdot5 \text{ m}$

$$V = \pi r^2 h \Rightarrow h = \frac{V}{\pi r^2} = \frac{523}{\pi(4\cdot5)^2} = 8\cdot2 \text{ m}$$

Formulae and Tables Book
Surface area and volume:
Cylinder [page 10]



$A = 2\pi rh$

$V = \pi r^2 h$

Marking Scheme Notes

Question 8 (c) (i) [Scale 5B (0, 3, 5)]

3: • Writes correct formula
• Treats as $\pi(r)^3$ with correct/consistent radius substituted
• Correct formula with correct/consistent substitution and stops
• Correct answer given in terms of $\pi \ldots \frac{243}{4}\pi$ or $60{\cdot}75\pi$

5: • Accept a correct/consistent answer without work

Question 8 (c) (ii) [Scale 10D (0, 2, 5, 8, 10)]

Note: Accept correct/consistent answer from (a)(ii) and (c)(i)

2: • Any correct or relevant work e.g. writes answers from (a)(ii) or (c)(i)
• Work towards setting up relevant equation with or without substitution

5: • Calculates the volume of the cylinder (523)
• Finishes with two errors… e.g. one incorrect volume and a transposition error

8: • Isolates h correctly and stops e.g. $[h = \frac{523}{\pi(4\cdot 5)^2}]$
• Finishes with only one error… e.g. index error, transposition error

Question 9 (70 marks)

Question 9 (a)

Categorical Nominal	
Categorical Ordinal	
Numerical Discrete	
Numerical Continuous	✓

Explanation: Heights are numerical measurements which can take on any real value.

Marking Scheme Notes

Question 9 (a) [Scale 5C (0, 3, 4, 5)]

3: • Tick in Numerical Continuous with no explanation

4: • A tick and an incomplete explanation for Numerical Continuous

5: • A tick and a correct explanation for Numerical Continuous
• A tick and a correct explanation for Numerical Discrete

Question 9 (b)

Table 1 (Girls)								
Height (cm)	145-150	150-155	155-160	160-165	165-170	170-175	175-180	180-185
Number of girls	15	48	80	112	125	81	29	10

(i)

Mid-interval value	f	fx
147·5	15	2212·5
152·5	48	7320·0
157·5	80	12 600·0
162·5	112	18 200·0
167·5	125	20 937·5
172·5	81	13 972·5
177·5	29	5147·5
182·5	10	1825·0
	500	82 215·0

$$\mu = \frac{\text{Sum of } (fx)}{\text{Sum of } f} = \frac{\sum fx}{\sum f} = \frac{82\ 215}{500} = 164 \cdot 43$$

Draw up a frequency distribution table by calculating mid-interval values.
Class 145-150:

$$\text{Mid-interval value} = \frac{145+150}{2} = 147 \cdot 5$$

Formulae and Tables Book
Statistics and probability:
Mean from a frequency table
[page 33]

$$\mu = \frac{\sum fx}{\sum f}$$

Marking Scheme Notes
Question 9 (b) (i) [Scale 20C (0, 10, 17, 20)]
10: • Work of merit
17: • Correctly calculates $\sum fx$ and fails to finish e.g. [82215]
• Finishes $\frac{\sum fx}{\sum f} = \frac{\Delta}{500}$ (with only one error).
Note: Δ = Candidate's $\sum fx$ worked
20: • Correct answer without work

(ii) Largest possible value for the range of heights = 185 – 145 = 40 cm

(iii) The median is the middle value when all the values are arranged in order.
50% have a height below 164·5 cm and 50% have a height above 164·5 cm.

Marking Scheme Notes
Question 9 (b) (ii) [Scale 5C (0, 3, 4, 5)]
3: • One end of range identified correctly
• Subtraction of two boundaries indicated e.g. 170−155
4: • Both ends of range identified correctly
5: • Correct answer without work

Question 9 (b) (iii) [Scale 5B (0, 3, 5)]
3: • Presents an incomplete explanation e.g. "Half are tall"
5: • Indicates "middle" of the sample

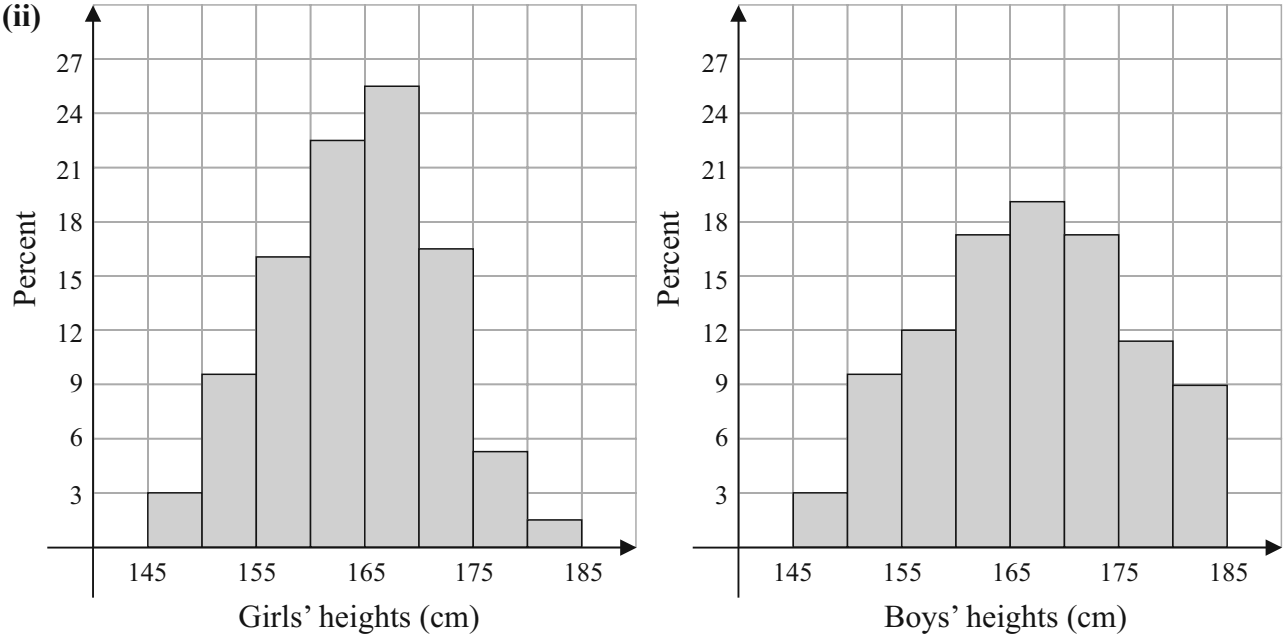
Question 9 (c)

(i) 15 girls out of 500 have heights between 145-150.

$\frac{15}{500} \times 100\% = 3\%$ of girls have heights between 145-150.

Table 2 (Girls, %)								
Height (cm)	145-150	150-155	155-160	160-165	165-170	170-175	175-180	180-185
Percentage of girls	3	9·6	16	22·4	25	16.2	5·8	2

(ii)



(iii) Yes - there are roughly twice as many boys as girls in the 175 to 180 cm category because the samples were the same size.

(iv) No. There are more boys with big heights. A bigger percentage of boys are over 165 cm. The area is bigger.

Marking Scheme Notes

Question 9 (c) (i) [Scale 5C (0, 3, 4, 5)]

3: • One/two correct percentage
• Attempt to find % e.g. Writes 100

4: • Any three correct percentages
• Decimal format used

Question 9 (c) (ii) [Scale 10B (0, 4, 10)]

4: • Any work of merit e.g. One correct block

Question 9 (c) (iii) [Scale 5B (0, 3, 5)]

3: • Indicates correct answer only
• Incomplete explanation

Question 9 (c) (iv) [Scale 5B (0, 3, 5)]

3: • Indicates correct answer
• Indicates correct answer and an incomplete explanation

Question 9 (d)

(i)

In any normal distribution with mean μ and standard deviation σ:

1. 68% of the data falls within 1σ of the mean μ.

2. 95% of the data falls within 2σ of the mean μ.

3. 99·7% of the data falls within 3σ of the mean μ.

$\mu = 166 \cdot 7$ cm

$\sigma = 8 \cdot 9$ cm

95% lie between $\mu \pm 2\sigma$

$= [166 \cdot 7 - 2 \times 8 \cdot 9,\ 166 \cdot 7 + 2 \times 8 \cdot 9]$

$= [148 \cdot 9 \text{ cm},\ 184 \cdot 5 \text{ cm}]$

(ii) The variation of the heights of the girls about their mean height of 165 cm is less than that of the boys about their mean height.

Marking Scheme Notes

Question 9 (d) (i) [Scale 5C (0, 3, 4, 5)]

3: • Writes a correct formula and stops
• Some work using σ [Interval as 157·8, or 175·6]

4: • One end of range identified correctly [148·9, or 184·5]
• Incorrect consistent error [Works with $\bar{x} \pm \sigma$]

Question 9 (d) (ii) [Scale 5B (0, 3, 5)]

3: • An incomplete interpretation e.g. Says different values/Subtracts values, with no interpretation

LC 2014 (Set E): Paper 1

Question 1 (25 marks)

Question 1 (a)

School blazers: $25 \times 30 = €750$
Trousers: $25 \times 20 = €500$
Cost price = €750 + €500 = €1250

Question 1 (b)

Selling Price per set: €89·95
Cost price per set = €30 + €20 = €50
Profit per set = Selling price per set – Cost price per set = €89·95 – €50 = €39·95

Question 1 (c)

Selling price: $22 \times$ €89·95 = €2248·75
$3 \times$ €89·95 × 0·8 = €215·88
€2194·78

Discount of 20%: Multiply original selling price by 0·8.

Profit = Selling price – Cost price = €2194·78 – €1250 = €944·78

$$\text{Mark up} = \frac{\text{Profit}}{\text{Cost price}} \times 100\% = \frac{744 \cdot 78}{1250} = 75 \cdot 6\%$$

Marking Scheme Notes

Question 1 (a) [Scale 10C (0, 3, 7, 10)]

3: • Finds cost of blazers or trousers
• Finds cost of 1 blazer and trouser set

7: • Finds cost of 25 of each

Question 1 (b) [Scale 5B (0, 2, 5)]

2: • Subtracts either 20 or 30 from 89·95
• Cost of 1 set = €50

Question 1 (c) [Scale 10D (0, 3, 5, 8, 10)]

3: • Finds SP of 22 sets
• Finds 20% of some figure

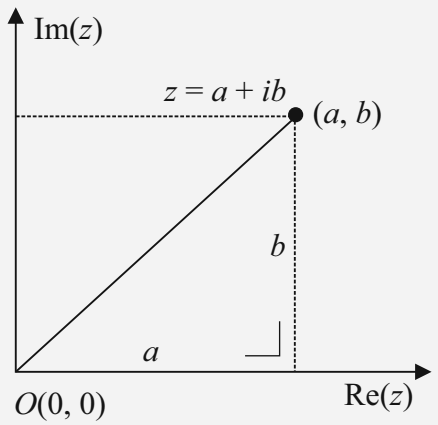
5: • Finds total selling price
• Finds total profit

8: • Correct expression for mark-up written but fails to finish or finishes incorrectly
• Incorrect total profit, from substantially correct work, but finishes correctly

Question 2 (25 marks)

Formula: Complex Numbers

Modulus $|z|$



$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Question 2 (a) (i)

$z_1 = 5 - i$

$z_2 = 4 + 3i$

$z_1 - z_2 = 5 - i - 4 - 3i = 1 - 4i$

Question 2 (a) (ii)

$|z_1 - z_2| = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17}$

$z_2 - z_1 = 4 + 3i - 5 + i = -1 + 4i$

$|z_2 - z_1| = \sqrt{(-1)^2 + 4^2} = \sqrt{1 + 16} = \sqrt{17}$

Question 2 (a) (iii)

$|z - w|$ and $|w - z|$ represents the distance between z and w which is a certain value regardless of the order in which you carry out the operation.

Marking Scheme Notes

Question 2 (a) (i) [Scale 10C (0, 3, 7, 10)]

3: • Correct substitution for z_1 or z_2 in expression

7: • Correct substitution for z_1 and z_2 in expression but error in finishing

Question 2 (a) (ii) [Scale 5C (0, 2, 3, 5)]

2: • Use of candidate's answer from (i)
• Correct modulus formula written
• $z_2 - z_1$ substituted

3: • One modulus correctly calculated

Question 2 (a) (iii) [Scale 5B (0, 2, 5)]

2: • Incomplete explanation but mentions distance or length or squaring
• $z - w = -(w - z)$ in words or similar

Question 2 (b)

Formula: Complex Numbers

Conjugates $\overline{z}$

$z = a + bi \Rightarrow \overline{z} = a - bi$

Multiplying a complex number by its conjugate:

$z\overline{z} = (a + bi)(a - bi) = a^2 + b^2$

$$z_1 = \frac{z_2}{z_3} \Rightarrow z_3 = \frac{z_2}{z_1}$$

$$z_3 = \frac{4 + 3i}{5 - i}$$

$$= \frac{4 + 3i}{5 - i} \times \frac{5 + i}{5 + i}$$ [Multiply above and below by the **conjugate** of the denominator.]

$$= \frac{20 + 4i + 15i + 3i^2}{25 + 5i - 5i - i^2}$$

$$= \frac{20 + 19i - 3}{25 + 1} = \frac{17 + 19i}{26}$$

$$= \frac{17}{26} + \frac{19}{26}i$$

Marking Scheme Notes
Question 2 (b) [Scale 5C (0, 2, 3, 5)]
2: • Terms rearranged
• Correct substitution
• $z_3 = z_1 z_2$ and continues
3: • Multiplication above and below by correct conjugate and some subsequent work
• Correct application of Real = Real, Im = Im
• $z_3 = \frac{z_1}{z_2}$ and continues

Question 3 (25 marks)

Question 3 (a) (i)

$2(4-3x)+12 = 7x-5(2x-7)$ ← Multiply out brackets

$8-6x+12 = 7x-10x+35$ ← Bring x terms to the left and numbers to the right.

$-6x-7x+10x = 35-8-12$ ← Add x terms and add numbers.

$-3x = 15$ ← Divide both sides by 3.

$\therefore x = \frac{15}{-3} = -5$

Question 3 (a) (ii)

Put $x = 5$ into the left hand side (**LHS**) of the equation and put $x = 5$ into the right-hand side (**RHS**) of the equation. The answer is verified if **LHS** = **RHS**.

LHS	**RHS**
$x = -5$	$x = -5$
$2(4-3x)+12$	$7x-5(2x-7)$
$= 2(4-3(-5))+12$	$= 7(-5)-5(2(-5)-7)$
$= 2(4+15)+12$	$= -35-5(-10-7)$
$= 2(19)+12$	$= -35-5(-17)$
$= 38+12$	$= -35+85$
$= 50$	$= 50$

Marking Scheme Notes
Question 3 (a) (i) [Scale 10C (0, 3, 7, 10)]
3: • Some correct attempt at solving
7: • Not more than 2 errors in solving

Question 3 (a) (ii) [Scale 5B (0, 2, 5)]
2: • Some correct substitution of answers from (i)
• No conclusion or incorrect conclusion if not equal

Question 3 (b)

$x + y = 7$.....(**1**)

$x^2 + y^2 = 25$.....(**2**)

$x + y = 7 \Rightarrow x = (7 - y)$ [Substitute equation (**1**) into equation (**2**).]

$x^2 + y^2 = 25$.....(**2**)

$(7 - y)^2 + y^2 = 25$ ← Expand the bracket.

$49 - 14y + y^2 + y^2 = 25$

$2y^2 - 14y + 24 = 0$ ← Divide each term by 2.

$y^2 - 7y + 12 = 0$ ← Factorise the quadratic.

$(y - 3)(y - 4) = 0$

$\therefore y = 3,\ 4$

Find the corresponding x values.

$y = 3 : x = (7 - y) = 7 - 3 = 4$

$y = 4 : x = (7 - y) = 7 - 4 = 3$

(3, 4) and (4, 3) are the solutions.

Marking Scheme Notes

Question 3 (b) [Scale 10D (0, 3, 5, 8, 10)]

3: • Some correct attempt at isolating x or y in linear equation
• Trial and Error into both equations (once only)

5: • Correctly substitutes into quadratic
• Trial and Error into both equations (more than once)

8: • Solves correctly for 1 variable only and stops or one (x, y) only
• Trial and Error into both equations leading to correct (x, y)

Question 4 (25 marks)

Question 4 (a)

$x^2 - x - 6 = 0$ ← Factorise the quadratic.

$(x + 2)(x - 3) = 0$ ← Put each bracket equal to zero and solve for x.

$x + 2 = 0 \Rightarrow x = -2$

$x - 3 = 0 \Rightarrow x = 3$

Solutions : $x = -2,\ 3$

Marking Scheme Notes

Question 4 (a) [Scale 15C (0, 5, 10, 15)]

5: • Some correct effort at factorising
• Correct quadratic formula written
• Trial and Error with no correct solution

10: • Correct factors found
• Solutions found from incorrect, but plausible, factors
• Correct substitution into quadratic formula
• Solutions found from substantially correct use of formula
• Trial and Error leading to one correct solution (verified)

Question 4 (b)

$f : x \mapsto +x^2 - x - 6 = (x+2)(x-3)$ This function is a quadratic curve with one bend.
As the coefficient of x^2 is positive, the graph is U-shaped.
This function crosses the x-axis at $x = -2$ and $x = 3$.

Answer: Graph **D**

Question 4 (c) (i)

From the graph write down the co-ordinates of $(x, g(x))$: (–2, 8), (–1, 5), (0, 0), (1, –1), (2, 0), (3, 3) and (4, 8).
To write the co-ordinates of $h(x)$ just add 2 to the second number of the co-ordinates of $g(x)$: (–2, 10), (–1, 7), (0, 2), (1, 1), (2, 2), (3, 5) and (4, 10).
The effect graphically of this transformation is to shift the graph vertically up by 2 units.

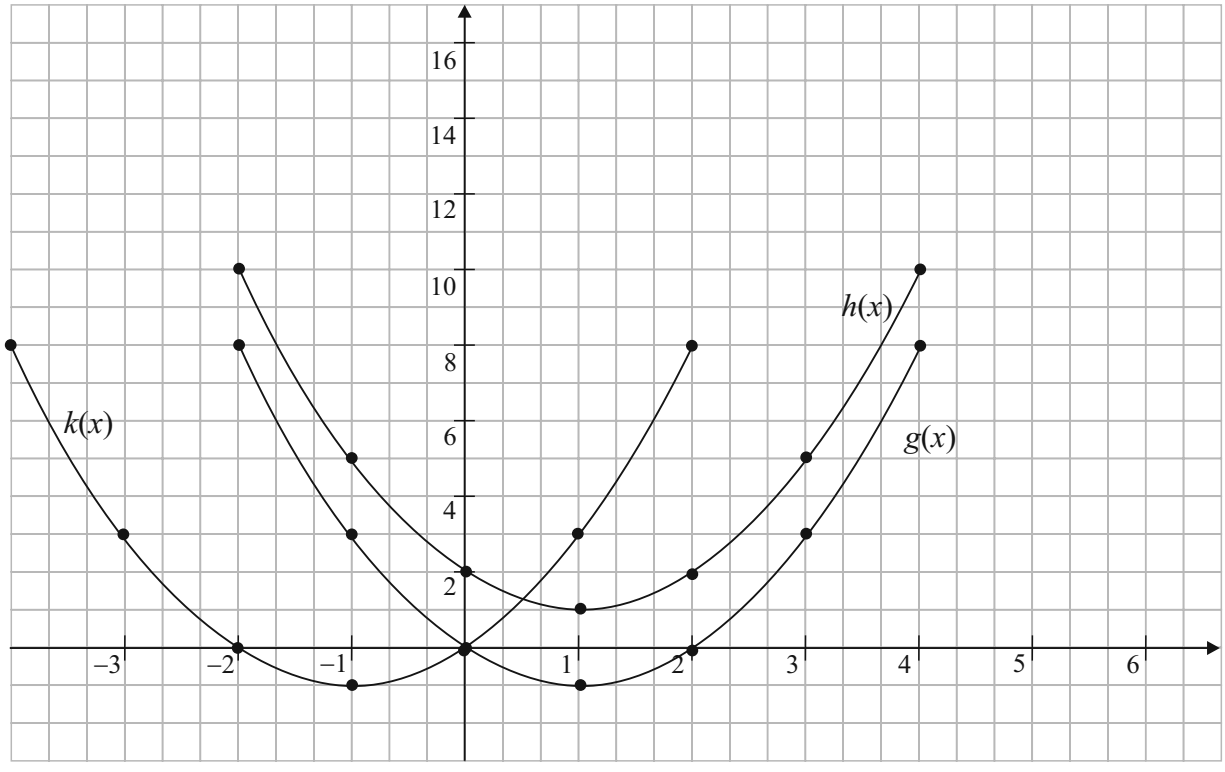
x	$g(x)$	$h(x) = g(x) + 2$
–2	8	10
–1	3	5
0	0	2
1	–1	1
2	0	2
3	3	5
4	8	10

Question 4 (c) (ii)

x	$x + 2$	$g(x)$	$k(x) = g(x + 2)$
–2	0	8	0
–1	1	3	–1
0	2	0	0
1	3	–1	3
2	4	0	8
3	5	3	
4	6	8	

For example $k(1) = g(1 + 2) = g(3) = 3$

The effect graphically of this transformation is to shift the graph horizontally left by 2 units.



EPL 104

Marking Scheme Notes
Question 4 (b) [Scale 5A (0, 5)]

Question 4 (c) [Scale 5C (0, 2, 3, 5)]
2: • 1 or 2 graphs of correct shape drawn
• Effort at evaluating $h(x)$ and/or $k(x)$
3: • Both drawn correctly but not labelled or incorrectly labelled
• $h(x) = g(x) - 2$ and/or $k(x) = g(x - 2)$ drawn and labelled
• 1 graph drawn correctly and labelled

Question 5 (25 marks)

Question 5 (a) (i)

$f : x \mapsto x^3 + 3x^2 - 9x + 5$

Cuts the y-axis or $f(x)$ axis: Put $x = 0$.

$f(0) = (0)^3 + 3(0)^2 - 9(0) + 5 = 5$

$\therefore$ (0, 5) are the co-ordinates of the point where f cuts the y-axis.

Question 5 (a) (ii)

$f : x \mapsto x^3 + 3x^2 - 9x + 5$

Cuts the x-axis: $y = 0$ or $f(x) = 0$.

$$f(-5) = (-5)^3 + 3(-5)^2 - 9(-5) + 5$$
$$= -125 + 3(25) + 45 + 5$$
$$= -125 + 75 + 45 + 5$$
$$= 0$$

Marking Scheme Notes
Question 5 (a) (i) (ii) [Scale 10C (0, 3, 7, 10)]
3: • Some correct substitution in either or both expressions
7: • One expression worked out correctly
• Both expressions worked out but with errors

Question 5 (b)

Find turning points (local maxiumum/minimum)
Put $\frac{dy}{dx} = 0$ and solve for x

$f : x \mapsto x^3 + 3x^2 - 9x + 5$

$f'(x) = 3x^2 + 6x - 9 = 0$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$\therefore x = 1, -3$

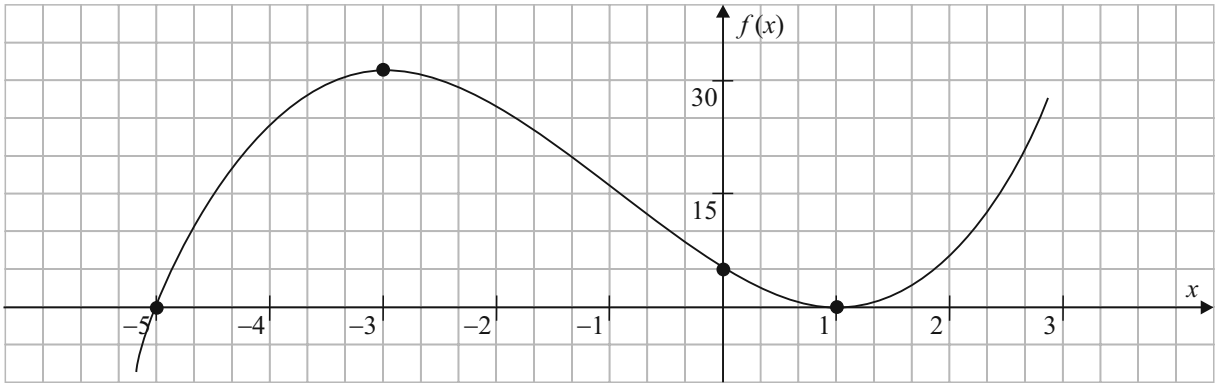
$$f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$$
$$= 1 + 3 - 9 + 5 = 0$$

$$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$$
$$= -27 + 27 + 27 + 5 = 32$$

Therefore, (1, 0) and (–3, 32) are the turning points.
(1, 0) is the local minimum and (–3, 32) is the local maximum.

Question 5 (c)

You know 4 points: (–5, 0), (–3, 32), (0, 5) and (1, 0).



Marking Scheme Notes

Question 5 (b) [Scale 10D (0, 3, 5, 8, 10)]

3: • Differentiates at least one term correctly
• States $f'(x) = 0$

5: • Correct differentiation and stops
• Correct structure to $f'(x) = 0$ but with errors, and continues
• Correct max and min found graphically

8: • Finds x values only of turning points
• Finds co-ordinates but with 2 or more errors

Question 5 (c) [Scale 5C (0, 2, 3, 5)]

2: • Plots 2 or 3 relevant points correctly
• Sketches the correct shape with no reference to relevant points

3: • Plots 4 relevant points correctly, but does not sketch a curve, or sketches an incorrectly shaped curve

Question 6 (25 marks)

Question 6 (a) (i)

To find the first 3 terms in the sequence replace n by 1, 2 and 3 to find T_1, T_2 and T_3.

$T_n = 15 - 2n$

$T_1 = 15 - 2(1) = 15 - 2 = 13$

$T_2 = 15 - 2(2) = 15 - 4 = 11$

$T_3 = 15 - 2(3) = 15 - 6 = 9$

Question 6 (a) (ii)

Write out the sequence: 13, 11, 9, 7, 5, 3, 1, –1, ...
The eighth term, T_8, is the first negative term in the sequence.

or

Find the value of n for which the general term, T_n, of the sequence is less than zero (negative).

$T_n = 15 - 2n < 0$

$15 < 2n$

$\frac{15}{2} < n$

$\therefore n > 7 \cdot 5$

Therefore, the eighth term, T_8, is the first negative term in the sequence.

Marking Scheme Notes

Question 6 (a) (i) [Scale 10C (0, 3, 7, 10)]

3: • Finds one term only
• Some correct substitution

7: • Finds 2 terms correctly
• Correct substitution for all 3 terms but not evaluated

Question 6 (a) (ii) [Scale 5C (0, 2, 3, 5)]

2: • Insufficient listing, but must list beyond T_3
• Some attempt at substitution
• Lets $T_n = 0$ or $T_n > 0$

3: • Stops at $n > 7\frac{1}{2}$ or $n = 7\frac{1}{2}$
• List method, but fails to identify term or number of the terms
• Sign error when solving inequality

Question 6 (b) (i)

You are being asked to add up the first n terms of the arithmetic sequence: 13, 11, 9, ... to find the general summing formula S_n.

First term $a = 13$

Common difference $d = 11 - 13 = -2$

$$S_n = \frac{n}{2}[2(13) + (n-1)(-2)]$$

$$= \frac{n}{2}[26 - 2n + 2]$$

$$= \frac{n}{2}[28 - 2n]$$

$$= n(14 - n)$$

Formulae and Tables Book
Sequences and series:
Arithmetic series or sequence
[page 22]

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Question 6 (b) (ii)

Put $S_n = 0$ and solve for n.

$S_n = 0 \Rightarrow (n)(14 - n) = 0$ ← Put each bracket equal to zero and solve for n.

$14 - n = 0 \Rightarrow n = 14$

$n = 0$ ← Ignore this solution - it is meaningless.

$\therefore n = 14$ ← Adding the first 14 terms gives a sum of zero.

Marking Scheme Notes

Question 6 (b) (i) [Scale 5C (0, 2, 3, 5)]

2: • Correct formula written
• Identifies a and/or d

3: • Correct formula substituted with 1 error

Question 6 (b) (ii) [Scale 5C (0, 2, 3, 5)]

2: • Some correct simplification of expression for S_n carried to (b)(ii)
• Solves $T_n = 0$ correctly

3: • Writes **correct** (quadratic) expression for $S_n = 0$

Question 7 (35 marks)

Question 7 (a)

It depreciates by 15% each year. Multiply the value of the car by 0·85 for each of the 4 years.

Value of car (1st July 2014) $= €20\,000 \times 0{\cdot}85 \times 0{\cdot}85 \times 0{\cdot}85 \times 0{\cdot}85$

$= €20\,000(0{\cdot}85)^4 \approx €10\,440$

Question 7 (b) (i)

Amount owed = Price of new car – trade-in amount

= €24 000 – €10 500 = €13 500

She borrows €13 500 at 11·5% for one year.

Amount repayable at end of year = €13 500 × 1·115 = €15 052·50

Question 7 (b) (ii)

Amount owed = Price of new car – trade-in amount

= €24 000 – €10 000 = €14 000

She pays back €4000 after 6 months which means she owes €10 000.

Amount repayable at end of year = $€10\,000 \times 1{\cdot}015^6 = €10\,934{\cdot}43$

Question 7 (b) (iii)

Buy Right Car Sales

Amount repaid = €15 052·50

Bargain Deals Car Sales

Amount repaid = €10 934·43 + €4000 = €14 934·43

Answer: She should choose *Bargain Deals Car Sales*.

Marking Scheme Notes

Question 7 (a) [Scale 10C (0, 3, 7, 10)]

3: • Writes correct formula without further work of merit
• Use of appreciation method
• Calculates 15% of an amount correctly

7: • Correct substitution into correct formula but errors in working out
• Calculates the amount at the end of each of 4 years, but with errors
• Calculates the amount at the end of each of 3 years, and stops

Question 7 (b) (i) [Scale 10C (0, 3, 7, 10)]

3: • Calculates the balance to be borrowed and stops
• Calculates 11·5% of an amount correctly

7: • Calculates interest only
• Calculates amount before deducting €10 500

Question 7 (b) (ii) [Scale 5C (0, 2, 3, 5)]

2: • Calculates amount to be repaid at end of 6 months
• Calculates 1·5% of an amount correctly
• Uses Simple Interest

3: • Correct substitution into correct formula but with errors
• Calculates 1·5% of an incorrect relevant figure correctly for 6 months
• Principal correct and calculates correctly for 3 or more months

Question 7 (b) (iii) [Scale 10B (0, 5, 10)]

5: • Correct conclusion with no justification
• Correct conclusion from incorrect total repayment(s)

Question 8 (40 marks)

Question 8 (a)

Area $A = 12 \text{ cm} \times 12 \text{ cm} = 144 \text{ cm}^2$

Question 8 (b)

Length of box = $(12 - 2h)$ cm

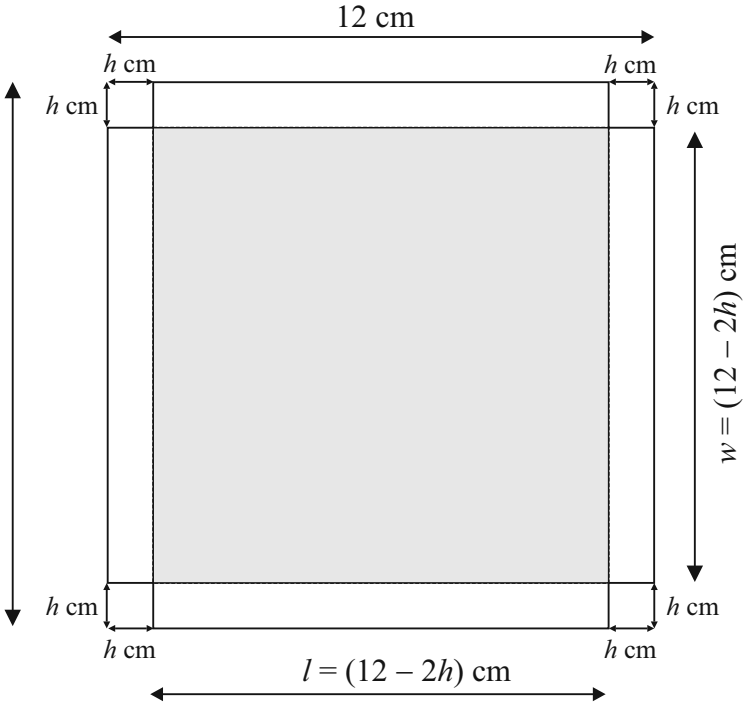
Width of box = $(12 - 2h)$ cm

Question 8 (c)

12 cm

Volume V = Length × Width × Height

$$V = (12-2h)(12-2h)h$$
$$= (144-24h-24h+4h^2)h$$
$$= (144-48h+4h^2)h$$
$$= 4h^3 - 48h^2 + 144h$$



Marking Scheme Notes

Question 8 (a) [Scale 5B (0, 2, 5)]

2: • 12×12 and stops

Question 8 (b) [Scale 10C (0, 3, 7, 10)]

3: • $2h$ written
• $12 -$ (incorrect h)
• $12 + 2h$

7: • 1 value correct
• Width = length (where length was of some merit e.g. $(12 - h)$ in both)

Question 8 (c) [Scale 10C (0, 3, 7, 10)]

3: • Correct formula written
• Some relevant attempt at multiplying terms

7: • $l \times w \times h$ fully substituted and stops
• Errors in finding product
Note: If l and w in (b) were without merit LPC at most

Question 8 (d)

$V = 4h^3 - 48h^2 + 144h \leftarrow$ Differentiate

$$\frac{dV}{dh} = 12h^2 - 96h + 144 = 0$$

$h^2 - 8h + 12 = 0 \leftarrow$ Factorise the quadratic.

$(h-2)(h-6) = 0$

$h - 2 = 0 \Rightarrow h = 2$ cm

$h - 6 = 0 \Rightarrow h = 6$ cm

Find turning points (local maxiumum/minimum)

Put $\frac{dy}{dx} = 0$ and solve for x

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Ignore the solution $h = 6$ cm, as this value of h would make the length and width of the box to be 0 cm.
ANSWER: 2 cm

Question 8 (e)

To find the maximum volume V_{Max} substitute the maximum height ($h = 2$ cm) into the formula for the volume V of the box.

$h = 2 : V_{\text{Max}} = 4(2)^3 - 48(2)^2 + 144(2) = 128 \text{ cm}^3$

MARKING SCHEME NOTES
Question 8 (d) [Scale 10D (0, 3, 5, 8, 10)]
3: • One or two terms differentiated correctly
• Value found by trial and error
5: • Correct differentiation and stops
• Correct structure to $f'(x) = 0$ but with errors, and continues
• Correct max found graphically
8: • Correct derivative = 0 and stops or error(s) in solving it

Question 8 (e) [Scale 5B (0, 2, 5)]
2: • Value of h from (d) substituted into $V(h)$
• More than one effort at trial and error
• $l \times w \times h$ with some correct or relevant substitution

QUESTION 9 (75 MARKS)

Question 9 (a)

Time, t	0	1	2	3	4	5	6	7	8	9	10
Height, h	0	9	16	21	24	**25**	**24**	**21**	**16**	9	0

$h = 10t - t^2$

$t = 0 : h = 10(0) - (0)^2 = 0$ m

$t = 1 : h = 10(1) - (1)^2 = 10 - 1 = 9$ m

$t = 2 : h = 10(2) - (2)^2 = 20 - 4 = 16$ m

$t = 3 : h = 10(3) - (3)^2 = 30 - 9 = 21$ m

$h = 10t - t^2$

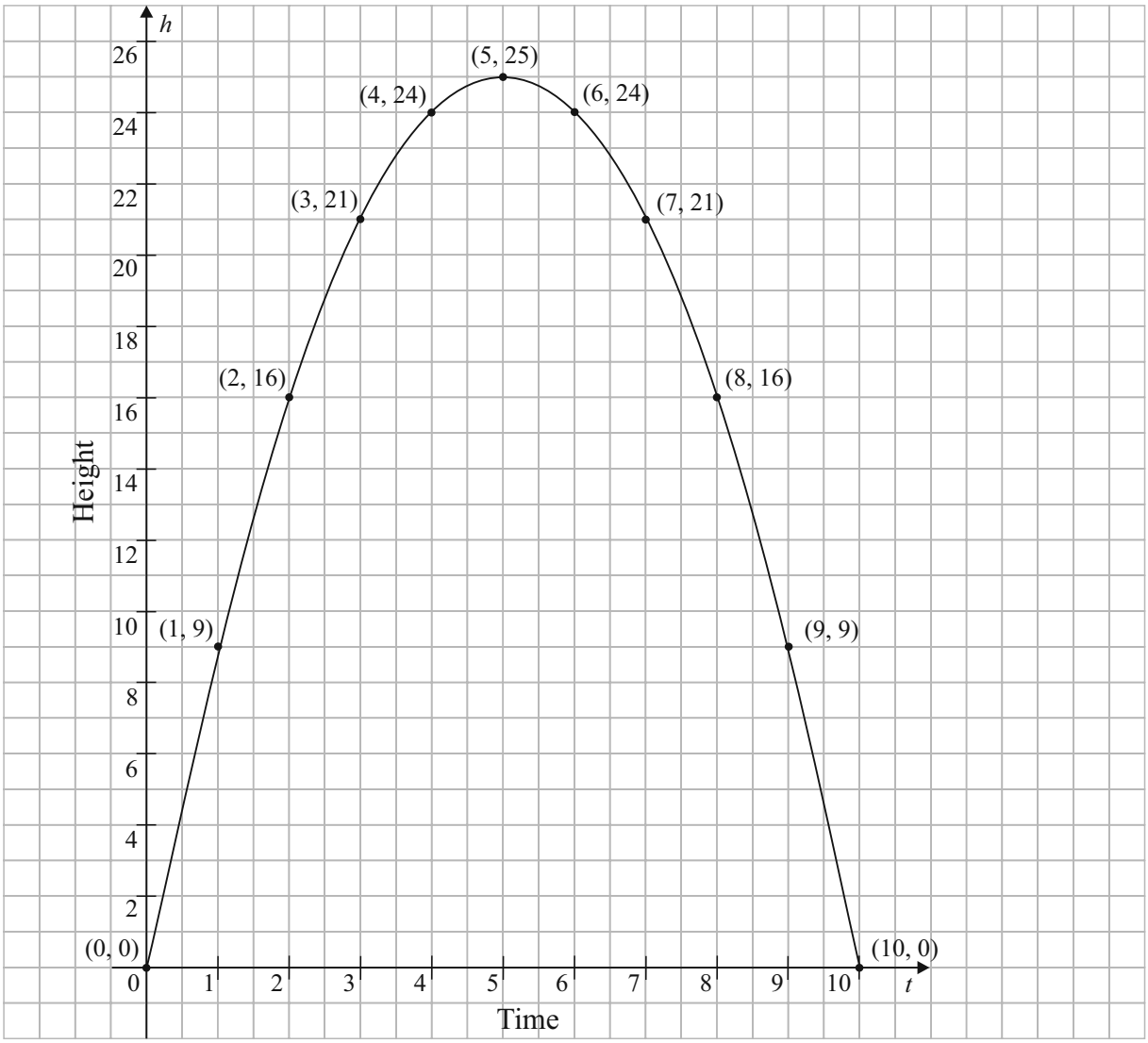
$t = 4 : h = 10(4) - (4)^2 = 40 - 16 = 24$ m

$t = 9 : h = 10(9) - (9)^2 = 90 - 81 = 9$ m

$t = 10 : h = 10(10) - (10)^2 = 100 - 100 = 0$ m

MARKING SCHEME NOTES
Question 9 (a) [Scale 15C (0, 5, 10, 15)]
5: • At least one correct value
• Substitutes correctly at least once
10: • 5 or 6 correct values

Question 9 (b)



Marking Scheme Notes

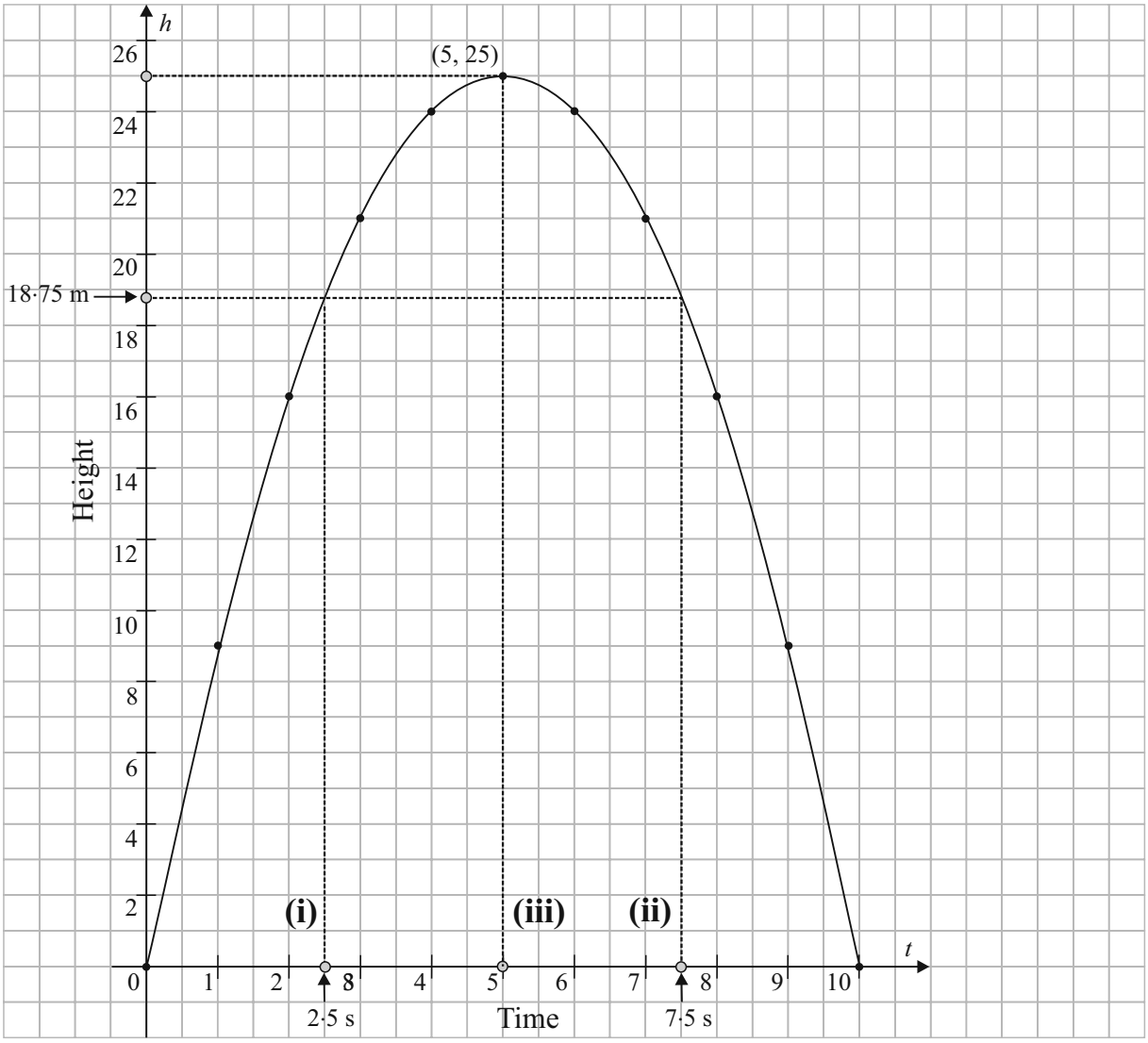
Question 9 (b) [Scale 15C (0, 5, 10, 15)]

5: • At least 1 point plotted correctly

10: • Points connected with straight edge
• At least 5 points plotted correctly and joined
• All points plotted correctly but not joined

Note: Answers to (c) (i) (ii) (iii) based on candidate's graph. Tolerance ±½ unit

Question 9 (c)



(i) Go to 2·5 s on the time axis. Draw a vertical line up until it meets the curve and then a horizontal line out to the height axis. Read off the height h.
$\therefore\ h = 18{\cdot}75$ m

(ii) Go to 18·75 m on the height axis. Draw a horizontal line across until it meets the curve for the second time. Draw a vertical line down to the time axis. Read off the time t.
$\therefore\ t = 7{\cdot}5$ s

(iii) Maximum point (5, 25)

Marking Scheme Notes
Question 9 (c) (i) [Scale 5B (0, 2, 5)]
2: • Uses $t = 2{\cdot}5$ to solve

Question 9 (c) (ii) [Scale 5B (0, 2, 5)]
2: • Correct h ordinate indicated on graph

Question 9 (c) (iii) [Scale 5B (0, 2, 5)]
2: • Only one ordinate given
• Coordinates reversed

Question 9 (d)

(i) $(6, 24) = (x_1, y_1), (7, 21) = (x_2, y_2)$

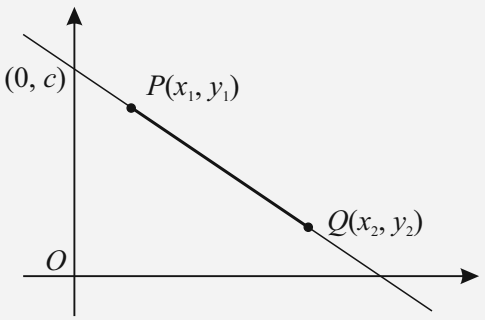
$$m = \frac{21-24}{7-6} = -3$$

(ii) Yes. For a one-second time interval, the rocket travels 5 m compared to 3 m in part **(i)**. It has a greater negative slope.

$(7, 21) = (x_1, y_1), (8, 16) = (x_2, y_2)$

$$m = \frac{16-21}{8-7} = -5$$

FORMULAE AND TABLES BOOK
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

MARKING SCHEME NOTES
Question 9 (d) (i) (ii) [Scale 10D (0, 3, 5, 8, 10)]

3: • Any work of merit e.g. correct formula given

5: • One slope calculated correctly
• Both slopes calculated but with errors
• Correct answer to (ii) but no slope in (i)

8: • Both slopes calculated correctly with incorrect or no conclusion

Question 9 (e)

(i) $h = 10t - t^2$

$$\frac{dh}{dt} = 10 - 2t$$

FORMULAE AND TABLES BOOK
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

(ii) $\frac{dh}{dt} = 0 \Rightarrow 10 - 2t = 0$

$10 = 2t$

$\therefore t = 5$ s

FIND TURNING POINTS (LOCAL MAXIMUM/MINIMUM)
Put $\frac{dy}{dx} = 0$ and solve for x

$h_{\text{Max}} = 10(5) - (5)^2 = 50 - 25 = 25$ m ← Put $t = 5$ s into the formula for height to find the maximum height h_{Max}

(iii) $v = \frac{dh}{dt} = 10 - 2t$

$t = 3 : v = 10 - 2(3)$

$= 10 - 6$

$= 4$ m/s

FORMULA: Velocity/Speed

$$\text{Velocity (Speed) } v = \frac{\text{Change in distance}}{\text{Change in time}} = \frac{ds}{dt}$$

Note: Distance in this case is height.

Marking Scheme Notes

Question 9 (e) (i) [Scale 5B (0, 2, 5)]

2: • 1 term differentiated correctly

Question 9 (e) (ii) [Scale 5C (0, 2, 3, 5)]

2: • $\frac{dh}{dt} = 0$

• Use of their $\frac{dh}{dt}$

3: • The candidate's value of t substituted into expression for h

• Solves their $\frac{dh}{dt} = 0$ correctly

Question 9 (e) (iii) [Scale 5B (0, 2, 5)]

2: • Recognition that speed = $\frac{dh}{dt}$

• 3 substituted into $\frac{dh}{dt}$ and stops

Question 9 (f)

You can find the slope of the tangent to a curve by differentiation.
Solve for t by putting the slope equal to 4.
Find the corresponding value of h by putting this value of t into the height formula.

$\frac{dh}{dt} = 10 - 2t = 2$

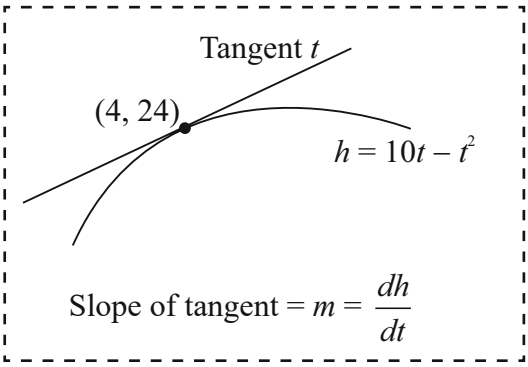
$10 - 2 = 2t$

$8 = 2t$

$\therefore t = 4$

$h = 10(4) - (4)^2 = 40 - 16 = 24$

The co-ordinates are (4, 24).



Marking Scheme Notes

Question 9 (f) [Scale 5C (0, 2, 3, 5)]

2: • Correct structure to $\frac{dh}{dt} = 2$

• Correct answer with no work or without calculus

3: • The candidate's value of t substituted into expression for h and stops or continues with errors

• Solves their $\frac{dh}{dt} = 2$ correctly

LC 2014 (Set E): Paper 2

Question 1 (25 marks)

Question 1 (a)

5 black cars, 9 red cars and 10 silver cars
Total = 24 cars

(i) $P(\text{Black car}) = \dfrac{\text{Number of black cars}}{\text{Number of cars}} = \dfrac{5}{24}$

$$P(\text{Event}) = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

(ii) $P(\text{Black or red car}) = \dfrac{\text{Number of black or red cars}}{\text{Number of cars}} = \dfrac{5+9}{24} = \dfrac{14}{24} = \dfrac{7}{12}$

Marking Scheme Notes
Question 1 (a) (i) (ii) [Scale 10C (0, 4, 7, 10)]
4: • Relevant work in one or both sections. e.g. either #E or #S correct
7: • One correct section
10: • Accept correct answers without work

Question 1 (b)

(i) $P(\text{Silver } \textbf{and then} \text{ a black car}) = \dfrac{10}{24} \times \dfrac{5}{23} = \dfrac{25}{276}$

There are 10 ways to pick a silver car from 24 cars **and then** (multiply) there are 5 ways to pick a black car from the remaining 23 cars.

(ii) $P(\text{Red } \textbf{and} \text{ a black car}) = \dfrac{9}{24} \times \dfrac{5}{23} \times 2 = \dfrac{15}{92}$

There are 9 ways to pick a red car from 24 cars **and** (multiply) there are 5 ways to pick a black car from the remaining 23 cars. However, you could have picked the red first and then the black or the other way around. Multiply your answer by 2 to take both options into account.

[Authors's note: This question should not have been asked. The syllabus states quite clearly that Ordinary Level students should deal only with the probability of independent events.]

Marking Scheme Notes
Question 1 (b) (i) [Scale 5C (0, 3, 4, 5)]
3: • Any correct #E or #S
• P(silver) $= \frac{10}{24}$ and/or P(black) $= \frac{5}{23}$
4: • $(\frac{10}{24} \times \frac{5}{23})$ and stops
• Calculates the probability with replacement. $(\frac{10}{24} \times \frac{5}{23} = \frac{50}{576}$ or $\frac{25}{288})$
• Correct answer without work

Question 1 (b) (ii) [Scale 5C (0, 3, 4, 5)]
3: • Any relevant work towards creating at least one correct probability
4: • $[(\frac{10}{24} \times \frac{5}{23}) + (\frac{5}{24} \times \frac{9}{23})]$ and stops
• Correct answer without work

Question 1 (c)

Non-diesel: 2 black cars, 7 red cars and 6 silver cars
Diesel: 3 black cars, 2 red cars and 4 silver cars
Number of diesel cars = 9
Number of red non-diesel cars or diesel cars = 7 + 9 = 16
Total = 24 cars

$$P(\text{Red } \textbf{or} \text{ a diesel car}) = \frac{\text{Number of red non-diesel cars or diesel cars}}{\text{Number of cars}} = \frac{16}{24} = \frac{2}{3}$$

Marking Scheme Notes
Question 1 (c) [Scale 5C (0, 3, 4, 5)]
3: • Any relevant work towards creating at least one correct probability
• Solution presented as: [P(red or diesel) $= \frac{9}{24} + \frac{9}{24} = \frac{18}{24}$ or $\frac{3}{4}$]
4: • Correct answer without work

Question 2 (25 marks)

Question 2 (a)

$$P(\text{Scoring}) = \frac{3}{4}$$

$$P(\text{Failing to score}) = 1 - \frac{3}{4} = \frac{1}{4}$$

Success and Failure
If there are 2 outcomes: Success *or* Failure
P(Success) = 1 – P(Failure)

Marking Scheme Notes
Question 2 (a) [Scale 10B (0, 5, 10)]
5: • Writes the correct #E or #S
• Any incomplete work with 1
10: • Accept correct answers without work

Question 2 (b)

$$P(\text{Scoring } \textbf{and then} \text{ scoring}) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

And/or
And: Multiply
P(A) and then P(B): P(A) × P(B)
Or: Add
P(A) or P(B): P(A) + P(B)

Marking Scheme Notes
Question 2 (b) [Scale 5C (0, 3, 4, 5)]
3: • Identifies the probability of scoring here
• Writes P(S, S,) as $(\frac{3}{4} + \frac{3}{4})$
4: • Fails to finish. [P(S, S) $= (\frac{3}{4} \times \frac{3}{4})$] or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
5: • Accept correct answers without work

Question 2 (c)

Scores = S, Fails to score = F

Different ways in which he can score exactly twice from 3 penalties: SSF, SFS, FSS (3 ways)

$$P(\text{S}) = \frac{3}{4}$$

$$P(\text{F}) = \frac{1}{4}$$

$$P(\text{S and S and F}) = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} \times 3 = \frac{27}{64}$$

Marking Scheme Notes
Question 2 (c) [Scale 5A (0, 5)]

5: • Accept correct answers without work

Question 2 (d)

This means that he fails to score on his first 4 penalties and scores on his fifth penalty: F, F, F, F, S

$$P(\text{F and F and F and F and S}) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{1024}$$

Marking Scheme Notes
Question 2 (d) [Scale 5C (0, 3, 4, 5)]

3: • Correctly identifies P(S) and/or P(M)
• Lists the outcomes as M,M,M,M,S, or writes as $(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{3}{4})$

4: • A fully correct substitution. i.e. $(\frac{1}{4})^4 \times \frac{3}{4}$

5: • Accept correct answers without work

Question 3 (25 marks)

Question 3 (a) (i)

c: $(x + 2)^2 + (y - 3)^2 = 100$

Centre $A(-2, 3)$

Radius $r = \sqrt{100} = 10$

Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r

$$(x-h)^2 + (y-k)^2 = r^2$$

Question 3 (a) (ii)

Show P is on c by substituting P into c:

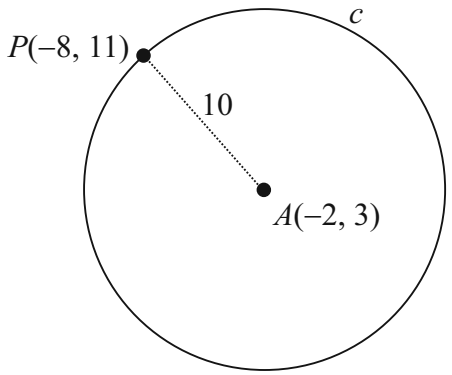
$$P(-8, 11) \in c : (x+2)^2 + (y-3)^2 = 100 ?$$

$$(-8+2)^2 + (11-3)^2$$

$$= (-6)^2 + (8)^2$$

$$= 36 + 64$$

$$= 100$$



or

Show that $|AP| = r$.

$A(-2, 3) = (x_1, y_1),\ P(-8, 11) = (x_2, y_2)$

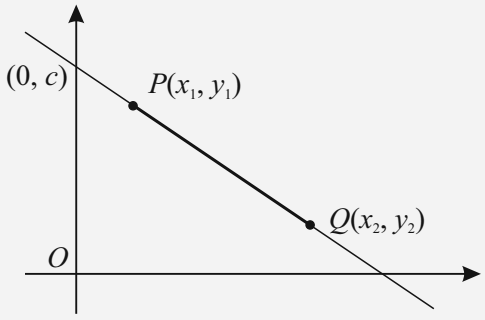
$$|AP| = \sqrt{(-8-(-2))^2 + (11-3)^2}$$
$$= \sqrt{(-6)^2 + (8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100} = 10 = r$$

Marking Scheme Notes
Question 3 (a) (i) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Relevant work in one section
e.g. Finds one correct co-ordinate of the centre A or works to $r^2 = 100$ and/or $\sqrt{100}$ and stops

4: • One section correct

Formulae and Tables Book
Co-ordinate geometry: Line



Length of PQ [page 18]

$$|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Slope of PQ [page 18]

$$m = \frac{y_2-y_1}{x_2-x_1}$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

Question 3 (b) (i)

$A(-2, 3) = (x_1, y_1),\ P(-8, 11) = (x_2, y_2)$

$$m = \frac{11-3}{-8-(-2)} = \frac{8}{-6} = -\frac{4}{3}$$

Marking Scheme Notes
Question 3 (b) (i) [Scale 10C (0, 4, 7, 10)]

4: • Identifies correct relevant formula, e.g. writes $\left(m = \frac{y_2-y_1}{x_2-x_1}\right)$
• Any correct or consistent substitution

7: • Formula substituted correctly
• Calculates slope with one incorrect substitution

10: • Accept for full credit answer as $\frac{8}{-6}$ without work

Question 3 (b) (ii)

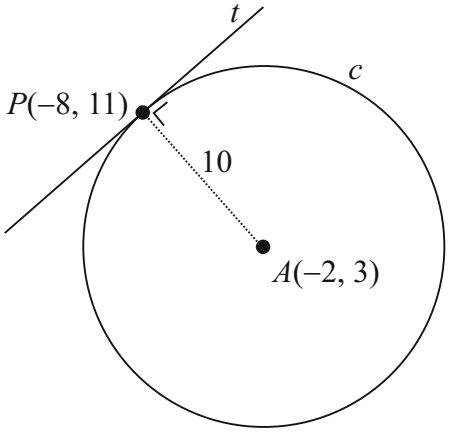
The tangent t is perpendicular to the radius $[AP]$.

$m = -\frac{4}{3} \Rightarrow m_\perp = \frac{3}{4}$

$$m = \frac{a}{b} \Rightarrow m_\perp = -\frac{b}{a}$$

Equation of t: $P(-8, 11) = (x_1, y_2),\ m = \frac{3}{4}$

$$y - 11 = \tfrac{3}{4}(x-(-8))$$
$$4(y-11) = 3(x+8)$$
$$4y - 44 = 3x + 24$$
$$3x - 4y + 68 = 0$$



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Marking Scheme Notes
Question 3 (b) (ii) [Scale 5C (0, 3, 4, 5)]

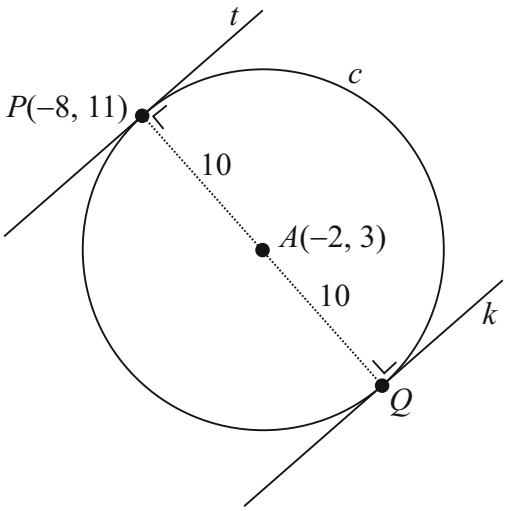
3: • Identifies correct relevant formula, e.g. writes $y - y_1 = m(x - x_1)$.
• Correct or consistent slope identified in this part and stops.

4: • A fully correct or consistent substitution. e.g. $[y - 11 = \frac{3}{4}(x + 8)]$.
• Calculates equation of line with one incorrect substitution.

Question 3 (c)

Q is obtained by passing P through A (central symmetry) and the same distance beyond A.

$P(-8, 11) \rightarrow A(-2, 3) \rightarrow Q(4, -5)$ [Add 6, Subtract 8]



Marking Scheme Notes
Question 3 (c) [Scale 5C (0, 3, 4, 5)]

3: • Writes slope of k equal to slope of t.
• Identifies correct relevant formula, e.g. writes $\left[x = \frac{x_1 + x_2}{2},\ y = \frac{y_1 + y_2}{2}\right]$.
• Work with a translation which indicates an understanding of the concept
• Any work of merit using algebraic method

4: • An incorrect translation correctly applied
• A correct graphical solution.
• Finds both points of intersection algebraically and stops

Question 4 (25 marks)

Question 4 (a)

(i) $|AB| = 5$

(ii) Area = ½(5)(7) = 17·5 $\boxed{\text{Area} = \frac{1}{2}bh}$

Marking Scheme Notes
Question 4 (a) (i) [Scale 10B (0, 5, 10)]

5: • Identifies correct relevant formula, e.g. writes $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and stops
• Any correct substitution into formula
• Calculates a distance with one incorrect substitution
• Incorrect reading of the scale diagram. (Distance = 10) i.e. Counting Boxes

10: • Accept correct answer without work

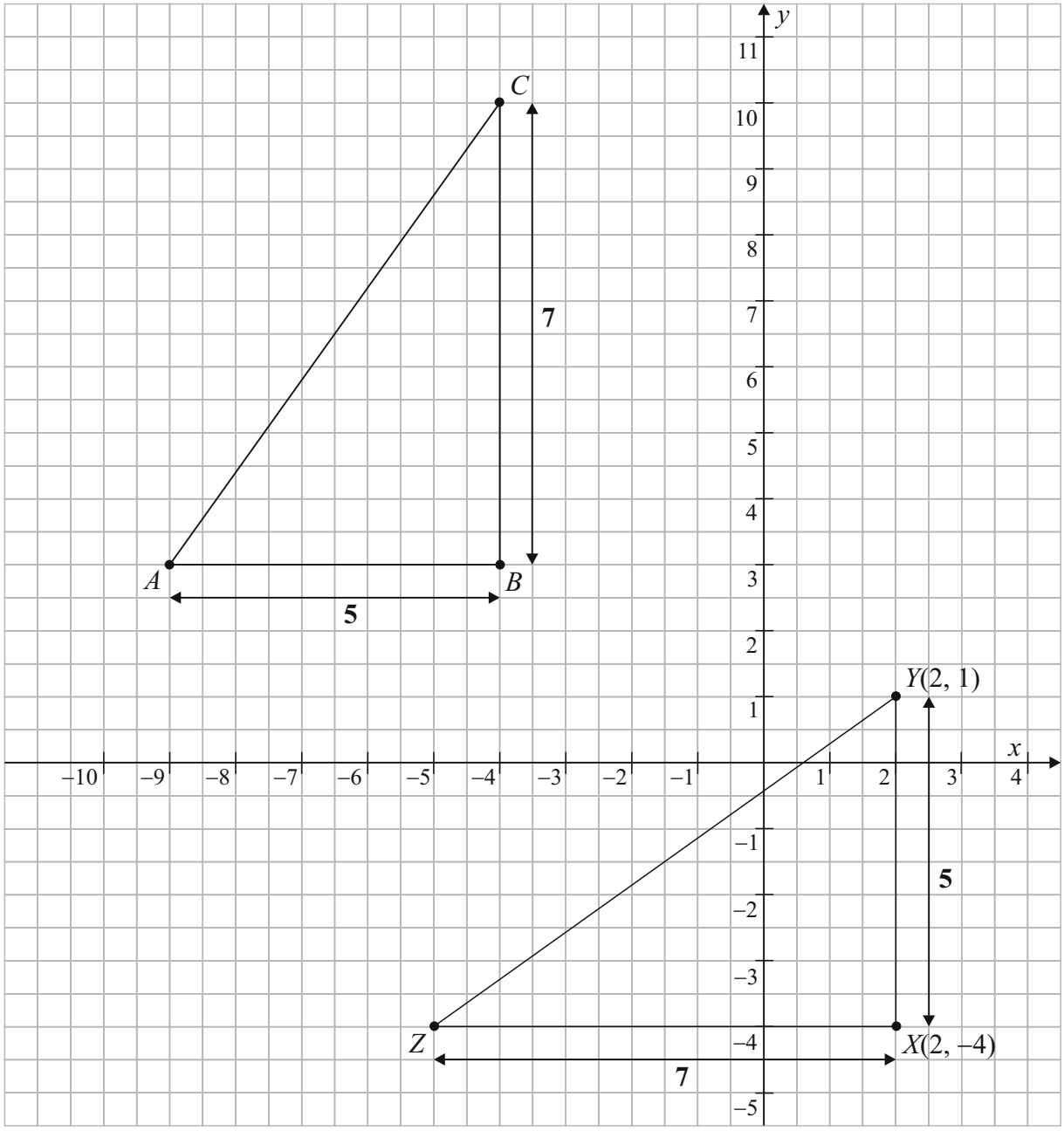
Question 4 (a) (ii) [Scale 10C (0, 4, 7, 10)]

4: • Writes a correct formula for the area of the triangle
• Any correct work towards the area of triangle calculation

7: • A fully correct substitution
• Calculates area with one incorrect substitution

10: • Accept correct answer without work

Question 4 (b) (i)



Question 4 (b) (ii)

$Z = (-5, -4)$

Triangles ABC and XYZ are congruent because (SAS):

$|AB| = |XY| = 5$

$|BC| = |XZ| = 7$

$|\angle ABC| = |\angle YXZ| = 90^{o}$

Marking Scheme Notes

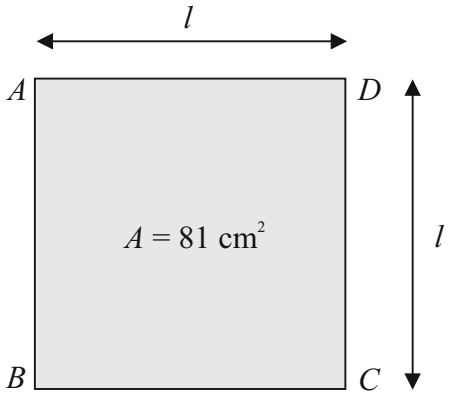
Question 4 (b) (i) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Relevant work in one section. e.g. Plots one or both points correctly
• Work of merit in at least one section

4: • One section correct

Question 5 (25 marks)

Question 5 (a)



$\text{Area} = l \times l = l^2 = 81 \text{ cm}^2$

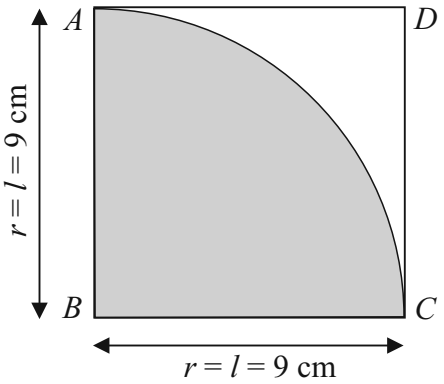
$|AD| = l = \sqrt{81} = 9 \text{ cm}$

Marking Scheme Notes
Question 5 (a) [Scale 5B (0, 3, 5)]
3: • Works to $x^2 = 81$ or $x = \sqrt{81}$ and stops
Note: A correct answer without work shown award full credit

Question 5 (b) (i)

The area of the sector shown is one-quarter the area of a full circle.

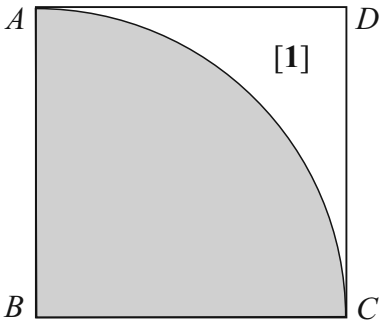
Area of a circle $A = \pi r^2$



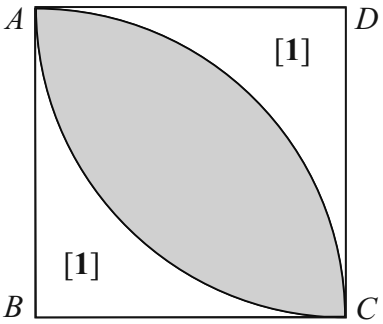
$|AC| = r = 9 \text{ cm}$

$\text{Area} = \frac{1}{4} \times \pi(9)^2 = 63{\cdot}6 \text{ cm}^2$

Question 5 (b) (ii)



Area of region [**1**]
= Area of square – Area of sector
= 81 – 63·6 = 17·4 cm²



Area of overlap
= Area of square – 2(Area of region [**1**])
= 81 – 2(17·4) = 46·2 cm²

Marking Scheme Notes
Question 5 (b) (i) [Scale 10C (0, 4, 7, 10)]
4: • Writes the correct formula for the sector area or area of circle
• Any correct or consistent substitution into formula
• Any work of merit e.g. $r = 9$ or Answer (a)
7: • Formula substituted correctly
• Calculates area of sector with one incorrect substitution
• Correct or consistent answer expressed in terms of π
10: • Accept correct answers without work

Question 5 (b) (ii) [Scale 5B (0, 3, 5)]
3: • Calculates a relevant area. e.g. Unshaded area worked as (81 – 63·64) or similar and stops
• Concept of shaded area by difference indicated but not fully developed
Note: Accept an answer consistent with previous work

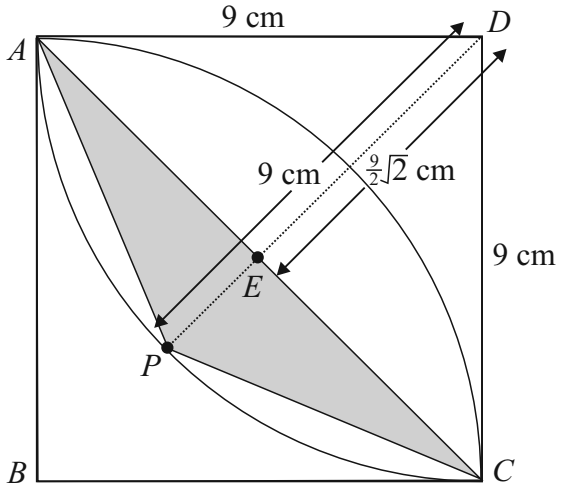
Question 5 (c)

E is the midpoint of $[AC]$, the point where the diagonals of the square intersect. Triangle ADC is a right-angled triangle. Apply Pythagoras' theorem.

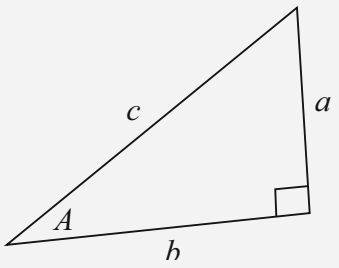
$$|AC|^2 = 9^2 + 9^2 = 81 + 81 = 162$$

$$\therefore |AC| = \sqrt{162} = 9\sqrt{2} \text{ cm}$$

$$|AE| = |ED| = \tfrac{9}{2}\sqrt{2} \text{ cm}$$



Formulae and Tables Book
Trigonometry (page 16)
Pythagoras' theorem



$$c^2 = a^2 + b^2$$

Area of $\Delta APC = \frac{1}{2}|AC||EP|$ Area $= \frac{1}{2}bh$

$$|AC| = 9\sqrt{2}$$

$$|EP| = 9 - \tfrac{9}{2}\sqrt{2}$$

$$\therefore \text{Area of } \Delta APC = \tfrac{1}{2}(9\sqrt{2})(9 - \tfrac{9}{2}\sqrt{2}) = 16 \cdot 8 \text{ cm}^2$$

Marking Scheme Notes
Question 5 (c) [Scale 5A (0, 5)]
5: • Correct answer
Note: Accept an answer consistent with previous work

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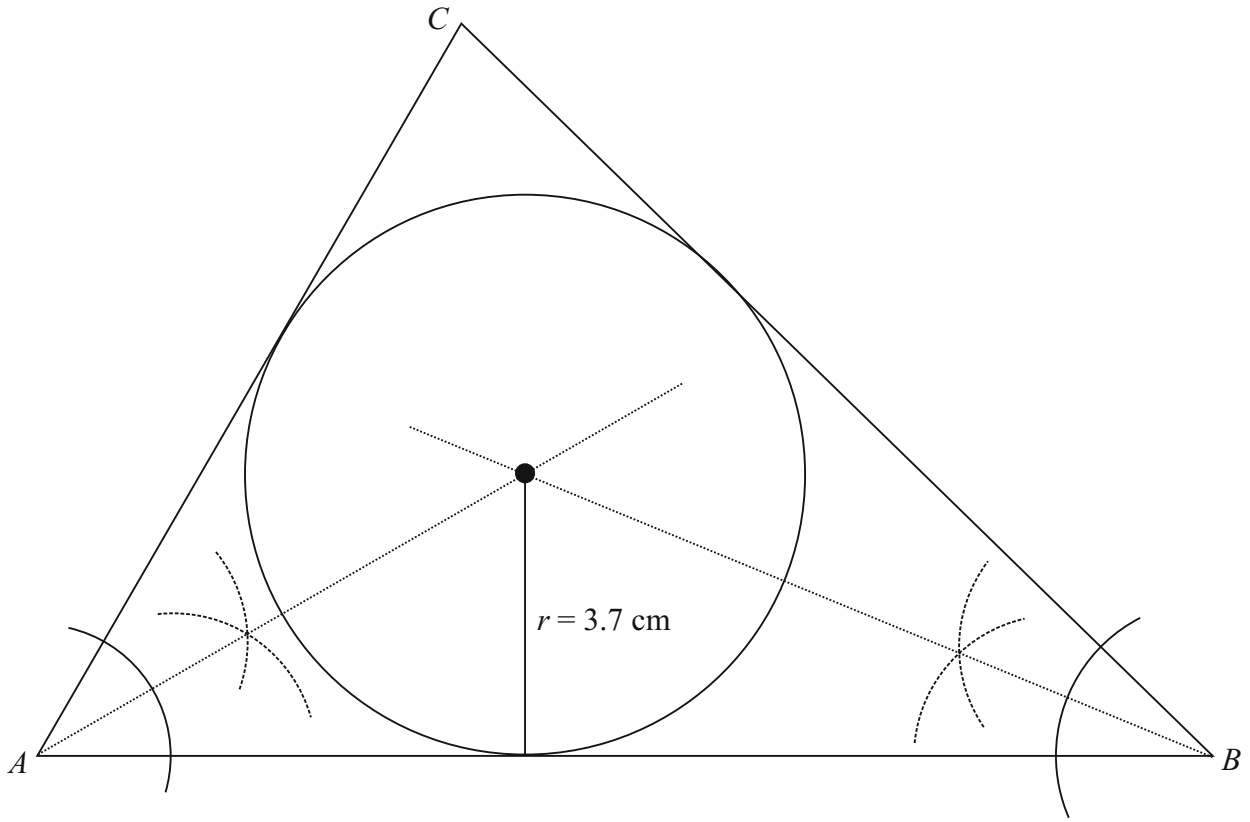
Question 6A (25 marks)

Question 6A (a) (i)



- Use your compass to bisect $\angle CAB$ and $\angle ABC$.
- The intersection of the angle bisectors is the incentre.
- Draw the incircle.

Question 6A (a) (ii)

Radius $r = 3{\cdot}7$ cm

Marking Scheme Notes

Question 6A (a) (i) [Scale 15B (0, 7, 15)]

7: • Any work of merit e.g. One arc drawn from a vertex

15: • An incircle constructed with at least one side of the triangle a tangent to the circle

Question 6A (a) (ii) [Scale 5B (0, 3, 5)]

3: • Accept length of radius in intervals $3{\cdot}3 \leq r < 3{\cdot}5$ and $3{\cdot}7 < r \leq 3{\cdot}9$

5: • Award full credit to radius in interval $3{\cdot}5 \leq r \leq 3{\cdot}7$

Question 6A (b)

$\angle MPN$ is a right angle as it stands on the diameter.

$|PM|^2 + |PN|^2 = |MN|^2$

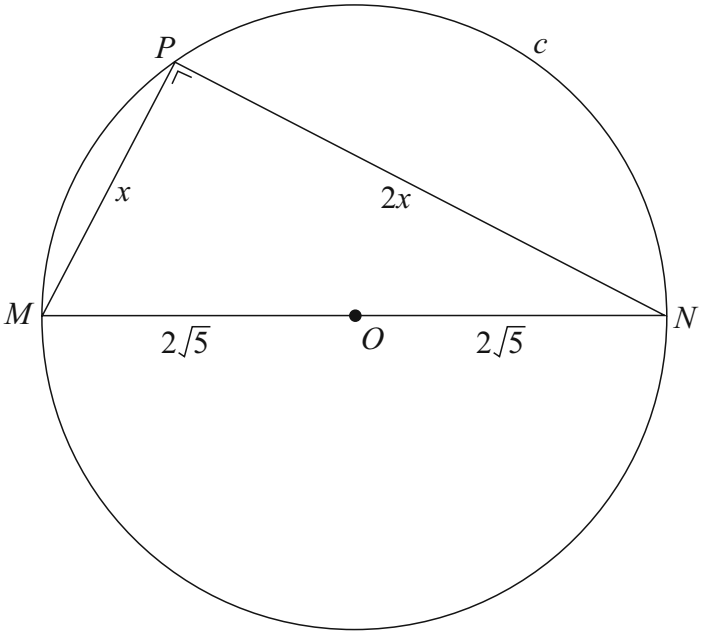
$x^2 + (2x)^2 = (4\sqrt{5})^2$

$x^2 + 4x^2 = 80$

$5x^2 = 80$

$x^2 = 16$

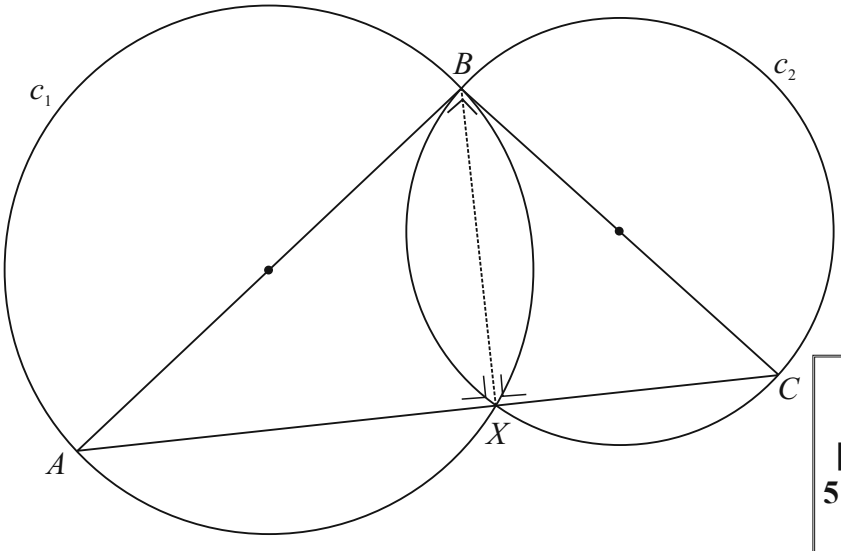
$x = \sqrt{16} = 4$ cm



Marking Scheme Notes
Question 6A (b) [Scale 5B (0, 3, 5)]

3: • Diameter calculated as $4\sqrt{5}$ or similar
• Any relevant work in the use of Pythagoras or recognition of angle standing on diameter at circumference measures 90°

Question 6B (25 marks)



$X \in c_1 \Rightarrow |\angle BXA| = 90^\circ$ [Angle standing on a diameter is a right angle]

$X \in c_2 \Rightarrow |\angle BXC| = 90^\circ$ [Angle standing on a diameter is a right angle]

$\therefore |\angle AXC| = 180^\circ$

$|\angle AXC|$ is a straight angle. Therefore, X is on the line AC.

Marking Scheme Notes
Question 6B
[Scale 25E (0, 5, 10, 15, 20, 25)]

5: • Any work of merit such as joining B to X
10: • Identifies diameter of c_1 or c_2, and one angle standing on semi-circle at circumference measuring 90°
15: • Identifies diameter of c_1 or c_2, and both angles standing on semi-circle at circumference measuring 90°
20: • Identifies $|\angle AXC| = 180^\circ$

Question 7 (70 marks)

Question 7 (a)

(i) Number of properties registered = 1·9 × 0·9 = 1·71 million

(ii) Owners refused to pay as a protest against Government austerity measures.

Question 7 (b)

There is 360° in a full circle (pie).
24·9% of homeowners have properties valued between €0 - €100 000.
The angle of the sector represented by the €0 - €100 000 band is 360° × 0·249 ≈ 90°. Round off all angles to the nearest degree.

€0 – €100 000 band:
360° × 0·249 ≈ 90°

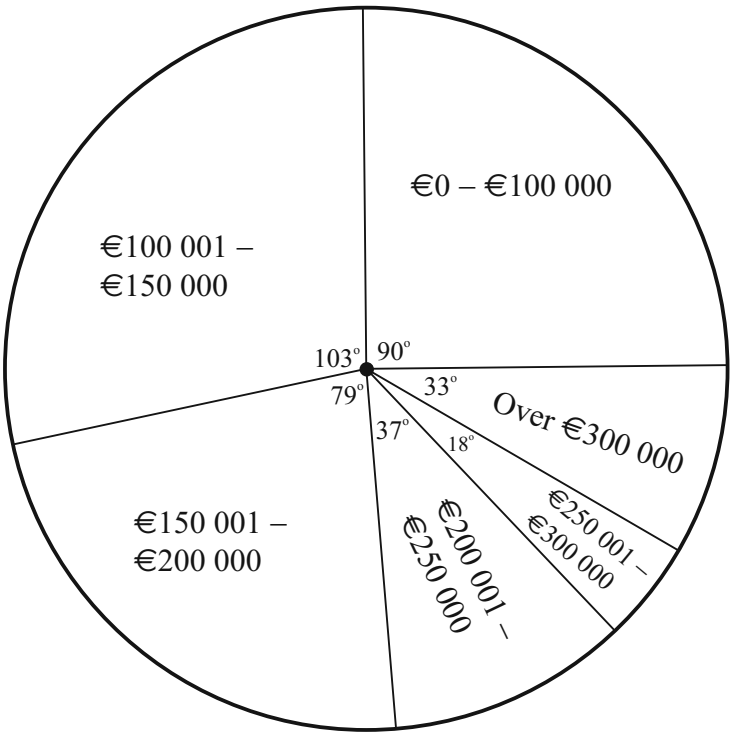
€100 001 – €150 000 band:
360° × 0·286 ≈ 103°

€150 001 – €200 000 band:
360° × 0·219 ≈ 79°

€200 001 – €250 000 band:
360° × 0·104 ≈ 37°

€250 001 – €300 000 band:
360° × 0·049 ≈ 18°

Over €300 000:
360° × 0·093 ≈ 33°



Marking Scheme Notes

Question 7 (a) (i) (ii) [Scale 10C (0, 4, 7, 10)]

4: • Any work of merit towards estimating the number of properties registered or suggesting a partially plausible reason. e.g. 90% expressed as 0·9

7: • One section correct

Question 7 (b)

Angle determination [Scale 5C (0, 3, 4, 5)]

3: • One sector angle correct
• More than one error in the calculation of sector angle
• Any work of merit. e.g. Correct work with percentage or use of 360

4: • One consistent error in the calculation of sector angle

Pie Chart [Scale 20B (0, 10, 20)]

10: • Any one sector indicated

Question 7 (c) (i)

Total number of registered homeowners $= 1 \cdot 71$ million $= 1\ 710\ 000$

€100 001 – €150 000 : Number of properties $= 1\ 710\ 000 \times 0 \cdot 249 = 425\ 790$

€100 001 – €150 000 : Number of properties $= 1\ 710\ 000 \times 0 \cdot 286 = 489\ 060$

€150 001 – €200 000 : Number of properties $= 1\ 710\ 000 \times 0 \cdot 219 = 374\ 490$

€200 001 – €250 000 : Number of properties $= 1\ 710\ 000 \times 0 \cdot 104 = 177\ 840$

€250 001 – €300 000 : Number of properties $= 1\ 710\ 000 \times 0 \cdot 049 = 83\ 790$

Over €150 000 : Number of properties $= 1\ 710\ 000 \times 0 \cdot 093 = 159\ 030$

Multiply the tax per property by the total number of properties to find the tax due.
€0 – €100 000: €45 × 425 790 = €19 160 550

Valuation Band	Tax per property	Number of properties	Total tax due (€)
€0 – €100 000	€45	425 790	19 160 550
€100 001 – €150 000	€112	489 060	**54 774 720**
€150 001 – €200 000	€157	**374 490**	**58 794 930**
€200 001 – €250 000	€202	**177 840**	**35 923 680**
€250 001 – €300 000	€247	**83 790**	**20 696 130**
Over €300 000	NA	**159 030**	NA

Marking Scheme Notes
Question 7 (c) (i)
Number of properties [Scale 5C (0, 3, 4, 5)]
3: • One correct entry.
4: • One consistent error in the calculation

Total tax due [Scale 10C (0, 4, 7, 10)]
4: • One correct entry
7: • One consistent error in the calculation

Question 7 (c) (ii)

Valuation Band	Tax per property	Number of properties	Total tax due (€)
€0 – €100 000	€45	425 790	**19 160 550**
€100 001 – €150 000	€112	489 060	**54 774 720**
€150 001 – €200 000	€157	374 490	**58 794 930**
€200 001 – €250 000	€202	177 840	**35 923 680**
€250 001 – €300 000	€247	83 790	**20 696 130**
Over €300 000	NA	159 030	NA

Total tax due = 19 160 550 + 54 774 720 + 58 794 930 + 35 923 680 + 20 696 130 = €189 350 010

Marking Scheme Notes
Question 7 (c) (ii) [Scale 5B (0, 3, 5)]
3: • Some addition of the correct values indicated. e.g. (19 160 550 + 54 774 720 + ….)
5: • Accept a correct or consistent answer without work

Question 7 (c) (iii)

Total tax due on those properties with a valuation over €300 000:
€241 000 000 – €189 350 010 = €51 649 990

Question 7 (c) (iv)

$$\text{Mean} = \frac{\text{Total tax}}{\text{Number of properties}} = \frac{51\,649\,990}{159\,030} \approx €325$$

Question 7 (c) (v)

Valuation Band	Tax per property	Number of properties	Total tax due (€)
€0 – €100 000	€45	425 790	19 160 550
€100 001 – €150 000	**€112**	**489 060**	**54 774 720**
€150 001 – €200 000	**€157**	**374 490**	**58 794 930**
€200 001 – €250 000	€202	177 840	35 923 680
€250 001 – €300 000	€247	83 790	20 696 130
Over €300 000	NA	159 030	NA

Find 20% of 489 060 properties: 489 060 × 0·2 = 97 812
€100 001 – €150 000: New number of properties in this band = 489 060 – 97 812 = 391 248
Tax take = 391 248 × €112 = €43 819 776

€150 001 – €200 000: New number of properties in this band = 374 490 + 97 812 = 472 302
Tax take = 472 302 × €157 = €74 151 414

New tax take = €43 819 776 + €74 151 414 = €117 971 190
Old tax take = €54 774 720 + €58 794 930 = €113 569 650
Extra tax = €117 971 190 – €113 569 650 = €4 401 540

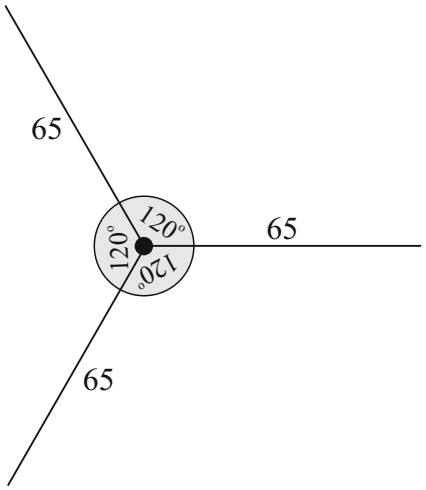
Marking Scheme Notes
Question 7 (c) (iii) [Scale 5B (0, 3, 5)]
3: • Writes 241 000 000 or Answer c(ii)
5: • Accept a correct or consistent answer without work

Question 7 (c) (iv) [Scale 5B (0, 3, 5)]
3: • Work of merit with any of the values [1 550 970, 159 030, 51 649 990]

Question 7 (c) (v) [Scale 5B (0, 3, 5)]
3: • Any work of merit in finding the number of properties changing band
e.g. (0·2)(489 060)
• Identifies the difference in the band valuation. i.e. (157 – 112)
• Fully correct substitution. i.e. [(0·2)(489 060) × 45]

Question 8 (50 marks)

Question 8 (a) (i)



Angle between two blades $= \frac{360^o}{3} = 120^o$

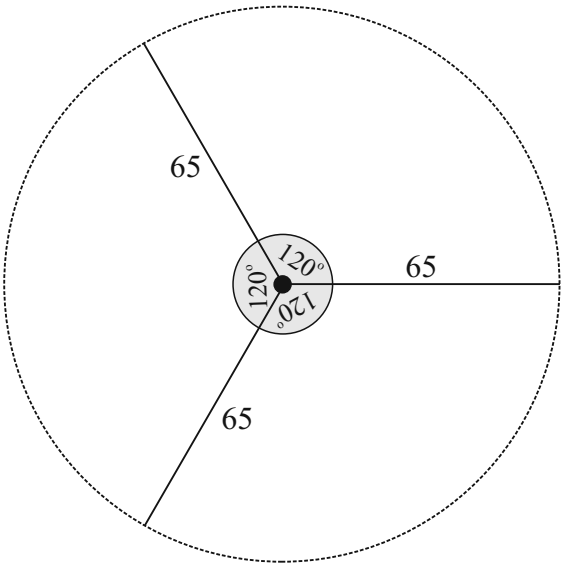
Marking Scheme Notes

Question 8 (a) (i) [Scale 10B (0, 5, 10)]

5: • Any work of merit. e.g. Joins tips of blades or draws circle on diagram

10: • Accept a correct answer without work (240°)

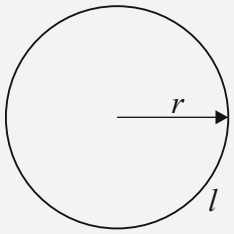
Question 8 (a) (ii)



Formulae and Tables Book

Length and area:

Circle [page 8]



$l = 2\pi r$

$A = \pi r^2$

Area $= \pi(65)^2 \approx 13\,273$ m^2

Marking Scheme Notes

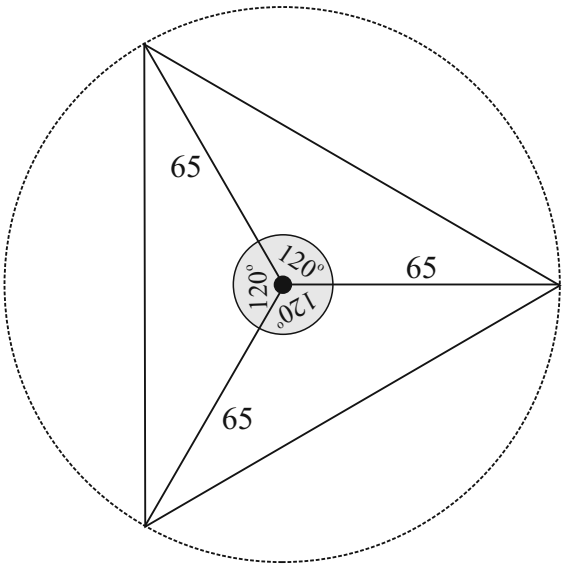
Question 8 (a) (ii) [Scale 10C (0, 4, 7, 10)]

4: • Writes the correct formula.
• Identifies the radius correctly ($r = 65$)

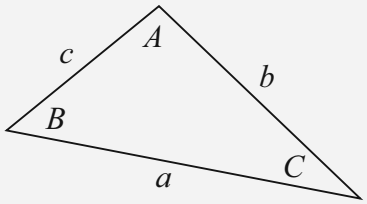
7: • A fully correct substitution
• Calculates the area of circle with one incorrect substitution
• Answer left in terms of π

10: • Accept a correct answer without work

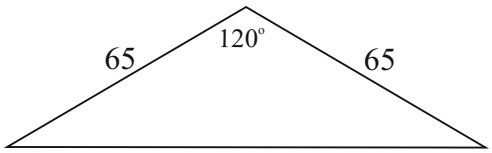
Question 8 (a) (iii)



Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Area: $\frac{1}{2}ab\sin C$



Area $= 3(\frac{1}{2}(65)(65)\sin 120^\circ) \approx 5488\ \text{m}^2$

Question 8 (a) (iv)

There are 15 rotations per minute, 60 minutes in an hour, 24 hours in a day, 365 days in a year, 25 years in a lifetime and it operates 31% of the time.

Number of rotations = $15 \times 60 \times 24 \times 365 \times 25 \times 0{\cdot}31 = 61\ 101\ 000 = 6{\cdot}11 \times 10^7$

Calculator [Casio *fx-85GT PLUS*]
Input $15 \times 60 \times 24 \times 365 \times 25 \times 0{\cdot}31$
Press =
Press Shift Mode
Press 7: Sci
Press 3

Marking Scheme Notes

Question 8 (a) (iii) [Scale 5C (0, 3, 4, 5)]

3: • Writes a correct formula for area of a triangle
• Some correct substitution into a correct relevant formula [Radius = 65, $\theta = 120^\circ$]
• Any work of merit such as a triangle drawn in this section with the tips of blades joined

4: • Fully correct or consistent substitution e.g. $3(\frac{1}{2}\times 65^2\sin 120^\circ)$
• Calculates the area of the triangle with one incorrect substitution
• Incorrect calculator mode (Once only) [Rad = 3680, Grad = 6027]

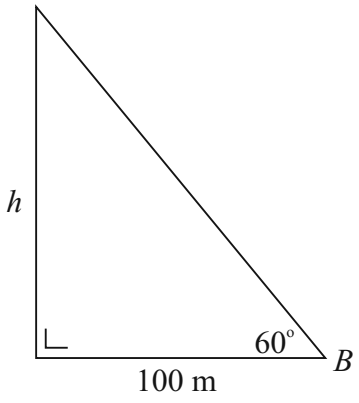
5: • Accept a correct answer without work

Question 8 (a) (iv) [Scale 10B (0, 5, 10)]

5: • Any work of merit in the determination of working life or number of rotations
e.g. 25(0·31) or 365(24)(60)15

10: • A correct answer

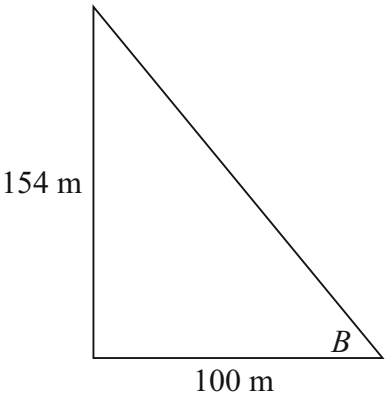
Question 8 (b)



$$\tan 60^\circ = \frac{h}{100}$$

$$\therefore h = 100 \tan 60^\circ \approx 173 \text{ m}$$

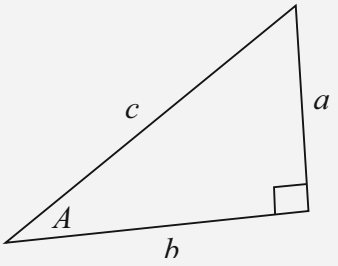
Question 8 (c) (i)



$$\tan B = \frac{154}{100}$$

$$\therefore B = \tan^{-1}\left(\frac{154}{100}\right) \approx 57^\circ$$

Formulae and Tables Book
Trigonometry: Right-angled triangle [page 16]



$$\sin A = \frac{a}{c},\ \cos A = \frac{b}{c},\ \tan A = \frac{a}{b}$$

Question 8 (c) (ii)

Formula: % errors

Absolute error in a quantity = |Measured value of quantity – Accepted value of quantity|

$$\text{Percentage error in a quantity} = \frac{\text{Absolute error in quantity}}{\text{Accepted value of quantity}} \times 100\%$$

$$\%\text{ error} = \left(\frac{60^\circ - 57^\circ}{57^\circ}\right) \times 100\% \approx 5 \cdot 3\%$$

Marking Scheme Notes

Question 8 (b) [Scale 5C (0, 3, 4, 5)]

3: • Any correct relevant work such as a trigonometric ratio or the Sine Rule identified.
• Any correct additional information identified in given triangle

4: • Correct substitution. i.e. [$\tan 60^\circ = \frac{h}{100}$]
• Calculates the height with one incorrect substitution

5: • Accept a correct answer without work

Question 8 (c) (i) [Scale 5C (0, 3, 4, 5)]

3: • Any work of merit involving Pythagoras Theorem, tan ratio or identification of a right angled triangle

4: • Correct substitution into formula or $|\angle B| = \tan^{-1}(\frac{154}{100})$.
• Calculates $|\angle B|$ with one incorrect substitution

5: Accept a correct answer without work

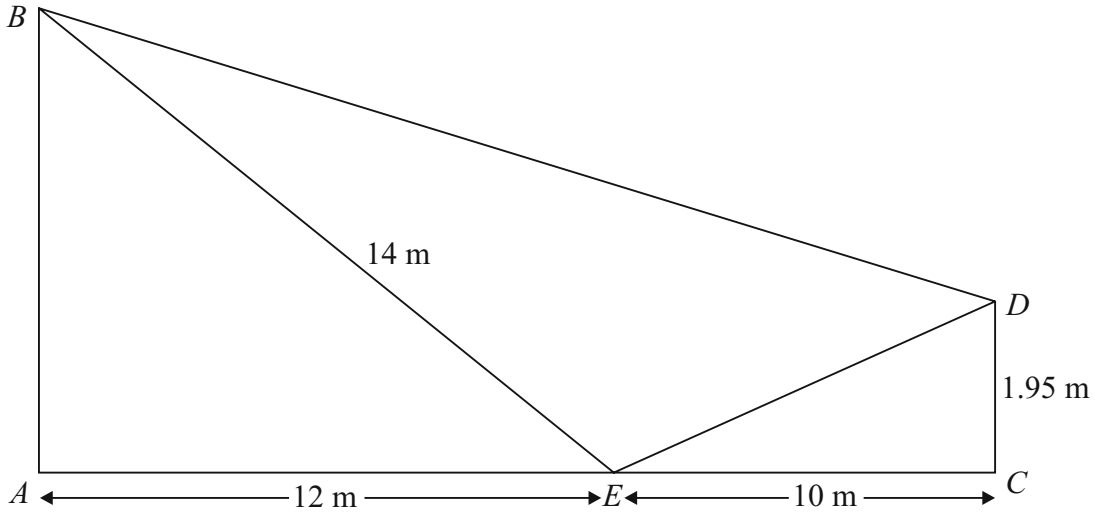
Question 8 (c) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Any work of merit. e.g. Calculates the size of the error or writes % error formula

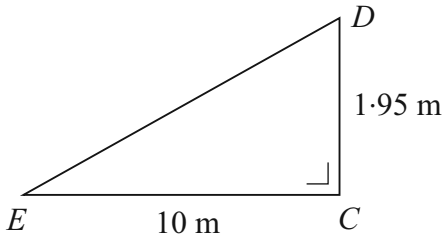
4: • A fully correct substitution $(\frac{3}{57} \times 100)$.
• Calculates the % error with one incorrect substitution

EPL 104

Question 9 (30 marks)



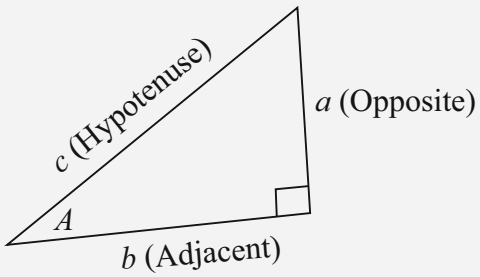
Question 9 (a) (i)



$|ED|^2 = 10^2 + 1{\cdot}95^2$

$\therefore |ED| = \sqrt{10^2 + 1{\cdot}95^2} \approx 10{\cdot}2 \text{ m}$

Formulae and Tables Book
Trigonometry: Right-angled triangle [page 16]



$$\sin A = \frac{a}{c},\ \cos A = \frac{b}{c},\ \tan A = \frac{a}{b}$$

$$c^2 = a^2 + b^2$$

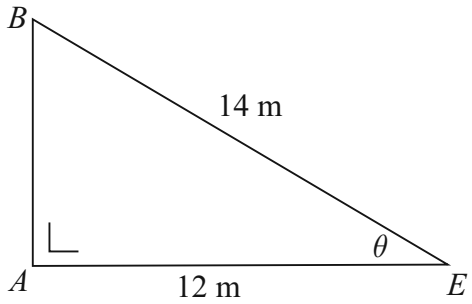
Marking Scheme Notes
Question 9 (a) (i) [Scale 10C (0, 4, 7, 10)]

4: • Any relevant work e.g. A correct relevant formula written e.g. Pythagoras, Cosine Rule, Sine Rule or part calculation $(10)^2 = 100$
Note: Accept use of information from (b) (i) in this part i.e. $|\angle CED| = 11^\circ$.

7: • A fully correct substitution
• Calculates $|ED|$ with one error in substitution

10: • Accept a correct answer without work

Question 9 (a) (ii)



Call $|\angle AEB|$ the angle θ.

$$\cos\theta = \frac{12}{14} \Rightarrow \theta = \cos^{-1}\left(\frac{12}{14}\right) \approx 31^\circ$$

Question 9 (a) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Indicates correct relevant information on the diagram
• Correct trigonometric ratio set up or similar work of merit

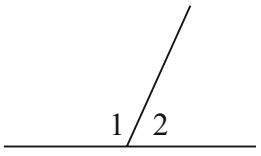
4: A fully correct substitution
• Calculates $|\angle AEB|$ with one error in substitution

10: • Accept a correct answer without work

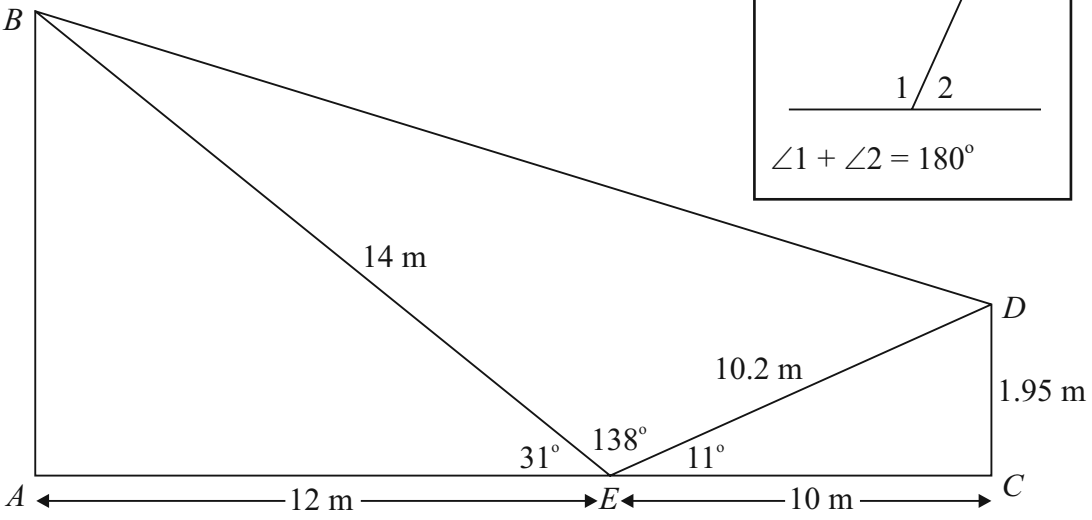
Question 9 (b) (i)

$|\angle DEB| = 180^o - 31^o - 11^o = 138^o$ [Straight angle (180^o)]

STRAIGHT ANGLE



$\angle 1 + \angle 2 = 180^o$



MARKING SCHEME NOTES

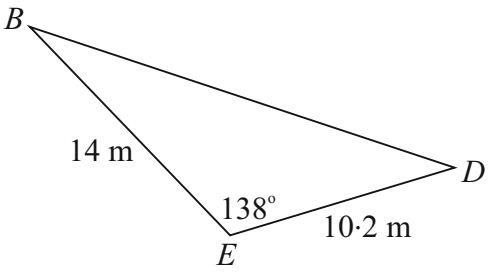
Question 9 (b) (i) [Scale 10C (0, 4, 7, 10)]

4: • Indicates correct or consistent relevant information on the diagram
• e.g. $|\angle CED| = 11^o$, $|\angle AEB| = 31^o$
• Any work of merit. e.g. Total angle measure in a straight line = 180°

7: • A fully correct or consistent substitution. i.e. 180° − (11° + 31°)
• Calculates $|\angle DEB|$ with one error in substitution

NOTE: Accept an answer consistent with previous work [See (a)(ii)]

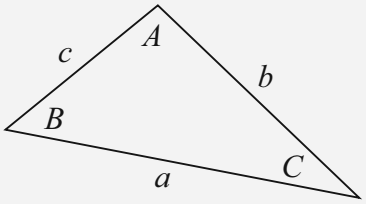
Question 9 (b) (ii)



$|BD|^2 = 14^2 + 10 \cdot 2^2 - 2(14)(10 \cdot 2)\cos 138^o$

$|BD| = \sqrt{14^2 + 10 \cdot 2^2 - 2(14)(10 \cdot 2)\cos 138^o} = 22 \cdot 6$ m

FORMULAE AND TABLES BOOK
Trigonometry of the triangle:
Cosine rule [page 16]



$a^2 = b^2 + c^2 - 2bc\cos A$

EPL 104

Marking Scheme Notes

Question 9 (b) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Indicates correct or consistent relevant information on the diagram.
• Identification or use of relevant trigonometric formula. e.g. Cosine Rule

4: • A fully correct or consistent substitution. i.e. $|DB|^2 = 14^2 + 10 \cdot 2^2 - 2(14)(10 \cdot 2)\cos 138^\circ$
• Calculates *BD* with one error in substitution
• Incorrect calculator mode. (once only)
Note: Accept an answer consistent with previous work

LC 2013 (Set F): Paper 1

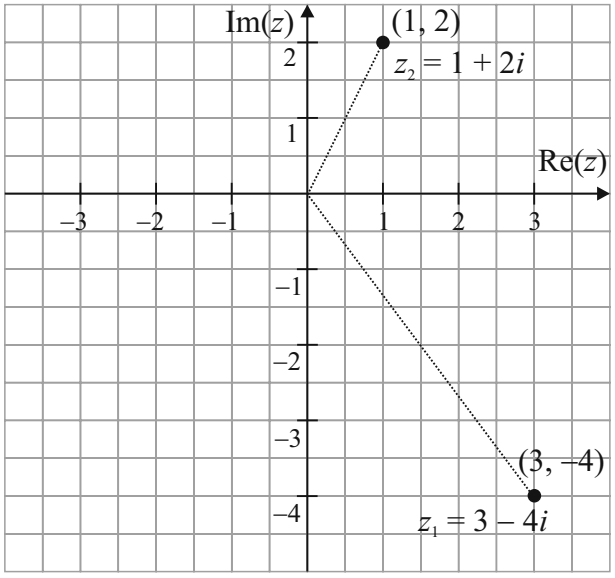
Question 1 (25 marks)

Question 1 (a)

$z_1 = 3 - 4i$

$z_2 = 1 + 2i$

z_1 and z_2 are plotted on the graph shown.



Question 1 (b)

Answer: Yes

Reason: The distance from the origin to z_1 is greater than the distance from the origin to z_2.

Marking Scheme Notes

Question 1 (a) [Scale 5C (0, 2, 4, 5)]

2: • A correct real or imaginary part plotted for either point
• Both numbers plotted correctly but real and imaginary axes interchanged

4: • Plots one point correctly

Question 1 (b) [Scale 5C (0, 2, 4, 5)]

2: • Correct answer given without a reason or reason given is without merit
• Answer not given, or incorrect, but some merit in the reason such as reference to distance

4: • Correct answer with a reason given that refers to distance
• A fully correct reason without an answer given

Question 1 (c)

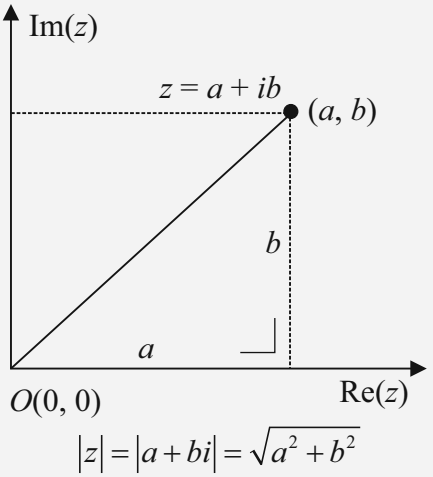
$$|z_1| = |3 - 4i|$$
$$= \sqrt{3^2 + (-4)^2}$$
$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

$$|z_2| = |1 + 2i|$$
$$= \sqrt{1^2 + 2^2}$$
$$= \sqrt{1 + 4} = \sqrt{5}$$

$$5 > \sqrt{5} \Rightarrow |z_1| > |z_2|$$

Formula: Complex Numbers

Modulus $|z|$



$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

EPL 104

Marking Scheme Notes
Question 1 (c) [Scale 5C (0, 2, 4, 5)]
2: • Correct modulus formula written.
• Each point joined to origin.
• Use of z_1 or z_2.
4: • One modulus value calculated correctly.
• Correct substitution into formula for both numbers but neither value correct.
Note: Both modulus values correct, without comparison shown, award full credit.

Question 1 (d)

$\frac{z_1}{z_2} = \frac{3-4i}{1+2i}$ [Multiply above and below by the conjugate of the denominator.]

$$= \frac{3-4i}{1+2i} \times \frac{1-2i}{1-2i}$$
$$= \frac{3-6i-4i+8i^2}{1-2i+2i-4i^2}$$
$$= \frac{3-10i-8}{1+4} = \frac{-5-10i}{5}$$
$$= -1-2i$$

Formula: Complex Numbers
Conjugates $\overline{z}$
$z = a+bi \Rightarrow \overline{z} = a-bi$
Multiplying a complex number by its conjugate:
$z\overline{z} = (a+bi)(a-bi) = a^2+b^2$

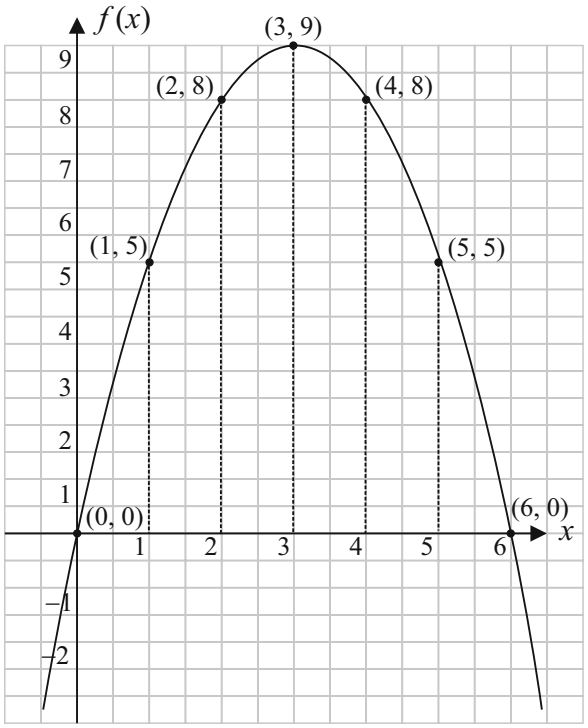
Marking Scheme Notes
Question 1 (b) [Scale 10C* (0, 3, 7, 10)]
3: • Correct substitution for z_1 and/or z_2
7: • Multiplication above and below by correct conjugate shown and some subsequent work
* Penalise one mark for not writing correct answer in the form $-1 - 2i$

Question 2 (25 marks)

Question 2 (a)

$f(x) = 6x - x^2$
$f(0) = 6(0) - 0^2 = 0 - 0 = 0$
$f(1) = 6(1) - 1^2 = 6 - 1 = 5$
$f(2) = 6(2) - 2^2 = 12 - 4 = 8$
$f(3) = 6(3) - 3^2 = 18 - 9 = 9$
$f(4) = 6(4) - 4^2 = 24 - 16 = 8$
$f(5) = 6(5) - 5^2 = 30 - 25 = 5$
$f(6) = 6(6) - 6^2 = 36 - 36 = 0$

x	0	1	2	3	4	5	6
$f(x)$	0	5	8	9	8	5	0



Question 2 (b)

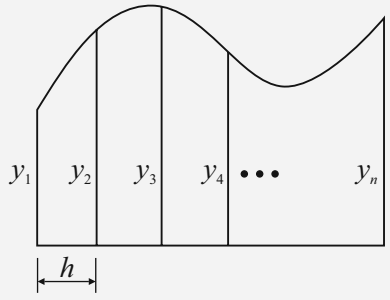
$h = 1$

$A = \frac{1}{2}[0 + 0 + 2(5 + 8 + 9 + 8 + 5)]$

$= \frac{1}{2}[2(35)]$

$= 35$

Formulae and Tables Book
Area approximations [page 12]



$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + ... + y_{n-1})]$$

Marking Scheme Notes
Question 2 (a) [Scale 10D (0, 2, 5, 8, 10)]
2: • One or two correct values only in the table
5: • Three or four correct values only in the table
8: • Five or six correct values in the table

Question 2 (b) [Scale 15C (0, 5, 10, 15)]
5: • Writes the correct trapezoidal rule, without further work of merit
• Simpson's rule with substantially correct substitution and calculation
10: • Trapezoidal rule with correct substitution
• Trapezoidal rule with incorrect substitution (maximum of 2) with correct calculation
• Correct answer without work shown
Note: Formula and correct answer, without work shown, award full credit

Question 3 (25 marks)
Question 3 (a)

Scientific Notation
$a \times 10^n, 1 \le a < 10, n \in \mathbb{Z}$
Ex. $2 \cdot 3 \times 10^{-5}$

149 597 871 km
= 149 597 871·0 km
= 1·49 597 871 × 100 000 000 km
= 1·49 597 871 × 10^8 km
≈ 1·5 × 10^8 km

Using your calculator
Input the number
Press **Shift** followed by **Mode**
Press **7**: Sci
Press **2** (two significant figures)
Press =

149 597 871 km $\approx 1 \cdot 5 \times 10^8$ km

Marking Scheme Notes
Question 3 (a) [Scale 5B* (0, 2, 5)]
2: • Either *a* (unrounded) or *n* correct
* Penalise one mark for incorrect or omitted round-off, provided full marks otherwise

Question 3 (b) (i)

	A	B	C	D	E	F	G
Number	2·1	$\sqrt{5}$	$\frac{243}{85}$	$\tan 70^{\circ}$	$\frac{3\pi}{4}$	250%	$\left(1+\frac{1}{10}\right)^{10}$
Decimal Number	2·10	2·24	2·86	2·75	2·36	2·50	2·59

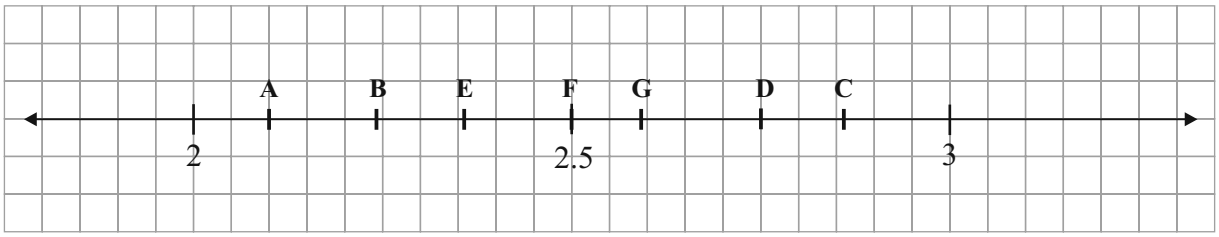
Using your calculator
Input the number
Press **Shift** followed by **Mode**
Press **6**: Fix
Press **2** (two decimal places)
Press =

Note: When you input the fraction $\frac{243}{85}$ and press the equal sign, the answer appears as a fraction.

Press the $S \Leftrightarrow D$ button to change this fraction to a decimal.

Press the $S \Leftrightarrow D$ button again to fix it to two decimal places.

Question 3 (b) (ii)



Marking Scheme Notes
Question 3 (b) (i) [Scale 5C (0, 2, 4, 5)]
2: • One or two numbers only (other than A) written correctly
4: • Three, four or five numbers only (other than A) written correctly

Question 3 (b) (ii) [Scale 5C (0, 2, 4, 5)]
2: • One or two numbers only plotted correctly
4: • Three or four numbers only plotted correctly

Question 3 (c)

$27^{2x} = 3^{x+10}$ ← Get the same base on each side.

$(3^3)^{2x} = 3^{x+10}$ $\boxed{(a^m)^n = a^{mn}}$

$3^{6x} = 3^{x+10}$ ← Equate the powers.

$\therefore 6x = x + 10$

$6x - x = 10$ ← Bring the x terms to the left.

$5x = 10$ ← Divide both sides by 2.

$x = \frac{10}{5} = 2$

Marking Scheme Notes
Question 3 (c) [Scale 10C (0, 3, 7, 10)]
3: • Some work of merit with indices
• Some work of merit in solving linear equation
• Solves the equation $2x = x + 10$ correctly
• Correct answer without work shown
7: • Correct equation in indices set up but fails to finish correctly
• Incorrect equation in indices but solves "equation of indices" correctly
Note: Correct answer by trial and improvement, shown for both sides, award full credit

Question 4 (25 marks)

Question 4 (a)

$R = (1 + 0 \cdot 015)^{12} \approx 1 \cdot 20$ ← Input these numbers into your calculator.

Question 4 (b)

The time period is in **months** as the **monthly** interest rate is given.

F = Future value
P = Present value = €800
i = Interest (monthly) = 0·015
t = Time = 15 months

$$F = P(1+i)^t$$
$$= 800(1 + 0 \cdot 015)^{15}$$
$$= €1000 \cdot 19$$

Formulae and Tables Book
Financial mathematics: Compound interest [page 30]

$$F = P(1 + i)^t$$

t = Time period (in years)
i = (Annual) rate of interest expressed as a decimal
F = Final value
P = Principal
Note: The time period can be months or weeks instead of years provided the interest rate is given for that time period.

Question 4 (c)

$€1 = £0 \cdot 8473$ ← Divide both sides by 0·8473 to find the value of £1 in euros.

$\frac{€1}{0 \cdot 8473} = £1$ ← Multiply both sides by 95 to find the value of £95 in euros.

$\frac{€1}{0 \cdot 8473} \times 95 \approx €112 \cdot 12 = £95$

Marking Scheme Notes

Question 4 (a) [Scale 5B* (0, 2, 5)]

2: • Some work of merit e.g. 1·015
• Multiplies by 12
• Accept $(1 \cdot 15)^{12} = 5 \cdot 3502...$ or $(1 \cdot 0015)^{12} = 1 \cdot 0181...$ with or without work shown
* Penalise one mark for incorrect or omitted round-off, provided full marks otherwise

Question 4 (b) [Scale 10C (0, 3, 7, 10)]

3: • Writes correct formula without further work of merit
• Some correct substitution into an incorrect formula, e.g. depreciation
• Calculates 1·5% of an amount correctly

7: • Correct substitution into correct formula but error in calculations
• Calculates the amount at the end of each of 15 months, but with minor errors

Question 4 (c) [Scale 10B* (0, 5, 10)]

5: • Writes $\frac{95}{0 \cdot 8473}$ and stops
• Gives an answer of €80·49 with or without work shown
Note: Correct answer without work shown, award full credit
* Penalise one mark for incorrect or omitted round-off, provided full marks otherwise

Question 5 (25 marks)

Question 5 (a)

$$y = 2x^3 - 3x^2 - 1$$

$$\frac{dy}{dx} = 6x^2 - 6x$$

> **Formulae and Tables Book**
> **Calculus: Derivatives** [page 25]
>
> $$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Question 5 (b)

Method 1: Multiply out the brackets and differentiate term by term.

$$y = (2x^2 + 3x + 1)(x^3 - x + 2)$$
$$= 2x^5 - 2x^3 + 4x^2 + 3x^4 - 3x^2 + 6x + x^3 - x + 2$$
$$= 2x^5 + 3x^4 - x^3 + x^2 + 5x + 2$$
$$\frac{dy}{dx} = 10x^4 + 12x^3 - 3x^2 + 2x + 5$$

Method 2: Use the product rule (this is not on the new course)

$$\frac{dy}{dx} = (2x^2 + 3x + 1)(3x^2 - 1) + (x^3 - x + 2)(4x + 3)$$
$$= 6x^4 - 2x^2 + 9x^3 - 3x + 3x^2 - 1 + 4x^4 + 3x^3 - 4x^2 - 3x + 8x + 6$$
$$= 10x^4 + 12x^3 - 3x^2 + 2x + 5$$

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$u = (2x^2 + 3x + 1) \Rightarrow \frac{du}{dx} = 4x + 3$$

$$v = (x^3 - x + 2) \Rightarrow \frac{dv}{dx} = 3x^2 - 1$$

Question 5 (c)

This is done using the quotient rule which is not on the new course.

$$\frac{dy}{dx} = \frac{(2x+5)3 - 3x(2)}{(2x+5)^2} = \frac{6x + 15 - 6x}{(2x+5)^2} = \frac{15}{(2x+5)^2}$$

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{15}{(2(0)+5)^2} = \frac{15}{5^2} = \frac{15}{25} = \frac{3}{5}$$

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$u = 3x \Rightarrow \frac{du}{dx} = 3$$

$$v = 2x + 5 \Rightarrow \frac{dv}{dx} = 2$$

> **Marking Scheme Notes**
> **Question 5 [Scale 25C (0, 8, 17, 25)]**
> This question required a range of skills, only some of which are on the syllabus for these candidates. Marking focuses only on the syllabus-related skills
> **8:** • Displays syllabus related skills, e.g. correct differentiation of one term only
> **17:** • Correct differentiation of two terms only
> • Correct coefficient for each term but error(s) in indices
> • Correct indices for each term but error(s) in coefficients
> **Note**: Consistently displays syllabus-related skills, e.g. three terms correctly differentiated, award full credit

Author's note: This question was a serious error by the SEC, including material from the old syllabus which is no longer on the new syllabus.

Question 6 (25 marks)

Question 6 (a)

$(x_1, y_1) = (2, 0), (x_2, y_2) = (-1, -3)$

$$m = \frac{-3-0}{-1-2} = \frac{-3}{-3} = 1$$

Question 6 (b) (i)

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

$f(x) = x^2 + 3x - 1$

$f'(x) = 2x + 3$

Question 6 (b) (ii)

$f'(-1) = 2(-1) + 3 = -2 + 3 = 1$

[The derivative at $x = -1$ is equal to the slope to the curve at $x = -1$.]

Question 6 (c)

Slope of $l_1 = m_1$
Slope of $l_2 = m_2$
$l_1 \perp l_2 \Rightarrow m_1 \times m_2 = -1$

Slope of $l_1 = m_1 = 1$

$\therefore$ Slope of $l_2 = m_2 = -1$

$f'(x) = m_2$

$2x + 3 = -1$

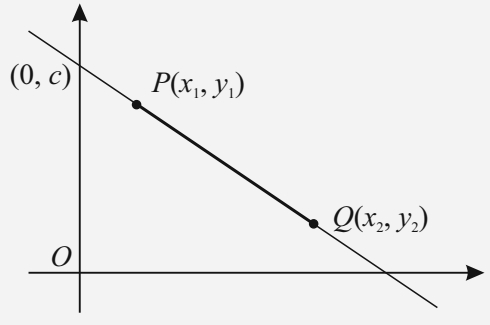
$2x = -4$

$\therefore x = -2$

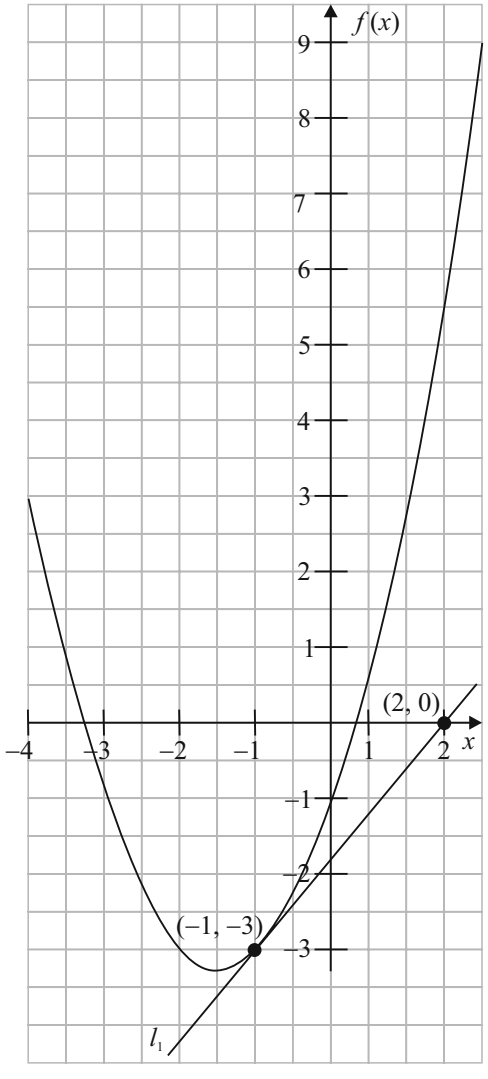
$f(-2) = (-2)^2 + 3(-2) - 1 = 4 - 6 - 1 = -3$

Point of contact: (–2, –3)

Formulae and Tables Book
Co-ordinate geometry: Line
Slope of PQ [page 18]



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



EPL 104

Marking Scheme Notes

Question 6 (a) [Scale 5C (0, 2, 4, 5)]

2: • Inverted slope formula with otherwise correct work
• Correct formula with errors in substitution

4: • Correct substitution with errors in simplification

Question 6 (b) (i) [Scale 10C (0, 3, 7, 10)]

3: • Correct differentiation of one term only

7: • Correct differentiation of two terms only
• Correct coefficient for each term but error(s) in indices
• Correct indices for each term but error(s) in coefficients

Question 6 (b) (ii) [Scale 5C (0, 2, 4, 5)]

2: • Shows $f(-1) = -3$
• Substitutes $x = -1$ into $f'(x)$ and stops
• Error in substitution into derivative

4: • Correct substitution into correct formula but error in calculations
• Correct substitution with errors in simplification

Question 6 (c) [Scale 5C (0, 2, 4, 5)]

2: • Attempt at finding slope of 2l
• Attempts to work with $f'(x)$
• Correct answer without work shown
• Finds the x co-ordinate only, graphically

4: • Finds the x co-ordinate only of the point of contact, using differentiation
• Correct answer obtained graphically

Question 7 (40 marks)

Question 7 (a)

Tank A: At 0 s there is a depth of 25 cm in the tank. Every 10 s the depth increases by 5 cm. Therefore, after 10 s the depth is 30 cm, after 20 s it is 35 cm and so on.

Tank B: At 0 s there is a depth of 10 cm in the tank. Every 10 s the depth increases by 7·5 cm. Therefore, after 10 s the depth is 17·5 cm, after 20 s it is 25 cm and so on.

Time (s)	0	10	20	30	40	50	60	70	80	90	100	110	120
Tank A (cm)	25	30	35	40	45	50	55	60	65	70	75	80	85
Tank B (cm)	10	17·5	25	32·5	40	47·5	55	62·5	70	77·5	85	92·5	100

Marking Scheme Notes

Question 7 (a) [Scale 10D (0, 2, 5, 8, 10)]

2: • Table for one tank with two correct entries

5: • Table for one tank correct
• Table for two tanks correct for 1 minute

8: • Four terms or less missing or incorrect, allowing for consistent terms
• Tables which begin with A(10) = 25 and B(10) = 10 and continue correctly
• Tables which begin with A(0) = 0 and B(0) = 0 and continue correctly
Note 1: Accept tables which are fully correct but do not include the values at $t = 0$
Note 2: Accept answers in (b), (c) and (d) based on candidate's tables

Question 7 (b)

Tank A:
After 120 s: depth = 85 cm;
After 130 s: depth = 90 cm;
After 140 s: depth = 95 cm;
After 150 s: depth = 100 cm = 1 m;
Time: 150 s = 2 minutes, 30 seconds

Tank B:
After 120 s: depth = 100 cm = 1 m
Time: 120 s = 2 minutes

Marking Scheme Notes
Question 7 (b) [Scale 5B* (0, 2, 5)]
2: • One correct answer.
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

Question 7 (c)

Tank A: What is the increase in depth every second? $\frac{5}{10}$ cm $= \frac{1}{2}$ cm

What is the depth after 20 seconds? $(25 + \frac{1}{2} \times 20)$ cm $=$ 35 cm

What is the depth after t seconds? $(25 + \frac{1}{2}t)$ cm

$$d = 25 + \tfrac{1}{2}t, \; [d = \text{depth (cm)}, \; t = \text{time (s)}]$$

Tank B: What is the increase in depth every second? $\frac{7{\cdot}5}{10}$ cm $= \frac{3}{4}$ cm

What is the depth after 20 seconds? $(10 + \frac{3}{4} \times 20)$ cm $=$ 25 cm

What is the depth after t seconds? $(10 + \frac{3}{4}t)$ cm

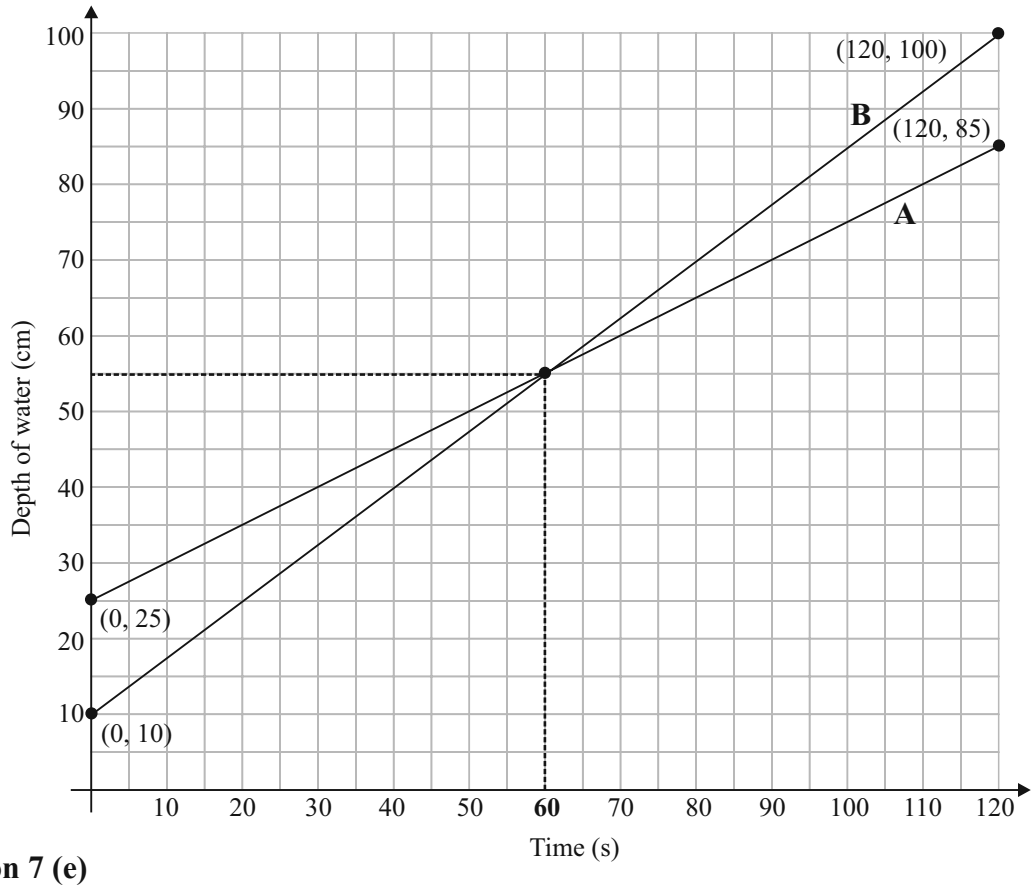
$$d = 10 + \tfrac{3}{4}t, \; [d = \text{depth (cm)}, \; t = \text{time (s)}]$$

Marking Scheme Notes
Question 7 (c) [Scale 5B (0, 2, 5)]
2: • Writes $T_n = a + (n - 1)d$
• Some work at setting up a formula

Question 7 (d)

Time (s)	**0**	10	20	30	40	50	60	70	80	90	100	110	**120**
Tank A (cm)	**25**	30	35	40	45	50	55	60	65	70	75	80	**85**
Tank B (cm)	**10**	17·5	25	32·5	40	47·5	55	62·5	70	77·5	85	92·5	**100**

The graphs are straight lines so you just need to pick 2 points from the table.
Tank A: (0, 25), (120, 85)
Tank B: (0, 10), (120, 100)



Question 7 (e)

You can find out from the graph the time when each tank has the same depth of water by finding out where the graphs intersect.

Answer: 60 seconds

Marking Scheme Notes

Question 7 (d) [Scale 10C (0, 3, 7, 10)]

3: • Correct point plotted for one line
• A line of some relevance drawn

7: • One graph fully correct or both graphs substantially correct
• Correct graphs drawn freehand
Note: Accept answers in (e) and (f) based on candidate's graph

Question 7 (e) [Scale 5B* (0, 2, 5)]

2: • Point of intersection identified
• Answer indicated on axis but value not written
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

Question 7 (f)

Tank A: $d = 25 + \frac{1}{2}t$

Tank B: $d = 10 + \frac{3}{4}t$

$\therefore 25 + \frac{1}{2}t = 10 + \frac{3}{4}t$

$25 - 10 = \frac{3}{4}t - \frac{1}{2}t$

$15 = \frac{1}{4}t$

$\therefore t = 60$ s

Or substitute $t = 60$ into each formula:

Tank A: $d = 25 + \frac{1}{2}t = 25 + \frac{1}{2}(60) = 55$ m

Tank B: $d = 10 + \frac{3}{4}t = 10 + \frac{3}{4}(60) = 55$ m

Marking Scheme Notes
Question 7 (f) [Scale 5C (0, 2, 4, 5)]
2: • Formulae written but not equated
4: • Error in solving equation
• Substitutes correctly into one formula only

Question 8 (60 marks)

Question 8 (a)

Year	1	2	3	4	5	6
Eoin's salary (€)	20 000	20 500	21 000	21 500	22 000	22 500
Peter's salary (€)	17 000	18 250	19 500	20 750	22 000	23 250

Question 8 (b)

Year	1 (2005)	2 (2006)	3 (2007)	4 (2008)	**5 (2009)**	6 (2010)
Eoin's salary (€)	20 000	20 500	21 000	21 500	**22 000**	22 500
Peter's salary (€)	17 000	18 250	19 500	20 750	**22 000**	23 250

Answer: 2009

Marking Scheme Notes
Question 8 (a) [Scale 5D (0, 4, 7, 11, 15)]
4: • One correct entry
7: • Errors in three or more terms, allowing for consistent errors
11: • Errors in two or less terms, allowing for consistent errors

Question 8 (b) [Scale 5A (0, 5)]
Note: Accept answers based on candidate's work

Question 8 (c)

(i) An arithmetic sequence is one in which the difference between any two successive terms is a constant.

(ii) Yes. A constant amount is added to his salary in any year to give his salary for the following year.

Marking Scheme Notes
Question 8 (c) (i) [Scale 5B (0, 2, 5)]
2: • Shows some understanding of an arithmetic sequence

Question 8 (c) (ii) [Scale 5B (0, 2, 5)]
2: • Correct answer without valid explanation
• Correct explanation without giving an answer

Question 8 (d)

$a = 20\,000$ ← This is the first term.
$d = 500$ ← This is the common difference.

$T_n = a + (n-1)d$
$= 20\,000 + (n-1)500$ ← Multiply out brackets.
$= 20\,000 + 500n - 500$
$= 19\,500 + 500n$ ← This is the general term.

Formulae and Tables Book
Sequences and series:
Arithmetic series or sequence [page 22]

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

a is the first term
d is the common difference

Marking Scheme Notes
Question 8 (d) [Scale 5B (0, 2, 5)]
2: • Correctly identifies a or d for Eoin's salary
• Correctly identifies a and d for Peter's salary
• Writes $T_n = a + (n-1)d$

Question 8 (e)

Year 2015: $n = 11$
$T_{11} = ?$
$T_n = 19\,500 + 500n$
$T_{11} = 19\,500 + 500(11) = €25\,000$

Question 8 (f)

$a = 17\,000$
$d = 1250$
$S_n = ?$

$S_n = \frac{n}{2}[2(17\,000) + (n-1)1250]$
$= \frac{n}{2}[34\,000 + 1250n - 1250]$
$= \frac{n}{2}[32\,750 + 1250n]$
$= n[16\,375 + 625n]$
$= 625n^2 + 16\,375n$

Marking Scheme Notes
Question 8 (e) [Scale 10C (0, 3, 7, 10)]
3: • Finds the correct value of n and stops
• Continues the pattern for one more term
• Attempts substitution into candidate's formula
• Some merit in work with the general term
7: • Continues pattern correctly but stops at the tenth year
• Substitutes $n = 10$ into formula and calculates answer

Question 8 (f) [Scale 5C (0, 2, 4, 5)]
3: • Writes $S_n = \frac{n}{2}[2a + (n-1)d]$ or other valid formula
• Identifies a or d for Peter's salary
• Identifies a or d for Eoin's salary
7: • A correct formula substituted, except for one error, for Peter
• A correct formula applied to Eoin's data, correctly

Question 8 (g)

$S_n = 625n^2 + 16375n$

$S_{11} = 625(11)^2 + 16375(11) = €255\ 750$

Question 8 (h)

The graph shows Peter's salary increasing constantly throughout the year which is not true. Peter's salary increases in steps at the end of each year and the graph does not reflect this.

Marking Scheme Notes

Question 8 (g) [Scale 5C (0, 2, 4, 5)]

Note: Accept candidate's answers from (f).

2: • Finds the correct value of n and stops.
• Continues the pattern for one more term
• Attempts substitution into candidate's formula
• Some merit in work with the S_n formula
• Uses Eoin's data incorrectly

4: • Continues the pattern correctly to 2015 but doesn't add the terms
• Continues the pattern incorrectly (stops at $n = 10$ or one error with consistent terms) but adds the terms
• Substitutes $n = 10$ into formula and calculates answer
• Substitutes correctly into formula but doesn't calculate answer
• Uses Eoin's data correctly

Question 8 (h) [Scale 5B (0, 2, 5)]

2: • A reason with some merit

Question 9 (50 marks)

Question 9 (a)

Production cost: $x = 20$

$C(x) = 5x^2 + 750x + 3000$

$C(20) = 5(20)^2 + 750(20) + 3000 = €20\ 000$

Total income: $R(x) = 1200x$

$R(20) = 1200(20) = €24\ 000$

Question 9 (b)

Profit: $R(20) - C(20) = €24\ 000 - €20\ 000 = €4000$

Marking Scheme Notes

Question 9 (a) [Scale 15C (0, 5, 10, 15)]

5: • Some correct substitution into either function

10: • Substantially correct substitution and calculation

Question 9 (b) [Scale 10B (0, 5, 10)]

Note: Accept candidate's work from previous section

5: • Any correct relevant step

Question 9 (c)

Call the profit function, $P(x)$. Subtract the cost function $C(x)$ from the revenue function $R(x)$ to find the profit function $P(x)$.

$$P(x) = R(x) - C(x)$$
$$= 1200x - (5x^2 + 750x + 3000) \quad \leftarrow \text{Multiply out brackets.}$$
$$= 1200x - 5x^2 - 750x - 3000 \quad \leftarrow \text{Add like terms.}$$
$$= -5x^2 + 450x - 3000$$

Marking Scheme Notes
Question 9 (c) [Scale 5B (0, 2, 5)]
2: • Relevant work such as writing $R(x) - C(x)$
• Mishandles subtraction

Question 9 (d)

Find the derivative of $P(x)$. Put the derivative equal to zero and solve for x. This value of x yields the maximum profit.

$$P(x) = -5x^2 + 450x - 3000$$
$$P'(x) = -10x + 450$$
$$P'(x) = 0 \Rightarrow -10x + 450 = 0$$
$$450 = 10x$$
$$\therefore x = 45$$

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Marking Scheme Notes
Question 9 (d) [Scale 5C (0, 2, 4, 5)]
2: • Relevant trial and improvement work
• Differentiation with error(s) and stops
4: • Correct differentiation without further work
• Substantially correct differentiation with attempt to solve $P'(x)$.

Question 9 (e)

$$P(x) = -5x^2 + 450x - 3000$$
$$x = 45:$$
$$P(45) = -5(45)^2 + 450(45) - 3000$$
$$= €7125$$

Question 9 (f)

$$C(x) = 5x^2 + 750x + 3000 = 11\,000$$
$$5x^2 + 750x - 8000 = 0$$
$$x^2 + 150x - 1600 = 0$$
$$(x - 10)(x + 160) = 0$$
$$\therefore x = 10$$

Marking Scheme Notes
Question 9 (e) [Scale 5B (0, 2, 5)]
Note: Accept candidate's answer from previous section
2: • Attempts to calculate $P(45)$
• Trial and improvement work, different from part (d)

Question 9 (f) [Scale 10D (0, 2, 5, 8, 10)]
2: • Recognises $C(x) = 11\,000$
• Substitutes a value for x into $C(x)$
5: • Substantially correct approach to solving quadratic equation
• Correct answer without any work
8: • Solves equation but fails to identify correct answer
• Substitutes $x = 10$ into $C(x)$ but fails to show answer is €11 000

LC 2013 (Set F): Paper 2

Question 1 (25 marks)

Question 1 (a)

Each player tosses more than 50% heads.

Question 1 (b)

Number of heads tossed = 109 + 238 + 291 = 638
Total number of tosses = 200 + 400 + 500 = 1100

$$P(\text{Event}) = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

$$P(\text{Head}) = \frac{\text{Number of heads tossed}}{\text{Total number of tosses}} = \frac{638}{1100} = 0{\cdot}58$$

Question 1 (c)

$P(\text{Head}) = 0{\cdot}58$

$P(\text{Tail}) = 1 - 0{\cdot}58 = 0{\cdot}42$

$P(\text{Head}$ **and then** Head **and then** $\text{Head})$

$= 0{\cdot}58 \times 0{\cdot}58 \times 0{\cdot}58 \approx 0{\cdot}195$

And/or
And: Multiply
P(A) and then P(B): $P(\text{A}) \times P(\text{B})$
Or: Add
P(A) or P(B): $P(\text{A}) + P(\text{B})$

$P(\text{Tail}$ **and then** $\text{Tail})$

$= 0{\cdot}42 \times 0{\cdot}42 \approx 0{\cdot}176$

$P(\text{Head}$ **and then** Head **and then** $\text{Head}) \neq P(\text{Tail}$ **and then** $\text{Tail})$

Joe's claim is not true.

Marking Scheme Notes

Question 1 (a) [Scale 5B (0, 3, 5)]

3: • States one or two players throw more heads than tails
• Gives an indication that 109 > 100 or 238 > 200

Question 1 (b) [Scale 10C (0, 4, 7, 10)]

4: • Finds the correct total number of heads or throws
• Calculates the correct probability for one player

7: • Finds the total number of heads and throws
• Calculates the probability using the totals but one element of the fraction incorrect
• Calculates the correct probability for each of the three players individually

Question 1 (c) [Scale 10C (0, 4, 7, 10)]

4: • Writes the correct probability for three heads or two tails
• Calculates the correct probability for three heads and two tails using a fair coin

7: • Calculates the correct probability for three heads and two tails but with an incorrect or omitted statement
• Fully correct answer using a fair coin

Question 2 (25 marks)

Question 2 (a)

(i) $P(\text{Green}) = \dfrac{\text{Number of green sectors}}{\text{Total number of sectors}} = \dfrac{2}{5}$

$$P(\text{Event}) = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

(ii) Possible outcomes of 3 successive spins of the spinner:
R: Red; G: Green
RRR
RRG, RGR, GRR
RGG, GRG, GGR
GGG

Question 2 (b)

GGG will yield €3 whereas RRR will yield nothing.

Player wins	€0	€1	€2	€3
Required outcomes	RRR	RRG RGR GRR	RGG GRG GGR	GGG

Question 2 (c)

Answer: **Yes**
A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: "success" or "failure".

Marking Scheme Notes
Question 2 (a) [Scale 10C (0, 4, 7, 10)]
4: • Relevant work in one section e.g. states two green sectors or gives GGG as a possible outcome
7: • One section correct
• Substantially correct work in both sections

Question 2 (b) [Scale 10C (0, 4, 7, 10)]
Note: Accept answers consistent with candidate's answers listed in (a)(ii)
4: • One correct or consistent outcome written
7: • One correct outcome for each category
• At least five outcomes in total from a (ii)

Question 2 (c) [Scale 5C (0, 3, 4, 5)]
3: • A correct response without an explanation of some merit
• An incorrect response but some merit in the explanation
4: • A correct response with one aspect of the explanation correct
• An incorrect response with a correct explanation

Question 3 (25 marks)

Question 3 (a)

Method 1: You can write the slope of a line from its equation written in the form $ax + by + c = 0$.

Finding the slope m from the equation of a line l:

$$l : ax + by + c = 0 \Rightarrow m = -\frac{a}{b}$$

$l : 3x + 2y + 18 = 0 \Rightarrow m = -\frac{3}{2}$

Method 2: You can write the equation of the line in the form $y = mx + c$ and read off m.

$3x + 2y + 18 = 0$

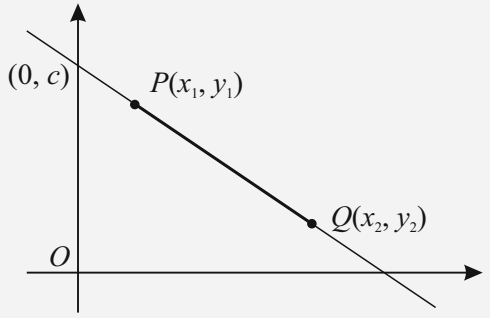
$2y = -3x - 18$

$y = -\frac{3}{2}x - 9$

$\therefore$ Slope $m = -\frac{3}{2}$

Formulae and Tables Book

Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

Question 3 (b)

To find the slope of a perpendicular line, invert the fraction and change the sign of the original slope.

Slope of l : $m_1 = -\frac{3}{2}$

Slope of k : $m_2 = \frac{2}{3}$

$l \perp k \Rightarrow m_1 \times m_2 = -1$ ← The product of perpendicular slope is –1.

Equation of k : $(x_1, y_1) = (7, 0)$, $m = \frac{2}{3}$

$y - 0 = \frac{2}{3}(x - 7)$

$y = \frac{2}{3}(x - 7)$ ← Multiply both sides of the equation by 3.

$3y = 2(x - 7)$ ← Multiply out brackets.

$3y = 2x - 14$ ← Bring all terms to the same side of the equation.

$0 = 2x - 3y - 14$

Marking Scheme Notes

Question 3 (a) [Scale 10C (0, 4, 7, 10)]

4: • Writes $y = mx + c$ and stops.
• Writes a relevant slope formula and stops
• Some correct work at transposing terms
• Finds one or two points on l

7: • Equation of the line written in the form $y = mx + c$ but slope not identified
• Finds the slope of l using two points, one of which is incorrect
Note: A correct answer without work shown, award full credit

Question 3 (b) [Scale 10C (0, 4, 7, 10)]

4: • Identifies correct relevant formula, e.g. $y - y_1 = m(x - x_1)$ and stops
• Writes correct or consistent slope for k
• Some correct work with (7, 0)

7: • Substitution into the line equation with one error
Note: A correct answer without work shown, award full credit

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Question 3 (c)

Solve the equations of l and k simultaneously to find the point of intersection (x, y).

$3x+2y+18=0....(\mathbf{1})(\times 3)$

$2x-3y-14=0....(\mathbf{2})(\times 2)$

$9x+6y+54=0$

$4x-6y-28=0$

$13x \quad +26=0 \Rightarrow 13x=-26 \Rightarrow x=-2$

$x=-2 : 3x+2y+18=0....(\mathbf{1})$

$3(-2)+2y+18=0$

$-6+2y+18=0$

$2y+12=0$

$2y=-12$

$y=-\frac{12}{2}=-6$

ANSWER: $(x, y) = (-2, -6)$

MARKING SCHEME NOTES

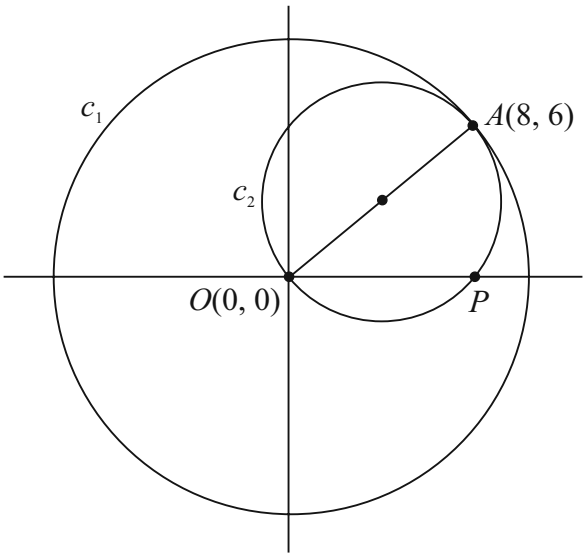
Question 3 (c) [Scale 5C (0, 3, 4, 5)]

3: • Relevant work towards equating coefficients in both equations
• Relevant work towards substituting for one variable using the other equation
• One line correctly graphed

4: • One correct or consistent co-ordinate found
• One or two algebraic errors leading to an incorrect point of intersection
• A correct structure to the solution with up to two errors
• A correct answer, obtained graphically

QUESTION 4 (25 MARKS)

Question 4 (a)



Find the radius r_1 of c_1 by finding the distance $|OA|$.

$O(0, 0) = (x_1, y_1),\ A(8, 6) = (x_2, y_2)$

$|OA| = r = \sqrt{(8-0)^2+(6-0)^2} = \sqrt{64+36} = \sqrt{100} = 10$

Equation of c_1:

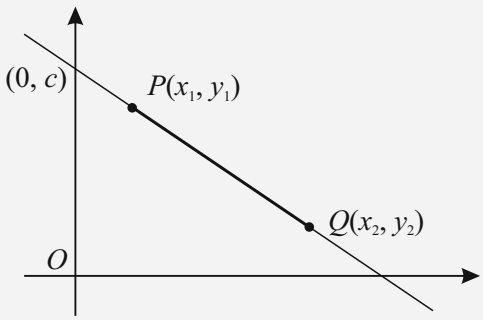
Centre $(0, 0)$, $r_1 = 10$ $\quad \boxed{x^2+y^2=r^2}$ Equation of circle with centre (0, 0).

$x^2+y^2=10^2$

$x^2+y^2=100$

FORMULAE AND TABLES BOOK

Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Length of PQ [page 18]

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of PQ [page 18]

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

Marking Scheme Notes
Question 4 (a) [Scale 5C (0, 3, 4, 5)]
3: • Writes a correct distance formula or formula for the equation of the circle
4: • Calculates the distance O to A
• Substitutes into correct circle formula with one error
• Calculates r^2 incorrectly
Note: A correct answer without work shown, award full credit

Question 4 (b)

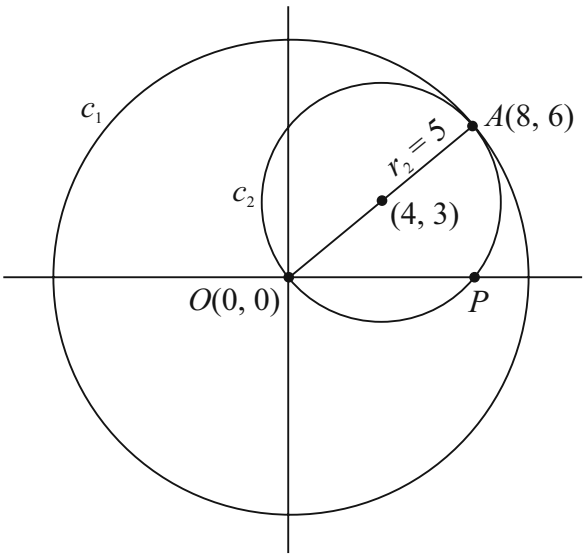
Equation of c_2:

The centre of c_2 is the midpoint of OA.

$O(0,\ 0) = (x_1,\ y_1),\ A(8,\ 6) = (x_2,\ y_2)$

$\text{Midpoint} = \left(\frac{0+8}{2}, \frac{0+6}{2}\right) = (4,\ 3)$

Radius $r_2 = \frac{1}{2}|OA| = 5$



Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r
$$(x-h)^2 + (y-k)^2 = r^2$$

Centre $(h,\ k) = (4,\ 3),\ r = 5$

$c_2 : (x-4)^2 + (y-3)^2 = 5^2$

$(x-4)^2 + (y-3)^2 = 25$

Marking Scheme Notes
Question 4 (b) [Scale 10D (0, 2, 5, 8, 10)]
2: • Writes a correct midpoint formula or formula for the equation of the circle
• Writes the midpoint or radius correctly
5: • Writes the midpoint and radius correctly
• Substitutes the centre or radius into the equation.
8: • Substitutes the centre and radius into the equation
• Substitutes into correct circle formula with one error
• Calculates r^2 incorrectly

Question 4 (c)

$c_2 : (x-4)^2 + (y-3)^2 = 25$

Cuts x-axis: Put $y = 0$

$\therefore (x-4)^2 + (0-3)^2 = 25$

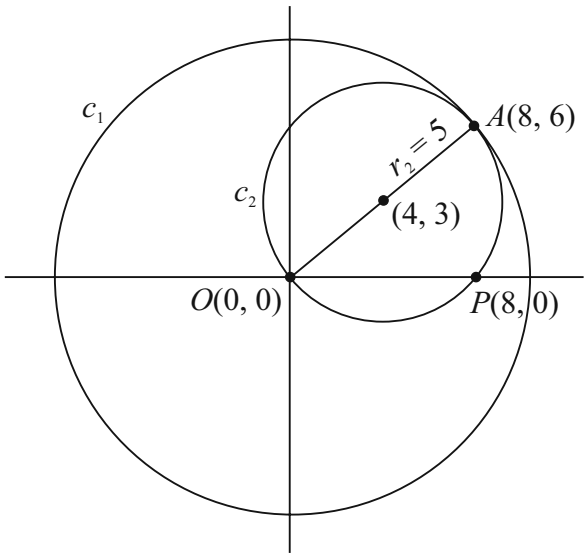
$(x-4)^2 + 9 = 25$

$(x-4)^2 = 25 - 9 = 16$

$x - 4 = \pm\sqrt{16} = \pm 4$

$\therefore x = 0, 8$

Answer: $P(8, 0)$



Marking Scheme Notes
Question 4 (c) [Scale 10C (0, 4, 7, 10)]

4: • Identifies that $y = 0$
• Some relevant substitution into the equation
• Correct answer without work shown

7: • Substantially correct work at solving for x, e.g. works incorrectly with $(x - 4)^2 = 16$
• Correct answer without the co-ordinates of P identified
Note: A correct answer with a correct geometrical explanation, award full credit

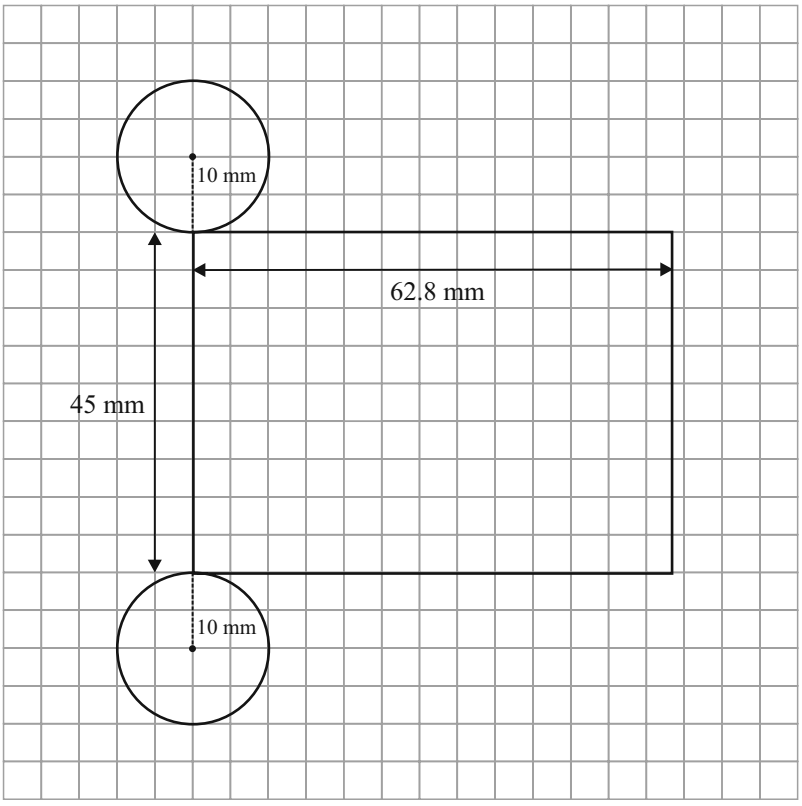
Question 5 (25 marks)

Question 5 (a)

Use a scale from the grid of 5 mm for each square.
The length of the rectangle is equal to the length of the circumference of the circle.

$$\boxed{l = 2\pi r}$$

$l = 2\pi(10) = 20\pi = 62 \cdot 8$ mm



Marking Scheme Notes
Question 5 (a)
[Scale 5C* (0, 3, 4, 5)]

3: • A rectangle or a circle drawn
• The length of the circle calculated

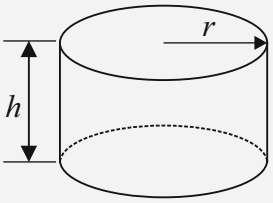
4: • A correct sketch drawn without dimensions
• A partially complete sketch drawn with some dimensions given
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

Question 5 (b)

Volume V of a cylinder: $r = 10$ mm, $h = 45$ mm

$V = \pi r^2 h = \pi(10)^2(45) = 4500\pi \text{ mm}^3$

FORMULAE AND TABLES BOOK
Surface area and volume:
Cylinder [page 10]



$A = 2\pi rh$

$V = \pi r^2 h$

Question 5 (c)

Volume of cylinder = Volume of sphere

$4500\pi = \frac{4}{3}\pi r^3$

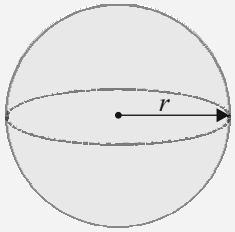
$\frac{3 \times 4500}{4} = r^3$

$3375 = r^3$

$r = \sqrt[3]{3375} = 15 \text{ mm}$

$A = 4\pi r^2 = 4\pi(15)^2 = 900\pi \text{ mm}^2$

FORMULAE AND TABLES BOOK
Surface area and volume:
Sphere [page 8]



$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

MARKING SCHEME NOTES

Question 5 (b) [Scale 10C* (0, 4, 7, 10)]

4: • Correct formula for the volume of a cylinder written and stops
• An incorrect relevant formula and some correct substitution.

7: • Formula substituted correctly without calculation or with incorrect calculation
• Some incorrect substitution into formula with correct calculation
• Correct answer not expressed in terms of π
* Penalise one mark for incorrect or omitted units, provided full marks otherwise
NOTE: A correct answer without work shown, award full credit

Question 5 (c) [Scale 10D* (0, 2, 5, 8, 10)]

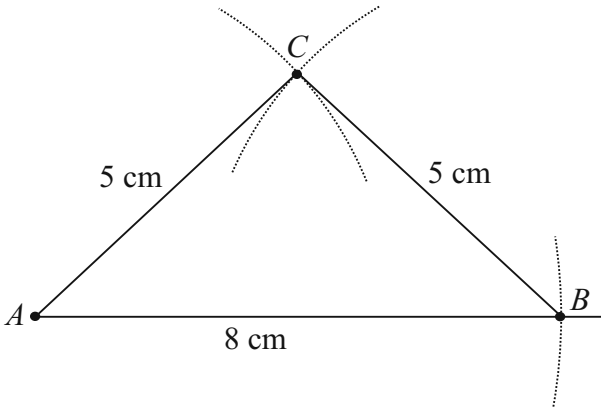
2: • Writes correct formula for the volume or surface area of a sphere
• Equates volume of cylinder with surface area of sphere

5: • Equates volume of cylinder with volume of sphere
• Equates volume of cylinder with surface area of sphere and correctly calculates r

8: • Calculates the radius of the sphere
• Calculates correctly an incorrect surface area
• Calculates the correct surface area with an incorrect radius
• Correct answer not expressed in terms of π
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

Question 6A (25 marks)

Question 6A (a)



Using a compass, measure out a length of 8 cm.

With the point of the compass on A, draw an arc $|AB|$ of length 8 cm.

Draw radii of 5 cm with the point of the compass placed at A and then B. Where the arcs intersect is point C.

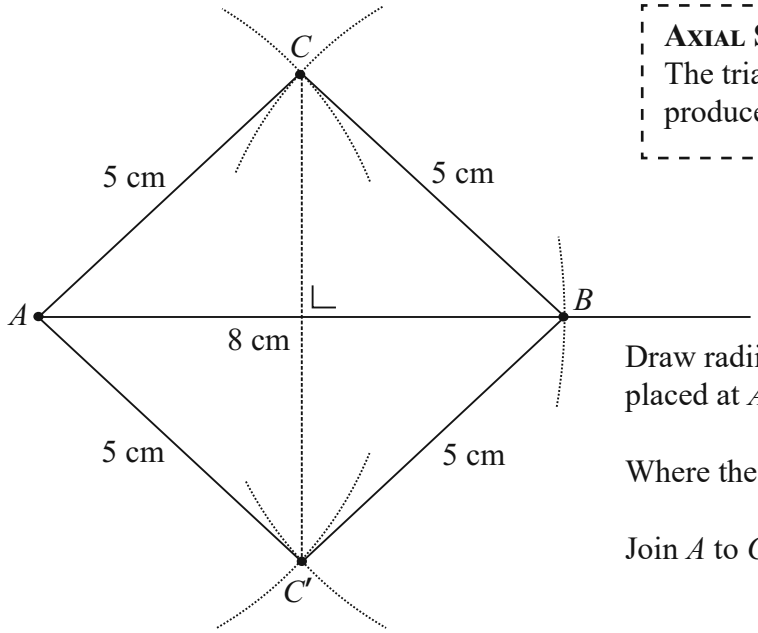
Join A to C and B to C to form triangle ABC.

Marking Scheme Notes
Question 6A (a) [Scale 10C (0, 4, 7, 10)]

4: • Point B or C correctly marked on the given line
• One arc drawn for a point not on the line
• Freehand triangle drawn with dimensions.

7: • Point B found and one arc for C drawn
• Correct triangle drawn but construction lines not shown
• Points B and C constructed but sides not drawn
• Triangle constructed correctly but measurements not exact

Question 6A (b)



Axial Symmetry
The triangle ABC folds over the line to produce triangle ABC'.

Draw radii of 5 cm with the point of the compass placed at A and then B.

Where the arcs intersect is point C'.

Join A to C' and B to C' to form triangle ABC'.

Marking Scheme Notes
Question 6A (b) [Scale 10C (0, 4, 7, 10)]

4: • Freehand image drawn
• Relevant work at constructing the image of C

7: • Correct triangle drawn but construction lines not shown

Question 6A (c)

The diagonals $[AB]$ and $[CC']$ of $AC'BC$ bisect one another.
Hence, $AC'BC$ is a parallelogram.

Marking Scheme Notes
Question 6A (c) [Scale 5B (0, 3, 5)]
3: • Some correct element in the candidate's justification or an incomplete justification

Question 6B (25 marks)

To prove: $\angle 1 + \angle 2 + \angle 3 = 90^\circ$

Proof:

$\Delta BQC : \angle 4 + \angle 2 + \angle 5 + 90^\circ = 180^\circ$

$\therefore \angle 4 + \angle 2 + \angle 5 = 90^\circ$

$\Delta PAC : \angle 3 + \angle 2 + \angle 5 + 90^\circ = 180^\circ$

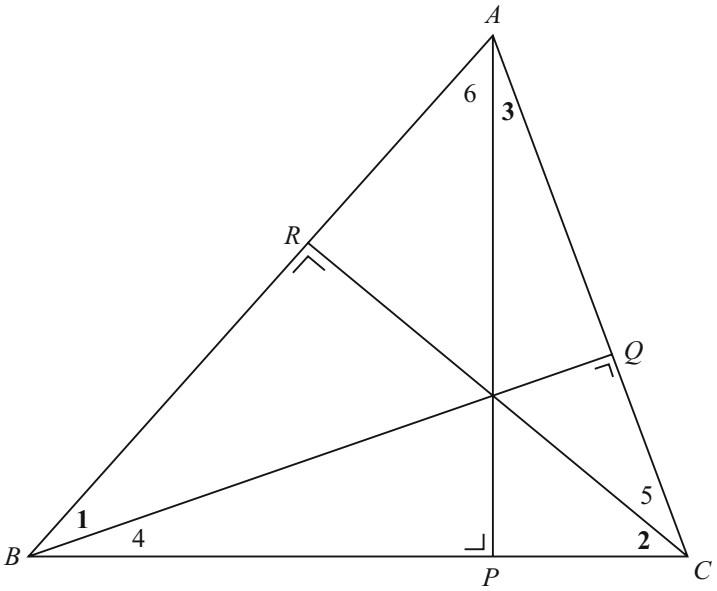
$\therefore \angle 3 + \angle 2 + \angle 5 = 90^\circ$

$\angle 4 + \angle 2 + \angle 5 = \angle 3 + \angle 2 + \angle 5 \Rightarrow \angle 4 = \angle 3$

$\Delta BCR : \angle 1 + \angle 4 + \angle 2 + 90^\circ = 180^\circ$

$\therefore \angle 1 + \angle 4 + \angle 2 = 90^\circ$ [But $\angle 4 = \angle 3$]

$\therefore \angle 1 + \angle 3 + \angle 2 = 90^\circ$



Marking Scheme Notes
Question 6B [Scale 25D (0, 6, 12, 19, 25)]
6: • Relevant work with the angle sum in one triangle
• Correct identification of vertically opposite angles being equal
12: • Similar work with two relevant triangles
19: • A correct answer other than one step omitted or incorrect

Question 7 (75 marks)

Question 7 (a)

Marriage rate												Death rate							
	9	8	7	7	6	6	5	5	3	3	4								
2	2	2	2	2	1	1	1	0	0	0	5								
											6	1	3	3	4	7	8		
											7	1	3	6	9				
											8	3	5	6	6	7	7	7	9
											9	0	0	0					
														Key: 7|6 = 76					

Question 7 (b)

The marriage rates range from 43 to 52 and are grouped at the top of the plot.
The death rates range from 61 to 90 and are grouped at the bottom of the plot.

Marking Scheme Notes

Question 7 (a) [Scale 15D (0, 4, 7, 11, 15)]

4: • A correct element of the plot entered
• Key given without entering any elements

7: • One side of the plot correct without the key
• Ten elements on each side correct, without the key
• Ten elements in total correct, with the correct key

11: • A correct plot without the key
• A plot with five or less elements incorrect with the correct key
• Correct plot and key but data not ordered

Question 7 (b) [Scale 5B (0, 3, 5)]

3: • A correct but incomplete difference given

Question 7 (c)

Marriage rate												Death rate							
	9	8	7	7	**6**	**6**	5	5	3	3	4								
2	2	2	2	**2**	**1**	1	1	0	0	**0**	5								
											6	1	3	3	4	7	8		
											7	1	3	6	9				
											8	3	5	6	6	7	7	7	9
											9	0	0	0					
															Key: 7|6 = 76				

Marriage rates: 43, 43, 45, 45, **46**, **46**, 47, 47, 48, 49, **50**, 50, 50, 51, 51, **51**, **52**, 52, 52, 52, 52

⇧ Lower quartile Q_1 ⇧ Median Q_2 ⇧ Upper quartile Q_3

Median = 50 (Middle value when the numbers are arranged in order)
Interquartile range = $Q_3 - Q_1 = 51.5 - 46 = 5.5$ [Get the mean of the 2 numbers for each quartile.]

Marking Scheme Notes
Question 7 (c) [Scale 10C (0, 4, 7, 10)]
4: • Describes the median or interquartile range
• Relevant work towards finding the median or the interquartile range
• Writes the range of the marriage rate
7: • One correct answer
• Substantially correct work in each part
• Correct answers using the data for the birth rate or the death rate

Question 7 (d) (i)

$$\text{Mean death rate} = \frac{1645}{21} = 78{\cdot}3$$

Formulae and Tables Book
Statistics and probability:
Mean from a list of n numbers
[page 33]

$$\mu = \frac{\text{Sum of all the numbers}}{\text{Number of numbers}} = \frac{\sum x}{n}$$

Deaths
90
89
87
90
86
90
87
86
85
87
83
79
76
73
71
68
67
64
63
63
61
1645

Marking Scheme Notes
Question 7 (d) (i) [Scale 10C* (0, 4, 7, 10)]
4: • Writes the formula for the mean
• Finds the value of n or adds some of the elements
7: • Correct substitution into the formula without calculation
• Incorrect substitution into the formula with calculation
* Penalise one mark for incorrect or omitted roundoff, provided full marks otherwise [Accept 78 in this context.]
Note: A correct answer without work shown, award full credit

Question 7 (d) (ii)

Mean = 78·3
Standard deviation = 10·3
Range of values within 1 standard deviation of the mean
= [78·3 – 10·3, 78·3 + 10·3] = [68, 88·6]

Marriage rate												Death rate							
	9	8	7	7	6	6	5	5	3	3	4								
2	2	2	2	2	1	1	1	0	0	0	5								
											6	1	3	3	4	7	**8**		
											7	**1**	**3**	**6**	**9**				
											8	**3**	**5**	**6**	**6**	**7**	**7**	**7**	9
											9	0	0	0					
															Key: 7|6 = 76				

List of death rates within 1 standard deviation of mean: 68, 71, 73, 76, 79, 83, 85, 86, 86, 87, 87, 87

Marking Scheme Notes
Question 7 (d) (ii) [Scale 5C (0, 3, 4, 5)]
3: • Finds one correct boundary
4: • Finds both correct boundaries without elements listed
• Finds one correct boundary with the correct elements

Question 7 (e)

Year: 2010

$165 \text{ births} \rightarrow 10\,000 \text{ people}$

$1 \text{ birth} \rightarrow \frac{10\,000}{165} \text{ people}$

$75\,174 \text{ births} \rightarrow \frac{10\,000}{165} \times 75\,174 = 4\,556\,000 \text{ people}$

Question 7 (f)

Year: 2010

$61 \text{ deaths} \rightarrow 10\,000 \text{ people}$

$\frac{61}{10\,000} \text{ deaths} \rightarrow 1 \text{ person}$

$\frac{61}{10\,000} \times 4\,556\,000 = 27\,791 \text{ deaths} \rightarrow 4\,556\,000 \text{ people}$

Question 7 (g)

The birth rates given are per 10 000 of the population. If the population in 2000 was greater than in 1990, more children could have been born in 2000 than in 1990 even though the birth rate in 2000 was lower.

Marking Scheme Notes
Question 7 (e) [Scale 5C (0, 3, 4, 5)]
3: • Identifies 165 as the birth rate or recognises the rate is per 10 000
4: • One correct outcome for each category
• Correct fraction without calculation
• Fraction with one element incorrect, correctly calculated

Question 7 (f) [Scale 5C (0, 3, 4, 5)]
3: • Identifies 61 as the death rate
• Carries forward the answer from (e)
4: • Correct fraction without calculation
• Fraction with one element incorrect, correctly calculated

Question 7 (g) [Scale 5B (0, 3, 5)]
3: • A plausible reason with some relevant reference to the data

Question 7 (h)

$$1990 \text{ ratio} = \frac{151}{90} = 1 \cdot 68$$

$$2010 \text{ ratio} = \frac{165}{61} = 2 \cdot 7$$

Prediction: The population of the country is expected to increase.
Reason: The increase in the ratio from 1990 to 2010 suggests that more children are being born for each person that dies.

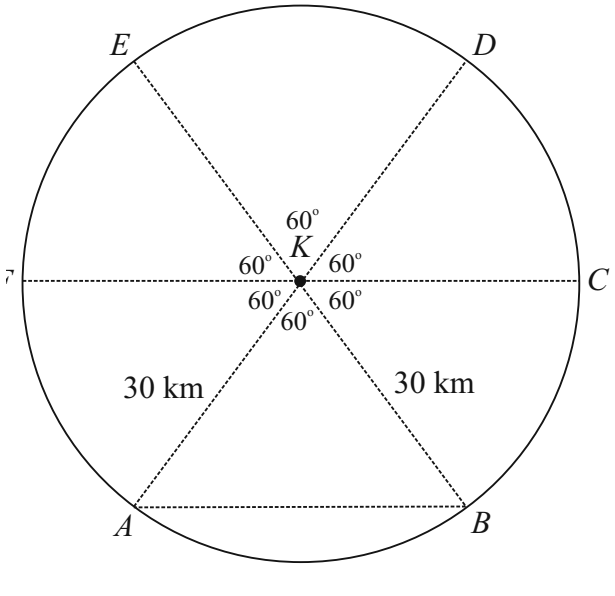
Question 7 (i)

There is a strong negative correlation.
With the increasing birth rate, the population is getting younger and the death rate is declining.

Marking Scheme Notes
Question 7 (h) [Scale 10C (0, 4, 7, 10)]
4: • One correct ratio found
• A prediction with some merit
• Two correct ratios based on years rather than events
7: • Two correct ratios found and a prediction or a reason
• One correct ratio with some merit in the prediction and reason
• A prediction with a reason, both having merit

Question 7 (i) [Scale 5B (0, 3, 5)]
3: • Relationship described as negative correlation
• Relevant reason about the population change

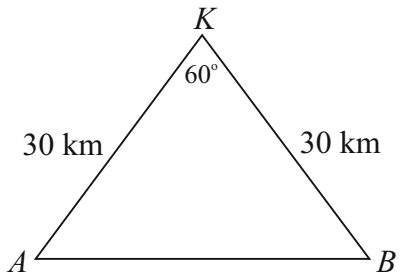
Question 8 (75 marks)
Question 8 (a) (i)



Marking Scheme Notes
Question 8 (a) (i) [Scale 5B (0, 3, 5)]
3: • Six equal sectors identified or notes the angle at the centre is 360^o
Note: A correct answer without work shown, award full credit

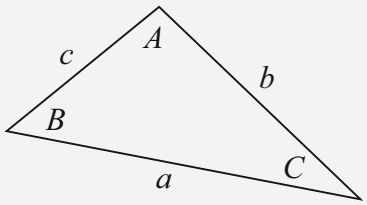
$|\angle BKA| = \frac{360^o}{6} = 60^o$ [The angle at the centre of the circle is 360^o. The angle BKA is one-sixth of this angle.]

Question 8 (a) (ii)



Area of $\Delta KAB = \frac{1}{2}(30)(30)\sin 60^\circ = 389\cdot 7 \text{ km}^2$

Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Area: $\frac{1}{2}ab\sin C$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Marking Scheme Notes
Question 8 (a) (ii) [Scale 10C* (0, 4, 7, 10)]
4: • Writes a correct relevant formula
• One or two correct substitutions into formula
7: • Correct or consistent substitution into formula
• Incorrect answer from an incorrect calculator mode
* Penalise one mark for incorrect or omitted units, provided full marks otherwise
Note: A correct answer without work shown, award full credit

Question 8 (a) (iii)

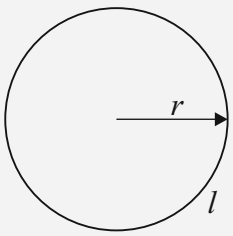
Each sector is one-sixth of the area of a circle.

Area of sector $KAB = \frac{1}{6}\pi(30)^2 = 471\cdot 24 \text{ km}^2$

$$\frac{\text{Area of triangle } KAB}{\text{Area of sector } KAB}\times 100\%$$

$$= \frac{389\cdot 7 \text{ km}^2}{471\cdot 24 \text{ km}^2}\times 100\% = 82\cdot 7\%$$

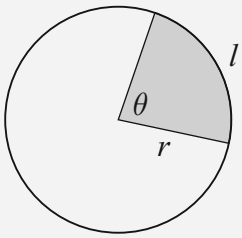
Formulae and Tables Book
Length and area: Circle [page 8]



$l = 2\pi r$

$A = \pi r^2$

Length and area: Arc/sector [page 9]



$l = 2\pi r\left(\frac{\theta}{360^\circ}\right)$

$A = \pi r^2\left(\frac{\theta}{360^\circ}\right)$

when θ is in degrees.

Question 8 (a) (iv)

$$|AB|^2 = 30^2 + 30^2 - 2(30)(30)\cos 60^\circ$$

$$|AB| = \sqrt{30^2 + 30^2 - 2(30)(30)\cos 60^\circ}$$

$$= 30 \text{ km}$$

Marking Scheme Notes
Question 8 (a) (iii) [Scale 10C (0, 4, 7, 10)]
4: • Writes a correct formula for area of sector
• Some correct substitution into a correct relevant formula
7: • Correct area of sector
• Fraction written with one error, calculated

Marking Scheme Notes
Question 8 (a) (iv) [Scale 10C* (0, 4, 7, 10)]
4: • Formula for the cosine rule written
• Some correct substitution
7: • A fully correct substitution, without complete and correct calculation
• Substantially correct work in substitution and calculation
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

Question 8 (a) (v)

The triangle KAB is an equilateral triangle as all angles are equal.

Marking Scheme Notes
Question 8 (a) (v) [Scale 5B (0, 3, 5)]
3: • Work of merit such as reference to two sides equal

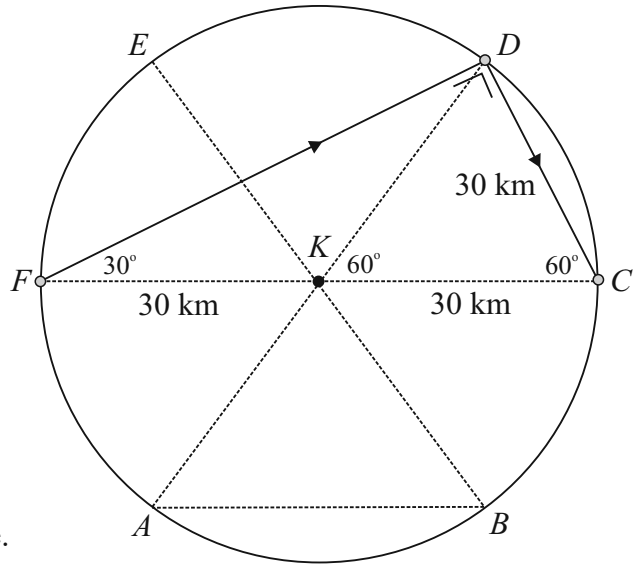
Question 8 (b) (i)

The perimeter is the length of the circumference of the circle.

$l = 2\pi r = 2\pi(30) \approx 188$ km

Question 8 (b) (ii)

Diagram shown.



Question 8 (b) (iii)

$|\angle FDC| = 90^\circ$

Theorem: The angle in a semicircle is a right-angle.

Marking Scheme Notes
Question 8 (b) (i) [Scale 5C* (0, 3, 4, 5)]
3: • Writes the correct formula.
4: • Correct substitution into formula without complete and correct calculation
• Incorrect substitution with correct calculation
• An answer of 189 km with work
* Penalise one mark for incorrect of omitted roundoff, provided full marks otherwise.
Note: A correct answer without work shown, award full credit

Question 8 (b) (ii) [Scale 5B (0, 3, 5)]
3: • One correct line drawn

Question 8 (b) (iii) [Scale 5B (0, 3, 5)]
3: • Relevant work such as $|\angle FDC|$ indicated correctly

Question 8 (b) (iv)

Triangle FDC is a right-angled triangle.

$$\cos 30^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{|FD|}{|FC|}$$

$$\therefore |FD| = |FC| \cos 30^\circ = 60 \cos 30^\circ = 51{\cdot}96 \text{ km}$$

$$\text{Total distance} = |FD| + |DC| = 51{\cdot}96 + 30 = 81{\cdot}96 \text{ km}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{81{\cdot}96 \text{ km}}{80 \text{ km/h}} = 1{\cdot}0245 \text{ h} = 61{\cdot}47 \text{ minutes}$$

Formulae and Tables Book
Speed:

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

$$v = \frac{s}{t}$$

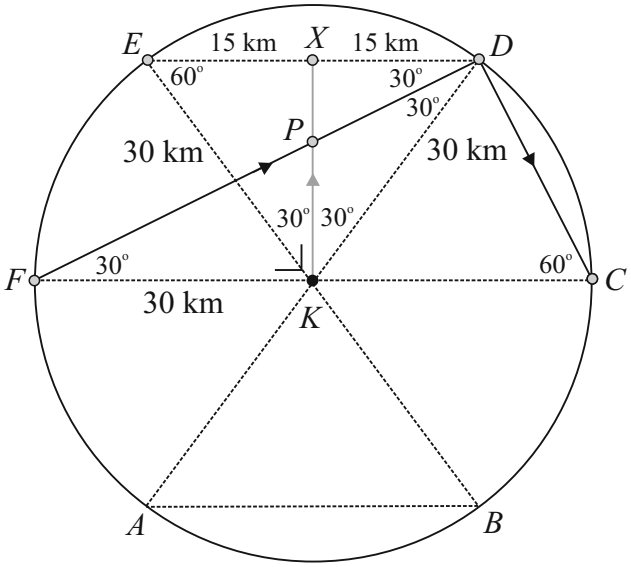
Marking Scheme Notes
Question 8 (b) (iv) [Scale 10C* (0, 4, 7, 10)]
4: • Calculates or gives one correct distance
7: • Calculates the total distance
• Sets up correct fraction to calculate time without complete and correct calculation
• Sets up incorrect fraction to calculate time with correct calculation
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

Question 8 (c) (i)

Consider the right-angled triangle KEX.

$$\sin 60^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{|KX|}{|EK|}$$

$$\therefore |KX| = |EK| \sin 60^\circ = 30 \sin 60^\circ = 25{\cdot}98 \text{ km}$$



Question 8 (c) (ii)

Call P, the point of intersection of the paths of the helicopter and lifeboat.
Consider the right-angled triangle FKP.

$$\tan 30^\circ = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{|PK|}{|FK|}$$

$$\therefore |PK| = |FK| \tan 30^\circ = 30 \tan 30^\circ = 17{\cdot}32 \text{ km}$$

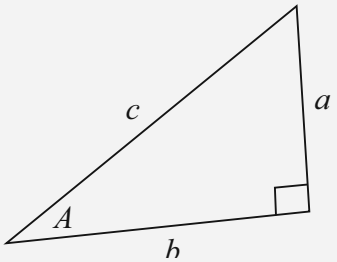
$$|PX| = |KX| - |PK| = 25{\cdot}98 - 17{\cdot}32 = 8{\cdot}66 \text{ km}$$

or consider triangle DPX.

$$\tan 30^\circ = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{|PX|}{|DX|}$$

$$\therefore |PX| = |DX| \tan 30^\circ = 15 \tan 30^\circ = 8{\cdot}66 \text{ km}$$

Formulae and Tables Book
Trigonometry: Right-angled triangle [page 16]



$$\sin A = \frac{a}{c},\ \cos A = \frac{b}{c},\ \tan A = \frac{a}{b}$$

Question 8 (c) (i) [Scale 5C* (0, 3, 4, 5)]

3: • Draws a line from K to X or notes additional relevant work on the diagram
• Correct trigonometric ratio set up
• Correct application of theorem of Pythagoras written

4: • A fully correct substitution, without complete and correct calculation
• One error in substitution with fully correct calculation
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

Question 8 (c) (ii) [Scale 5C* (0, 3, 4, 5)]

3: • Indicates the required point on the diagram
• Correct trigonometric ratio set up or similar work of merit.

4: • A fully correct substitution, without complete and correct calculation
• One error in substitution with fully correct calculation
* Penalise one mark for incorrect or omitted units, provided full marks otherwise

LC 2012 (Set G): Paper 1

Question 1 (25 marks)

Question 1 (a)

Gross weekly wage = €510
Income tax: €510 × 0·2 = €102 ← Get 20% of the gross wage by multiplting by 0·2.
Weekly tax credits = €63 ← Subtract this tax credit from income tax to find the tax payable.
Income tax payable = €102 – €63 = €39

Question 1 (b)

USC

First €193 at 2%: €193 × 0·02	= €3·86
Second €115 at 4%: €115 × 0·04	= €4·60
Rest €202 at 7%: €202 × 0·07	= €14·14
Total USC Charge	= €22·60

Question 1 (c)

PRSI

Total Deductions = €76·92 ← This includes income tax + USC + PRSI.
Income tax payable = €39
USC Charge = €22·60
PRSI = ?
PRSI = €76·92 – €39 – €22·60 = €15·32

Marking Scheme Notes

Question 1 (a) [Scale 10C (0, 6, 8, 10)]

6: • Error in calculating gross tax **and** misuse of tax credit
• Calculates 20% of some figure

8: • Error in calculating gross tax or misuse of tax credit

Question 1 (b) [Scale 10C (0, 6, 8, 10)]

6: • One percentage calculated correctly
• Adds 193 and 115 and stops
• Any correct relevant step

8: • Calculates two of the required percentages correctly but fails to finish

Question 1 (c) [Scale 5B (0, 3, 5)]

3: • Adds the tax and the USC but fails to finish
• Subtracts answer to part (a) or part (b) from total deductions
• Any correct relevant step

Question 2 (25 marks)

Question 2 (a)

A rational number **can** be written as a fraction of integers.
An irrational number **cannot** be written as a fraction of integers. The decimal form goes on forever with no overall pattern.

$a = \sqrt{2} = 1 \cdot 414213562.....$ [Irrational, no overall pattern]

$a - 1 = \sqrt{2} - 1 = 0 \cdot 414213562.....$ [Irrational, no overall pattern]

$(-a)^2 = (-\sqrt{2})^2 = 2$ [Rational]

$(a-2)^2 = (\sqrt{2}-2)^2 = (\sqrt{2}-2)(\sqrt{2}-2) = 2 - 4\sqrt{2} + 4 = 6 - 4\sqrt{2}$ [Irrational]

$1 + a^2 = 1 + (\sqrt{2})^2 = 1 + 2 = 3$ [Rational]

Number	Rational	Irrational
a		✓
$a-1$		✓
$(-a)^2$	✓	
$(a-2)^2$		✓
$1+a^2$	✓	

Marking Scheme Notes
Question 2 (a) [Scale 10C (0, 6, 8, 10)]
6: • Correctly identifies 1 or 2 numbers
8: • Correctly identifies 3 or 4 numbers

Question 2 (b)

$a = \sqrt{2} = 1 \cdot 414213562...$

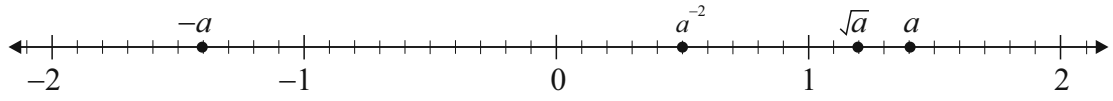
$a = \sqrt{2} = 1 \cdot 41$

$-a = -\sqrt{2} = -1 \cdot 41$

$\sqrt{a} = \sqrt{\sqrt{2}} = 1 \cdot 19$

$a^{-2} = \frac{1}{a^2} = \frac{1}{(\sqrt{2})^2} = \frac{1}{2} = 0 \cdot 5$

Marking Scheme Notes
Question 2 (b) [Scale 10C (0, 6, 8, 10)]
6: • Correctly shows 1 or 2 numbers (see note)
• No numbers correctly shown but a and $-a$ equidistant from zero
8: • Correctly shows 3 numbers
Note: Treat as one error if a and $-a$ are incorrectly shown but are equidistant from the origin



Question 2 (c)

Replace x by its root and show it satisfies the equation.

$x^2 - 6x + 7 = 0$

$(3-\sqrt{2})^2 - 6(3-\sqrt{2}) + 7$

$= (3-\sqrt{2})(3-\sqrt{2}) - 6(3-\sqrt{2}) + 7$

$= 9 - 3\sqrt{2} - 3\sqrt{2} + 2 - 18 + 6\sqrt{2} + 7$

$= 0$

Marking Scheme Notes
Question 2 (c) [Scale 5C (0, 3, 4, 5)]
3: • Some correct substitution for x in the given equation
• Some correct substitution into quadratic formula
4: • Fails to finish from $x = \frac{6 \pm \sqrt{8}}{2}$
• Correct substitution for x in the given equation but fails to finish
• Use of decimals after correct substitution

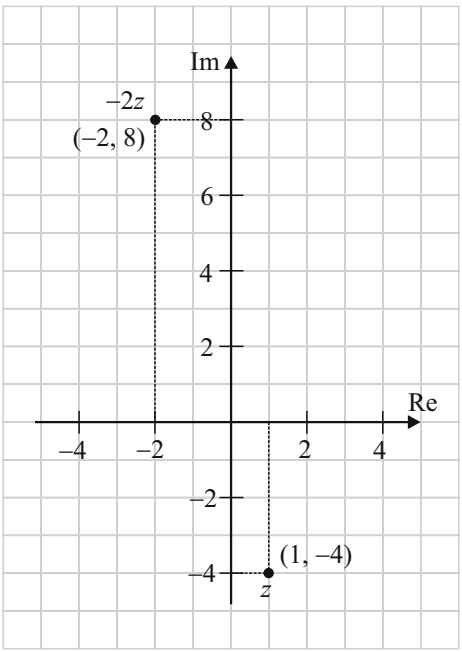
EPL 104

Question 3 (25 marks)

Question 3 (a)

$z = 1 - 4i$

$-2z = -2(1 - 4i) = -2 + 8i$



Marking Scheme Notes
Question 3 (a) [Scale 15C (0, 8, 13, 15)]

8: • Plots z only
• Correct real or imaginary part plotted for either point
• Correct substitution for z

13: • Plots $-2z$ correctly only
• Distribution error in finding $-2z$ but plots z and in correct $-2z$ correctly (b), (c), (d)

Question 3 (b)

Show the left-hand side (LHS) is equal to the right-hand side (RHS).

LHS

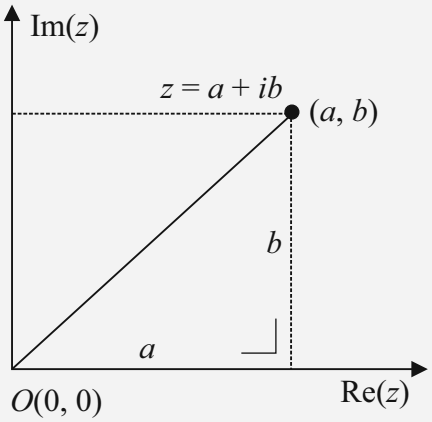
$$2|z| = 2|1 - 4i|$$
$$= 2\sqrt{1^2 + (-4)^2}$$
$$= 2\sqrt{1 + 16}$$
$$= 2\sqrt{17}$$

RHS

$$|-2z| = |-2 + 8i|$$
$$= \sqrt{(-2)^2 + (8)^2}$$
$$= \sqrt{4 + 64}$$
$$= \sqrt{68} = \sqrt{4 \times 17}$$
$$= 2\sqrt{17}$$

$\therefore\ 2|z| = |-2z|.$

Formula: Complex Numbers
Modulus $|z|$



$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Question 3 (c)

$-2z$ is twice as far from the origin as z.

Question 3 (d)

$|z + k| = 5$ ← Replace z by $1 - 4i$

$|1 - 4i + k| = 5$ ← Gather up the real and imaginary parts.

$|(k + 1) - 4i| = 5$ ← Apply the modulus formula.

$\sqrt{(k+1)^2 + (-4)^2} = 5$ ← Square both sides.

$(k + 1)^2 + 16 = 25$

$(k + 1)^2 = 9$ ← Square root both sides.

$(k + 1) = \pm\sqrt{9} = \pm 3$

$\therefore k = -4,\ 2$

Question 3 (b) (c) (d) [Scale 10C (0, 6, 8, 10)]

6: • Substitutes for z at least once
• Some correct substitution into the modulus formula
• Modulus formula correctly stated
• Joins z and/or $-2z$ to the origin
• Any correct relevant statement for (c)

8: • At least one of (b), (c) or (d) fully correct
• Work of merit in at least 2 parts

Question 4 (25 marks)

Question 4 (a)

$\frac{1}{2}(7x-2)+5=2x+7$ ← Multiply across by 2 to get rid of fractions.

$1(7x-2)+10=4x+14$ ← Multiply out brackets.

$7x-2+10=4x+14$ ← Bring x terms to the left and numbers to the right.

$7x-4x=14+2-10$ ← Add x terms and numbers.

$3x=6$ ← Divide both sides by 3.

$$\therefore x=\frac{6}{3}=2$$

Question 4 (b)

$\frac{2}{3x-4}-\frac{1}{2x+1}=\frac{1}{2}$ ← Multiply across by $2(3x-4)(2x+1)$ to get rid of fractions.

$$\frac{2(3x-4)(2x+1)2}{(3x-4)}-\frac{2(3x-4)(2x+1)1}{(2x+1)}=\frac{2(3x-4)(2x+1)1}{2}$$

$4(2x+1)-2(3x-4)=(3x-4)(2x+1)$

$8x+4-6x+8=6x^2+3x-8x-4$

$2x+12=6x^2-5x-4$

$0=6x^2-5x-4-2x-12$

$0=6x^2-7x-16$

$a=6,\ b=-7,\ c=-16$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(6)(-16)}}{2(6)}$$

$$=\frac{7\pm\sqrt{49+384}}{12}=\frac{7\pm\sqrt{433}}{12}$$

$=-1\cdot 2,\ 2\cdot 3$

Formulae and Tables Book
Algebra: Roots of the quadratic equation
$ax^2 + bx + c = 0$ [page 20]

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Question 4 [Scale 25D (0, 6, 13, 20, 25)]
6: • Any correct relevant step
13: • Correct structure to the method but with at least two major errors in solving
20: • Clear proficiency in solving equations but with one major error

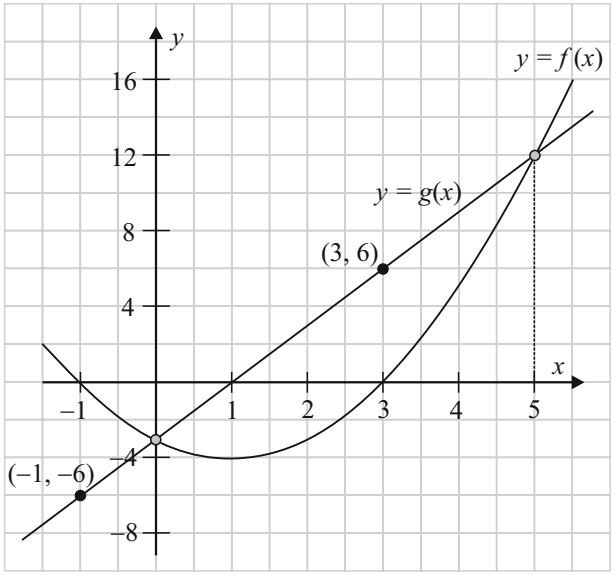
EPL 104

Question 5 (25 marks)

Question 5 (a)

$g(x)$ is a straight line. Therefore, you need just 2 points from the line to draw it. You are given those 2 points.

Points on $g(x)$: $(-1, -6)$ and $(3, 6)$



Question 5 (b)

Look at where the two graphs intersect. This occurs at $x = 0$ and $x = 5$.

$x = 0, 5$

Marking Scheme Notes

Question 5 (a) [Scale 15C (0, 8, 13, 15)]

8: • Plots correct x or correct y coordinate of either point
• Plots one point only
• Plots $(-6, -1)$ or $(6,3)$

13: • Plots both points correctly but fails to join
• Joins a correctly plotted point to an incorrectly plotted point

Question 5 (b) [Scale 5C (0, 3, 4, 5)] (Accept candidate's graph from (a))

3: • Only one x value given
• Marks one or both points of intersection on graph
• States answer is -3 or 12

4: • Writes answer as $(0, -3)$ and $(5, 12)$
• Gives answers as -3 and 12

Question 5 (c)

-1 and 3 are elements of $g(x)$.

$g(x) = ax + b$

$g(-1) = -6 \Rightarrow a(-1) + b = -6$

$-a + b = -6 \ldots\ldots (\mathbf{1})$

$g(3) = 6 \Rightarrow a(3) + b = 6$

$3a + b = 6 \ldots\ldots (\mathbf{2})$

Solve the 2 equations simultaneously.

$-a + b = -6 \ldots\ldots (\mathbf{1})(\times -1)$

$3a + b = 6 \ldots\ldots (\mathbf{2})$

$a - b = 6$

$3a + b = 6$

$4a \quad = 12 \Rightarrow a = 3$

Replace a by its value in Equation (**1**).

$-(3) + b = -6 \ldots\ldots (\mathbf{1})$

$b = -6 + 3 = -3$

$\therefore g(x) = ax + b = 3x - 3$

-1 and 3 are elements of $f(x)$.

$f(x) = x^2 + px + q$

$f(-1) = 0$

$f(-1) = (-1)^2 + p(-1) + q = 0$

$1 - p + q = 0$

$-p + q = -1$....(**1**)

$f(3) = 0$

$f(3) = (3)^2 + p(3) + q = 0$

$9 + 3p + q = 0$

$3p + q = -9$....(**2**)

Solve the 2 equations simultaneously.

$-p + q = -1$....(**1**)$(\times -1)$

$3p + q = -9$....(**2**)

$p - q = 1$

$3p + q = -9$

$4p \quad = -8 \Rightarrow p = -2$

Replace p by its value in Equation (**1**).

$-(-2) + q = -1$....(**1**)

$2 + q = -1$

$q = -3$

$\therefore f(x) = x^2 + px + q = x^2 - 2x - 3$

Equate the 2 functions and solve for x:

$f(x) = g(x)$

$x^2 - 2x - 3 = 3x - 3$ ← Bring all terms to the left-hand side.

$x^2 - 5x = 0$ ← Factorise by taking out x in common.

$x(x - 5) = 0$ ← Put each bracket equal to zero and solve for x.

$\therefore x = 0, 5$ ← These are the same answers as part **(b)** which used a graphical method.

Marking Scheme Notes
Question 5 (c) [Scale 5C (0, 3, 4, 5)]
3: • Any correct relevant step
• Solves for a, b, p or q
4: • Solves correctly for a, b, p and q but fails to finish
• Correct structure to the method but with some errors

Question 6 (25 marks)

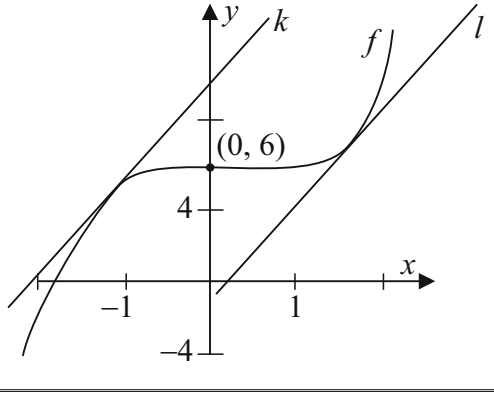
Question 6 (a)

Cuts the y-axis: Put $x = 0$

$f(x) = x^3 - x^2 - x + 6$

$f(0) = (0)^3 - (0)^2 - (0) + 6 = 6$

y intercept: (0, 6)



Marking Scheme Notes
Question 6 (a) [Scale 10C (0, 4, 7, 10)]
4: • Writes down answer by reading from graph
• Any correct differentiation and stops
• Indicates the local maximum on the graph
7: • Writes answer as 6 or (6, 0)

EPL 104

Question 6 (b)

$f(x) = x^3 - x^2 - x + 6$

$f'(x) = 3x^2 - 2x - 1 = 0$ ← Factorise the quadratic.

$(3x + 1)(x - 1) = 0$ ← Solve for x.

$\therefore x = -\frac{1}{3}, 1$ ← $x = 1$ was given.

$f(-\frac{1}{3}) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) + 6$

$= \frac{167}{27}$

$\therefore (-\frac{1}{3}, \frac{167}{27})$ is the maximum turning point.

FIND TURNING POINTS
(LOCAL MAXIMUM/MINIMUM)
Put $\frac{dy}{dx} = 0$ and solve for x

FORMULAE AND TABLES BOOK
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Question 6 (c)

Finding the slope m from the equation of a line l:

$$l : ax + by + c = 0 \Rightarrow m = -\frac{a}{b}$$

Equation of k: $4x - y + 9 = 0$
Slope of k: $m = 4$

Slope of l: $m = 4$ [Parallel lines have the same slopes.]

Find the slope of the tangent to the function f by finding its derivative $f'(x)$. Put $f'(x) = 4$ and solve for x.

$f'(x) = 4$

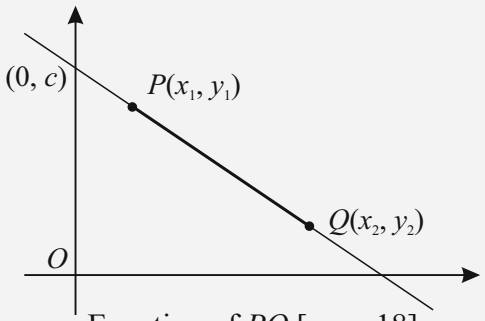
$3x^2 - 2x - 1 = 4$

$3x^2 - 2x - 5 = 0$ ← Factorise the quadratic.

$(3x - 5)(x + 1) = 0$

$x = \frac{5}{3}, \not{-1}$ [The line l is in the first quadrant so x is positive.]

FORMULAE AND TABLES BOOK
Co-ordinate geometry: Line



Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

MARKING SCHEME NOTES
Question 6 (b) [Scale 10C (0, 4, 7, 10)]

4: • Writes down answer by reading from graph
• Any correct differentiation and stops
• Indicates the local maximum on the graph

7: • Correct structure to answer but with some errors
• Puts first derivative (without errors) = 0 and stops

Question 6 (c) [Scale 5C (0, 3, 4, 5)]

3: • Attempts to find slope of k
• Writes down first derivative of f
• States slope of k = slope of l (or equivalent statement)

4: • Fails to solve $3x^2 - 2x - 1 = 4$ (or candidate's equivalent answer)

Question 7 (50 marks)

Question 7 (a)

Clark's rule
$A = 125$ mg per day
$W = 30$ kg
$C = ?$

$$C = \left(\frac{W}{68}\right) \times A = \left(\frac{30}{68}\right) \times 125 \approx 55 \text{ mg per day}$$

Question 7 (b) (i)

Young's rule 1
Let x = Child's age
C = Child dose
A = Adult dose

$$C = \frac{\text{Child's age}}{\text{Child's age} + 12} \times \text{Adult dose} = \frac{x}{(x+12)} \times A$$

Question 7 (b) (ii)

Young's rule 2
Let x = Child's age
C = Child dose
A = Adult dose

$$C = \frac{\text{Adult dose}}{\frac{(\text{Child's age} + 12)}{\text{Child's age}}} = \frac{A}{\frac{(x+12)}{x}}$$

Question 7 (b) (iii)

Young's rule 3
Let x = Child's age
C = Child dose
A = Adult dose

$$C = \frac{\text{Adult dose} \times \text{Child's age}}{(\text{Child's age} + 12)} = \frac{A \times x}{(x+12)}$$

Question 7 (c)

All three formulae equate to $C = \frac{Ax}{x+12}$.

Marking Scheme Notes
Question 7 (a) [Scale 10B* (0, 6, 10)]
6: • Some correct substitution
Note: Penalise once only for omission of units in parts (a), (d) and (f)
* Failure to round: penalty is one mark (provided otherwise full marks)

Question 7 (b) (i) [Scale 5B (0, 3, 5)]
3: • Partially correct formula
• Fail to state meaning of letters used in formula
[Needs to be stated once only in part (b)]

Question 7 (b) (ii) [Scale 5B (0, 3, 5)]
3: • Partially correct formula

Question 7 (b) (iii) [Scale 5B (0, 3, 5)]
3: • Partially correct formula

Question 7 (c) [Scale 5B (0, 3, 5)]
3: • Any reasonable explanations with error(s)

Question 7 (d)

Young's rule
$A = 150$ mg per day
$x = 6$ years
$C = ?$

$$C = \frac{Ax}{x+12} = \frac{150 \times 6}{6+12} = 50 \text{ mg per day}$$

Marking Scheme Notes
Question 7 (d) [Scale 5B* (0, 3, 5)]
3: • Some correct substitution
* Accept candidate's rule from (b)

Question 7 (e)

Young's rule
$A = 150$ mg per day
$x = ?$
$C = 30$ mg per day [One-fifth of the adult dose.]

$$C = \frac{Ax}{x+12} \Rightarrow 30 = \frac{150x}{x+12}$$
$$1 = \frac{5x}{x+12}$$
$$x + 12 = 5x$$
$$12 = 5x - x$$
$$12 = 4x$$
$$\therefore x = 3 \text{ years}$$

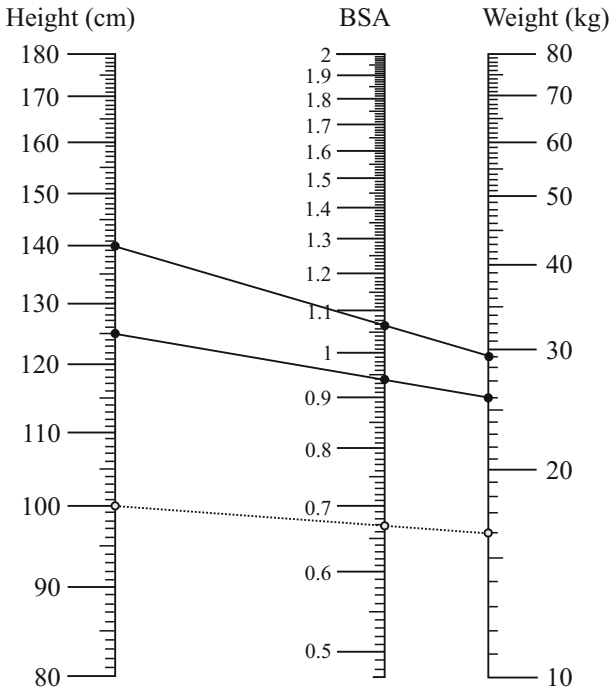
Marking Scheme Notes
Question 7 (e) [Scale 5C (0, 3, 4, 5)]
3: • Finds one fifth of a dose of medicine
• Attempt at setting up a relevant equation
4: • Correct equation but error(s) in solving
• Correct answer without work

Question 7 (f)

On the chart mark in the height and the weight. Draw a straight line between them and read off the BSA measure.

Adult dose = 200 mg per day
Height = 125 cm
Weight = 26 kg
BSA = 0·94 m^2
Child dose = ?

$$\text{Child dose} = \frac{\text{Child's BSA in } m^2}{1 \cdot 73} \times \text{Adult dose}$$
$$= \frac{0 \cdot 94}{1 \cdot 73} \times 200 = 109 \cdot 8 \text{ mg per day}$$



Marking Scheme Notes
Question 7 (f) [Scale 5C* (0, 3, 4, 5)]
3: • States BSA from chart and stops
• Substitutes for A
• Correct height or weight indicated on chart
4: • Substitutes BSA into rule and calculates dose with errors

Question 7 (g)

Information given:

1. the child is nine years old

2. Clark's rule and Young's rule both give a dose of 90 mg per day

3. the BSA rule gives a dose of 130 mg per day.

Child's age $x = 9$ years
Child dose $C = 90$ mg per day

Use Young's rule first: $C = \dfrac{Ax}{x+12}$

$$\therefore 90 = \frac{A(9)}{(9)+12} \Rightarrow 90 = \frac{9A}{21}$$

$$\therefore A = \frac{90 \times 21}{9} = 210 \text{ mg per day}$$

Therefore, you have worked out the adult dose A is 210 mg per day.

Use Clark's Rule: $C = \left(\dfrac{W}{68}\right) \times A$

Child's age $x = 9$ years
Child dose $C = 90$ mg per day
Adult dose $A = 210$ mg per day
Weight of child $W = ?$

$$90 = \left(\frac{W}{68}\right) \times 210$$

$$\frac{90 \times 68}{210} = W$$

$$\therefore W = 29.1 \text{ kg}$$

Use BSA rule:

$$\text{Child's dose} = \frac{\text{Child's BSA in m}^2}{1.73} \times \text{Adult dose}$$

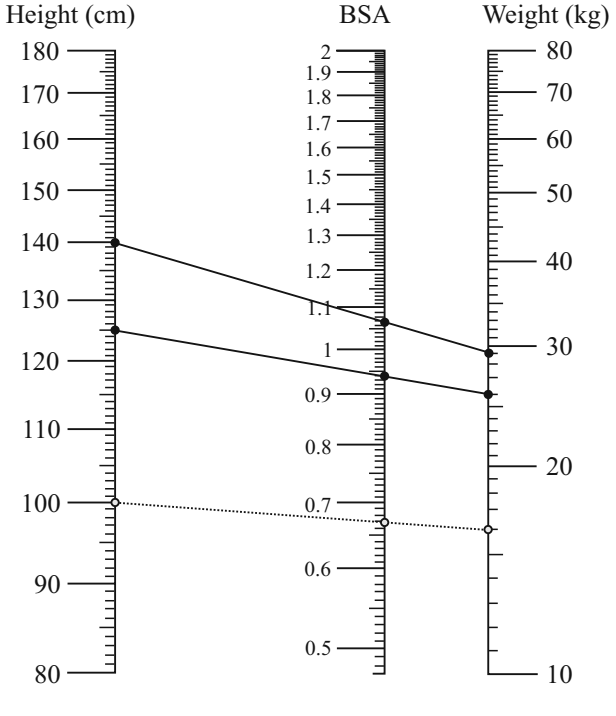
Child dose $C = 210$ mg per day
Adult dose $A = 210$ mg per day
Child's BSA = ?

$$130 = \frac{\text{BSA}}{1.73} \times 210$$

$$\frac{130 \times 1.73}{210} = \text{BSA}$$

$$\therefore \text{BSA} = 1.07 \text{ m}^2$$

Go to the chart. Mark in the following values:
Weight = 29·1 kg, BSA = 1·07 m^2
Draw a line through these points and read off the height of the child.
Height = 140 cm



Marking Scheme Notes

Question 7 (g) [Scale 5C (0, 3, 4, 5)]

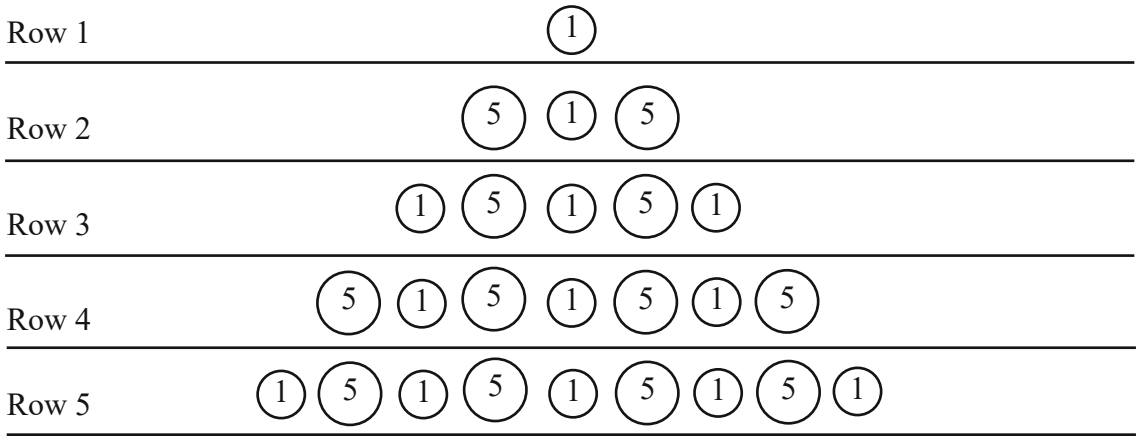
3: • Some correct substitution into a relevant rule

4: • Correct weight only
• Incorrect weight found but then finds height using correct method

EPL 104

Question 8 (50 marks)

Question 8 (a)



Question 8 (b)

Row number n	*Number of 1-cent coins*	*Number of 5-cent coins*	*Total number of coins in the row*	*Total value of the coins in the row*
1	1	0	1	1
2	1	2	3	11
3	3	2	5	13
4	**3**	**4**	**7**	**23**
5	**5**	**4**	**9**	**25**
6	**5**	**6**	**11**	**35**
7	**7**	**6**	**13**	**37**

Question 8 (c)

(i) If n is odd, row n has $\boldsymbol{n}$ 1-cent coins and $\boldsymbol{(n-1)}$ 5-cent coins.
(ii) If n is even, row n has $\boldsymbol{(n-1)}$ 1-cent coins and $\boldsymbol{n}$ 5-cent coins.

Marking Scheme Notes
Question 8 (a) [Scale 10B (0, 6, 10)]
6: • Error in pattern

Question 8 (b) [Scale 10C (0, 6, 8, 10)]
6: • At least one correct entry
• Error(s) in more than one column (but at least one entry correct)
8: • Error(s) in only one column

Question 8 (c) [Scale 5C (0, 3, 4, 5)]
3: • 1 space only correctly filled
• Correct sentence regarding a specific odd/even case
• Correct sentence(s) but not in terms of n
4: • 2 or 3 spaces correctly filled

Question 8 (d)

If n is even, row n has $\boldsymbol{(n-1)}$ 1-cent coins and $\boldsymbol{n}$ 5-cent coins.
Therefore, row 40 has $\boldsymbol{(40-1)}$ 1-cent coins and **40** 5-cent coins.
Total number of coins = 79

Question 8 (e)

Total value = $39 \times 1 + 40 \times 5 = 39 + 200 = 239$ c = €2·39

Marking Scheme Notes

Question 8 (d) [Scale 15C (0, 8, 13, 15)]

8: • Attempt at continuing pattern beyond row 7
• Writes $a + (n-1)d$ with partial or no substitution
• Correct number of 1-cent coins or correct number of 5-cent coins

13: • Correct substitution into $a + (n-1)d$ but fails to finish correctly
• Correct number of 1-cent coins and correct number of 5-cent coins but no/incorrect addition
• Correct answer without work

Question 8 (e) [Scale 5C (0, 3, 4, 5)]

3: • Correct value of 5-cent coins or correct value of 1-cent coins
• Relevant formula with some substitution
• Recognition of sequence of even rows

4: • Correct value of 5-cent coins and correct value of 1-cent coins but no/incorrect addition
• Correct answer without work
• 40(1) + 39(5) = 235

Question 8 (f)

n even:
$$(n-1)\times 1 + n\times 5 = 337$$
$$n - 1 + 5n = 337$$
$$6n - 1 = 337$$
$$6n = 338$$
$$\therefore n = \frac{338}{6} = 56\tfrac{1}{3}$$

This solution is not possible as n has to be a whole number.

n odd:
$$n\times 1 + (n-1)\times 5 = 337$$
$$n + 5n - 5 = 337$$
$$6n - 5 = 337$$
$$6n = 342$$
$$\therefore n = \frac{342}{6} = 57$$

Answer: 57th row

Question 8 (g)

Odd rows: $n = 1, 3, 5, 7, \ldots\ldots, 39$ (20 rows)
Values in each odd row: 1, 13, 25, 37,.....
$a = 1$, $d = 12$, $n = 20$

$$S_{20} = \tfrac{20}{2}[2(1) + (20-1)(12)] = 10[2 + 19(12)] = 2300$$

Even rows: $n = 2, 4, 6, 8, \ldots\ldots, 40$ (20 rows)
Values in each odd row: 11, 23, 35,.....
$a = 11$, $d = 12$, $n = 20$

$$S_{20} = \tfrac{20}{2}[2(11) + (20-1)(12)] = 10[22 + 19(12)] = 2500$$

Total value = 2300 c + 2500 c = 4800 c = €48·00

Formulae and Tables Book
Sequences and series: Arithmetic series or sequence [page 22]

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

a is the first term
d is the common difference

Marking Scheme Notes
Question 8 (f) (g) [Scale 5C (0, 3, 4, 5)]

3: • Correct relevant statement involving 337
• Attempt at listing
• Any correct relevant step
• Writes $\frac{n}{2}[2a + (n-1)d]$ with some substitution for (g)
• Solves $(n - 1)(1) + n(5) = 337$ and stops
• Correct number of 1 cent coins and 5 cent coins but fail to find correct value
• Correct answer to one part without work

4: • Correct answer to both parts without work
• One part correct with work shown
• Work of merit for both parts

Question 9 (50 marks)

Question 9 (a) (i)

Differentiate V with respect to t to find the rate at which the value of the investment changes.

$$V = 100 + 45t - 1{\cdot}5t^2$$

$$\frac{dV}{dt} = 45 - 2(1{\cdot}5)t = 45 - 3t \leftarrow \text{Replace } t \text{ by 6 in the derivative.}$$

$$\left(\frac{dV}{dt}\right)_{t=6} = 45 - 3(6) = 45 - 18 = €27 \text{ per month}$$

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Marking Scheme Notes
Question 9 (a) (i) [Scale 5C* (0, 3, 4, 5)]

3: • Substitutes $t = 6$ into V formula
• Effort at differentiation

4: • Correct derivative with or without substitution for t

Question 9 (a) (ii)

$$\left(\frac{dV}{dt}\right)_{t=18} = 45 - 3(18) = 45 - 54 = -€9 \text{ per month}$$

After 18 months the value of the investment is decreasing by €9 per month. The negative sign tells me it is decreasing.

Question 9 (a) (iii)

Replace t by 24 in the original equation.

$$V = 100 + 45t - 1 \cdot 5t^2$$
$$= 100 + 45(24) - 1 \cdot 5(24)^2$$
$$= €316$$

Question 9 (a) (iv)

Maximise the function by putting the derivative equal to zero and solving for t.

$$\frac{dV}{dt} = 0 \Rightarrow 45 - 3t = 0$$
$$45 = 3t$$
$$\therefore t = 15$$

FIND TURNING POINTS
(LOCAL MAXIMUM/MINIMUM)
Put $\frac{dy}{dx} = 0$ and solve for x

$V = 100 + 45t - 1 \cdot 5t^2$ ← Replace t by 15 to find the maximum value of the investment.
$= 100 + 45(15) - 1 \cdot 5(15)^2$
$= €437 \cdot 50$

MARKING SCHEME NOTES

Question 9 (a) (ii) [Scale 5C (0, 3, 4, 5)]

3: • Substitutes $t = 18$ into V formula
• Finds first derivative and stops
• States investment was decreasing but incorrect justification

4: • First derivative with substitution for t but subsequent error(s)
• Correct justification but fails to state that investment was decreasing
• Uses substitution of $t = 17$, 18 and 19 into V formula but no conclusion

5: • Correct answer with justification, by calculus or otherwise

Question 9 (a) (iii) [Scale 25C (0, 8, 20, 25)]

8: • Some correct substitution into formula for V
• Substitutes $t = 24$ into expression for $\frac{dV}{dt}$

20: • Correct substitution but errors in evaluating worth

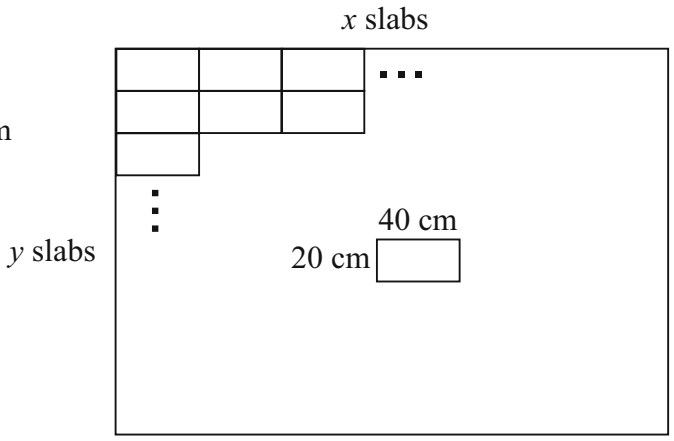
Question 9 (a) (iv) [Scale 5C (0, 3, 4, 5)]

3: • Writes down first derivative
• Solves for t and stops
• Substitutes a value for t ($\neq 15$) into V formula

4: • Substitutes correct value for t into V formula with error(s) in calculations
• Uses substitution of $t = 14$, 15 and 16 into V formula with no conclusion
• Correct answer without justification of using $t = 15$

Question 9 (b) (i)

Length = $40x$ cm
Width = $20y$ cm
Perimeter $P = 2(40x + 20y) = (80x + 40y)$ cm



Question 9 (b) (ii)

$P = 64$ m $= 6400$ cm

$80x + 40y = 6400$

$2x + y = 160$

$\therefore\ y = 160 - 2x$

Question 9 (b) (iii)

Differentiate A with respect to x and maximise the area by putting the derivative equal to zero.

$A = 40x \times 20y = 800xy$

$= 800x(160 - 2x)$

$= 128\ 000x - 1600x^2$

$\frac{dA}{dx} = 128\ 000 - 3200x = 0$

$128\ 000 = 3200x$

$\therefore\ x = 40$

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Question 9 (b) (iv)

$x = 40$

$y = 160 - 2x = 160 - 2(40) = 160 - 80 = 80$

Number of slabs $= xy = (40)(80) = 3200$

Marking Scheme Notes
Question 9 (b) [Scale 10C (0, 6, 8, 10)]

6: • States Perimeter = 2 length + 2 breadth (or equivalent)
• Perimeter = $2x + 2y$
• Perimeter = $40x + 20y$
• Changes 64 metres to centimetres
• Changes $80x + 40y$ to $0{\cdot}8x + 0{\cdot}4y$
• Correct expression for area in terms of x and y or in terms of x only
[Accept candidate's answer from (ii)]
• Some correct differentiation
• Substitutes value for x from part (iii) into a relevant formula

8: • Correct structure to answer with some errors
• At least 2 parts fully correct (allowing for candidate error in an earlier part)

LC 2012 (Set G): Paper 2

Question 1 (25 marks)

Question 1 (a)

S = Sample place
B: Boy, G: Girl
S = {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}
$\#S = 8$ ← This is the number of elements in the sample space.

Question 1 (b)

You need to know the number of boys and the number of girls in the school.

Marking Scheme Notes
Question 1 (a) [Scale 10C (0, 4, 7, 10)]
4: • Any work of merit e.g. one correct outcome other than BGG
7: • An almost correct response such as one/two outcomes missing or extra outcomes

Question 1 (b) [Scale 5B (0, 2, 5)]
2: • Any work of merit e.g. the number of boys or the number of girls

Question 1 (c)

$$P(\text{Event}) = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

S = {BBB, BBG, BGB, GBB, **GGB**, GBG, BGG, GGG}

$$P(\text{GGB}) = \frac{\text{Girl and then a girl and then a boy (G, G, B)}}{\text{Total number of outcomes}} = \frac{1}{8}$$

Marking Scheme Notes
Question 1 (c) [Scale 5B (0, 2, 5)]
2: • Any work of merit e.g. correct numerator or denominator in fraction format
e.g. $P(2 \text{ girls and a boy}) = \frac{3}{8}$
5: • Correct answer without work shown

Question 1 (d)

E = {BBG, BGB, GBB, GGB, GBG, BGG, GGG} ← Each element has at least 1 girl.
$\#(E) = 7$

$$P(\text{At least 1 Girl}) = \frac{7}{8}$$

E = {BBB, BBG, BGB, GBB} ← Each element has 3 boys or 2 boys and a girl.
$\#(E) = 4$

$$P(\text{3 boys or 2 boys and a girl}) = \frac{4}{8} = \frac{1}{2}$$

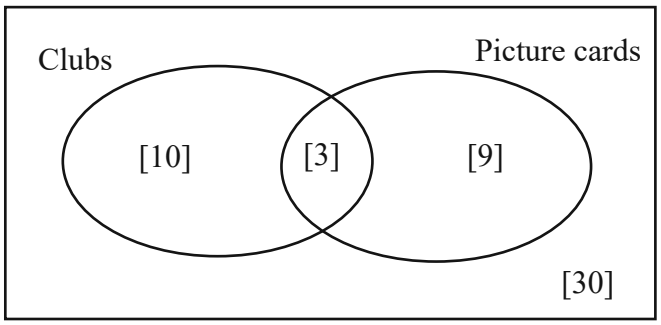
Niamh is more likely to be right as the probability of at least 1 girl is $\frac{7}{8}$ which is greater than the probability of 3 boys or 2 boys and 1 girl which is $\frac{4}{8}$.

Marking Scheme Notes
Question 1 (d) [Scale 5C (0, 2, 4, 5)]
2: • Any work of merit
e.g. outcomes for Niamh or Peter given or a correct answer with no work shown
4: • An almost correct response such as the outcomes for both Niamh and Peter given
• Answer with one element missing

Question 2 (25 marks)

Question 2 (a)

There are 13 clubs in the deck.
There are 12 picture cards (4 kings, 4 queens, 4 jacks).
There are 3 picture cards that are also clubs (king of clubs, queen of clubs, jack of clubs).
There are 30 cards that are neither picture cards nor clubs.
There are 22 cards ([10] + [3] + [9]) that are club or picture cards.



Marking Scheme Notes
Question 2 (a) [Scale 5C (0, 2, 4, 5)]
2: • Any work of merit such as one correct element
• States investment was decreasing but incorrect justification
4: • An almost correct response such as two or three correct elements

Question 2 (b)

$$P(\text{Event}) = \frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

(i) $P(\text{King of Clubs}) = \dfrac{\text{Number of kings of clubs}}{\text{Total number of cards}} = \dfrac{1}{52}$

(ii) $P(\text{Club or Picture card}) = \dfrac{\text{Number of clubs or picture cards}}{\text{Total number of cards}} = \dfrac{22}{52} = \dfrac{11}{26}$

(iii) On the first pick there are 30 cards that are not clubs or picture cards to be picked from 52 cards **and then** on the second pick there are 29 cards that are not clubs or picture cards to be picked from 51 cards.

And/or
And: Multiply
P(A) and then P(B): $P(A) \times P(B)$
Or: Add
P(A) or P(B): $P(A) + P(B)$

$$P(\text{Not a club or picture, Not a club or picture}) = \frac{30}{52} \times \frac{29}{51} = 0 \cdot 33$$

Marking Scheme Notes

Question 2 (b) (i) [Scale 10B (0, 5, 10)]

5: • Any work of merit such as correct or consistent numerator or denominator in fraction format

10: • Correct answer without work shown

Question 2 (b) (ii) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit such as a correct or consistent partial probability e.g. $\frac{10}{52}$.

4: • Almost correct or consistent response e.g. $= \frac{13}{52} + \frac{9}{52}$ and stops

5: • Correct answer without work shown

Question 2 (b) (iii) [Scale 5C* (0, 2, 4, 5)]

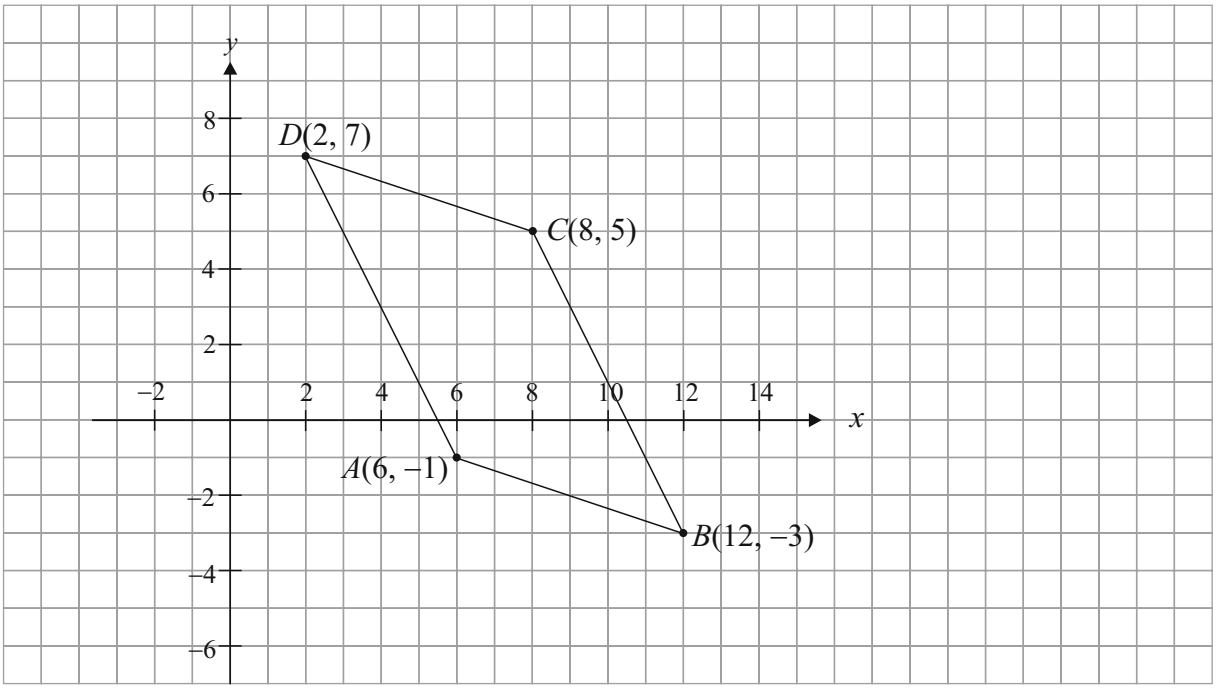
2: • Any work of merit such as a correct or consistent numerator or denominator in fraction format

4: • Almost correct or consistent response e.g. $= \frac{30}{52} \times \frac{29}{51}$ and stops

5: • Correct answer without work shown

Question 3 (25 marks)

Question 3 (a)



Marking Scheme Notes

Question 3 (a) [Scale 15C (0, 5, 10, 15)]

5: • Any work of merit such as showing knowledge of plotting a point

10: • An almost correct response such as two or three points plotted correctly or a consistent error in plotting all four points

Question 3 (b)

First method: Show that opposite sides are parallel by showing the slopes of opposite lines are equal.

Second method: Show that the diagonals bisect each other by showing the midpoint of $[AC]$ equals the midpoint of $[DB]$.

Question 3 (c)

Use the second method.

$A(6, -1) = (x_1, y_1)$, $C(8, 5) = (x_2, y_2)$

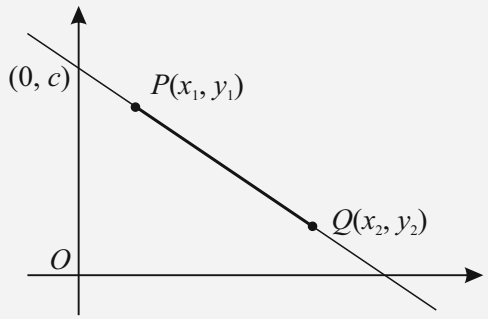
$$\text{Mid-point of } [AC] = \left(\frac{6+8}{2}, \frac{-1+5}{2}\right) = (7, 2)$$

$B(12, -3) = (x_1, y_1)$, $D(2, 7) = (x_2, y_2)$

$$\text{Mid-point of } [BD] = \left(\frac{12+2}{2}, \frac{-3+7}{2}\right) = (7, 2)$$

Therefore, $ABCD$ is a parallelogram.

Formulae and Tables Book
Co-ordinate geometry: Line



Midpoint of PQ [page 18]

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Marking Scheme Notes

Question 3 (b) [Scale 5B (0, 2, 5)]

2: • Any work of merit e.g. incomplete statement(s) with some merit

Question 3 (c) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit such as identification of a relevant formula

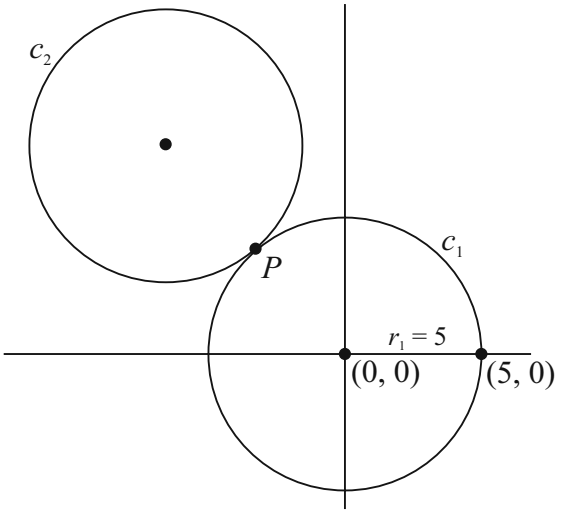
4: • Almost correct response such as one pair of lines shown as parallel or a correct response using a method not described in (b)

Question 4 (25 marks)

Question 4 (a)

Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r

$$(x-h)^2 + (y-k)^2 = r^2$$



Circle c_1: Centre $(0, 0) = (h, k)$, radius $r_1 = 5$

$$c_1 : (x-0)^2 + (y-0)^2 = 5^2$$

$$\therefore x^2 + y^2 = 25$$

Marking Scheme Notes

Question 4 (a) [Scale 10C (0, 4, 7, 10)]

4: • Any work of merit such as identification of a relevant formula

7: • An almost correct response e.g. mishandles r^2

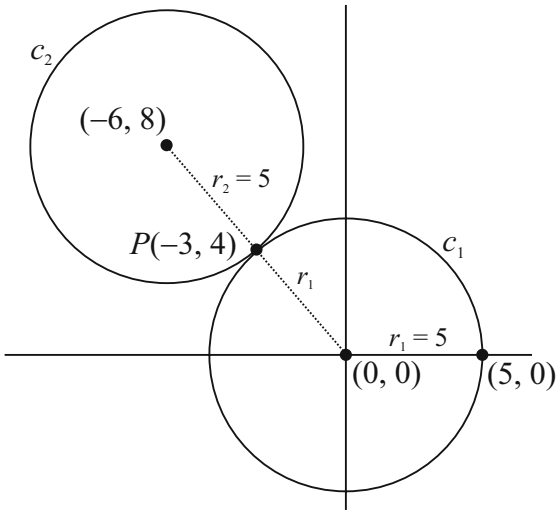
Question 4 (b)

Circle c_1: Centre (0, 0), radius $r_1 = 5$

$P(-3, 4) \in c_1$?

$x^2 + y^2 = (-3)^2 + (4)^2 = 9 + 16 = 25$

$\therefore P(-3, 4) \in c_1$



Question 4 (c)

$(0, 0) \to (-3, 4) \to (-6, 8)$ [Subtract 3, Add 4]

Circle c_2: Centre (−6, 8), radius $r_2 = 5$

$c_2 : (x-(-6))^2 + (y-8)^2 = 5^2$

$\therefore (x+6)^2 + (y-8)^2 = 25$

Marking Scheme Notes

Question 4 (b) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit such as a correct substitution

4: • Almost correct response such as required conclusion not given

Question 4 (c) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit e.g. use of translation indicated

4: • Almost correct response such as correct centre and radius indicated

• Correct answer without work shown

Question 4 (d)

Equation of t:

Point $P(-3, 4)$, slope m (perpendicular to OP)

Slope of OP: $O(0, 0) = (x_1, y_1)$, $P(-3, 4) = (x_2, y_2)$

$$m_1 = \frac{4-0}{-3-0} = -\frac{4}{3}$$

Slope of t: To find the perpendicular slope, invert m_1 and change its sign.

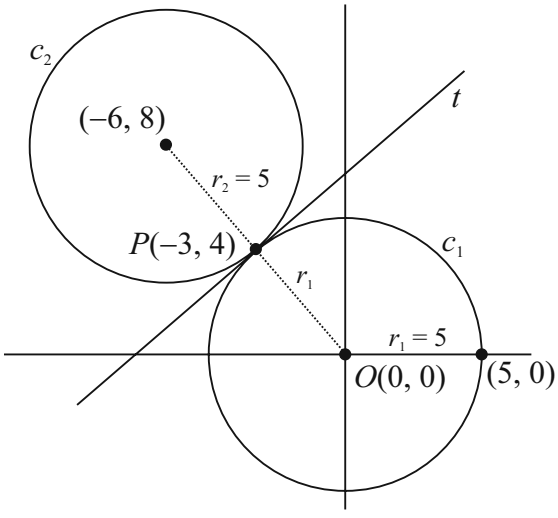
$$m_2 = \frac{3}{4}$$

Equation of t: $y - 4 = \frac{3}{4}(x-(-3))$

$$4(y-4) = 3(x+3)$$

$$4y - 16 = 3x + 9$$

$$0 = 3x - 4y + 25$$



Marking Scheme Notes

Question 4 (d) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit such as identification of a relevant formula

4: • An almost correct response such as slope of tangent found correctly but fails to finish.

• Correct answer without work shown

Formulae and Tables Book

Co-ordinate geometry: Line

Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

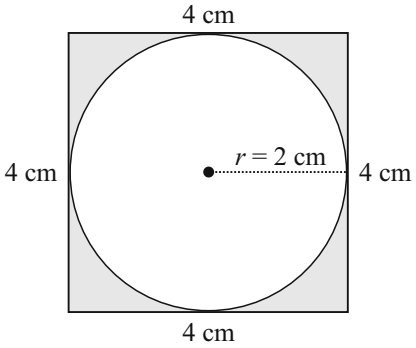
Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

Question 5 (25 marks)

Question 5 (a) (i)



Area of square: $A = 16 \text{ cm}^2$

$l = ?$

$16 = l^2 \Rightarrow l = \sqrt{16} = 4 \text{ cm}$

$\therefore r = 2 \text{ cm}$

Question 5 (a) (ii)

Area of circle: $r = 2$ cm

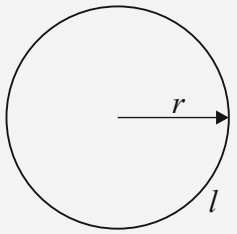
$A = \pi(r)^2 = 12{\cdot}6 \text{ cm}^2$

Area of shaded area

= Area of square – Area of circle

$= 16 \text{ cm}^2 - 12{\cdot}6 \text{ cm}^2 = 3{\cdot}4 \text{ cm}^2$

Formulae and Tables Book
Length and area:
Circle [page 8]



$l = 2\pi r$

$A = \pi r^2$

Marking Scheme Notes

Question 5 (a) (i) [Scale 5B* (0, 2, 5)]

2: • Any work of merit e.g. leaves diameter = 4 or works with $l = \frac{16}{4}$.

5: • Correct answer without work shown

Question 5 (a) (ii) [Scale 5B* (0, 2, 5)]

2: • Any work of merit such as identification of r in this part

5: • Correct answer without work shown

Question 5 (b) (i)

Vol. of candle = Vol. of cylinder + Vol. of cone

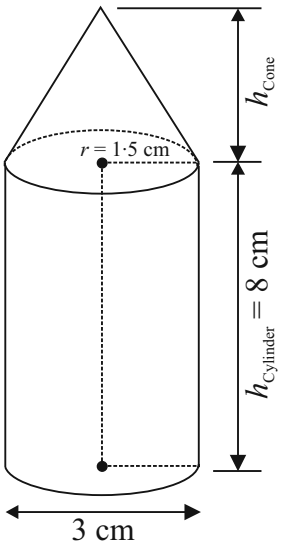
Volume of candle = $21\pi \text{ cm}^3$

Volume of cylinder:

$r = 1{\cdot}5$ cm, $h_{\text{Cylinder}} = 8$ cm

Volume of cone:

$r = 1{\cdot}5$ cm, $h_{\text{Cone}} = ?$



$V_{\text{Candle}} = V_{\text{Cylinder}} + V_{\text{Cone}}$

$21\pi = \pi(1{\cdot}5)^2(8) + \frac{1}{3}\pi(1{\cdot}5)^2 h_{\text{Cone}}$

$21\pi = 18\pi + 0{\cdot}75\pi h_{\text{Cone}}$

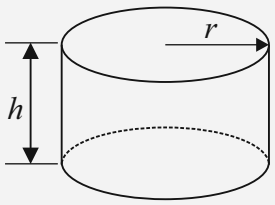
$21 = 18 + 0{\cdot}75 h_{\text{Cone}}$

$3 = 0{\cdot}75 h_{\text{Cone}}$

$\therefore h_{\text{Cone}} = 4 \text{ cm}$

Total height of candle = 8 cm + 4 cm = 12 cm

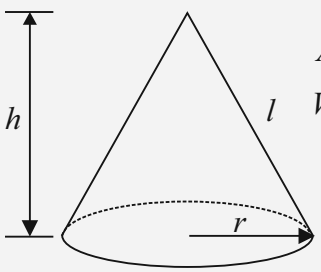
Formulae and Tables Book
Surface area and volume:
Cylinder [page 10]



$A = 2\pi rh$

$V = \pi r^2 h$

Cone [page 10]



$A = \pi rl$

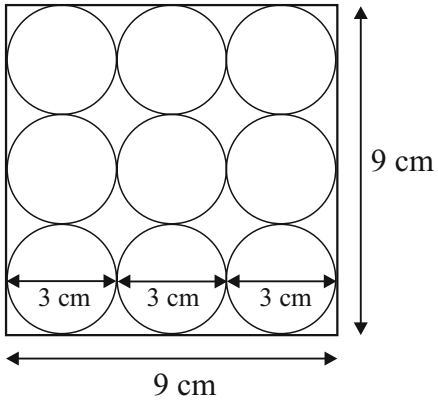
$V = \frac{1}{3}\pi r^2 h$

SEC Set G
2012 P2

Question 5 (b) (ii)

Length l of the box = 3 × 3 cm = 9 cm
Breadth b of the box = 3 × 3 cm = 9 cm
Height h of the box = Height of candle = 12 cm

$\therefore$ Volume of box = $l \times b \times h = 9 \times 9 \times 12 = 972$ cm^3



Marking Scheme Notes

Question 5 (b) (i) [Scale 10D* (0, 2, 5, 8, 10)]

2: • Any work of merit such as r identified as 1·5

5: • Further work of merit e.g. correct substitution for r and h or volume of cylinder or cone correct

8: • An almost correct response such as h isolated

Question 5 (b) (ii) [Scale 5B* (0, 2, 5)]

2: • Any work of merit e.g. finds base or height of the box

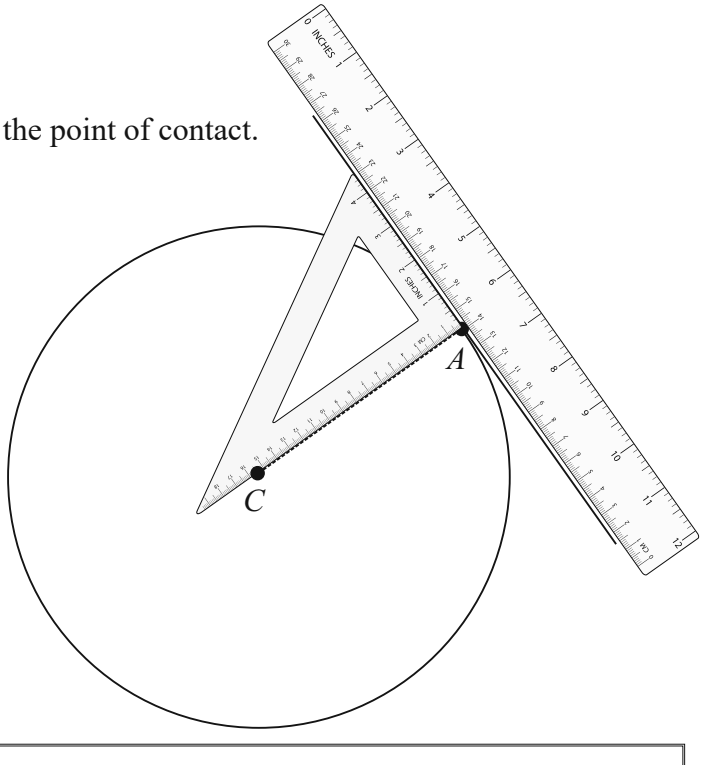
Question 6A (25 marks)

Question 6A (a) (i)

The tangent is perpendicular to the radius at the point of contact.

Question 6A (a) (ii)

Place a set square along the line AC with its corner at A as shown.
Place a ruler along the shorter edge of the set square.
Remove the set square and draw a line through A along the edge of the rule.
This line is a tangent to the circle.



Marking Scheme Notes

Question 6A (a) (i) [Scale 5B (0, 2, 5)]

2: • Any work of merit such as an incomplete statement with radius or diameter or point of contact mentioned

Question 6A (a) (ii) [Scale 5B (0, 2, 5)]

2: • Any work of merit e.g. any incorrect line through the point of contact

Question 6A (b)

The **circumcentre** *O* is the intersection of the **perpendicular bisectors** (also called the **mediators**) of the sides.

Find the perpendicular bisector of side *AB*. Place the point of the compass at *A* and draw an arc longer than the half-way point of the side to the left and right *AB* as shown.

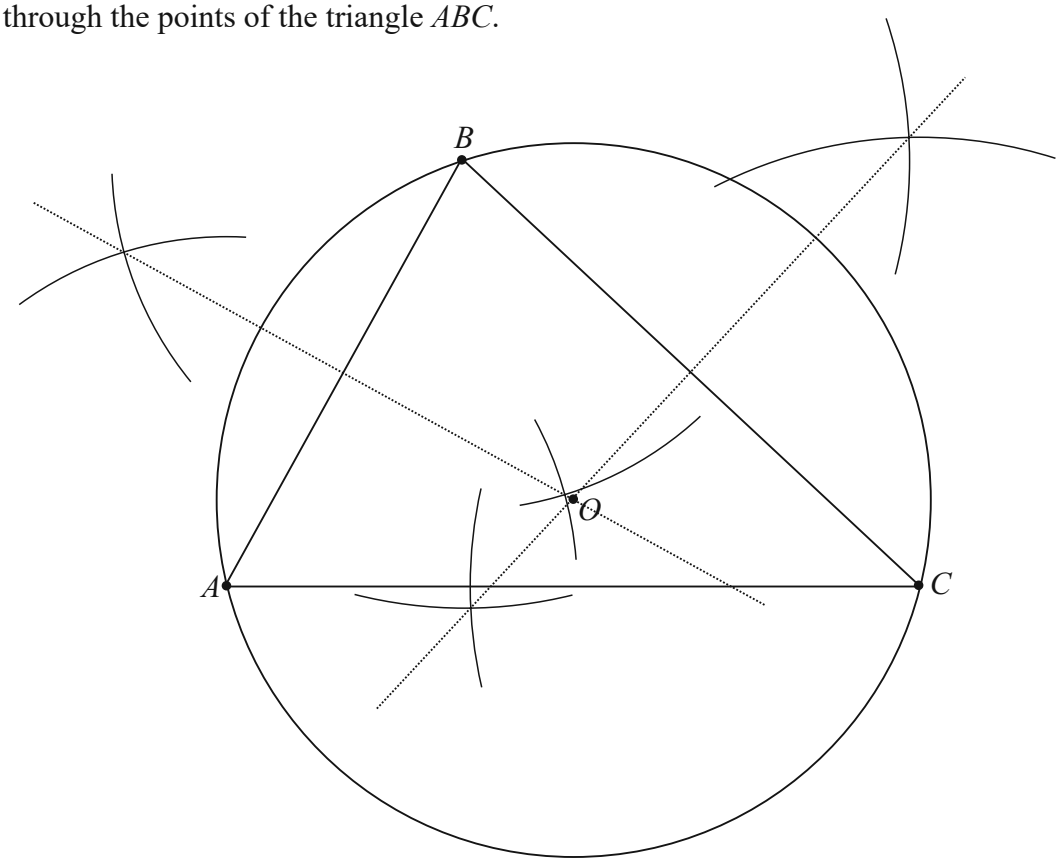
Leaving the compass at the same width, place the point of the compass at *B* and draw arcs that intersect the previous arcs.

Draw a line through the points where both sets of arcs intersect. This line is a **perpendicular bisector**.

Draw the perpendicular bisector of side *BC* in the same way.

The intersection of the two perpendicular bisectors is the circumcentre *O*.

Place the point of the compass at *O* and move your pencil out to *A* and draw the circumcircle which passes through the points of the triangle *ABC*.



Marking Scheme Notes

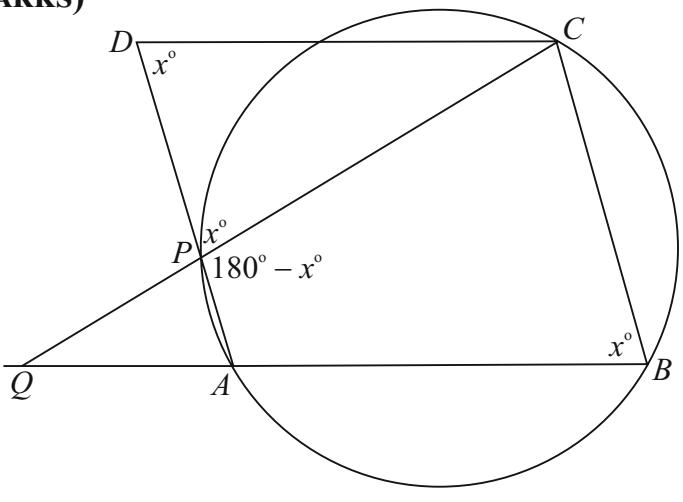
Question 6A (b) [Scale 15D (0, 4, 7, 11, 15)]

4: • Any work of merit such as an arc drawn from any vertex of the triangle
• Midpoint of a triangle side indicated or finds the incentre or centroid

7: • Further work of merit such as one perpendicular bisector of the triangle side constructed correctly

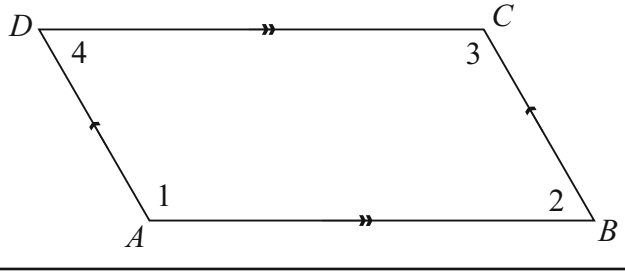
11: • An almost correct response where the circumcentre is established correctly but no circle drawn or both mediators and circle drawn correctly but no evidence of construction

Question 6B (25 marks)

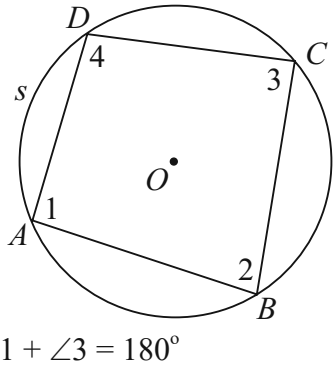


Properties of parallelograms

1. Opposite sides are equal in length: $|AB| = |DC|$ and $|AD| = |BC|$.
2. Opposite sides are parallel: $AB \parallel DC$ and $AD \parallel BC$.
3. Opposite angles are equal in measure: $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

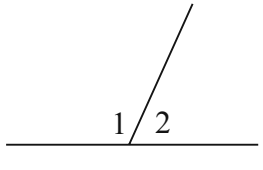


Cyclicquadrilateral



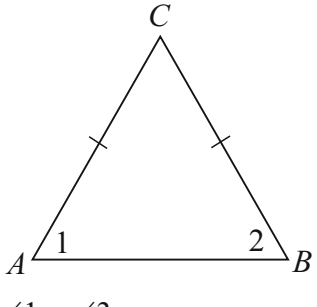
$\angle 1 + \angle 3 = 180^o$
$\angle 2 + \angle 4 = 180^o$

Straight angle



$\angle 1 + \angle 2 = 180^o$

Isosceles triangle



$\angle 1 = \angle 3$
$|AC| = |BC|$

Let $|\angle CDP| = x^o$

$|\angle CDP| = |\angle CBA|$ [Opposite angles of a parallelogram]

$|\angle CBA| + |\angle CPA| = 180^o$ [Opposite angles of a cyclic quadrilateral add up to 180^o]

$\therefore |\angle CPA| = 180^o - x^o$

$|\angle CPD| = x^o$ [Straight angle]

$\therefore |\angle CPD| = |\angle CDP|$

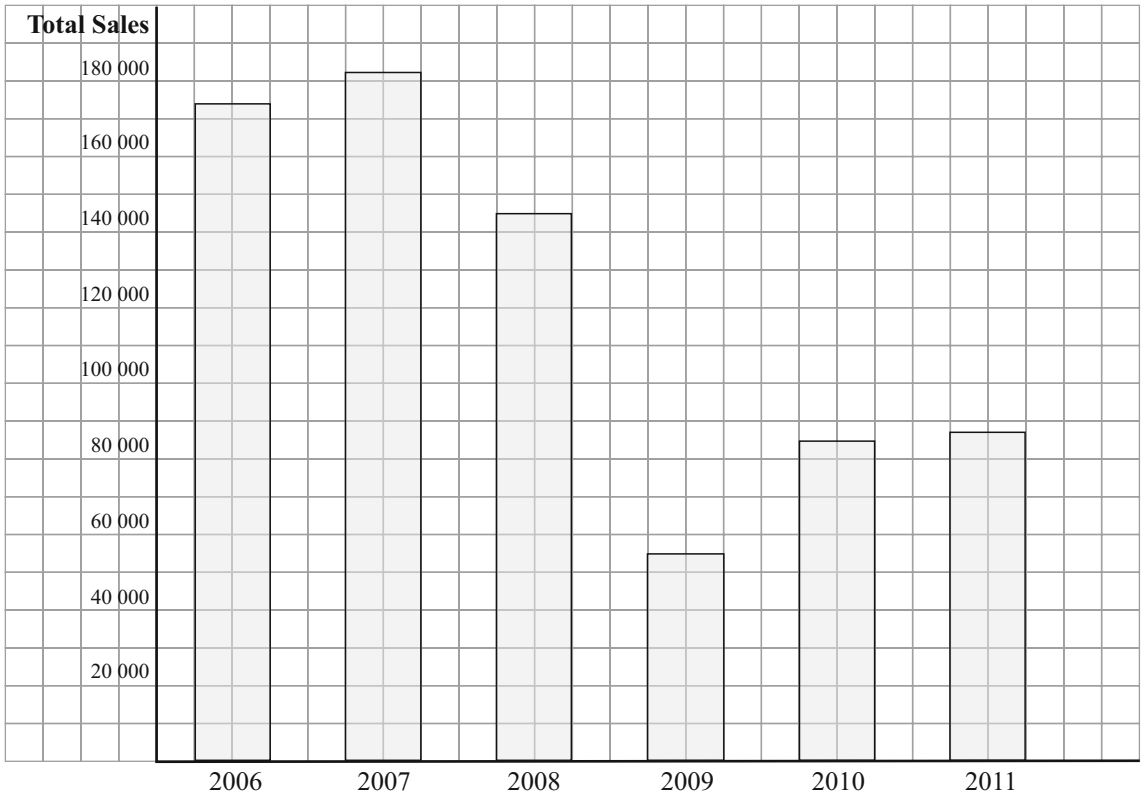
$\therefore \Delta CDP$ is an isosceles triangle

$\therefore |CD| = |CP|$

Marking Scheme Notes
Question 6B [Scale 25C (0, 8, 17, 25)]
8: • Any one correct step
17: • One or two steps missing or incorrect

Question 7 (75 marks)
Question 7 (a) (i)

Number of cars sold	
Year	Annual total
2006	173 273
2007	180 754
2008	146 470
2009	54 432
2010	84 907
2011	86 932



Marking Scheme Notes
Question 7 (a) (i) [Scale 10C (0, 4, 7, 10)]
4: • Any work of merit such as a rectangle for one year represented correctly or a correct scaled axis shown
7: • An almost correct response such as one or two incorrect rectangles or scales omitted

Question 7 (a) (ii)

Formulae and Tables Book
Statistics and probability:
Mean from a list of n numbers
[page 33]

$$\mu = \frac{\text{Sum of all the numbers}}{\text{Number of numbers}} = \frac{\sum x}{n}$$

Number of cars sold	
Year	Annual total
2006	173 273
2007	180 754
2008	146 470
2009	54 432
2010	84 907
2011	86 932
	726 768

$$\text{Mean } \mu = \frac{173\,273 + 180\,754 + 146\,470 + 54\,432 + 84\,907 + 86\,932}{6} = \frac{726\,768}{6} = 121\,128$$

Question 7 (a) (iii)

$$\%\text{ increase} = \frac{86\,932 - 54\,432}{54\,432} \times 100\%$$
$$= 59 \cdot 71\%$$

Number of cars sold	
Year	Annual total
2006	173 273
2007	180 754
2008	146 470
2009	**54 432**
2010	84 907
2011	**86 932**

Marking Scheme Notes
Question 7 (a) (ii) [Scale 10C (0, 4, 7, 10)]
4: • Any work of merit e.g. at least two correct elements identified
7: • An almost correct response such as formula for mean with fully correct substitution.
10: • 121 128 without work

Question 7 (a) (iii) [Scale 5C (0, 2, 4, 5)]
2: • Any work of merit e.g. a correct relevant figure identified 86 932 or 54 432
4: • An almost correct response such as one step missing
5: • Correct answer without work shown

Question 7 (a) (iv)

Aoife's argument does not recognise that this increase comes from a very low base, and that sales had been much better before 2009.

Paul's argument does not recognise that, although sales are lower now than in 2007, they have recovered a lot since their lowest point in 2009.

Question 7 (a) (v)

Sales fell dramatically from 2007 to 2009; they have recovered a lot since then, but are still much lower than they were at the start.

Marking Scheme Notes

Question 7 (a) (iv) [Scale 5B (0, 2, 5)]

2: • Any work of merit which includes any mention of the year 2009, a decline in sales or an increase in sales

Question 7 (a) (v) [Scale 5B (0, 2, 5)]

2: • Any work of merit which includes any mention of an increase of sales in year 2007, a decline in sales in years 2008/2009 or an increase in sales in years 2010/2011

Question 7 (b)

(i) Highest quarterly sales are in the first quarter and decrease significantly in each subsequent quarter.

(ii) People like to buy new cars early in the year so that they have a new year number plate.

(iii) First quarter sales in 2012 are about one-third of the range between those quarters in 2010 and 2011. Assuming the same proportion of sales gives an estimate of 85 500 for annual sales.

Marking Scheme Notes

Question 7 (b) (i) [Scale 5B (0, 2, 5)]

2: • Any work of merit such as a correct relevant observation that is incomplete

Question 7 (b) (ii) [Scale 5A (0, 5)]

0: • No reason given

Question 7 (b) (iii) [Scale 5B (0, 2, 5)]

2: • Any work of merit such as an estimate within the range of values 72 000 to 90 000 without justification or any work with 36 081

Question 7 (c) (i)

Year: 2011

Petrol:	23 246
Diesel:	61 730
Other:	1 956
Total:	86 932

Petrol: $\frac{23\ 246}{86\ 932} \times 100\% = 26 \cdot 7\%$

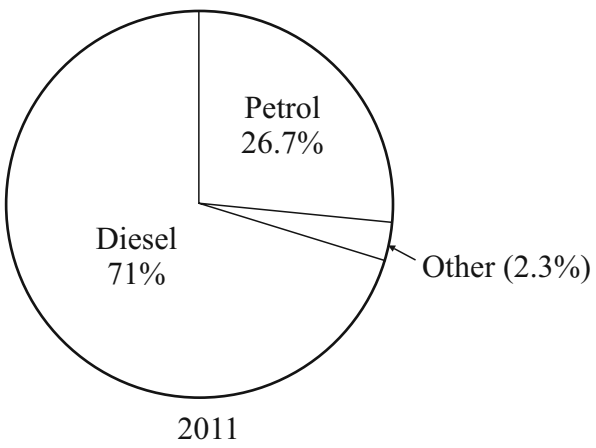
$\frac{23\ 246}{86\ 932} \times 360^{o} = 96^{o}$

Diesel: $\frac{61\ 730}{86\ 932} \times 100\% = 71\%$

$\frac{61\ 730}{86\ 932} \times 360^{o} = 256^{o}$

Other: $\frac{1956}{86\ 932} \times 100\% = 2.3\%$

$\frac{1956}{86\ 932} \times 360^{o} = 8^{o}$



Question 7 (c) (ii)

Answer: **D**

Marking Scheme Notes

Question 7 (c) (i) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit such as finding any or all of the correct percentages or fractions.

4: • An almost correct response such as all angles indicated but drawn incorrectly or not drawn

• Correct answer without work shown

Question 7 (c) (ii) [Scale 5B (0, 2, 5)]

2: • Selects B or C

Question 7 (d)

Diesel engines	Petrol engines
117, 125, 120, 125, 134, 110, 118, 114, 119, 119, 116, 107,	139, 133, 150, 157, 138, 159, 129, 138, 134, 129, 129, 136.

(i)

Diesel engines						Stem	Leaf	Petrol engines					
						7	10						
9	9	8	7	6	4	0	11						
				5	5	0	12	9	9	9			
						4	13	3	4	6	8	8	9
							14						
							15	0	7	9			

(ii) Yes. The diesel engines grouped at the top of the plot have a smaller median value.

(iii) No. The emissions for the petrol engines are *more* spread out than for the diesel ones. The range for the petrol engines is greater than that for the diesel engines.

Marking Scheme Notes

Question 7 (d) (i) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit e.g. a correct element of stem-leaf plot indicated

4: • An almost complete response such as an incomplete plot with one/two elements missing or incorrect or no key indicated

Question 7 (d) (ii) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit such as an answer with no reference to stem-leaf or central tendency

4: • An almost complete response such as reference to median/mean but no conclusion

Question 7 (d) (iii) [Scale 5C (0, 2, 4, 5)]

2: • Any work of merit such as an answer with no reference to stem-leaf or measure of variability

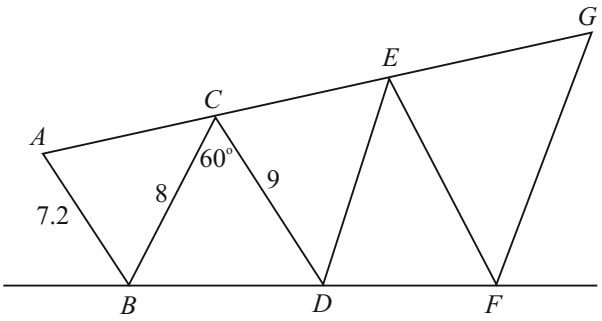
4: • An almost complete response such as reference to range or measurement of variability but no conclusion

Question 8 (75 marks)

Question 8 (a) (i)

A triangle in which the three sides have different lengths

Question 8 (a) (ii)



Formula: Enlargements

Scale factor $k = \dfrac{|\text{Image length}|}{|\text{Object length}|}$

$k^2 = \dfrac{|\text{Image area}|}{|\text{Object area}|}$

$k = \dfrac{|\text{Image length}|}{|\text{Object length}|} = \dfrac{|CD|}{|AB|} = \dfrac{9}{7 \cdot 2} = 1 \cdot 25$ ← Triangle CDE is the image of the object ABC.

$|ED| = k|BC| = 1 \cdot 25(8) = 10$ m

$|FG| = k|ED| = 1 \cdot 25(10) = 12 \cdot 5$ m

Marking Scheme Notes

Question 8 (a) (i) [Scale 5B (0, 2, 5)]

2: • Any work of merit such as any triangle drawn with no measurement of sides indicated or defines an isosceles triangle

5: • A triangle drawn with three different lengths indicated

Question 8 (a) (ii) [Scale 15D* (0, 4, 7, 11, 15)]

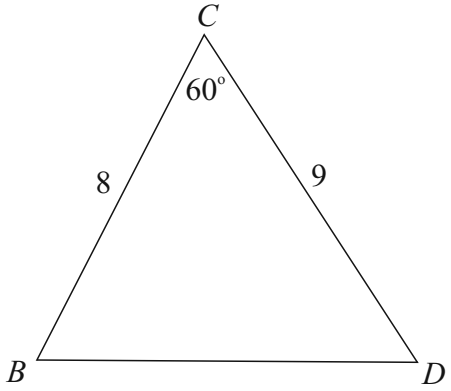
4: • Any work of merit e.g. an indication that [FG] is an enlargement of [BC] or [ED] or work towards scale factor

7: • Further work of merit such as scale factor found

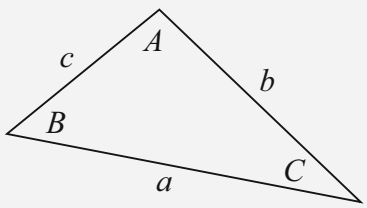
11: • An almost complete response such as scale factor applied to [BC] to get [ED] correctly or fails to finish e.g. leaves $|FG|$ as $1 \cdot 25^2 \times 8$ or $1 \cdot 25 \times 10$

15: • Correct answer without work shown

Question 8 (a) (iii)



Formulae and Tables Book
Trigonometry of the triangle:
Cosine rule [page 16]



$a^2 = b^2 + c^2 - 2bc\cos A$

$|BD|^2 = 8^2 + 9^2 - 2(8)(9)\cos 60^\circ = 73$

$\therefore |BD| = \sqrt{73} = 8 \cdot 544$ m

Marking Scheme Notes
Question 8 (a) (iii) [Scale 15D* (0, 4, 7, 11, 15)]
4: • Any work of merit such as identifying the cosine rule as a method of solution
7: • Further work of merit such as significant correct substitution into the cosine formula
11: • An almost complete response such as cosine rule worked to $\sqrt{73}$.
• Correct answer without work shown

Question 8 (a) (iv)

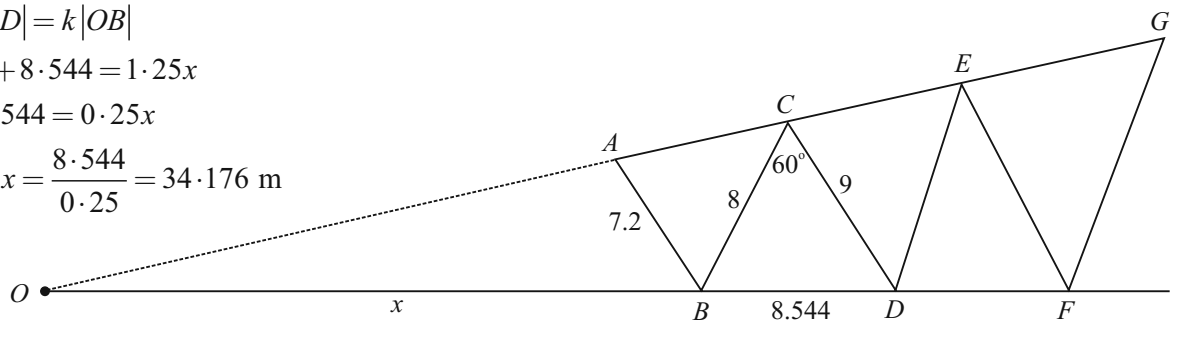
If you work the lines AG and BF back they will meet at O, the centre of the enlargement.
Triangle ODC is the image of triangle OBA.
Call x, the distance OB.

$|OD| = k|OB|$

$x + 8 \cdot 544 = 1 \cdot 25x$

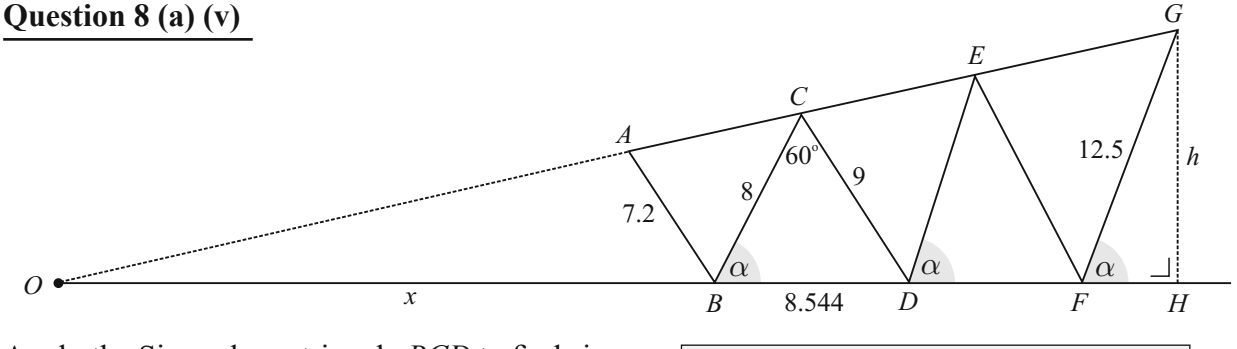
$8 \cdot 544 = 0 \cdot 25x$

$\therefore x = \dfrac{8 \cdot 544}{0 \cdot 25} = 34 \cdot 176$ m



Marking Scheme Notes
Question 8 (a) (iv) [Scale 5B* (0, 2, 5)]
2: • Any work of merit such as an effort at finding or explaining the centre of enlargement or an effort at the use of the scale factor to find the centre of enlargement
• Correct answer without work shown

Question 8 (a) (v)

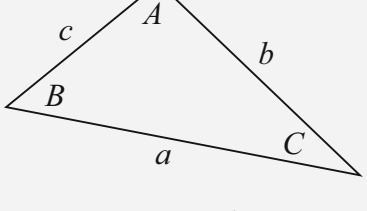


Apply the Sine rule on triangle BCD to find $\sin\alpha$:

$$\frac{\sin\alpha}{9} = \frac{\sin 60^\circ}{8 \cdot 544}$$

$$\sin\alpha = \frac{9\sin 60^\circ}{8 \cdot 544}$$

Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Sine Rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

EPL 104

Triangle *GHF* is a right-angled triangle. Find $\sin\alpha$:

$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

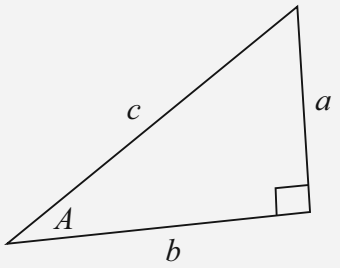
$$\sin\alpha = \frac{h}{12\cdot5}$$

$$\therefore \frac{9\sin 60^\circ}{8\cdot544} = \frac{h}{12\cdot5}$$

$$\therefore h = \frac{12\cdot5\times9\sin 60^\circ}{8\cdot544} = 11\cdot4 < 11\cdot6 \text{ m}$$

The plan meets the condition.

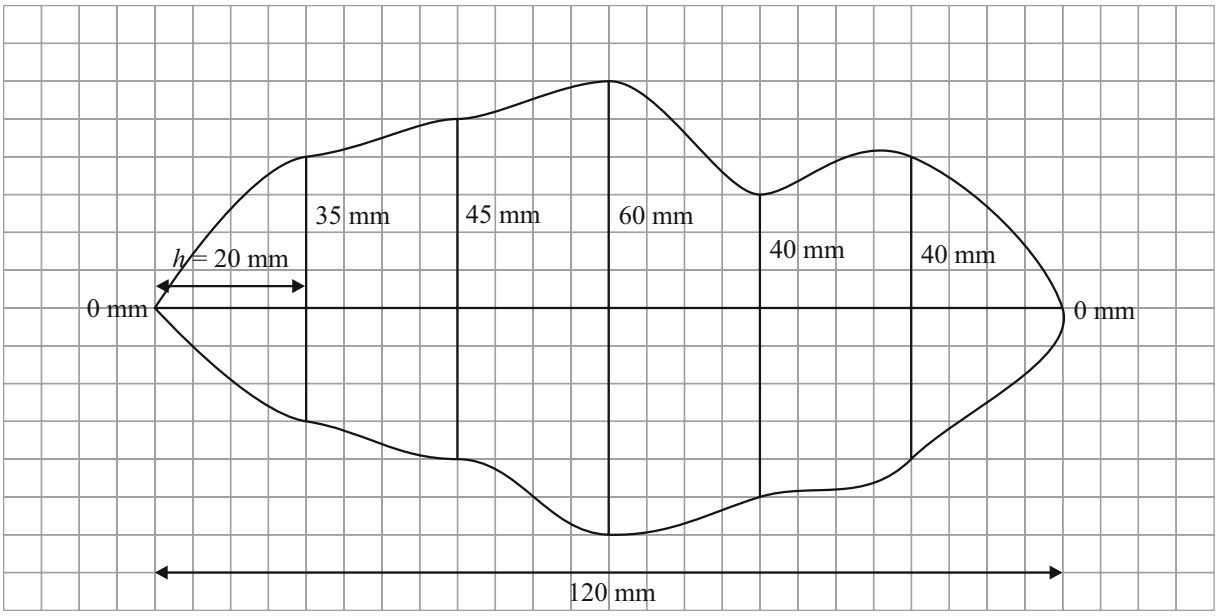
Formulae and Tables Book
Trigonometry: Right-angled triangle [page 16]



$$\sin A = \frac{a}{c},\ \cos A = \frac{b}{c},\ \tan A = \frac{a}{b}$$

Marking Scheme Notes
Question 8 (a) (v) [Scale 10D (0, 2, 5, 8, 10)]
2: • Any work of merit e.g. vertical height indicated (from point *G* or in triangles *CBD*, *EDF*) or correct relevant formula identified or $|FG| = 12\cdot5$
• Correct answer without work shown
5: • Further work of merit such as α or $\sin\alpha$ found
8: • An almost complete response whereby *h* is found but no conclusion is given

Question 8 (b) (i)



Marking Scheme Notes
Question 8 (b) (i) [Scale 10C* (0, 4, 7, 10)]
4: • Any work of merit such as one or two correct measurements shown
• If (b) (i) not done but at least one correct measurement is shown in (b) (ii)
7: • An almost complete response such as an incomplete plot with one/two elements missing or incorrect or no key indicated

Question 8 (b) (ii)

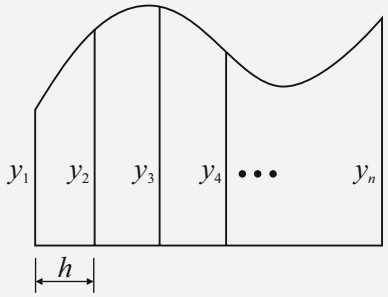
$h = 20$ mm

$A = \frac{20}{2}[0 + 0 + 2(35 + 45 + 60 + 40 + 40)]$

$= 10[440]$

$= 4400$ mm^2

Formulae and Tables Book
Area approximations [page 12]



$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + ... + y_{n-1})]$$

Marking Scheme Notes
Question 8 (b) (ii) [Scale 15D* (0, 4, 7, 11, 15)]

4: • Any work of merit such as a correct substitution of h, F + L or OTHERS
• Correct answer without work shown

7: • Further work of merit such as two correct substitutions of h, F + L or OTHERS.
• One correct substitution followed by some correct calculation

11: • An almost correct response such as fully correct substitution followed by an error in calculation
• Two correct substitutions followed by fully correct calculation

Sample Paper 2014 (Set H): Paper 1

Question 1 (25 marks)

Formulae and Tables Book
Indices and logs [page 21]

$$a^p \times a^q = a^{p+q}$$
$$\frac{a^p}{a^q} = a^{p-q}$$
$$(a^p)^q = a^{pq}$$
$$a^0 = 1$$
$$a^{-p} = \frac{1}{a^p}$$
$$a^{\frac{1}{2}} = \sqrt{a}$$

Question 1 (a)

$6^{-2} = \frac{1}{6^2}$ ← Move down to turn into a positive power.

$= \frac{1}{36}$

$81^{\frac{1}{2}} = \sqrt{81}$ ← Square root is the same as a number to the power of a half

$= 9$

Question 1 (b)

Scientific Notation
$a \times 10^n, 1 \le a < 10, n \in \mathbb{Z}$
Ex. $2 \cdot 3 \times 10^{-5}$

$2^{24} = 1.68 \times 10^7$ ← Input 2^{24} into your calculator.

Casio
Input 2^{24}.
Press Shift followed by Mode
Press 7: Sci
Press 3 (For 3 significant figures)

Question 1 (c)

$\frac{(a\sqrt{a})^3}{a^4}$ ← Use the power rules to write a to its correct power.

$= \frac{(a^1 \times a^{\frac{1}{2}})^3}{a^4}$ ← Add the powers in the bracket.

$= \frac{(a^{\frac{3}{2}})^3}{a^4}$ ← Multiply the powers on top of the fraction.

$= \frac{a^{\frac{3}{2} \times 3}}{a^4} = \frac{a^{\frac{9}{2}}}{a^4}$ ← Subtract powers.

$= a^{\frac{9}{2}-4} = a^{\frac{1}{2}} = \sqrt{a}$

Question 1 (d)

$49^x = 7^{2+x}$ ← Get the same base 7 on both sides.

$(7^2)^x = 7^{2+x}$

$7^{2x} = 7^{2+x}$ ← Equate the powers once you have the same base on each side.

$\therefore 2x = 2 + x$

$2x - x = 2$

$x = 2$

Question 2 (25 marks)

Question 2 (a)

$P = €5000$
$i = 0.06$
$t = 8$ years
$F = ?$

$F = 5000(1 + 0 \cdot 06)^8 = €7969 \cdot 24$

Formulae and Tables Book
Financial mathematics: Compound interest [page 30]

$$F = P(1 + i)^t$$

t = Time period (in years)
i = (Annual) rate of interest expressed as a decimal
F = Final value
P = Principal
Note: The time period can be months or weeks instead of years provided the interest rate is given for that time period.

Question 2 (b)

$€5000 \times 1 \cdot 2 = €6000$ ← Multiply €5000 by 1·2 to increase investment by 20%.

$P = €5000$
$i = ?$
$t = 8$ years
$F = €6000$

$F = P(1 + i)^t$ ← Use the folrmula to find i.

$6000 = 5000(1 + i)^8$ ← Divide both sides by 5000.

$$\frac{6000}{5000} = (1 + i)^8$$

$1 \cdot 2 = (1 + i)^8$ ← Use your calculator to get the eighth root on each side.

$1 \cdot 2^{\frac{1}{8}} = 1 + i$

$1 \cdot 023 = 1 + i$

$\therefore i = 0 \cdot 023 = 2 \cdot 3\%$ ← Multiply i by 100 to turn this number into an annual interest rate AER.

Question 3 (25 marks)

Question 3 (a)

$u = 2 + 3i$, $v = -1 + i$

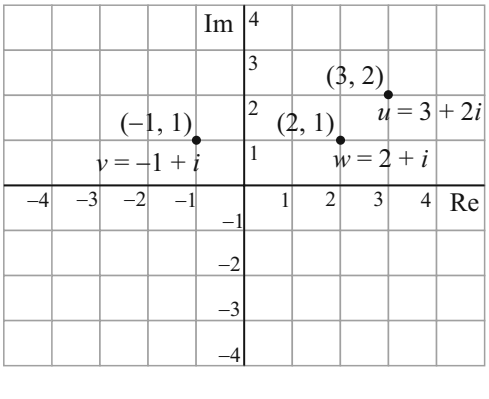
$w = u - v - 2$

$= (3 + 2i) - (-1 + i) - 2$ ← Get rid of brackets.

$= 3 + 2i + 1 - i - 2$ ← Add numbers and add i's.

$= 2 + i$

Question 3 (b)



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Question 3 (c)

$u = 2 + 3i$, $v = -1 + i$, $w = 2 + i$

$\frac{2u+v}{w}$ ← Replace u, v and w by their complex numbers.

$= \frac{2(3+2i)+(-1+i)}{2+i}$ ← Get rid of brackets.

$= \frac{6+4i-1+i}{2+i}$ ← Add numbers and add i's.

$= \frac{5+5i}{2+i}$ ← Multiply above and below by the conjugate of the bottom.

$= \frac{5+5i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{10-5i+10i-5i^2}{4-2i+2i-i^2}$

$= \frac{10+5i-5(-1)}{4-\cancel{2i}+\cancel{2i}-(-1)}$

$= \frac{10+5i+5}{4+1}$

$= \frac{15+5i}{5}$

$= 3+i$

FORMULA: Complex Numbers

Conjugates $\overline{z}$

$z = a + bi \Rightarrow \overline{z} = a - bi$

Multiplying a complex number by its conjugate:

$z\overline{z} = (a+bi)(a-bi) = a^2 + b^2$

QUESTION 4 (25 MARKS)

Question 4 (a)

$x^2 - 6x - 23 = 0$ ← Write down the values of a, b and c.

$a = 1$, $b = -6$, $c = -23$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-23)}}{2(1)}$ ← Input into your calculator as shown.
For ± chooose either + or - but put ± back into the answer.

$= \frac{6 \pm \sqrt{36+92}}{2}$

$= \frac{6 \pm \sqrt{128}}{2}$

$= \frac{6 \pm \sqrt{64 \times 2}}{2}$

$= \frac{6 \pm 8\sqrt{2}}{2}$

$= 3 \pm 4\sqrt{2}$

FORMULAE AND TABLES BOOK

Algebra: Roots of the quadratic equation

$ax^2 + bx + c = 0$ [page 20]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Question 4 (b)

$2r - s = 10 \Rightarrow 2r - 10 = s$.....(**L**) ← Write the linear equation (**L**) in terms of s.

$rs - s^2 = 12$....(**Q**)

$r(2r-10) - (2r-10)^2 = 12$ ← Replace s in the quadratic equation (**Q**).

$r(2r-10) - (2r-10)(2r-10) = 12$

$2r^2 - 10r - [4r^2 - 20r - 20r + 100] = 12$

$2r^2 - 10r - [4r^2 - 40r + 100] - 12 = 0$

$2r^2 - 10r - 4r^2 + 40r - 100 - 12 = 0$

$-2r^2 + 30r - 112 = 0$

$r^2 - 15r + 56 = 0$ ← Factorise the quadratic.

$(r-7)(r-8) = 0$

$\therefore r = 7, 8$

$r = 7 : s = 2(7) - 10 = 14 - 10 = 4$ ← Solve for s by substituting r into the linear equation (**L**).

$r = 8 : s = 2(8) - 10 = 16 - 10 = 6$

Answers: $r = 7$, $s = 4$ and $r = 8$, $s = 6$.

Question 5 (25 marks)

Question 5 (a)

$f(x) = 2^x$

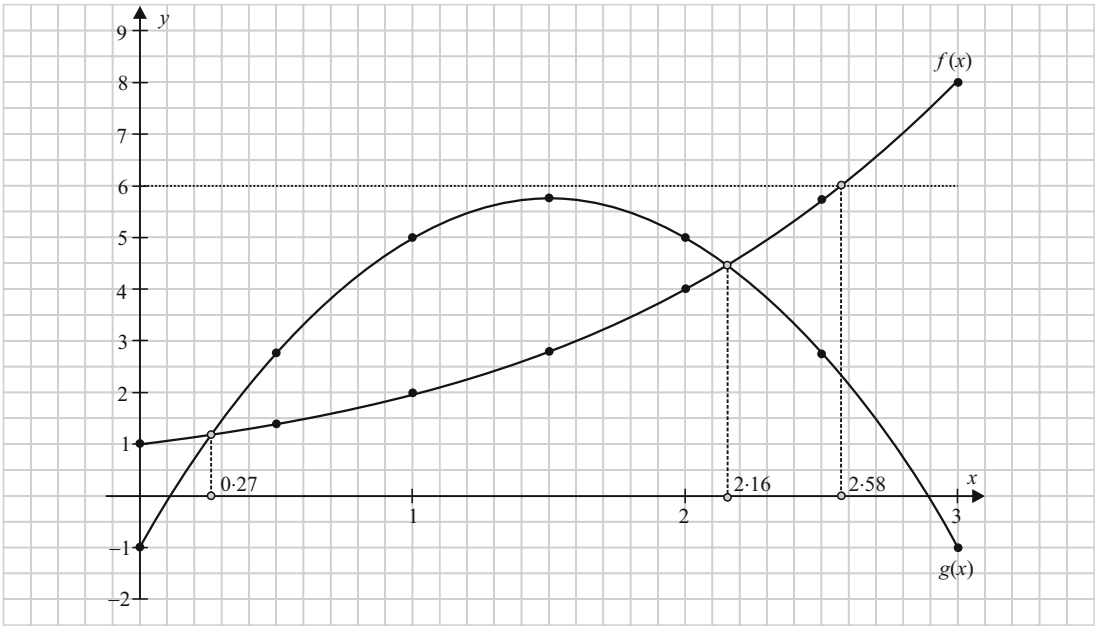
$g(x) = 9x - 3x^2 - 1$

Use your calculator to generate the values in the table correct to 2 decimal places.

Ex. $f(2 \cdot 5) = 2^{2 \cdot 5} = 5 \cdot 66$

$g(2 \cdot 5) = 9(2 \cdot 5) - 3(2 \cdot 5)^2 - 1 = 2 \cdot 75$

x	0	0.5	1	1·5	2	2·5	3
$f(x)$	1	1·41	2	2·83	4	5·66	8
$g(x)$	–1	2·75	5	5·75	5	2·75	–1



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Question 5 (b)

$f(x) = g(x)$ ← Put $f(x) = g(x)$ to get the required equation.

$2^x = 9x - 3x^2 - 1$

$2^x + 3x^2 - 9x + 1 = 0$

$x = 0{\cdot}27,\ 2{\cdot}16$ ← $f(x) = g(x)$ is where the 2 graphs intersect. Read off these values from the graph.

Question 5 (c)

$2^k = 6$

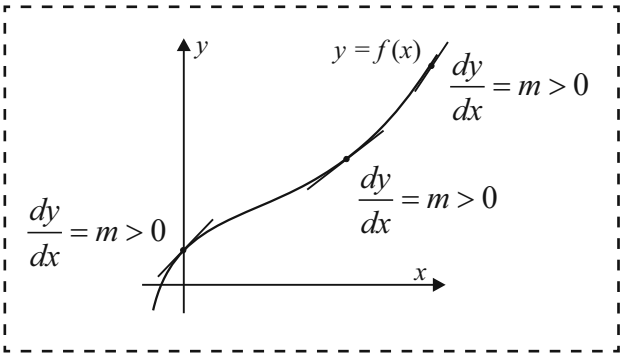
$f(k) = 6$ ← Draw in the horizontal line $f(x) = 6$ and find out where it intersects the $f(x)$.

$k = 2 \cdot 58$

Question 6 (25 marks)

Answer: **C**

Justification:
The graph of the cubic function shown is always increasing. Therefore, its slope m is always positive. When you differentiate a function you are finding its slope.



Example

Cubic function: $y = x^3 + x^2 + x + 1$

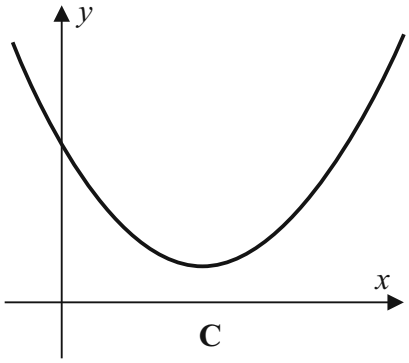
$$\frac{dy}{dx} = 3x^2 + 2x + 1 > 0$$

Quadratic Function: $y = 3x^2 + 2x + 1 > 0$

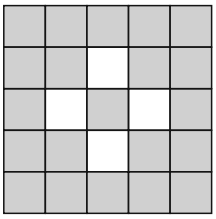
This quadratic function is positive for all values of x.

The y value is positive for all values of x.

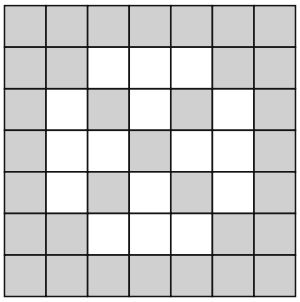
When you plot the function the graph will be above the x-axis.



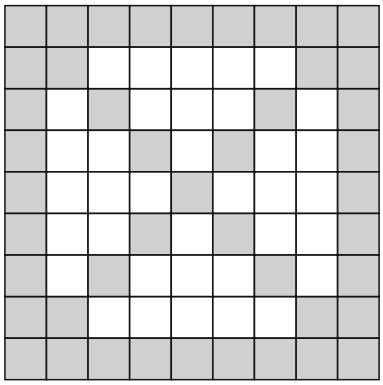
Question 7 (50 marks)



1st pattern



2nd pattern



3rd pattern

Question 7 (a)

Count the number of tiles in the first 3 patterns. Notice that there is a constant difference of 12 between each pattern. Therefore, add 12 each time to get the number of tiles in the 4th and 5th patterns.

Pattern	1	2	3	4	5
Number of tiles	21	33	45	**57**	**69**

Question 7 (b)

First term $a = 21$, Common difference $d = 12$

$$T_n = 21 + (n-1)(12)$$
$$= 21 + 12n - 12$$
$$= 12n + 9$$

Formulae and Tables Book
Sequences and series:
Arithmetic series or sequence
[page 22]

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

a is the first term
d is the common difference

Question 7 (c)

The number of tiles in the tenth pattern is T_{10}.
Replace n by 10 in the general term T_n.

$n = 10$

$T_{10} = 12(10) + 9 = 120 + 9 = 129$

Question 7 (d)

$T_n = 399$ ← Put the general term T_n equal to 399 and solve for n.

$12n + 9 = 399$

$12n = 399 - 9 = 390$

$n = \frac{390}{12} = 32 \cdot 5$ ← With 399 tiles she can go up to the 32nd pattern.

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Question 7 (e)

You are asked to add the number of tiles in the first n patterns: $21 + 33 + 45 + ... + n$

$a = 21$, $d = 12$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2}[2(21) + (n-1)(12)]$$
$$= \frac{n}{2}[42 + 12n - 12]$$
$$= \frac{n}{2}[12n + 30]$$
$$= n[6n + 15]$$

Question 7 (f)

$S_n = 399$ ← Put S_n equal to 399 and solve for n.

$n[6n + 15] = 399$

$6n^2 + 15n - 399 = 0$

$2n^2 + 5n - 133 = 0$ ← Factorise the quadratic.

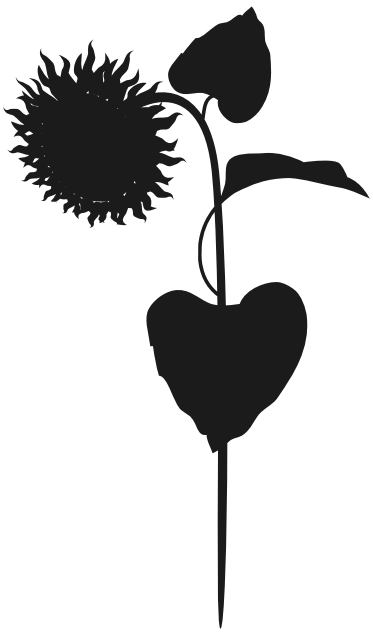
$(2n + 19)(n - 7) = 0$ ← Put each bracket equal to zero and solve for n.

$\therefore n = 7, \cancel{-\frac{19}{2}}$ ← Ignore the fractional solution as n is a natural number.

She can make the first 7 patterns with her 399 tiles.

Question 8 (50 marks)

Question 8 (a)



Plant 1

Height (day 1) = 16 cm

Height (day 2) = 20 cm (it grows 4 cm each day)

Plant 2

Height (day 1) = 24 cm

Height (day 2) = 27·5 cm (it grows 3·5 cm each day)

Day	Height of Plant 1 (cm)	Height of Plant 2 (cm)
1	16	24
2	20	27·5
3	24	31
4	28	34·5
5	32	38
6	36	41·5
7	40	45

Question 8 (b)

The heights of each plant form an arithmetic sequence.
n is the number of the term in the sequence which is also the day number.
T_n is the value of the nth term which is also the height h of the plant.

Plant 1: $a = 16$, $d = 4$

$T_n = 16 + (n - 1)(4)$
$= 16 + 4n - 4$
$= 4n + 12$
$\therefore\ h = 4n + 12$

Plant 2: $a = 24$, $d = 3{\cdot}5$

$T_n = 24 + (n - 1)(3{\cdot}5)$
$= 24 + 3{\cdot}5n - 3{\cdot}5$
$= 3{\cdot}5n + 20{\cdot}5$
$\therefore\ h = 3{\cdot}5n + 20{\cdot}5$

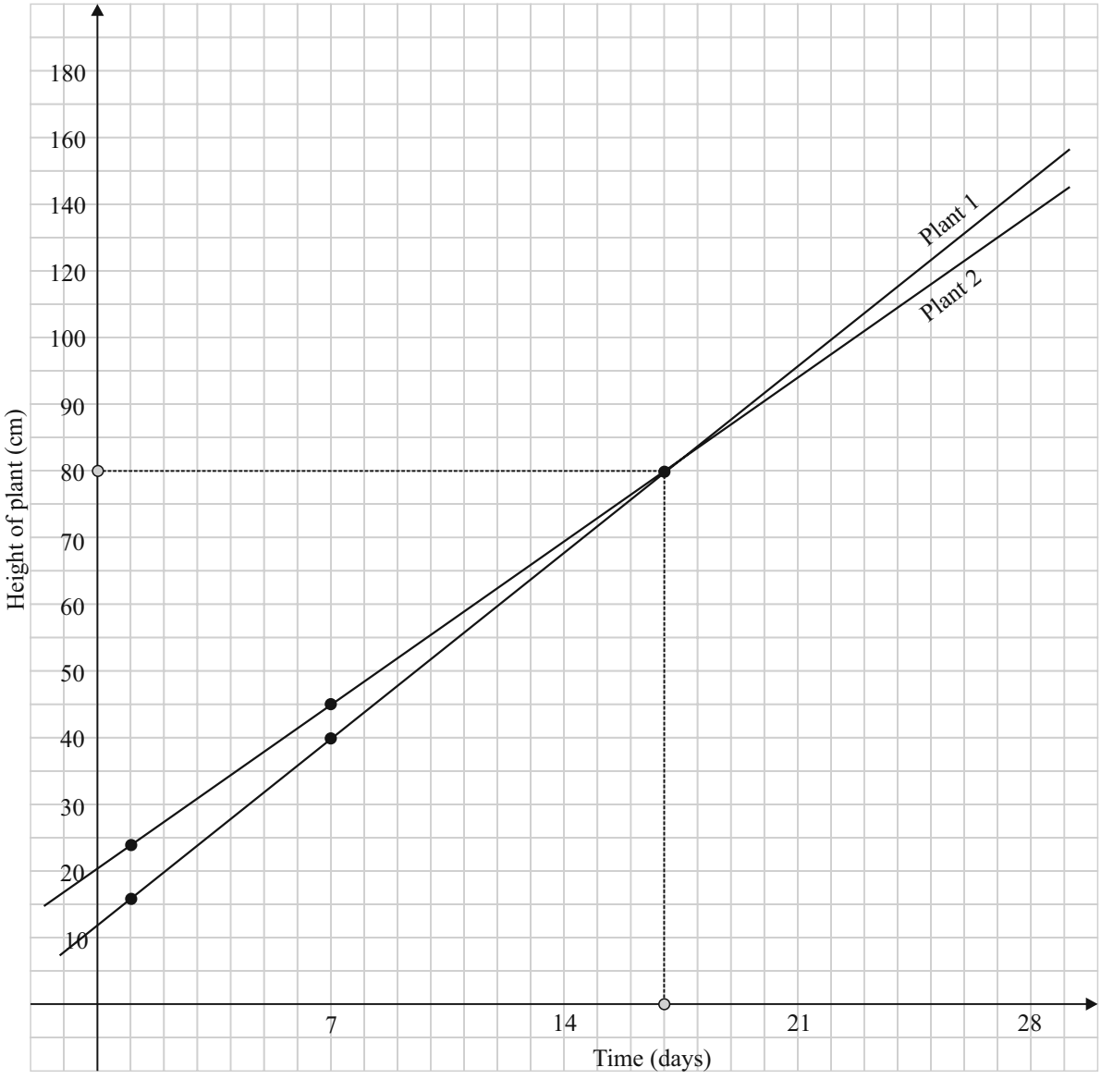
Formulae and Tables Book
Sequences and series: Arithmetic series or sequence [page 22]

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

a is the first term
d is the common difference

Question 8 (c)



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Question 8 (d)

(i) Point of intersection = (17, 80)

(ii) The point of intersection means that after 17 days both plants have the same height of 80 cm.

Question 8 (e)

Replace n by 17 in each formula and show you get an answer of $h = 80$.

Plant 1:

$h = 4n + 12$

$h = 4(17) + 12 = 68 + 12 = 80$

Plant 2:

$h = 3 \cdot 5n + 20.5$

$h = 3 \cdot 5(17) + 20 \cdot 5 = 59 \cdot 5 + 20 \cdot 5 = 80$

Question 8 (f)

Using algebra will always give you the exact answer.

Question 8 (g)

John's model has assumed that each plant grows by the same amount each day which means the graphs will be straight lines. Weather conditions (sun and rain) may alter the rates of growth which means that the graph may not be perfectly straight which will affect the point of intersection.

Question 9 (50 marks)

Question 9 (a) (i)

Replace N by 100 in the formula to work out the yield of wheat Y.

$N = 100 \text{ kg per hectare}$

$Y = 7000 + 32N - 0 \cdot 1N^2$

$= 7000 + 32(100) - 0 \cdot 1(100)^2$

$= 7000 + 3200 - 1000$

$= 9200 \text{ kg per hectare}$

Question 9 (a) (ii)

To find the maximum yield differentiate Y with respect to N, put it equal to zero and solve for N.

$Y = 7000 + 32N - 0 \cdot 1N^2$

$\frac{dY}{dN} = 32 - 0 \cdot 2N = 0$

$\therefore 32 = 0 \cdot 2N$

$N = \frac{32}{0 \cdot 2} = 160 \text{ kg per hectare}$

> Find turning points (local maxiumum/minimum)
>
> Put $\frac{dy}{dx} = 0$ and solve for x

Question 9 (a) (iii)

Replace N by 160 in the formula to work out the maximum yield of wheat Y.

$N = 160$ kg per hectare

$Y = 7000 + 32N - 0{\cdot}1N^2$

$= 7000 + 32(160) - 0{\cdot}1(160)^2$

$= 9560$ kg per hectare

Question 9 (a) (iv)

Cost of production = €1300

Maximum yield of wheat per hectare = 9560 kg = 9·56 tonne

1000 kg = 1 tonne 9560 kg = 9·56 tonne

Price of left-over straw = €75

Price of wheat = €160 per tonne

Income = 160 × 9·56 + 75 = €1606·60

Profit = Income – Cost = €1606·60 – €1300 = €304·60

Question 9 (b) (i)

Firstly, find out how long it takes the marble to hit the ground by putting the height $h = 0$ and solving for the time t.

Hits the ground: $h = 0$

$h = 44{\cdot}1 - 4{\cdot}9t^2$

$0 = 44{\cdot}1 - 4{\cdot}9t^2$

$4{\cdot}9t^2 = 44{\cdot}1$

$t^2 = \dfrac{44{\cdot}1}{4{\cdot}1} = 9$

$t = \sqrt{9} = 3$ s

Differentiate the height h with respect to time t to find the speed v. Then replace t in your answer by 3 s to find the speed when it hits the ground.

$h = 44{\cdot}1 - 4{\cdot}9t^2$

$v = \dfrac{dh}{dt} = -2 \times 4{\cdot}9t = -9{\cdot}8t$

$\left(\dfrac{dh}{dt}\right)_{t=3} = -9{\cdot}8(3) = -29{\cdot}4\,\text{m/s}$

$v = \dfrac{dh}{dt}$

Formulae and Tables Book
Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

Answer: Speed = 29·4 m/s

QUESTION 9 (b) (ii)

$$v = 29\cdot4\,\text{m/s} = \frac{29\cdot4\ \text{m}}{1\ \text{s}} = \frac{\frac{29\cdot4}{1000}\ \text{km}}{\frac{1}{3600}\ \text{h}} = 105\cdot84\ \text{km/h}$$

1000 m = 1 km 3600 s = 1 hour

Or

$$\begin{aligned} &29\cdot4\ \text{m/s} \\ &= 29\cdot4\times60\ \text{m/min} \\ &= 29\cdot4\times60\times60\ \text{m/h} \\ &= \frac{29\cdot4\times60\times60}{1000}\ \text{km/h} \\ &= 105\cdot84\ \text{km/h} \end{aligned}$$

Sample Paper 2014 (Set H): Paper 2

Question 1 (25 marks)

Question 1 (a)

Fundamental Principle of Counting

If there are a ways (outcomes) to do one operation and then b ways to do a second operation and then c ways to do a third operation then the number of ways of doing the first operation and then doing the second operation and then doing the third operation is given by $a \times b \times c$.

Question 1 (b)

There are 5 ways to arrange the first object.
Once this is arranged, there are 4 ways to arrange the second object.
Once this is arranged, there are 3 ways to arrange the third object.
Once this is arranged, there are 2 ways to arrange the fourth object.
Once this is arranged, there is 1 way to arrange the last object.

$n!$ = Number of arrangement of n different objects

1st object		2nd object		3rd object		4th object		5th object	
☐	and then	☐	and then	☐	and then	☐	and then	☐	
5	×	4	×	3	×	2	×	1	= 5! = 120

Question 1 (c)

The 5 novels need to be together. There are 5! ways of arranging the 5 novels.
The 3 poetry books need to be together. There are 3! ways of arranging the 3 poetry books.
There are 2 ways of arranging these 2 groups of books.

N		N		N		N		N
5	×	4	×	3	×	2	×	1

P		P		P
3	×	2	×	1

Number of arrangements $= 5! \times 3! \times 2 = 1440$

Question 2 (25 marks)

Question 2 (a)

X	$P(X)$	$XP(X)$
1	0·25	0·25
2	0·25	0·50
3	0·15	0·45
4	0·15	0·60
5	0·1	0·50
6	0·1	0·60
		$\sum XP(X) = 2{\cdot}9$

$$E(X) = \sum XP(X)$$

Multiply the probability of obtaining each number ($P(X)$) by the number (X) to get $XP(X)$. Then sum all the individual $XP(X)$'s.

Expected value $E(X) = 2{\cdot}9$ (biased die)

Question 2 (b)

X	$P(X)$	$XP(X)$
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	1
		$\sum XP(X) = \frac{21}{6} = 3{\cdot}5$

With a fair die the probabability of obtaining each number is one-sixth.

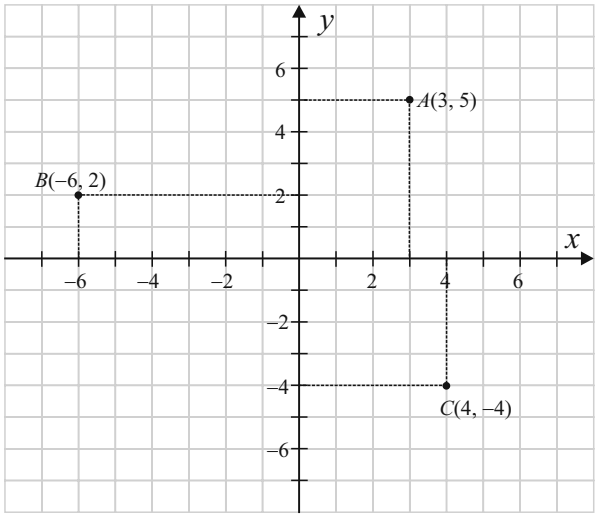
Expected value $E(X) = 3{\cdot}5$ (fair die)

It cost €3 to play a game at the funfair. You win the number of euro shown on the die.
With the biased die you expect to win €2·90. However, given that you paid €3 to play the game, your net loss will be 10 cent.
With the fair die you expect to win €3·50. However, given that you paid €3 to play the game, your net win will be 50 cent.
"If you play the game many times with a fair die, you will win an average of **50 c** per game, but if you play with the biased die you will lose an average of **10 c** per game."

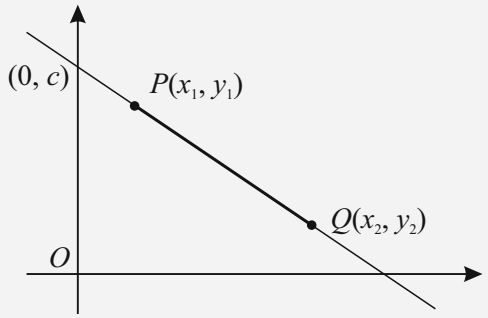
Question 3 (25 marks)

Question 3 (a)

Points: $A(3, 5)$, $B(-6, 2)$, $C(4, -4)$



Formulae and Tables Book
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$
$$y = mx + c$$

Area of triangle OPQ [page 18]

$$\frac{1}{2}|x_1y_2 - x_2y_1|$$

QUESTION 3 (b)

Slope m of AB: $A(3, 5) = (x_1, y_1)$, $B(-6, 2) = (x_2, y_2)$

$$m = \frac{2-5}{-6-3} = \frac{-3}{-9} = \frac{1}{3}$$

Equation of AB: Point $(x_1, y_1) = (3, 5)$, slope $m = \frac{1}{3}$

$y - 5 = \frac{1}{3}(x - 3)$ ← Multiply across by 3.

$3(y - 5) = 1(x - 3)$ ← Multiply out brackets.

$3y - 15 = x - 3$ ← Bring all terms to the same side.

$x - 3y + 12 = 0$

QUESTION 3 (c)

There are two ways to find the area of a triangle:

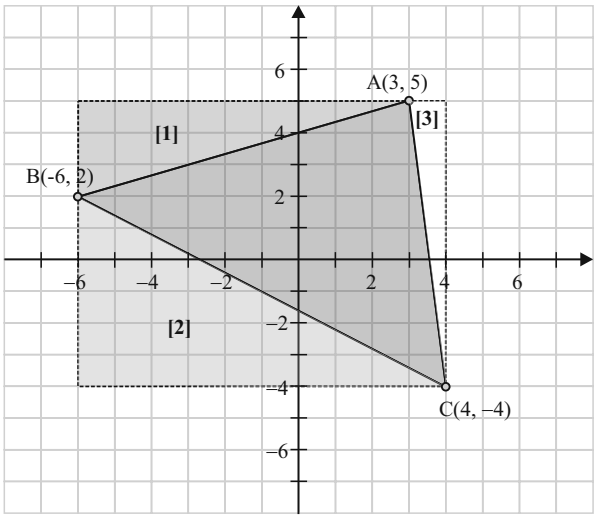
1. Use the formula. $\boxed{A = \frac{1}{2}|x_1y_2 - x_2y_1|}$

To use this formula one of the points must be at (0, 0). Translate $A(3, 5)$ to (0, 0) by subtracting 3 from the x-coordinate and 5 from the y-coordinate. Do the same to the other two points.

$A(3, 5)$	$\rightarrow (0, 0)$	(x_1, y_1)	(x_2, y_2)
$B(-6, 2)$	$\rightarrow (-9, -3)$	$\downarrow \downarrow$	$\downarrow \downarrow$
$C(4, -4)$	$\rightarrow (1, -9)$	$(-9, -3)$	$(1, -9)$

$$A = \frac{1}{2}|(-9)(-9) - (1)(-3)| = \frac{1}{2}|81 + 3| = \frac{1}{2}(84) = 42$$

2. Use a graphical method.



Draw a rectangle around the three points: A, B and C.
Three right-angled triangles are formed called [1], [2] and [3]. You can work out their area using the formula:

$$\boxed{A = \frac{1}{2}bh}$$

Area of triangle ABC = Area of Rectangle − [Area Δ1 + Area Δ2 + Area Δ3]

$$= (10)(9) - [\tfrac{1}{2}(9)(3) + \tfrac{1}{2}(10)(6) + \tfrac{1}{2}(1)(9)]$$
$$= 90 - \tfrac{1}{2}[27 + 60 + 9]$$
$$= 90 - \tfrac{1}{2}[96]$$
$$= 90 - 48$$
$$= 42$$

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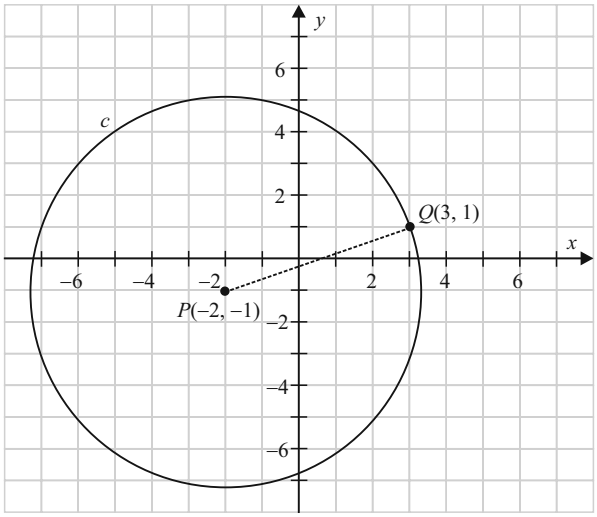
Question 4 (25 marks)

Question 4 (a)



Plot points $P(-2, -1)$ and $Q(3, 1)$.
Draw the circle c with centre P passing through Q.

Question 4 (b)

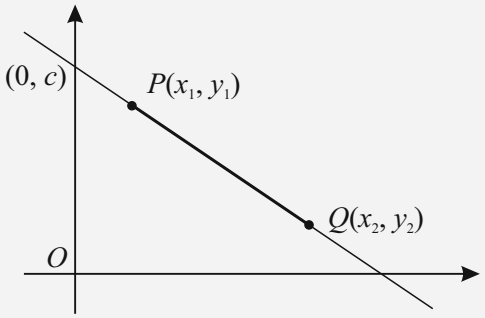
The radius r of c is the length of $|PQ|$.
$P(-2, -1) = (x_1, y_1)$, $Q(3, 1) = (x_2, y_2)$

$$r = |PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-(-2))^2 + (1-(-1))^2}$$
$$= \sqrt{5^2 + 2^2}$$
$$= \sqrt{25 + 4}$$
$$= \sqrt{29}$$

Circle c: Centre $(-2, -1) = (h, k)$, radius $r = \sqrt{29}$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-(-2))^2 + (y-(-1))^2 = (\sqrt{29})^2$$
$$(x+2)^2 + (y+1)^2 = 29$$

Formulae and Tables Book
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Length of PQ [page 18]

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Formulae and Tables Book
Co-ordinate geometry: Circle [page 19]
Given centre (h, k) and radius r

$$(x-h)^2 + (y-k)^2 = r^2$$

Question 4 (c)

Slope m_1 of PQ:
$P(-2, -1) = (x_1, y_1)$, $Q(3, 1) = (x_2, y_2)$

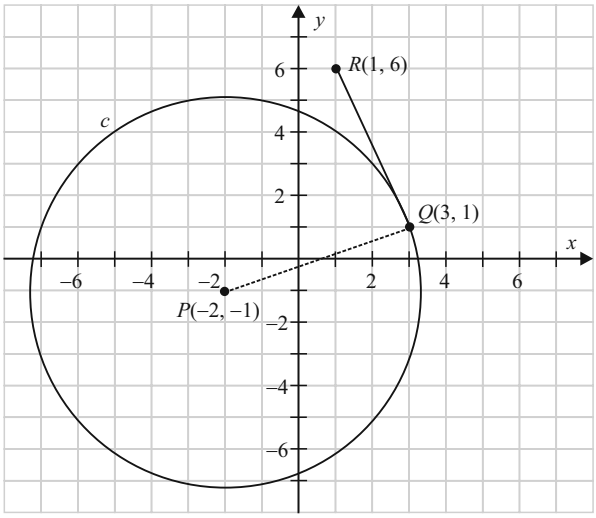
$$m_1 = \frac{1-(-1)}{3-(-2)} = \frac{1+1}{3+2} = \frac{2}{5}$$

Slope m_2 of QR:
$R(1, 6) = (x_1, y_1)$, $Q(3, 1) = (x_2, y_2)$

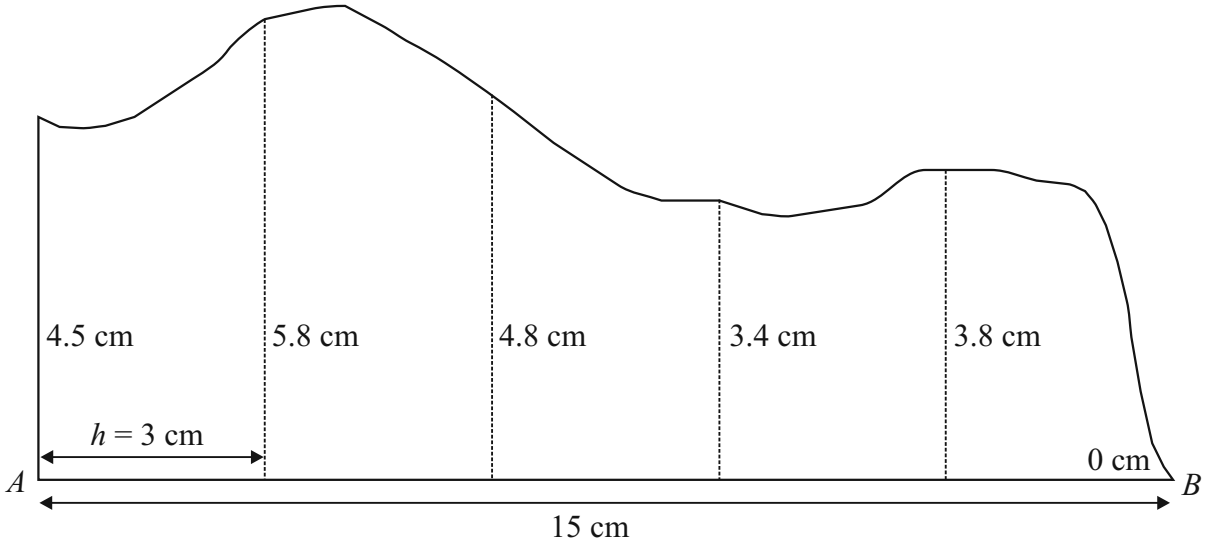
$$m_2 = \frac{6-1}{1-3} = \frac{5}{-2} = -\frac{5}{2}$$

$$m_1 \times m_2 = (\tfrac{2}{5})(-\tfrac{5}{2}) = -1$$

PQ is perpendicular to QR as the product of their slopes is equal to -1. Therefore, RQ is a tangent to c.



Question 5 (25 marks)

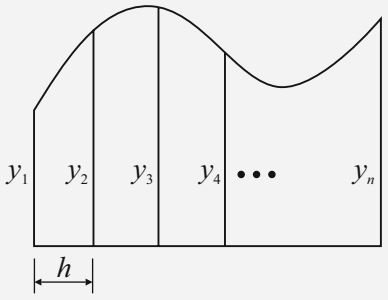


$h = 3$ cm

$A \approx \frac{3}{2}[4 \cdot 5 + 0 + 2(5 \cdot 8 + 4 \cdot 8 + 3 \cdot 4 + 3 \cdot 8)]$

$= 60 \text{ cm}^2$

Formulae and Tables Book
Area approximations [page 12]



$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + ... + y_{n-1})]$$

Question 6A (25 marks)

Question 6A (a)

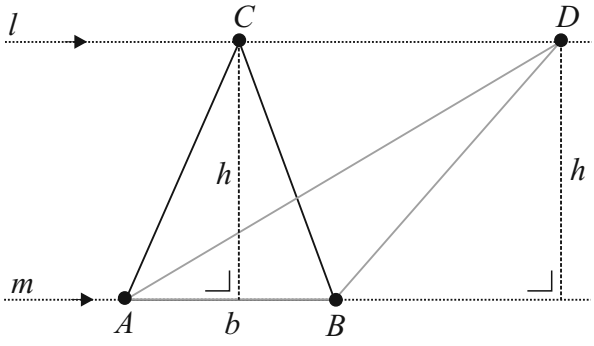
Explanation: A theorem formed by interchanging the hypothesis and conclusion of a given theorem.

Question 6A (b)

Statement: 'Congruent triangles have equal areas.'

Converse: 'Triangles with equal areas are congruent.'

This statement is false.



Draw two parallel lines l and m.
Draw a line $[AB]$ which forms the base of two triangles, ABC and ABD.
Each triangle drawn has the same base b and height h and therefore, each triangle has the same area. However, it is obvious that these triangles are **not** congruent.

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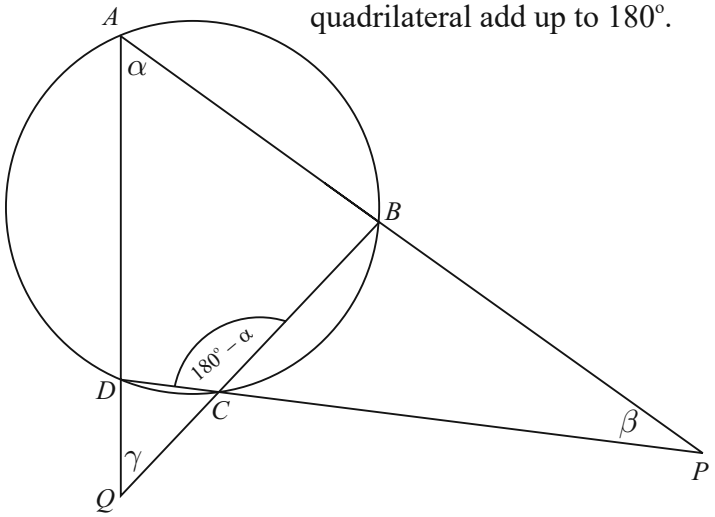
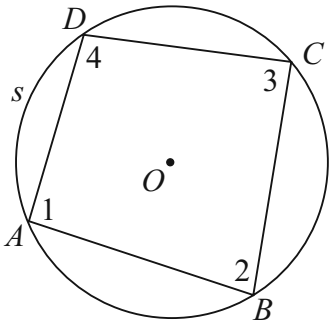
Question 6B (25 marks)

$|\angle DCB| = 180^o - \alpha$ ← Opposite angles of a cyclic quadrilateral add up to 180^o.

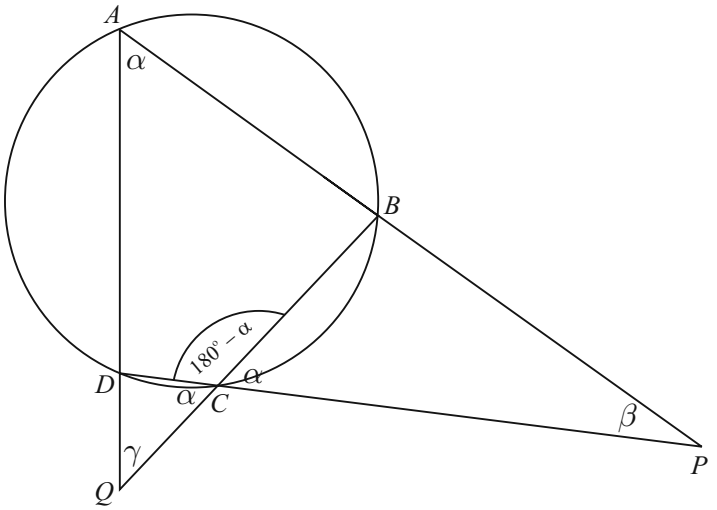


Cyclicquadrilateral

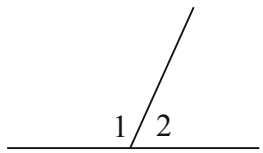


$\angle 1 + \angle 3 = 180^o$
$\angle 2 + \angle 4 = 180^o$

$|\angle DCQ| = |\angle PCB| = \alpha$ ← Straight angle



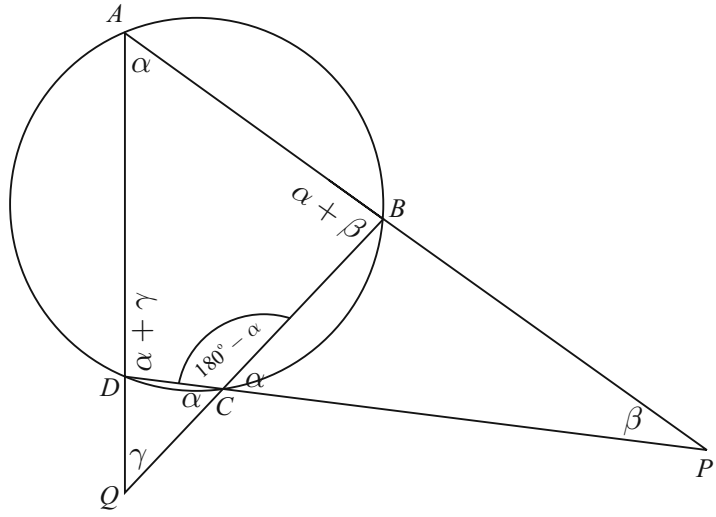
Straight angle



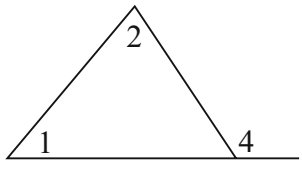
$\angle 1 + \angle 2 = 180^o$

$|\angle ADC| = \alpha + \gamma$ ← This exterior angle is equal to the sum of the 2 interior angles in ΔDCQ.

$|\angle ABC| = \alpha + \beta$ ← This exterior angle is equal to the sum of the 2 interior angles in ΔBPC.



Exterior angles



$\angle 4 = \angle 1 + \angle 2$

$\alpha + \beta + \alpha + \gamma = 180^o$ ← Opposite angles of a cyclic quadrilateral add up to 180°.

$2\alpha + \beta + \gamma = 180^o$

$\beta + \gamma = 180^o - 2\alpha$

QUESTION 7 (75 MARKS)

Question 7 (a)

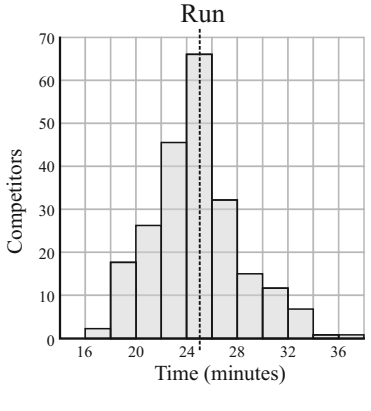
(i) The swim event takes competitors from 10 to 32 minutes to complete.
The cycle event takes competitors from 30 to 60 minutes to complete.
The run event takes competitors from 16 to 38 minutes to complete.
Obviously, the cycle event takes the longest to complete on average.

(ii) In all three histograms, the times are grouped into intervals of **2** minutes.

(iii) The time of the fastest person in the swim was between **10** and **12** minutes.
The fastest person completes the swim event in the shortest time.

(iv) The median time for the run is approximately **25** minutes.
The median line in a histogram is that line which divides the area of the histogram in half. Using your eye this line appears to be along the highest bar in the middle. This is along the class interval 24-26 minutes.

$\therefore \text{Median} = \frac{26+24}{2} = 25$

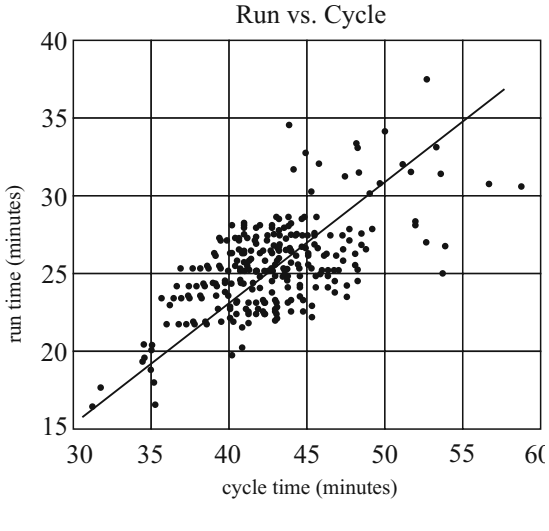


(v) The event in which the times are most spread out is the **cycle**.
The swim event takes competitors from 10 to 32 minutes to complete.
The cycle event takes competitors from 30 to 60 minutes to complete.
The run event takes competitors from 16 to 38 minutes to complete.
The cycle event has the largest spread taking 30 minutes between the slowest and fastest.

Question 7 (b) (i)

B

Draw a line of best fit through the points. The correlation coefficient *r* is positive as the slope of the line is positive (it rises from left to right). A value of $r = 1$ means that the points are on the line exactly. A low value of *r* means the points are spread all over the place. Option B seems like a good choice as the points are clustered moderately well about the best line fit.

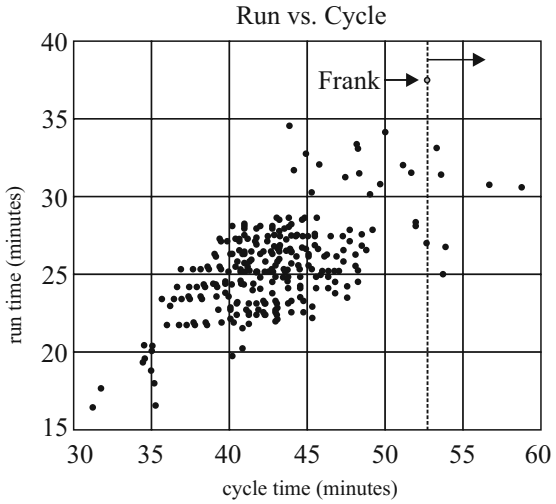


EPL 104

Question 7 (b) (ii)

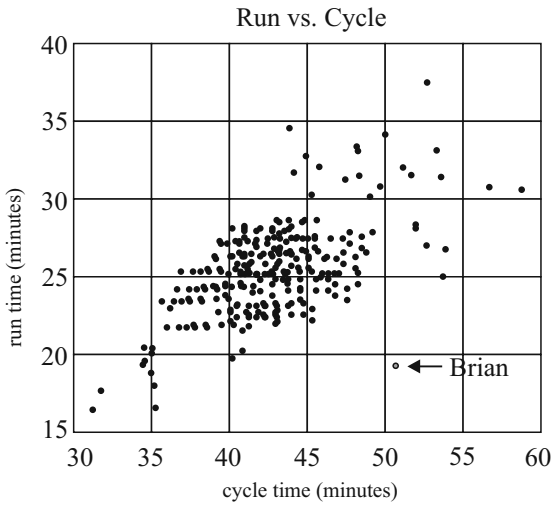
Answer: **6**

Frank took over 35 minutes to finish the run. However, there are 6 people to the right of him on the graph who had slower cycle times.



Question 7 (b) (iii)

Brian would be an outlier which is a point that is very different to the other points on the graph. He had a very fast run but a very slow cycle.



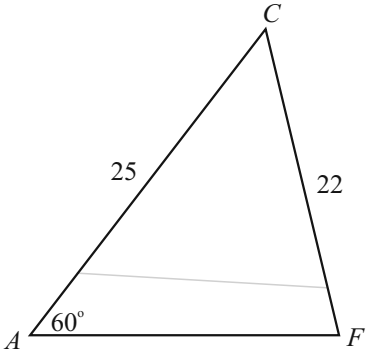
Question 7 (c)

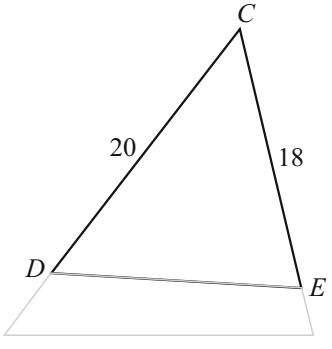
(i) Range of the females = 29·7 – 13·4 = 16·3 minutes
Range of the males = 23·0 – 14·9 = 8·1 minutes
The female distribution is more spread out. It is more skewed.

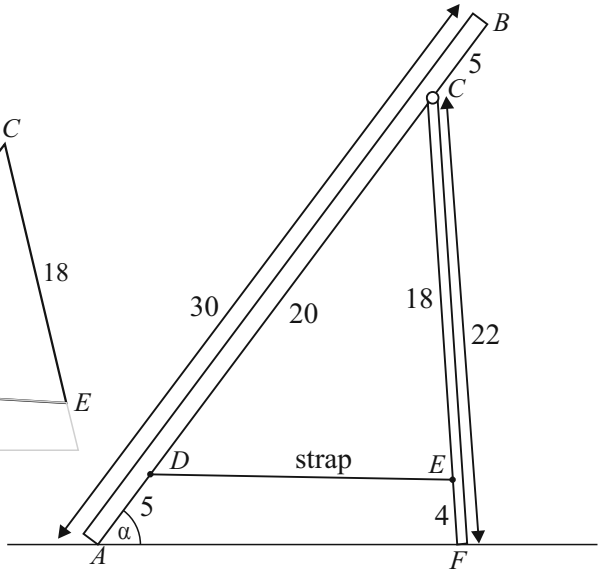
(ii) Yes, the shapes are more or less the same. The only difference is in the range. Maybe with a larger sample, her theory may be right.

Question 8 (45 marks)

Question 8 (a)

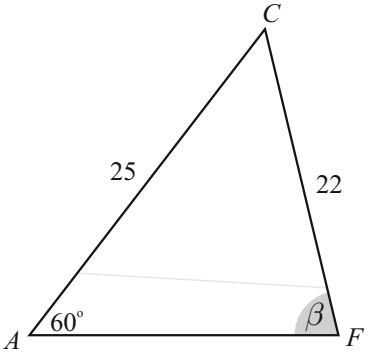




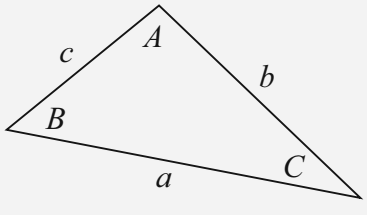


Question 8 (b)

Call β the measure of $|\angle CFA|$. Use the Sine Rule to find this angle as you are given 2 sides and a non-included angle.



Formulae and Tables Book
Trigonometry of the triangle:
[page 16]



Area: $\frac{1}{2}ab\sin C$

Sine Rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Cosine rule: $a^2=b^2+c^2-2bc\cos A$

$\dfrac{\sin\beta}{25}=\dfrac{\sin 60^\circ}{22}$ ← Multiply both sides by 22.

$\therefore \sin\beta=\dfrac{25\sin 60^\circ}{22}$ ← Find the angle β by finding the inverse sine of the right-hand side.

$\beta=\sin^{-1}\left(\dfrac{25\sin 60^\circ}{22}\right)$

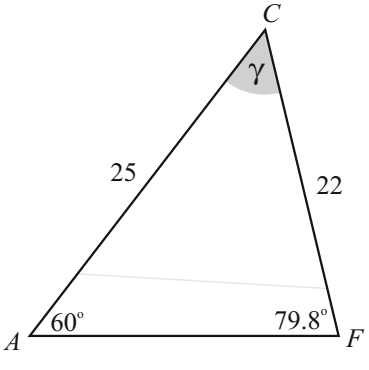
$=79\cdot 8^\circ$

Question 8 (c)

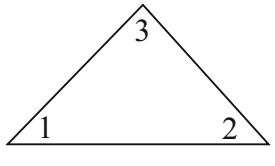
Call γ the measure of $|\angle ACF|$.

$\gamma+60^\circ+79\cdot 78^\circ=180^\circ$

$\gamma=180^\circ-60^\circ-79\cdot 8^\circ=40\cdot 2^\circ$



3 angles in a triangle



$\angle 1+\angle 2+\angle 3=180^\circ$

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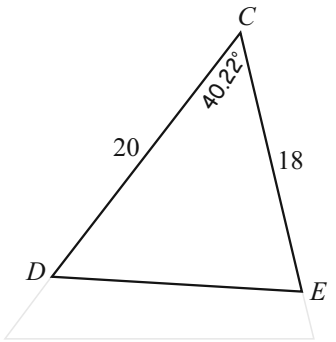
Question 8 (d)

Use the Cosine Rule to find $|DE|$. Use the Cosine Rule anytime you are given 2 sides and an included angle.



$$|DE|^2 = 20^2 + 18^2 - 2(20)(18)\cos 40\cdot 2^\circ$$

$$|DE| = \sqrt{20^2 + 18^2 - 2(20)(18)\cos 40\cdot 2^\circ} = 13\cdot 2 \text{ cm}$$

Question 9 (30 marks)

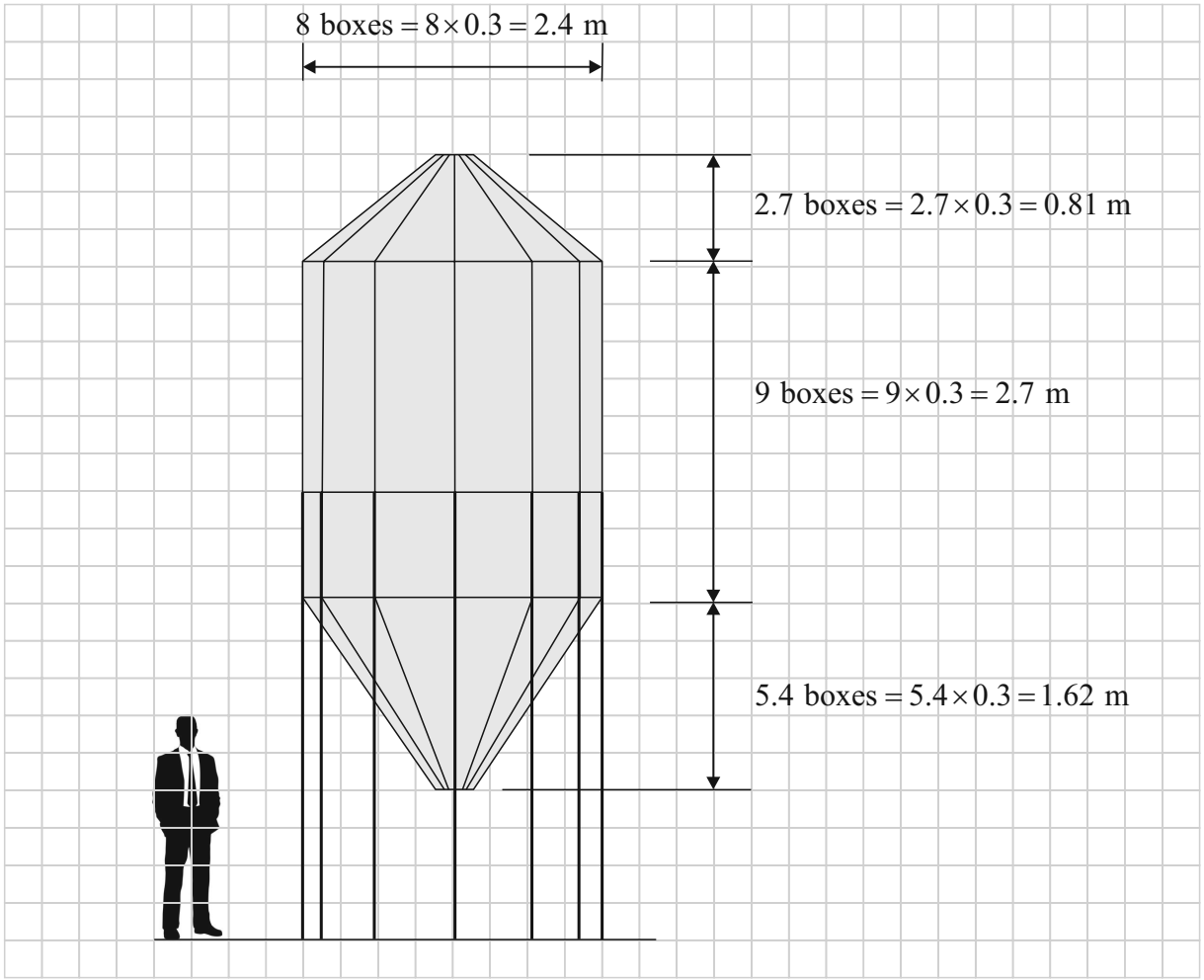
Question 9 (a)

Average height of an adult man = 1·7 m ← You were actually supposed to know this.

Question 9 (b)

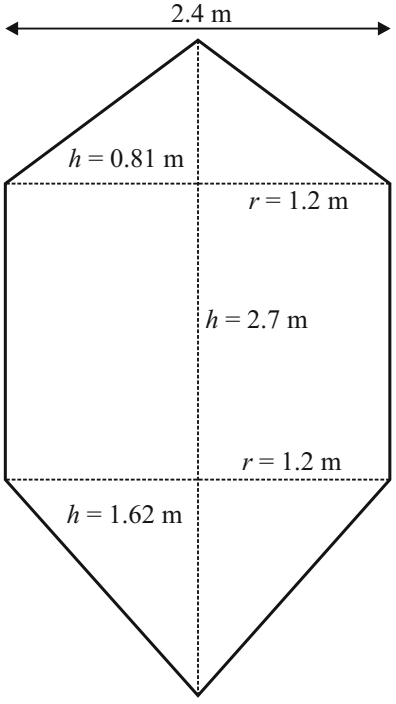
6 boxes = 1·78 m ← The man's height fills 6 square boxes.
1 box = 0·3 m ← A single square represents a measurement of side 0·3 m.

Count the number of boxes below and multiply by 0·3 m.

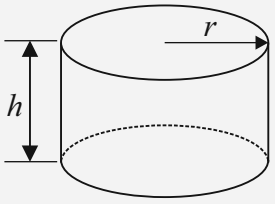


Question 9 (c)

The shape of the tank is made up of two cones and a cylinder.



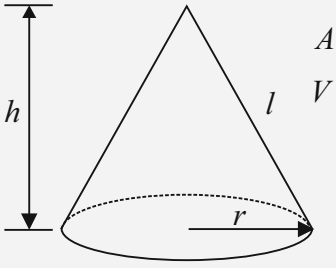
Formulae and Tables Book
Surface area and volume:
Cylinder [page 10]



$A = 2\pi rh$

$V = \pi r^2 h$

Cone [page 10]



$A = \pi rl$

$V = \frac{1}{3}\pi r^2 h$

Top Cone: $V = \frac{1}{3}\pi(1\cdot2)^2(0\cdot81) = 1\cdot22 \text{ m}^3$

Cylinder: $V = \frac{1}{3}\pi(1\cdot2)^2(1\cdot62) = 2\cdot44 \text{ m}^3$

Bottom Cone: $V = \pi(1\cdot2)^2(2\cdot7) = 12\cdot21 \text{ m}^3$

Total Volume = $15\cdot87 \text{ m}^3$

Sample Paper 1: Paper 1

Question 1 (25 marks)

Question 1 (a)

The value of the house has depreciated by 25%, which means that it is now 75% of its original value.

€450 000 × 0·75 = €337 500

Question 1 (b)

The contents have a value of 95% of the original value after 1 year, and 95% of this value after 2 years, and so on.

$€150\,000 \times 0{\cdot}95^5 = €116\,067$

Question 1 (c) (i)

Value in 2007:

€450 000 + €150 000 = €600 000

Question 1 (c) (ii)

Value in 2012:

€337 500 + €116 067 = €453 567

Question 1 (d)

Decrease in value of contents and building = €600 000 – €453 567 = €146 433

$$\%\text{ decrease} = \frac{146\,433}{600\,000} \times 100\% = 24{\cdot}4\%$$

Question 1 (e)

$€780 \times 0{\cdot}244 = €190{\cdot}32$ [The premium should decrease by 24·4%.]

$\therefore\ €780 - €190{\cdot}32 = €589{\cdot}68$

Question 2 (25 marks)

Question 2 (a)

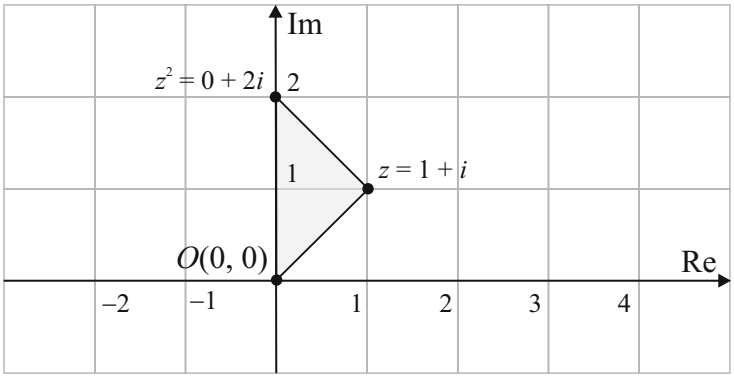
$z = 1 + i$

$z^2 = (1+i)(1+i)$

$= 1 + i + i + i^2$

$= 1 + i + i - 1$

$= 0 + 2i$



$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2}(1)(2) = 1$

Question 2 (b)

$z^4 = (z^2)^2 = (2i)^2 = 4i^2 = -4$

$z^{32} = (z^4)^8 = (-4)^8 = 65\,536$

Question 3 (25 marks)

Question 3 (a)

$f(x) = ax + b$

$f(0) = 1 \Rightarrow a(0) + b = 1$

$b = 1$

$f(-2) = -5 \Rightarrow -2a + 1 = -5$

$6 = 2a$

$\therefore a = 3$

$f(x) = 3x + 1$

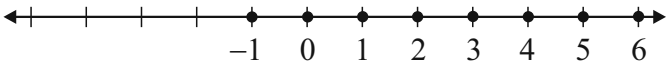
Question 3 (b)

$f(x) \geq -2,\ x \in \mathbb{Z}$

$3x + 1 \geq -2$

$3x \geq -3$

$x \geq -1$



Question 3 (c)

$\frac{4^{\frac{3}{2}}}{\sqrt{2}} = \frac{(2^2)^{\frac{3}{2}}}{2^{\frac{1}{2}}} = \frac{2^3}{2^{\frac{1}{2}}} = 2^{3-\frac{1}{2}} = 2^{\frac{5}{2}}$

$4^{f(x)} = \frac{4^{\frac{3}{2}}}{\sqrt{2}}$

$4^{3x+1} = 2^{\frac{5}{2}}$

$(2^2)^{3x+1} = 2^{\frac{5}{2}}$

$2^{6x+2} = 2^{\frac{5}{2}}$

$\therefore 6x + 2 = \frac{5}{2}$

$6x = \frac{5}{2} - 2 = \frac{1}{2}$

$\therefore x = \frac{1}{12}$

Formulae and Tables Book
Indices and logs [page 21]

$a^p \times a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{2}} = \sqrt{a}$

Question 4 (25 marks)

Question 4 (a)

$\frac{3(x+3)}{4} - \frac{2(x-3)}{3} = \frac{x+1}{2}$ [Multiply by 12.]

$9(x+3) - 8(x-3) = 6(x+1)$

$9x + 27 - 8x + 24 = 6x + 6$

$27 + 24 - 6 = 6x - 9x + 8x$

$45 = 5x$

$\therefore x = 9$

Question 4 (b)

$4x^2 - 8x - 21 = 0$

$(2x+3)(2x-7) = 0$

$\therefore x = -\frac{3}{2}, \frac{7}{2}$

$4(k+\frac{1}{2})^2 - 8(k+\frac{1}{2}) - 21 = 0$

$\therefore (k+\frac{1}{2}) = -\frac{3}{2} \Rightarrow k = -\frac{3}{2} - \frac{1}{2} = -2$

$\therefore (k+\frac{1}{2}) = \frac{7}{2} \Rightarrow k = \frac{7}{2} - \frac{1}{2} = 3$

Question 4 (c)

$\sqrt{x+1} = 5$ [Square both sides.]

$x + 1 = 25$

$\therefore x = 24$

Check

$x = 24 : \sqrt{x+1} = \sqrt{24+1} = \sqrt{25} = 5$

Question 5 (25 marks)

Question 5 (a)

$g(x) = -2x + d$

$(4, 0) \in g(x) \Rightarrow -2(4) + d = 0$

$-8 + d = 0$

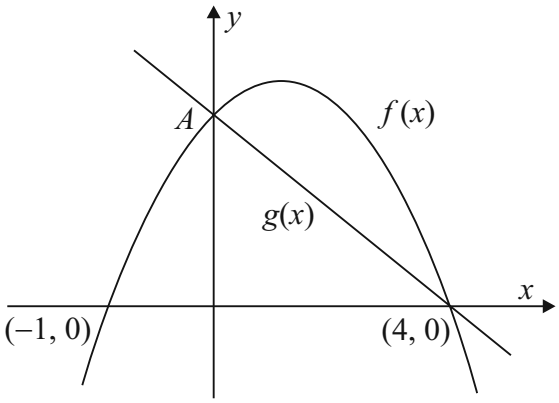
$\therefore d = 8$

Question 5 (b)

$g(x) = -2x + 8$

$x = 0 : g(x) = -2(0) + 8 = 8$

$\therefore A(0, 8)$



Question 5 (c)

$f(x) = ax^2 + bx + c$

$(0, 8) \in f(x) \Rightarrow a(0)^2 + b(0) + c = 8$

$\therefore c = 8$

$f(x) = ax^2 + bx + 8$

$(-1, 0) \in f(x) \Rightarrow a(-1)^2 + b(-1) + 8 = 0$

$\therefore a - b = -8....(\mathbf{1})$

$(4, 0) \in f(x) \Rightarrow a(4)^2 + b(4) + 8 = 0$

$16a + 4b + 8 = 0$

$\therefore 4a + b = -2....(\mathbf{2})$

$(\mathbf{1}) + (\mathbf{2}) : 5a = -10 \Rightarrow a = -2$

$-2 - b = -8....(\mathbf{1}) \Rightarrow b = 6$

$\therefore f(x) = -2x^2 + 6x + 8$

Question 5 (d)

$f(x) = -2x^2 + 6x + 8$

$f'(x) = -4x + 6$

$f'(x) = 0 \Rightarrow -4x + 6 = 0$

$\therefore x = \frac{3}{2}$

$f(\frac{3}{2}) = -2(\frac{3}{2})^2 + 6(\frac{3}{2}) + 8 = \frac{25}{2}$

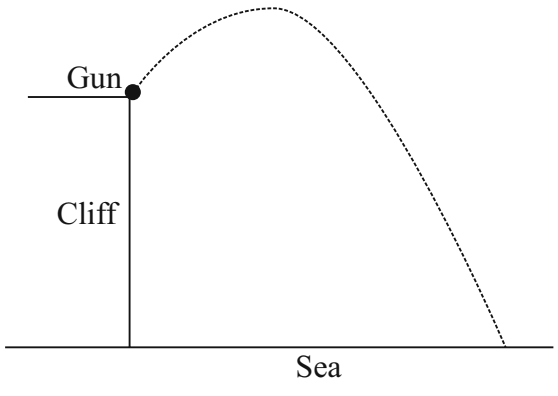
Local maximum: $(\frac{3}{2}, \frac{25}{2})$

Question 6 (25 marks)

Question 6 (a)

$h = 80 + 30t - 5t^2$

$t = 0 : h = 80 + 30(0) - 5(0)^2 = 80 \text{ m}$



Question 6 (b)

$h = 80 + 30t - 5t^2$

$t = 4 : h = 80 + 30(4) - 5(4)^2$

$= 80 + 120 - 80$

$= 120 \text{ m}$

Question 6 (c)

$h = 80 + 30t - 5t^2$

$h = 0 : 0 = 80 + 30t - 5t^2$

$5t^2 - 30t - 80 = 0$

$t^2 - 6t - 16 = 0$

$(t + 2)(t - 8) = 0$

$\therefore t = 8 \text{ s}$

Question 6 (d)

$h = 80 + 30t - 5t^2$

$\frac{dh}{dt} = 30 - 10t$

$\frac{dh}{dt} = 0 \Rightarrow 30 - 10t = 0$

$30 = 10t$

$\therefore t = 3 \text{ s}$

$h = 80 + 30(3) - 5(3)^2$

$= 80 + 90 - 45$

$= 125 \text{ m}$

Question 7 (50 marks)

Question 7 (a)

1 + 1 = 2
2 + 1 = 3
3 + 2 = 5
5 + 3 = 8
8 + 5 = 13
13 + 8 = 21

Next 3 terms of the Fibonacci Sequence:
8, 13, 21

First 5 terms of the Fibonacci Sequence:
1, 1, 2, 3, 5

Question 7 (b)

1 + 1 + 2 + 3 + 5 + 8 = 4(5)
20 = 20

Question 7 (c)

4, –2, 2, 0, 2, 2, 4, 6

Question 7 (d)

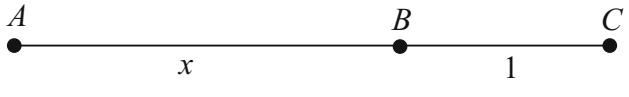
$x, y, x + y, x + 2y, 2x + 3y, 3x + 5y$

$x + y + x + y + x + 2y + 2x + 3y + 3x + 5y$
$= 4(2x + 3y)$

$8x + 12y = 8x + 12y$

Question 7 (e) (i)

$\frac{|AB| + |BC|}{|AB|} = \frac{|AB|}{|BC|} \Rightarrow \frac{x+1}{x} = \frac{x}{1}$



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Question 7 (e) (ii)

$$\frac{x+1}{x} = x$$

$$x+1 = x^2$$

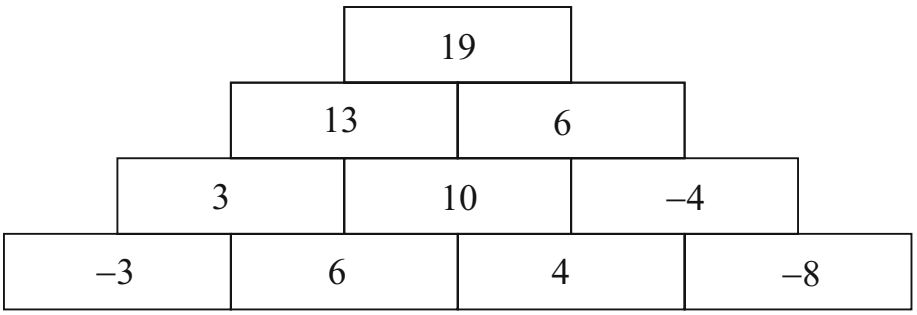
$$x^2 - x - 1 = 0 \qquad \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$a = 1,\ b = -1,\ c = -1$$

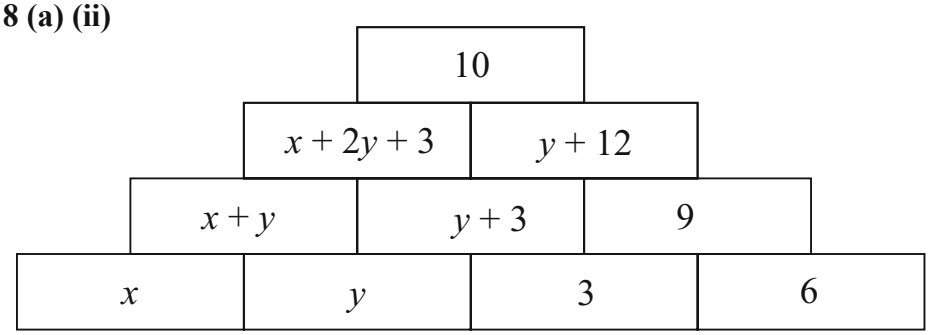
$$\therefore x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} = 1 \cdot 618034$$

Question 8 (50 marks)

Question 8 (a) (i)



Question 8 (a) (ii)



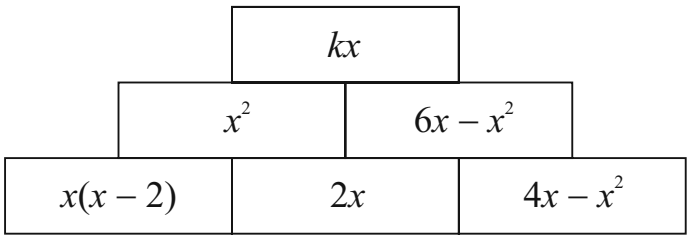
$$10 = x + 2y + 3 + y + 12$$
$$10 = x + 3y + 15$$
$$-5 = x + 3y$$

$$x + 3y = -5 \ (\times -2)$$
$$2x - y = 4$$

$$-2x - 6y = 10$$
$$2x - y = 4$$
$$-7y = 14 \Rightarrow y = -2$$

$$x + 3(-2) = -5$$
$$x - 6 = -5$$
$$\therefore x = 1$$

Question 8 (a) (iii)



$$kx = x^2 + 6x - x^2$$
$$kx = 6x$$
$$\therefore k = 6$$

Question 8 (b) (i)

The container has 40% acid and 60% water.

Volume of acid = $2 \times 0{\cdot}4 = 0{\cdot}8$ litres

Volume of water = $2 \times 0{\cdot}6 = 1{\cdot}2$ litres

Question 8 (b) (ii)

The container has 35% acid and 65% water.

Volume of acid = $0{\cdot}35x$ litres

Volume of water = $0{\cdot}65x$ litres

Question 8 (c) (i)

$x + y = 12$

Question 8 (c) (ii)

Bottle 1: Volume of acid = $0{\cdot}5x$

Bottle 2: Volume of acid = $0{\cdot}2y$

Mixture: Volume of acid = $12 \times 0{\cdot}3 = 3{\cdot}6$

$0{\cdot}5x + 0{\cdot}2y = 3{\cdot}6$

Question 8 (c) (iii)

$x + y = 12 \ (\times -0{\cdot}2)$

$0{\cdot}5x + 0{\cdot}2y = 3{\cdot}6$

$$-0{\cdot}2x - 0{\cdot}2y = -2{\cdot}4$$
$$0{\cdot}5x + 0{\cdot}2y = 3{\cdot}6$$
$$0{\cdot}3x \qquad = 1{\cdot}2 \Rightarrow x = 4 \text{ litres}$$

$x + y = 12$

$4 + y = 12$

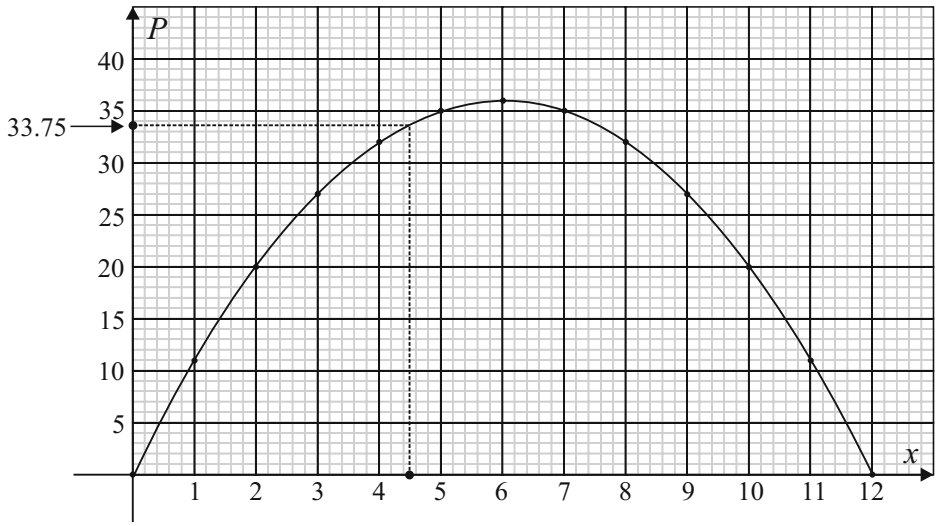
$\therefore y = 8$ litres

Question 9 (50 marks)

Question 9 (a)

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	12	11	10	9	8	7	6	5	4	3	2	1	0
$P = xy$	0	11	20	27	32	35	36	35	32	27	20	11	0

Question 9 (b)



Question 9 (c)

Maximum value of $P = 36$ ($x = 6$, $y = 6$)

Question 9 (d)

At $x = 4.5$, $P = 33.75$ (using graph)

$$P = xy \Rightarrow y = \frac{P}{x} = \frac{33.75}{4.5} = 7.5 \text{ (using algebra)}$$

Question 9 (e)

$$x + y = 12 \Rightarrow y = 12 - x$$

Question 9 (f)

$$P = xy = x(12 - x) = 12x - x^2$$

Question 9 (g)

$$P = 12x - x^2$$

$$\frac{dP}{dx} = 12 - 2x$$

$$\frac{dP}{dx} = 0 \Rightarrow 12 - 2x = 0$$

$$12 = 2x$$

$$\therefore x = 6$$

$$y = 12 - x = 12 - 6 = 6$$

Question 9 (h)

You obtained the same maximum value by using the graph or using calculus.

Sample Paper 1: Paper 2

Question 1 (25 marks)

Question 1 (a)

January		June
9	7	5 7 7 8 9
8 8 6 6 5 5 5 4 2 1 0	8	0 2 2 3 3 3 4 5 6
3 1	9	

Question 1 (b)

January		June
9	7	5 7 7 8 9
8 8 6 6 [5 5] 5 4 2 1 0	8	0 [2 2] 3 3 3 4 5 6
3 1	9	

Median in January = 85 kg
Median in June = 82 kg

Question 1 (c)

$$\%\text{ decrease in median weight} = \frac{3}{85} \times 100\% = 3.5\%$$

Question 2 (25 marks)

Question 2 (a) (i)

Possible outcomes: Head, Tail

Question 2 (a) (ii)

The coin is biased if the probability of tossing a head or a tail is **not** equal to 0·5.

Question 2 (b) (i)

Group *B*, because they tossed the coin more times than Group *A*, and so it is more likely that their estimate is closer to the true probability.

Question 2 (b) (ii)

$$\text{Expected number of heads} = 200 \times 0.44 = 88$$

Question 2 (b) (iii)

$$\text{Expected number of heads} = 500 \times 0.46 = 230$$

Total number of heads = 88 + 230 = 318

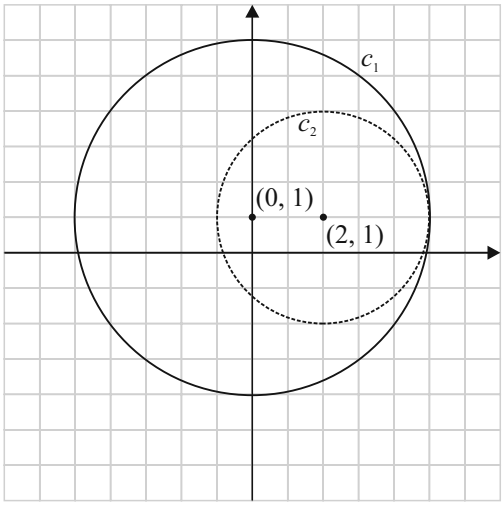
$$P(\text{Head}) = \frac{318}{700} = 0.454$$

Question 2 (c)

P(Head) = 0·454
P(2 Heads) = 0·454 × 0.454 = 0·206
Note: This is incorrect in the Sample Papers book.

Question 3 (25 marks)

Question 3 (a)



Question 3 (b)

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

Centre (0, 1), $r = 5$

$(x-0)^2+(y-1)^2=5^2$

$x^2+(y-1)^2=25$

Question 3 (c)

$A(4, 4) \in c_1$?

$4^2+(4-1)^2=4^2+3^2=16+9=25$

A is on c_1 as it satisfies the equation of the circle.

$B(3, 5)$ is also on c_1.

$\therefore 3^2+(5-1)^2=3^2+4^2=9+16=25$

Question 3 (d)

Circle c_2: Centre(2, 1), $r = 3$

$(x-2)^2+(y-1)^2=9$

Question 4 (25 marks)

Question 4 (a)

$l: 4x-3y+12=0$

x-intercept: Put $y=0 \Rightarrow 4x-3(0)+12=0$

$4x=-12$

$x=-3$

$\therefore A(-3, 0)$

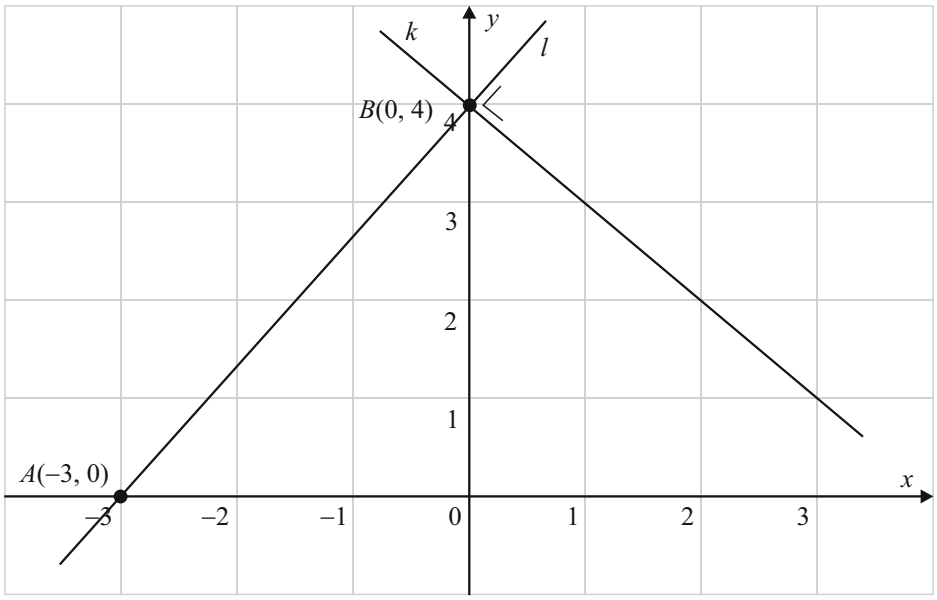
y-intercept: Put $x=0 \Rightarrow 4(0)-3y+12=0$

$-3y=-12$

$y=4$

$\therefore B(0, 4)$

Question 4 (b)



Question 4 (c)

Slope of l: $A(-3, 0)$, $B(0, 4)$

$$m = \frac{4-0}{0-(-3)} = \frac{4}{3} \qquad \boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Equation of k: Point $B(0, 4)$, slope $m^{\perp} = -\frac{3}{4}$

$$y - 4 = -\tfrac{3}{4}(x - 0) \qquad \boxed{y - y_1 = m(x - x_1)}$$
$$4y - 16 = -3x$$
$$3x + 4y - 16 = 0$$

Question 4 (d)

$$(4t, -2t) \in k \Rightarrow 3(4t) + 4(-2t) - 16 = 0$$
$$12t - 8t = 16$$
$$4t = 16$$
$$\therefore t = 4$$

Question 5 (25 marks)

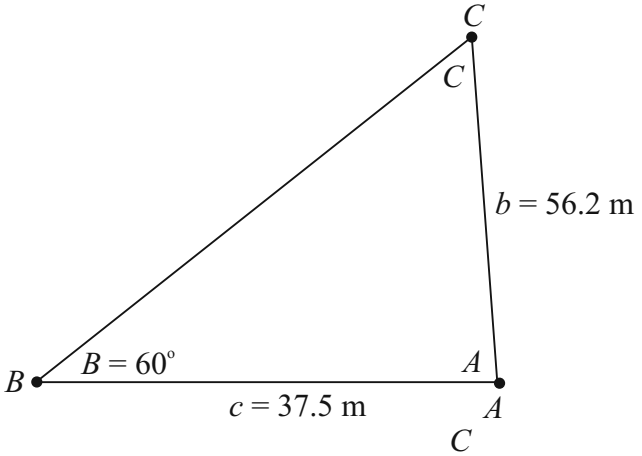
Question 5 (a) (i)

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

$$\frac{\sin C}{37.5} = \frac{\sin 60^\circ}{56.2}$$

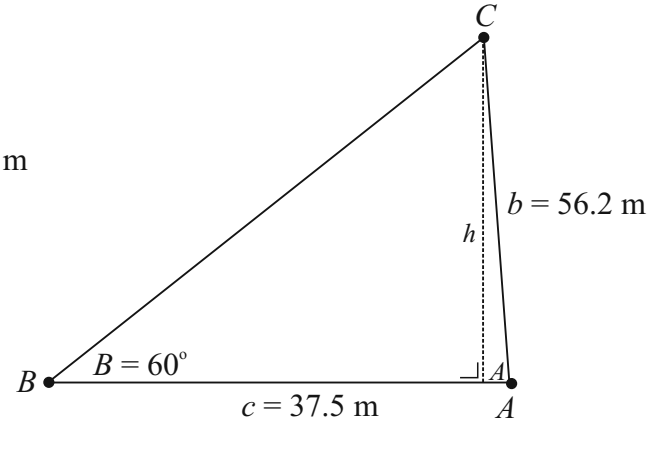
$$\therefore C = \sin^{-1}\left(\frac{37.5 \sin 60^\circ}{56.2}\right) = 35.3^\circ$$

$$\left|\angle CAB\right| = A = 180^\circ - 35.3^\circ - 60^\circ = 84.7^\circ$$



Question 5 (a) (ii)

$$\sin A = \frac{h}{56.2} \Rightarrow h = 56.2 \sin 84.7^\circ = 55.96 \text{ m}$$



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Question 5 (b)

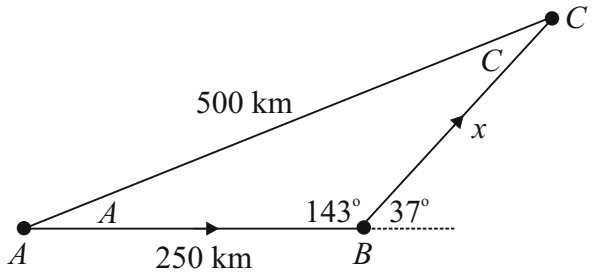
$$\frac{\sin C}{250} = \frac{\sin 143^\circ}{500}$$

$$\therefore C = \sin^{-1}\left(\frac{250\sin 143^\circ}{500}\right) = 17.5^\circ$$

$$A = 180^\circ - 17.5^\circ - 143^\circ = 19.5^\circ$$

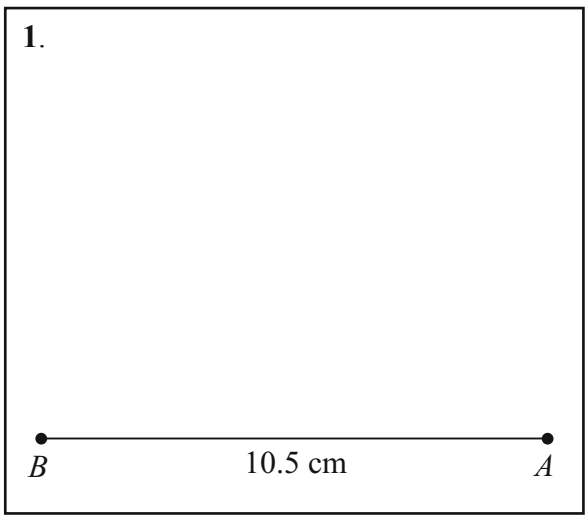
$$\frac{x}{\sin 19.5^\circ} = \frac{500}{\sin 143^\circ}$$

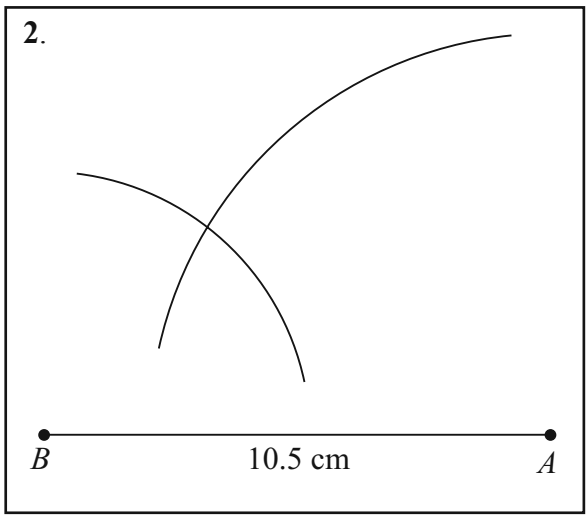
$$\therefore x = \frac{500\sin 19.5^\circ}{\sin 143^\circ} = 277.3 \text{ km}$$

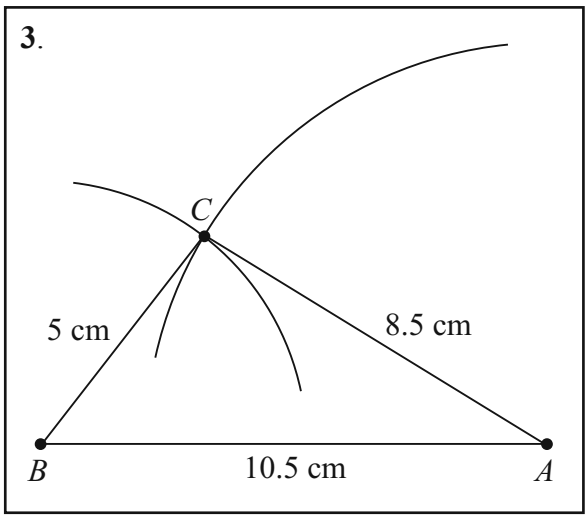


Question 6 (25 marks)

Question 6 (a)





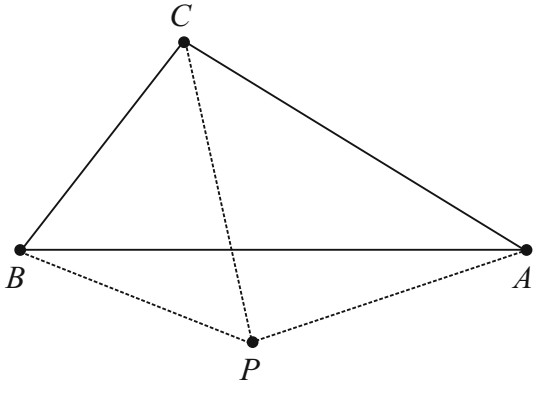


1. Draw a line AB of length 10.5 cm.

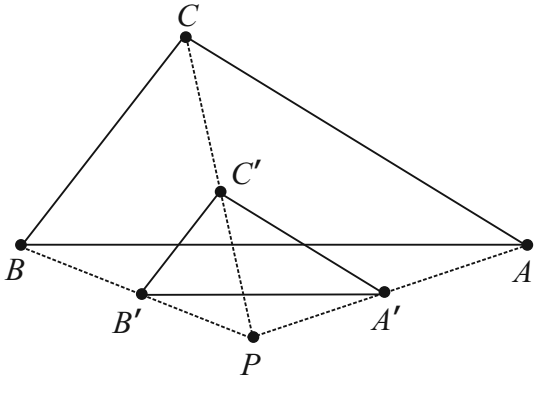
2. Use a compass to draw arcs of 5 cm and 8.5 cm.

3. Join A and B to the point of intersection of the arcs to form triangle ABC.

Question 6 (b)



1. Mark in a point P and draw lines from P to A, B and C.



2. Measure the length of each of these lines with a ruler. Multiply these distances by 0·4 to give points A', B' and C'.

Question 6 (c)

$k = 0.4$

Image area $= 3.36$ cm^2

Object area $= ?$

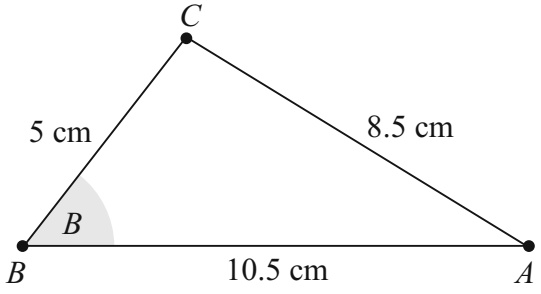
$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}$$

$$0.4^2 = \frac{3.36}{\text{Object area}} \Rightarrow \text{Object area} = \frac{3.36}{0.4^2} = 21 \text{ cm}^2$$

$\text{Area} = \frac{1}{2}ac\sin B$

$21 = \frac{1}{2}(8.5)(10.5)\sin B$

$\therefore \sin B = \frac{42}{(5)(10.5)} = \frac{4}{5}$



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Question 7 (75 marks)

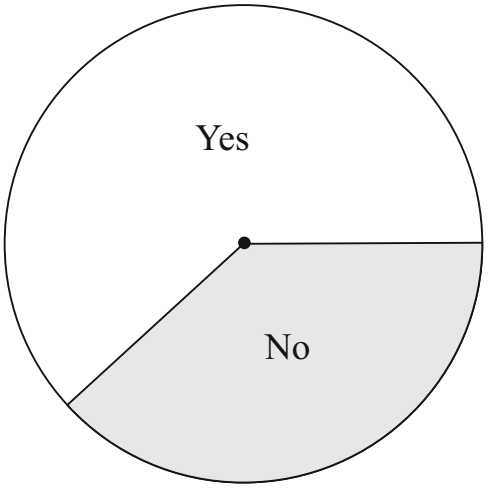
Question 7 (a) (i)

60.3% voted yes. Therefore, 39.7% voted no.

$60.3\% \rightarrow 955\ 091$

$1\% \rightarrow \dfrac{955\ 091}{60.3}$

$39.7\% \rightarrow \dfrac{955\ 091}{60.3} \times 39.7 = 628\ 808$



Question 7 (a) (ii)

Total number of voters = 955 091 + 628 808 = 1 583 899

Yes vote: $\dfrac{60.3}{100} \times 360^{\circ} \approx 217^{\circ}$

No vote: $360^{\circ} - 217^{\circ} = 143^{\circ}$

Question 7 (b) (i)

People gave multiple answers to questions.

Question 7 (b) (ii)

28% of the 2000 people gave one of the reasons for voting 'No' as 'Anti-Government'.

$2000 \times 0.28 = 560$ people

Question 7 (b) (iii)

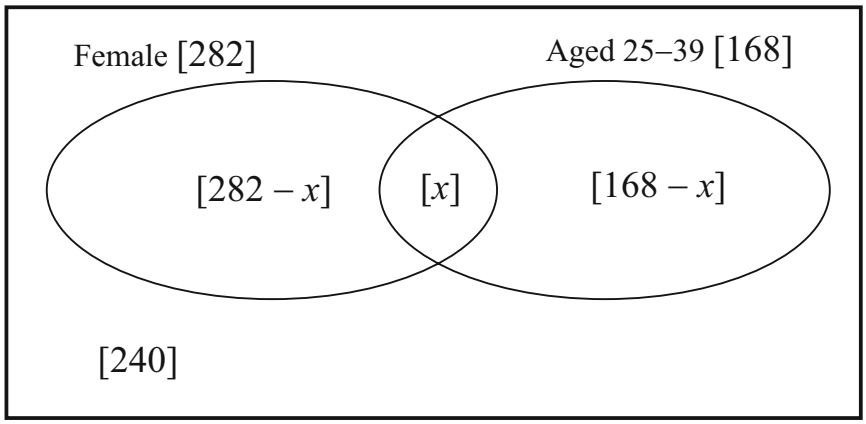
Pro-Eu: 18%
Dependent on EU: 16%
The maximum number of people that could have voted for **both** of these positions is 16%.
$2000 \times 0.16 = 320$ people

The minimum number of people is zero. In other words, none of the 18% who were 'pro-EU' were also 'Dependent on EU' and vice versa.

Question 7 (c) (i)

'Yes' voters: $1000 \times 0.6 = 600$
Females who voted 'Yes': $600 \times 0.47 = 282$
25–39-year-olds who voted 'Yes': $600 \times 0.28 = 168$
Males not aged 28-39 years old who voted 'Yes': $600 \times 0.40 = 240$

'Yes' voters: [600]



Question 7 (c) (ii)

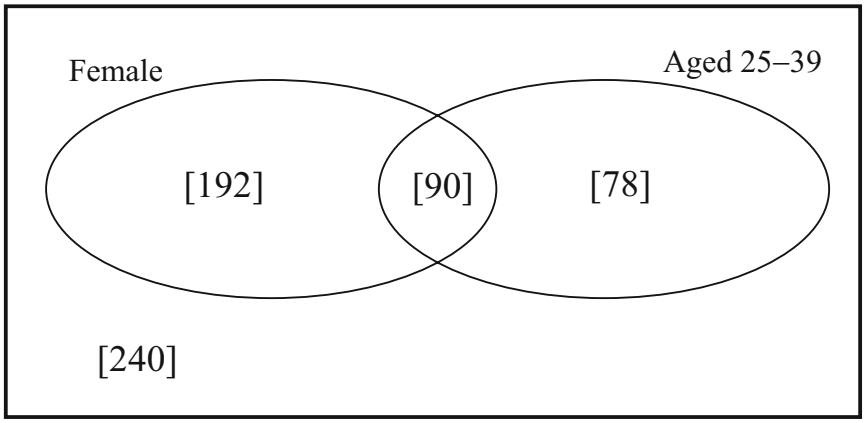
$282 - x + x + 168 - x + 240 = 600$

$690 - x = 600$

$690 - 600 = x$

$\therefore x = 90$

'Yes' voters: [600]



Question 7 (c) (iii)

P(Females aged 25–39 who voted 'Yes') $= \frac{90}{600} = \frac{3}{20}$

Question 7 (d) (i)

Expected number of females aged between 25 and 39 years of age $= 955\ 091 \times \frac{3}{20} = 143\ 264$

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Question 7 (d) (ii)

Advantages of Telephone Survey

1. High Accessibility

Over 95% of the Irish population has a phone at their homes.

2. Good Quality Control

Trained interviewers can put the questions to the respondents in a uniform manner, promoting accuracy and precision in eliciting responses.

3. Anonymous Respondents

The telephone survey approach provides perhaps the highest level of anonymity for respondents who wish to keep their opinions confidential.

Disadvantages of Telephone Survey

1. Time-Constrained Interviews

Since telephone surveys may interrupt the personal time of the respondents, interviews conducted via phone are to be no longer than 15 minutes.

2. Hard-to-Reach Respondents

Many people use call screening to accept only calls that they are expecting.

3. Unseen Product

In market research, it is more ideal to conduct a face-to-face interview/survey rather than a telephone survey.

Question 7 (e)

Percentage turnout: $\frac{38\,157}{81\,587} \times 100\% = 46.8\%$

Valid poll: 38 157 – 187 = 37 970

No votes: 37 970 – 24 015 = 13 955

% no vote: $\frac{13\,955}{37\,970} \times 100\% = 36.75\%$

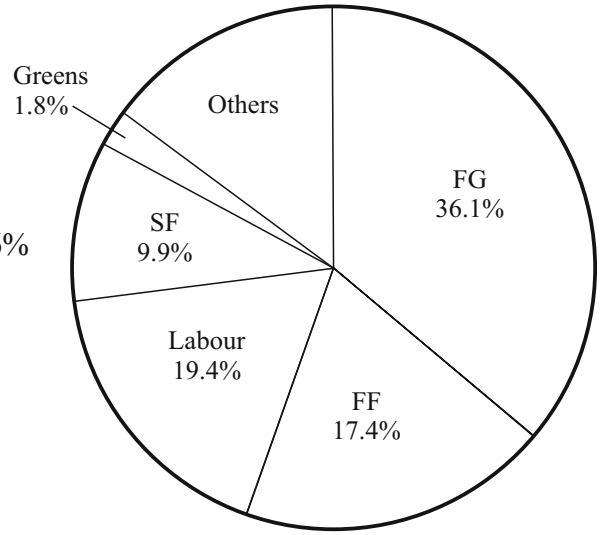
Question 7 (f) (i)

Date	Polling Company	FG	Lab	FF	SF	Others	Greens
07/01/2011	Red C	35%	21%	14%	14%	12%	4%
19/12/2010	Red C	34%	23%	17%	14%	10%	2%
16/12/2010	MRBI	30%	25%	17%	15%	11%	2%
03/12/2010	Red C	32%	24%	13%	16%	11%	3%
21/11/2010	Red C	33%	27%	17%	11%	8%	3%
	Average	32.8	24	15.6	14	10.4	2.8

Question 7 (f) (ii)

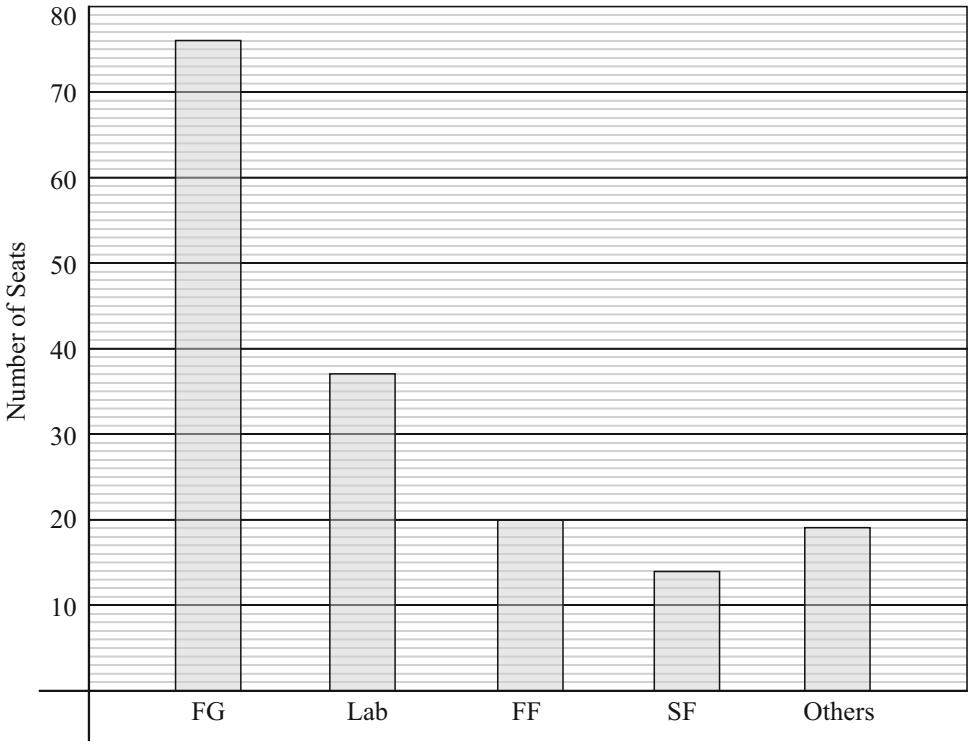
Others = 100% – 36.1% – 17.4% – 19.4% – 9.9% – 1.8% = 15.4%

Question 7 (f) (iii)



Error in FG's percentage = 36.1% – 32.8% = 3.3%
Error in FF's percentage = 17.4% – 15.6% = 1.8%
Error in Labour's percentage = 24% – 19.4% = 4.6%

Question 7 (g) (i)



Party	FG	Lab	FF	SF	Others
Number of seats	76	37	20	14	19

Total number of seats = 166

Question 7 (g) (ii)

$36.1\% \rightarrow 76$ seats

$1\% \rightarrow \frac{76}{36.1} = 2.11$ seats

$1.8\% \rightarrow 2.11 \times 1.8 = 3.8$ (4 seats)

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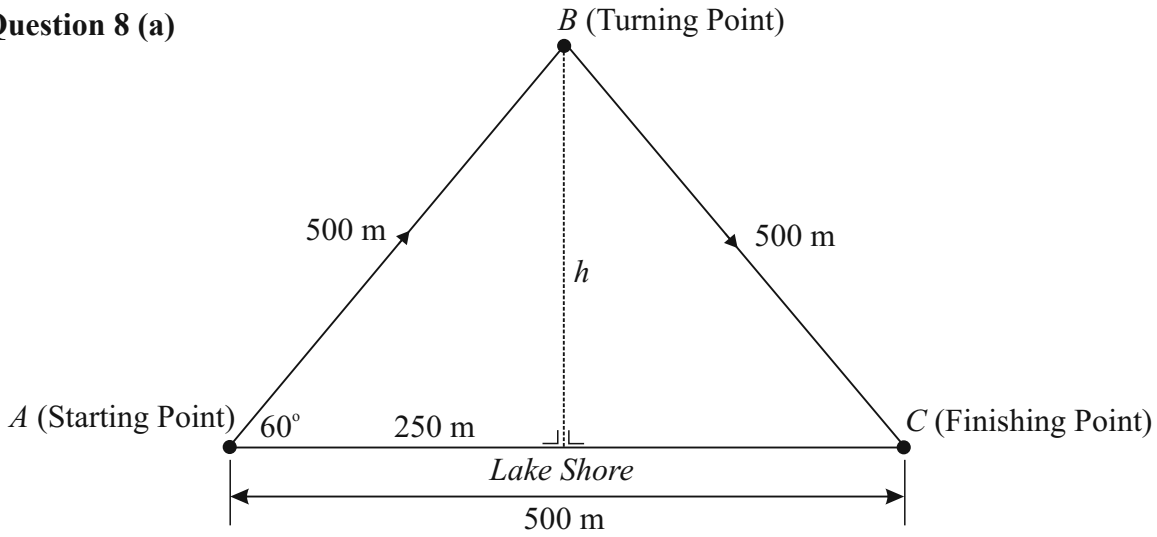
Question 8 (75 marks)

Question 8 (a)



(i) $|AB| = 500$ m, $|BC| = 500$ m,

(ii) Equilateral triangle

(iii) All angles in an equilateral triangle are 60°.

(iv) $250^2 + h^2 = 500^2$

$\therefore h = \sqrt{500^2 - 250^2} = 433$ m

(v) Area $= \frac{1}{2}bh = \frac{1}{2}(500)(433) = 108\ 250\ \text{m}^2$

or

Area $= \frac{1}{2}ab\sin C = \frac{1}{2}(500)(500)\sin 60^\circ = 108\ 253\ \text{m}^2$

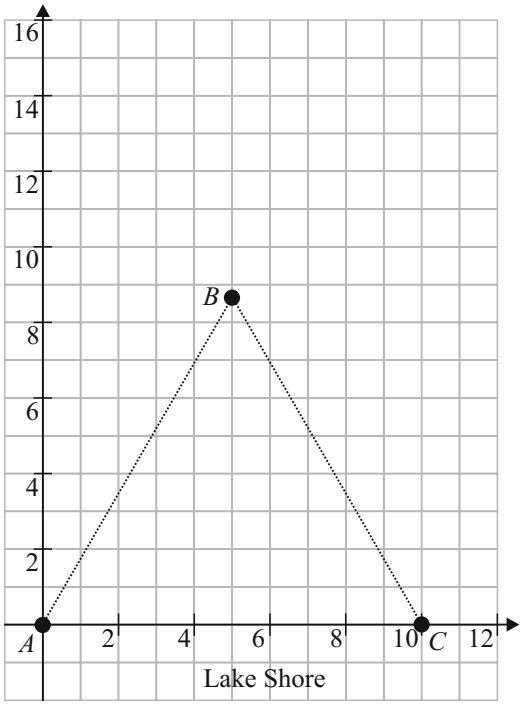
Question 8 (b)

Divide each distance by 50 to find its coordinates.

Starting point: $A(0, 0)$

Turning point: $B(5, 8.7)$

Finishing point: $C(10, 0)$



Question 8 (c) (i)

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Category 1: $1.5 \text{ km/h} = \frac{1 \text{ h}}{t} \Rightarrow t = \frac{1}{1.5} \text{ h} = \frac{2}{3} \text{h} = \frac{2}{3} \times 60 \text{ mins} = 40 \text{ mins}$

Category 2: $1.4 \text{ km/h} = \frac{1 \text{ h}}{t} \Rightarrow t = \frac{1}{1.4} \text{ h} = \frac{1}{1.4} \times 60 \text{ mins} \approx 43 \text{ mins}$

Category 3: $1.3 \text{ km/h} = \frac{1 \text{ h}}{t} \Rightarrow t = \frac{1}{1.3} \text{ h} = \frac{1}{1.3} \times 60 \text{ mins} \approx 46 \text{ mins}$

	Average Speed	Time
Category 1	1.5 km/h	40 mins
Category 2	1.4 km/h	43 mins
Category 3	1.3 km/h	46 mins

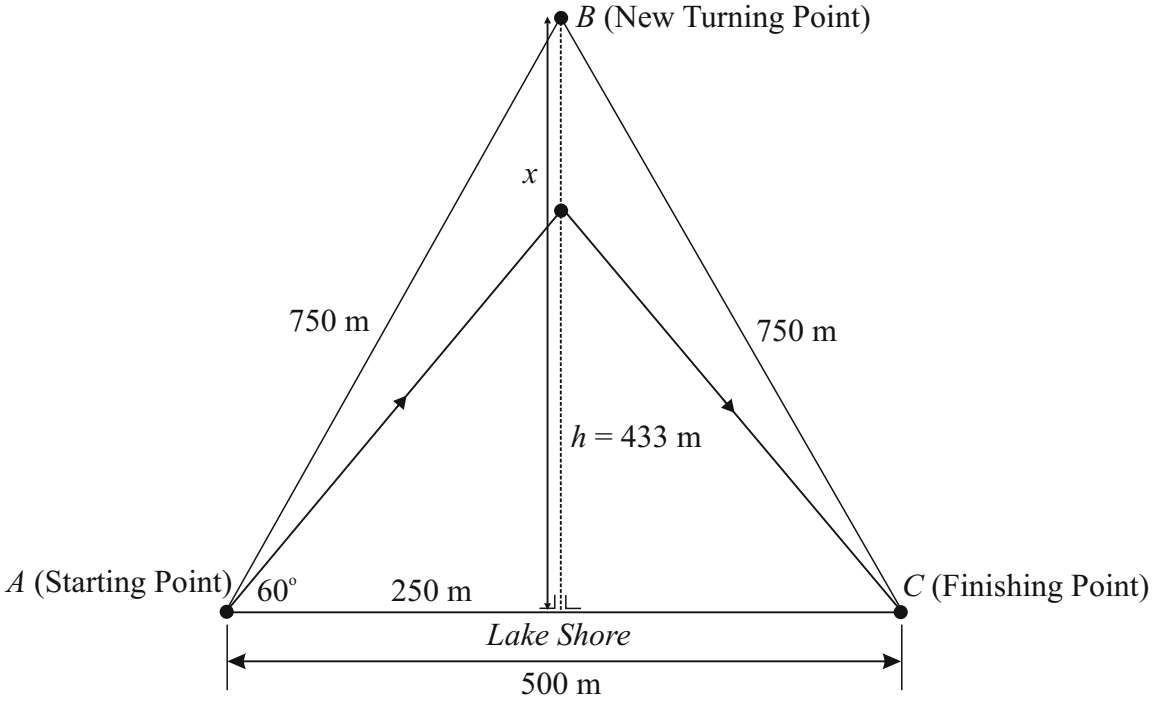
Question 8 (c) (ii)

Category 3 starts first and category 1 starts last. The race will be exciting, with a close finish between all swimmers.

Question 8 (c) (iii)

Starting time of race: 3:00 p.m.
Second starting time: 3:03 pm
Third starting time: 3:06 pm

Question 8 (d)



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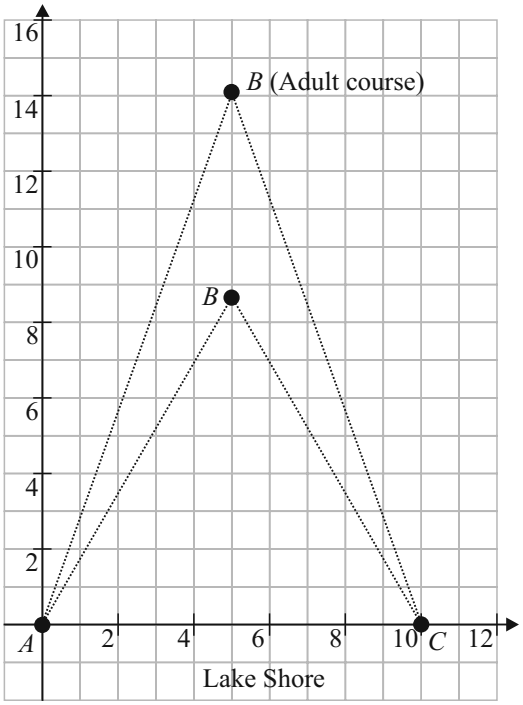
$250^2 + x^2 = 750^2$

$\therefore x = \sqrt{750^2 - 250^2} = 707$ m

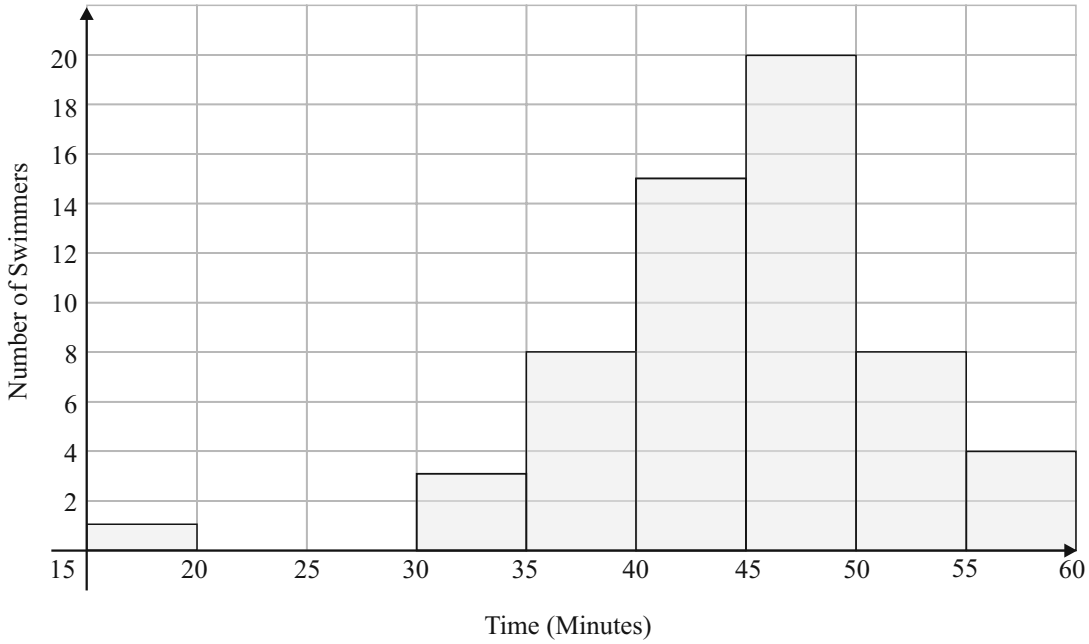
Extra distance $= 707 - 433 = 274$ m

New turning point: $B(5, 14.1)$



Question 8 (e) (i)

Time (Minutes)	Number of swimmers
15–20	1
20–25	0
25–30	0
30–35	3
35–40	8
40–45	15
45–50	20
50–55	8
55–60	4



(ii) An outlier is a result that is very different to the other results. It could have arisen because a top-class, professional swimmer took part.

(iii) The histogram has normal distribution if the outlier is excluded. Most of the swimmers were average swimmers who probably swam for recreation.

Sample Paper 2: Paper 1

Question 1 (25 marks)

Question 1 (a) (i)

$$f(x)=\frac{x-3}{2}$$

$$f(1)=\frac{1-3}{2}=\frac{-2}{2}=-1$$

$$f(2)=\frac{2-3}{2}=\frac{-1}{2}=-\frac{1}{2}$$

$$f(5)=\frac{5-3}{2}=\frac{2}{2}=1$$

Question 1 (a) (ii)

$$g(x)=\frac{2x-1}{3}$$

$$g(1)=\frac{2(1)-1}{3}=\frac{1}{3}$$

$$g(2)=\frac{2(2)-1}{3}=\frac{3}{3}=1$$

$$g(5)=\frac{2(5)-1}{3}=\frac{9}{3}=3$$

Question 1 (b)

$$f(x)+g(x)=\frac{x-3}{2}+\frac{2x-1}{3}$$

$$=\frac{3(x-3)+2(2x-1)}{6}$$

$$=\frac{3x-9+4x-2}{6}$$

$$=\frac{7x-11}{6}$$

$$f(x)+g(x)=4$$

$$\therefore \frac{7x-11}{6}=4$$

$$7x-11=24$$

$$7x=24+11=35$$

$$x=\frac{35}{7}=5$$

Verify your answer: $f(5)+g(5)=1+3=4$

Question 1 (c)

$$6(f(x)+g(x))+11=x^2+12$$

$$6\left(\frac{7x-11}{6}\right)+11=x^2+12$$

$$7x-11+11=x^2+12$$

$$7x=x^2+12$$

$$x^2-7x+12=0$$

$$(x-3)(x-4)=0$$

$$\therefore x=3,\ 4$$

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Question 2 (25 marks)

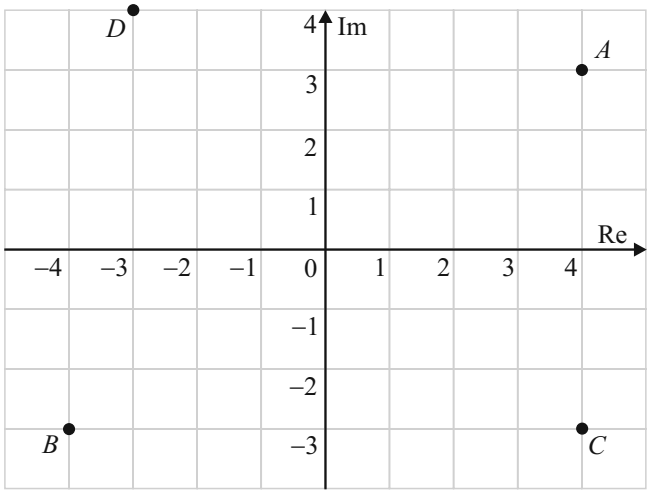
Question 2 (a)

$z = 4 + 3i$ $(A(4, 3))$

(i) $-z = -4 - 3i$ $(B(-4, -3))$

(ii) $\overline{z} = 4 - 3i$ $(C(4, -3))$

(iii) $iz = 4i + 3i^2 = -3 + 4i$ $(D(-3, 4))$



(iv) $O(0, 0)$, $A(4, 3)$

Slope of $OA = \dfrac{3-0}{4-0} = \dfrac{3}{4}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$O(0, 0)$, $D(-3, 4)$

Slope of $OD = \dfrac{4-0}{-3-0} = -\dfrac{4}{3}$

$\dfrac{3}{4} \times -\dfrac{4}{3} = -1 \Rightarrow OA \perp OD$

Perpendicular lines: $m_1 \times m_2 = -1$

Question 2 (b) (i)

$w = 3 + 4i$

$w^2 = (3 + 4i)^2 = (3 + 4i)(3 + 4i)$

$= 9 + 12i + 12i + 16i^2$

$= 9 + 24i - 16$

$= -7 + 24i$

Question 2 (b) (ii)

$w = 3 + 4i \Rightarrow \overline{w} = 3 - 4i$

$(\overline{w})^2 = (3 - 4i)^2 = (3 - 4i)(3 - 4i)$

$= 9 - 12i - 12i + 16i^2$

$= 9 - 24i - 16$

$= -7 - 24i$

Question 2 (b) (iii)

$\dfrac{25}{w} = \dfrac{25}{3 + 4i}$

$= \dfrac{25}{(3 + 4i)} \times \dfrac{(3 - 4i)}{(3 - 4i)}$ [Multiply above and below by the conjugate of the denominator.]

$= \dfrac{25(3 - 4i)}{9 + 12i - 12i - 16i^2}$

$= \dfrac{25(3 - 4i)}{9 + 16} = \dfrac{25(3 - 4i)}{25}$

$= 3 - 4i = \overline{w}$

Question 3 (25 marks)

Question 3 (a)

Mass of sun $= 2.2\times10^{30}$ kg

Mass of earth $= 6\times10^{24}$ kg

$$\frac{6\times10^{24}}{2.2\times10^{30}}\times100\% = 2.73\times10^{-4}\%$$

Question 3 (b)

$$49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$$

$$64^{\frac{2}{3}} = (64^{\frac{1}{3}})^2 = 4^2 = 16$$

Question 3 (c)

$$\frac{a\sqrt{a}\times(2a)^3}{\sqrt{a^3}} = \frac{a^1\times a^{\frac{1}{2}}\times 8a^3}{(a^3)^{\frac{1}{2}}}$$

$$= \frac{8a^{\frac{9}{2}}}{a^{\frac{3}{2}}}$$

$$= 8a^{\frac{9}{2}-\frac{3}{2}}$$

$$= 8a^3$$

Question 3 (d)

$$25^{x+1} = 5^{x-1}$$

$$(5^2)^{x+1} = 5^{x-1}$$

$$5^{2x+2} = 5^{x-1}$$

$$\therefore 2x+2 = x-1$$

$$2x-x = -1-2$$

$$\therefore x = -3$$

Formulae and Tables Book
Indices and logs [page 21]

$$a^p \times a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{2}} = \sqrt{a}$$

Question 4 (25 marks)

Question 4 (a)

A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

List: 2, 3, 5, 7, 11, 13, 17

Question 4 (b)

Proper divisors of 6: 1, 2, 3

Proper divisors of 18: 1, 2, 3, 6, 9

2	18
3	9
3	3
	1

Question 4 (c)

$6 = 3 + 2 + 1$ (Perfect number)

$18 \neq 1+2+3+6+9$

Question 4 (d)

$n = 2^{p-1}(2^p - 1)$

$p = 2$:

$n = 2^1(2^2 - 1) = 2(3) = 6$

$6 = 1+2+3$

$n = 2^{p-1}(2^p - 1)$

$p = 3$:

$n = 2^2(2^3 - 1) = 4(8-1) = 4(7) = 28$

2	28
2	14
7	7
	1

Proper divisors of 28: 1, 2, 4, 7, 14

$28 = 1 + 2 + 4 + 7 + 14$

Question 5 (25 marks)

Question 5 (a)

$f(x) = x^3 - 6x^2 + 11x - 6$

$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

$f(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

Question 5 (b)

x	0·5	1·0	1·5	2·0	2·5	3·0	3·5
$y = f(x)$	−1·875	0	0·375	0	−0·375	0	1·875

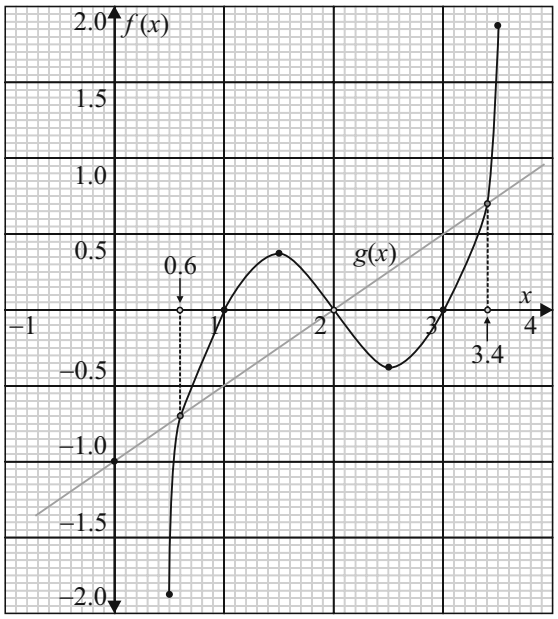
Question 5 (c) & (d)

$g(x)$ is a straight line. Find two points on this line so it can be drawn.

$g(x) = x - 2$

$g(1) = 1 - 2 = -1$

$g(2) = 2 - 2 = 0$



Question 5 (e)

$f(x) = g(x)$

$x^3 - 6x^2 + 11x - 6 = x - 2$

$x^3 - 6x^2 + 10x - 4 = 0$

Find solutions at which the two functions intersect.

$x = 0{\cdot}6,\ 2,\ 3{\cdot}4$

Question 6 (25 marks)

Question 6 (a)

$y = 2x^3 - 3x^2 - 12x + 4$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 6x - 12 = 0$ [The slope of the tangent is zero. Put the derivative equal to zero and solve for x.]

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$\therefore x = -1,\ 2$

$x = -1 : y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 4$

$= -2 - 3 + 12 + 4 = 11$

$x = 2 : y = 2(2)^3 - 3(2)^2 - 12(2) + 4$

$= 16 - 12 - 24 + 4 = -16$

Turning points: $(-1, 11)$, $(2, -16)$

Question 6 (b) (i)

$f(x) = x^2 + kx + 10$

$f'(x) = 2x + k$

Question 6 (b) (ii)

$f'(3) = 0 \Rightarrow 2(3) + k = 0$

$6 + k = 0$

$\therefore k = -6$

Question 6 (b) (iii)

$f(x) = x^2 - 6x + 10$

$f'(x) = 2x - 6$

$f(0) = 10 \Rightarrow (0, 10) \in t$

$f'(0) = m = -6$

Equation of t: Point (0, 10), $m = -6$

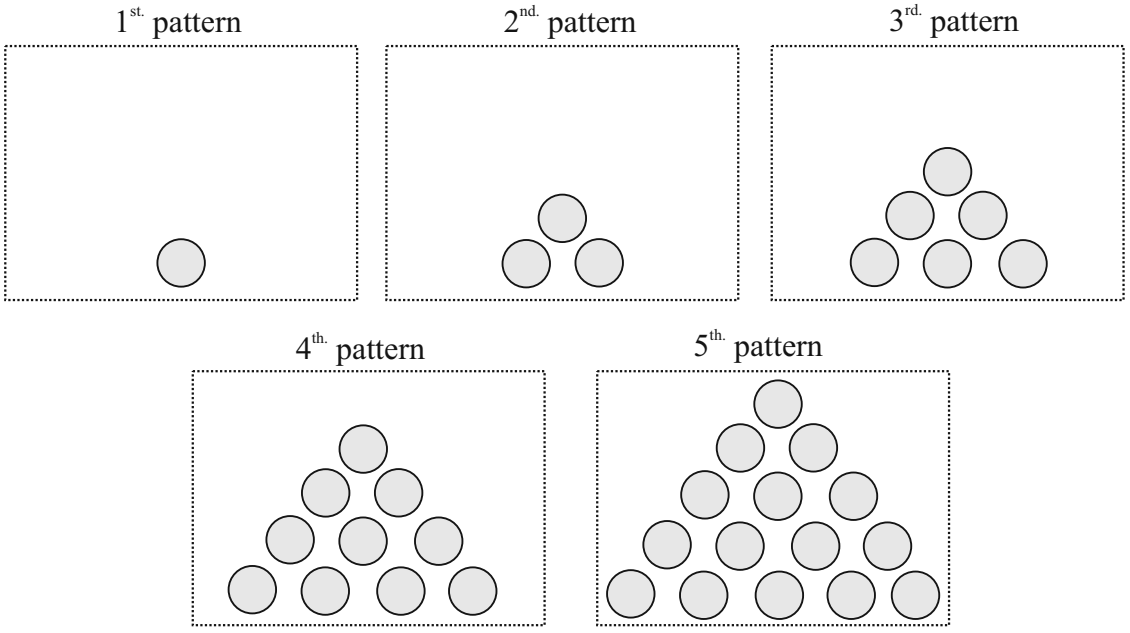
$y - 10 = -6(x - 0)$ $\boxed{y - y_1 = m(x - x_1)}$

$y - 10 = -6x$

$6x + y - 10 = 0$

QUESTION 7 (50 MARKS)

Question 7 (a)



Question 7 (b)

Pattern	Number of counters
1st.	1
2nd.	3 = 1 + 2
3rd.	6 = 1 + 2 + 3
4th.	10 = 1 + 2 + 3 + 4
5th.	15 = 1 + 2 + 3 + 4 + 5

Question 7 (c)

15^{th} triangular number = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = 120

Question 7 (d) (i)

$S_n = 1 + 2 + 3 + ... + n$

$a = 1,\ d = 1$

$S_n = \frac{n}{2}[2(1) + (n-1)1] = \frac{n}{2}[2 + n - 1]$

$\therefore S_n = \frac{n}{2}(n+1)$

Question 7 (d) (ii)

$S_{15} = \frac{15}{2}(15+1) = \frac{15}{2}(16) = 15 \times 8 = 120$

Question 7 (d) (iii)

$S_{100} = \frac{100}{2}(100+1) = 50(101) = 5050$

Question 7 (e)

$666 = \frac{n}{2}(n+1)$

$1332 = n(n+1)$

$1332 = n^2 + n$

$0 = n^2 + n - 1332$

$0 = (n-36)(n+37)$

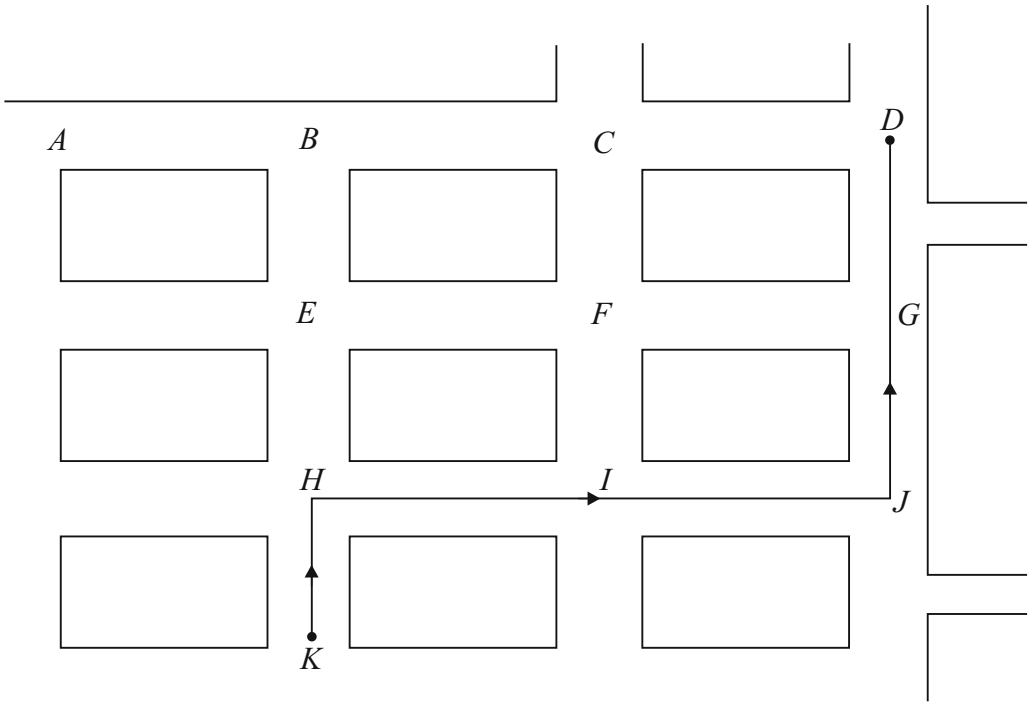
$n = 36$

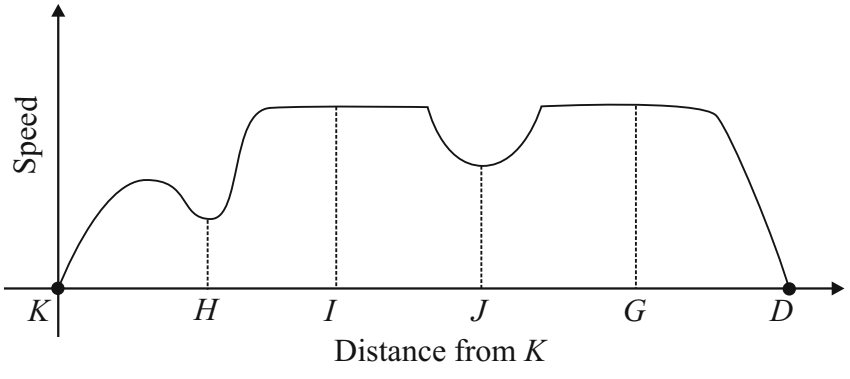
Finding the factors of 1332:

2	1332
2	666
3	333
3	111
	37

Question 8 (50 marks)

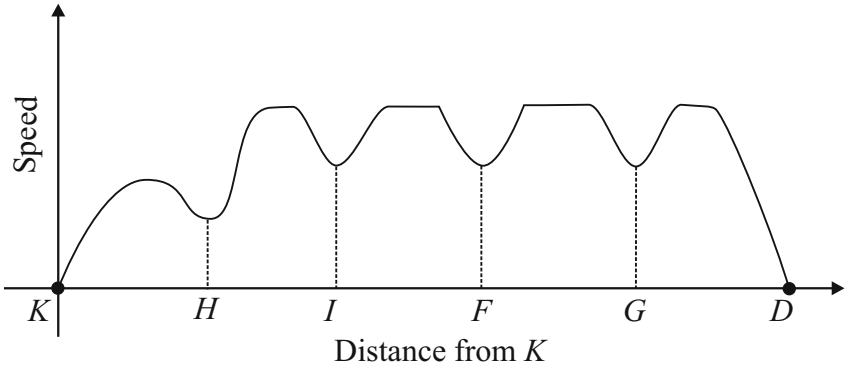
Question 8 (a) (i)



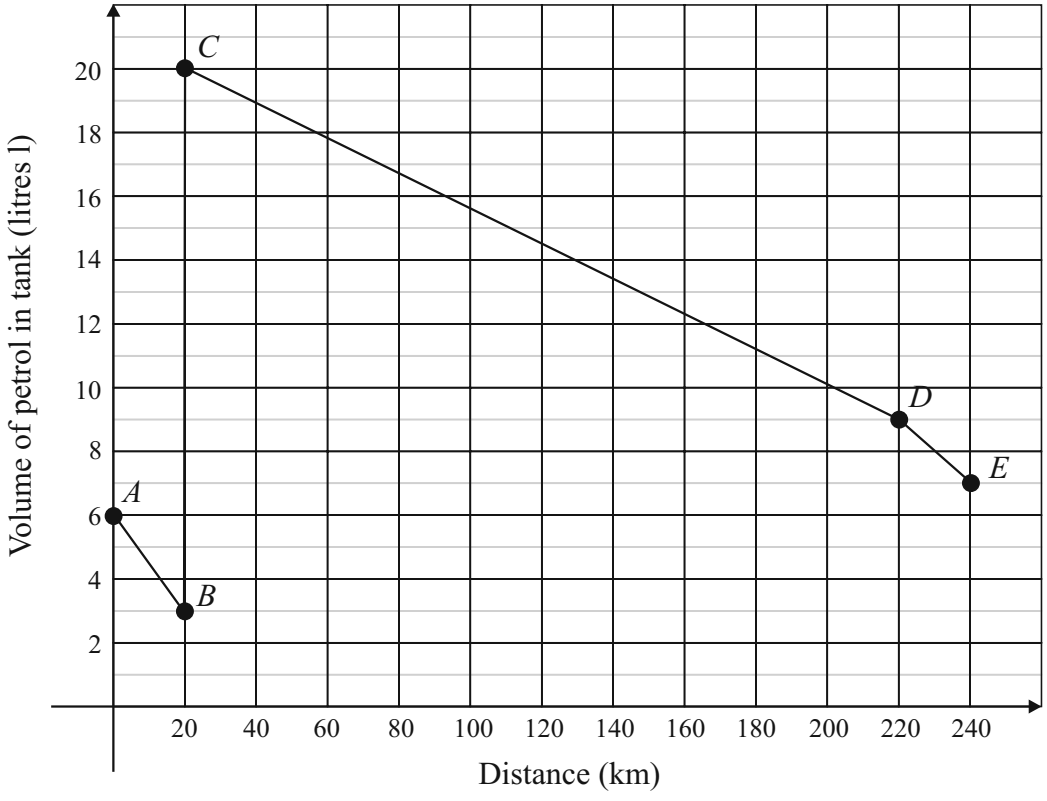


Route taken: *KHIJGD*

Question 8 (a) (ii)



Question 8 (b) (i)



AB: Car is travelling in town for 20 km. There is a rapid decline in petrol from 6 l to 3 l.
BC: Car pulls into a petrol station and fills the tank with petrol from 3 l to 20 l.
CD: Car travels 200 km on the motorway using only 11 l of petrol.
DE: Car is travelling through town using up 2 l of petrol in 20 km.

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Question 8 (b) (ii)

Volume = 6 litres

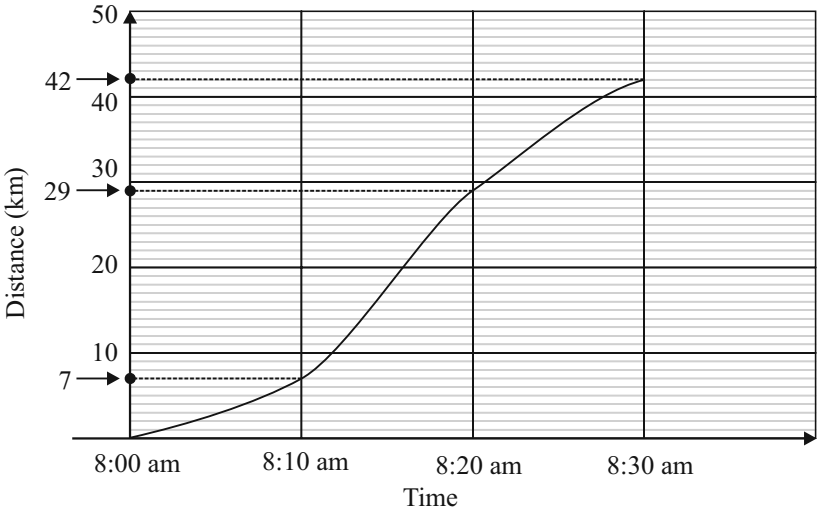
Question 8 (b) (iii)

$$\text{Consumption rate} = \frac{\text{Number of litres of petrol}}{\text{Distance travelled (km)}}$$

$$AB: \text{Consumption rate} = \frac{6-3}{20-0} = \frac{3}{20} = 0.15 \text{ l/km}$$

$$CD: \text{Consumption rate} = \frac{20-9}{220-20} = \frac{11}{200} = 0.055 \text{ l/km}$$

Question 8 (c) (i)



$$\text{Average Speed} = \frac{7-0}{\frac{1}{6}} = \frac{7 \text{ km}}{\frac{1}{6} \text{ h}} = 42 \text{ km/h}$$

$$\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}$$

Question 8 (c) (ii)

He accelerates up to a maximum speed as he turns onto the motorway. The car covers a greater distance in a shorter time as it speeds up. His average speed on the motorway is 132 km/h.

$$\text{Average Speed} = \frac{29-7}{\frac{1}{6}} = \frac{22 \text{ km}}{\frac{1}{6} \text{ h}} = 132 \text{ km/h}$$

Question 8 (c) (iii)

$$\text{Average Speed} = \frac{42-29}{\frac{1}{6}} = \frac{13 \text{ km}}{\frac{1}{6} \text{ h}} = 78 \text{ km/h}$$

Question 8 (c) (iv)

Yes, he broke the speed limit on the motorway where he was travelling on average at 132 km/h. The speed limit is 120 km/h.

Question 8 (c) (v)

$$\text{Average Speed} = \frac{42-0}{\frac{1}{2}} = \frac{42 \text{ km}}{\frac{1}{2} \text{ h}} = 84 \text{ km/h}$$

QUESTION 9 (50 MARKS)

Question 9 (a)

Distance (km)	1	2	3	4	5	6	7	8	9	10
Price (€)	7	9	11	13	15	17	19	21	23	25

Question 9 (b)

$p = 5 + 2d$

Question 9 (c)

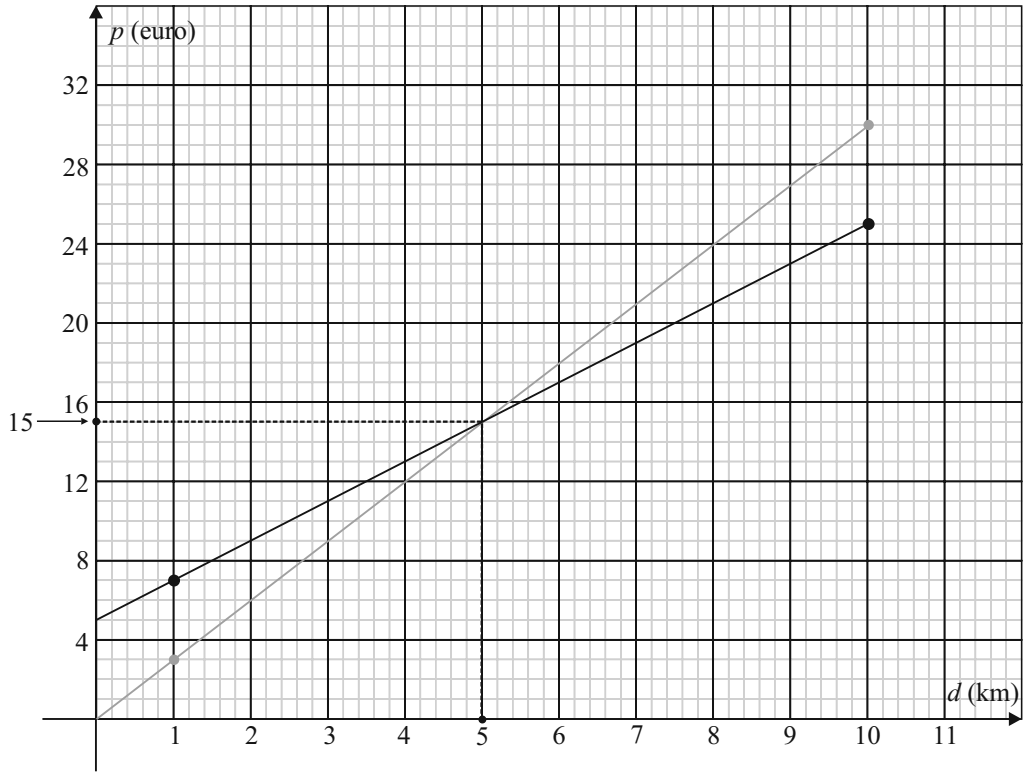
$$\begin{aligned} p &= 5 + 2d \\ &= 5 + 2(15 \cdot 5) \\ &= 5 + 31 \\ &= €36 \end{aligned}$$

Question 9 (d)

Fixed charge = €0

Distance (km)	1	2	3	4	5	6	7	8	9	10
Price (€)	3	6	9	12	15	18	21	24	27	30

Question 9 (e)



Question 9 (f)

Graph: $d = 5$ km , $p = €15$

Algebra: $5 + 2d = 3d$

$\therefore d = 5$ km

$p = 3d = €15$

Question 9 (g)

Average distance per job = 5.6 km

Total distance travelled $= 15 \times 5.6 = 84$ km

Income from fares $p = 15(5 + 2(5 \cdot 6)) = €243$

Petrol consumption: 7 litres $\rightarrow 100$ km

$$\frac{7}{100} \text{ litres} \rightarrow 1 \text{ km}$$

$$\frac{7}{100} \times 84 \text{ litres} \rightarrow 84 \text{ km}$$

$$= 5.88 \text{ litres}$$

Cost of petrol = $5 \cdot 88 \times 1 \cdot 45 = €8 \cdot 53$

Profit = €243 – €25 – €8.53 = €209.47

Sample Paper 2: Paper 2

Question 1 (25 marks)

Question 1 (a)

$l : x + 3y = 0 \Rightarrow m_1 = -\frac{1}{3}$

$k : ax + y = 0 \Rightarrow m_2 = -a$

$m_1 \times m_2 = -1 \Rightarrow (-\frac{1}{3})(-a) = -1$

Perpendicular lines: $m_1 \times m_2 = -1$

$\frac{1}{3}a = -1$

$\therefore a = -3$

Question 1 (b)

$(0, 0) \in l : x + 3y = 0?$

$(0) + 3(0) = 0$

$(0, 0) \in k : -3x + y = 0?$

$-3(0) + (0) = 0$

Question 1 (c)

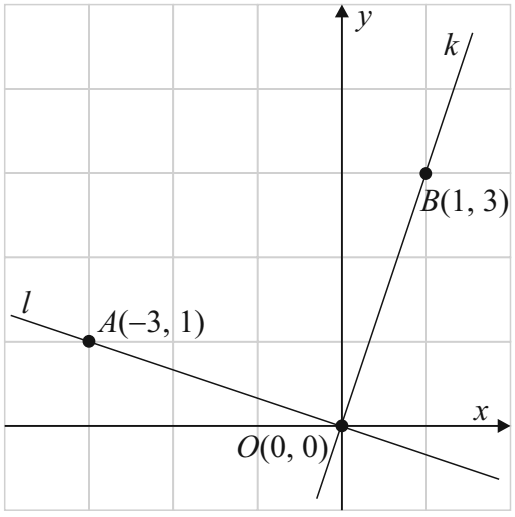
$A(-3, 1) \in l : x + 3y = 0?$

$(-3) + 3(1) = -3 + 3 = 0$

$B(1, 3) \in k : -3x + y = 0?$

$-3(1) + (3) = -3 + 3 = 0$

Question 1 (d)



Question 1 (e)

Find D by a translation:

$O(0, 0) \rightarrow B(1, 3)$

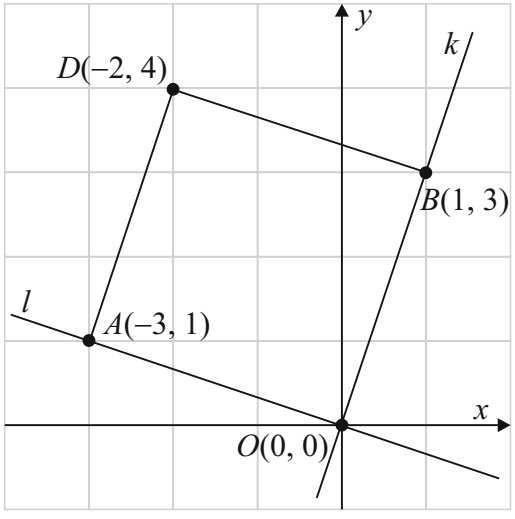
$A(-3, 1) \rightarrow D(-2, 4)$

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$O(0, 0)$, $B(1, 3)$

$|OB| = \sqrt{(3-0)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$

Area of $AOBD = \sqrt{10} \times \sqrt{10} = 10$



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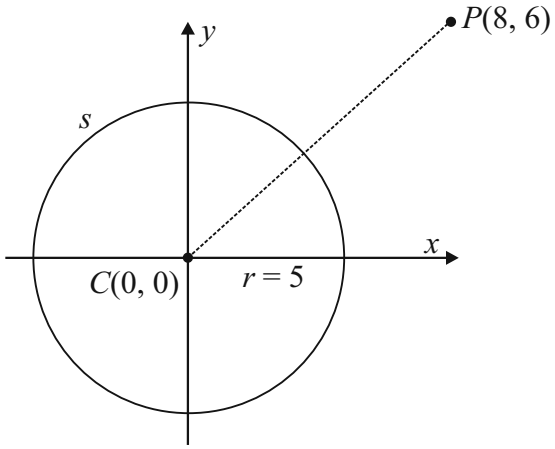
Question 2 (25 marks)

Question 2 (a)



Circle: $x^2 + y^2 = r^2$
Centre(0, 0), Radius $= r$

Circle s: Centre (0, 0), $r = 5$

Question 2 (b)

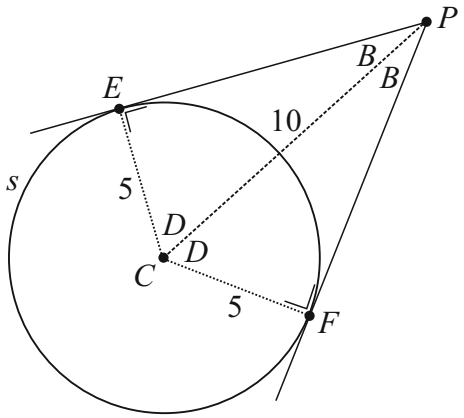
$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$C(0, 0)$, $P(8, 6)$

$$|CP| = \sqrt{(8-0)^2 + (6-0)^2} = \sqrt{64+36} = \sqrt{100} = 10$$

Question 2 (c)

Triangles PCF and PCE are congruent right-angled triangles. The hypotenuse is CP.



$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin B = \frac{5}{10} = \frac{1}{2} \Rightarrow B = \sin^{-1}(\tfrac{1}{2}) = 30^\circ$$

$$\angle B + \angle D + 90^\circ = 180^\circ$$

$$30^\circ + \angle D = 90^\circ$$

$$\therefore \angle D = 60^\circ$$

Question 2 (d)

$r = 6400$ km, $\theta = 2(\angle D) = 120^\circ$

$$l = 2\pi r\left(\frac{\theta}{360^\circ}\right)$$

$$l = 2\pi(6400)\left(\frac{120^\circ}{360^\circ}\right) = 13404 \text{ km}$$

$$|CP| = 2r \Rightarrow h = r$$

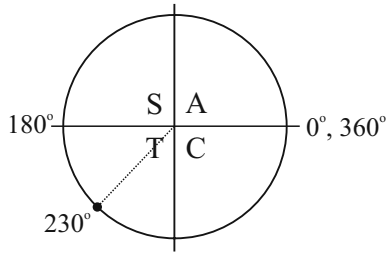
Height $h = 6400$ km

QUESTION 3 (25 MARKS)

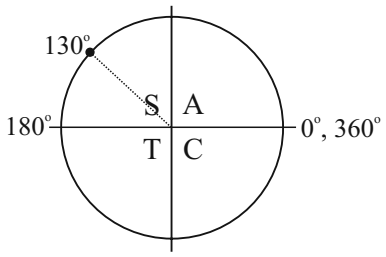
Question 3 (a)

$\sin 50^\circ = 0.766$
$\cos 50^\circ = 0.643$
$\tan 50^\circ = 1.192$

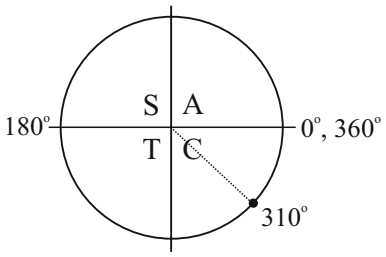
Question 3 (b)



$$\begin{aligned}&\sin 230^\circ\\ &= \sin(180^\circ + 50^\circ)\\ &= -\sin 50^\circ\\ &= -0.766\end{aligned}$$



$$\begin{aligned}&\cos 130^\circ\\ &= \cos(180^\circ - 50^\circ)\\ &= -\cos 50^\circ\\ &= -0.643\end{aligned}$$



$$\begin{aligned}&\tan 310^\circ\\ &= \tan(360^\circ - 50^\circ)\\ &= -\tan 50^\circ\\ &= -1.192\end{aligned}$$

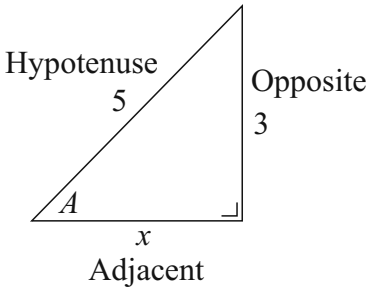
Question 3 (c) (i)

$$\sin A = \frac{3}{5} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$x^2 + 3^2 = 5^2$$

$$x^2 = 25 - 9 = 16$$

$$x = \sqrt{16} = 4$$



$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{4}{5}$$

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{3}{4}$$

Question 3 (c) (ii)

$\cos(180^\circ + A) = -\cos A = -\frac{4}{5}$

$\tan(180^\circ + A) = +\tan A = \frac{3}{4}$

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Question 4 (25 marks)

Question 4 (a)

(i) Modal class: 8–10

(ii) Median class: 8–10

(iii) $\text{Mean} = \dfrac{5\times2+7\times4+9\times8+11\times5+13\times1}{20} = 8.9$

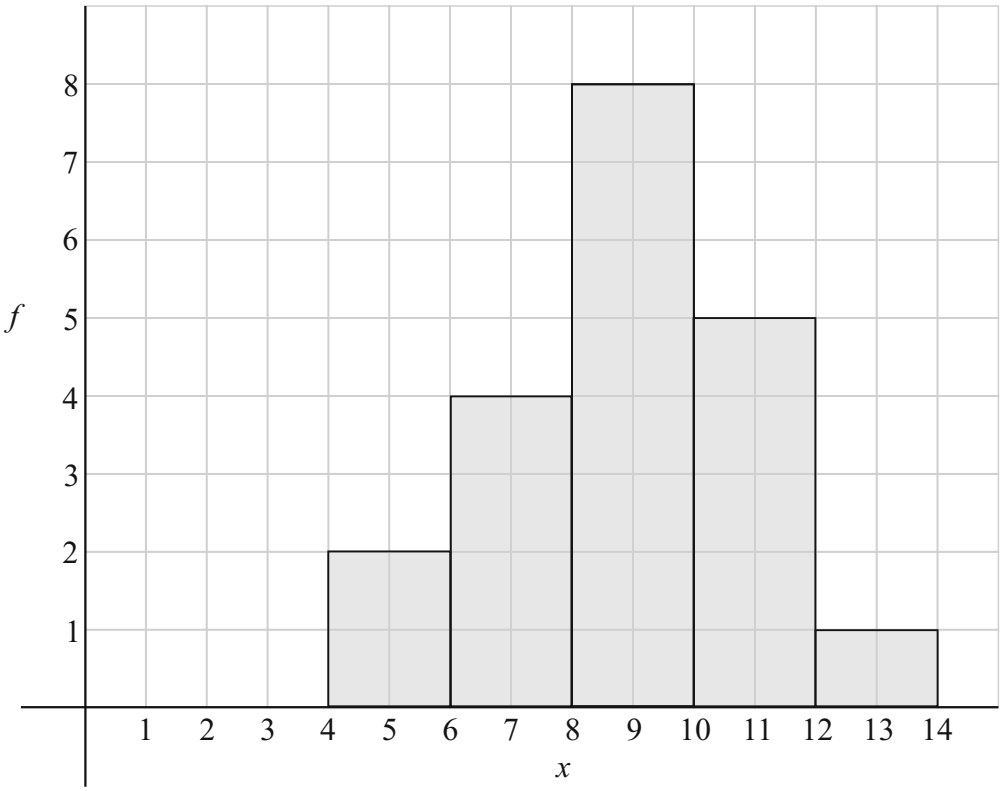
x	f
4–6	2
6–8	4
8–10	**8**
10–12	5
12–14	1
	20

$10^{\text{th.}}$ and $11^{\text{th.}}$ results in here

(iv) Range = Mid-interval value of highest interval – Mid-interval value of lowest interval

$\text{Range} = 13 - 5 = 8$

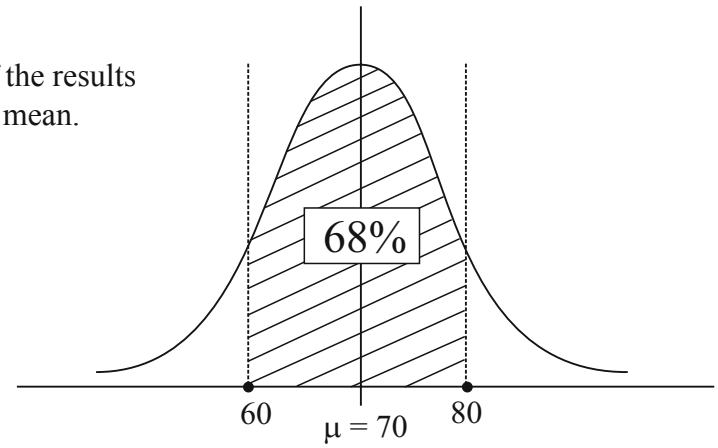
(v)



Question 4 (b)

According to the empirical rule, 68% of the results are within one standard deviation of the mean.

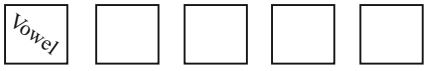
Lowest C: 60
Highest C: 80



Question 5 (25 marks)

Question 5 (a) (i)

$2 \times 4 \times 3 \times 2 \times 1 = 48$



Number of arrangements = 48

There are 2 ways to fill the first box (an A or E).
Once this box has been filled, there are 4 ways to fill the second box.
Once the first 2 boxes have been filled, there are 3 ways to fill the third box, and so on.

Question 5 (a) (ii)

$4 \times 3 \times 2 \times 1 = 24$

2 ways

Number of arrangements = $24 \times 2 = 48$

Glue the 2 vowels together and treat them as a single unit.
There are 4 ways to fill the first box.
Once this box has been filled, there are 3 ways to fill the second box, and so on.
The number of arrangements needs to be multiplied by 2, as the two vowels can be swapped.

Question 5 (b) (i)

$$P(\text{Double}) = \frac{\text{Number of doubles}}{\text{Number of outcomes}} = \frac{6}{36} = \frac{1}{6}$$

Question 5 (b) (ii)

$$E(\text{Doubles}) = \frac{1}{6} \times 1872 = 312$$

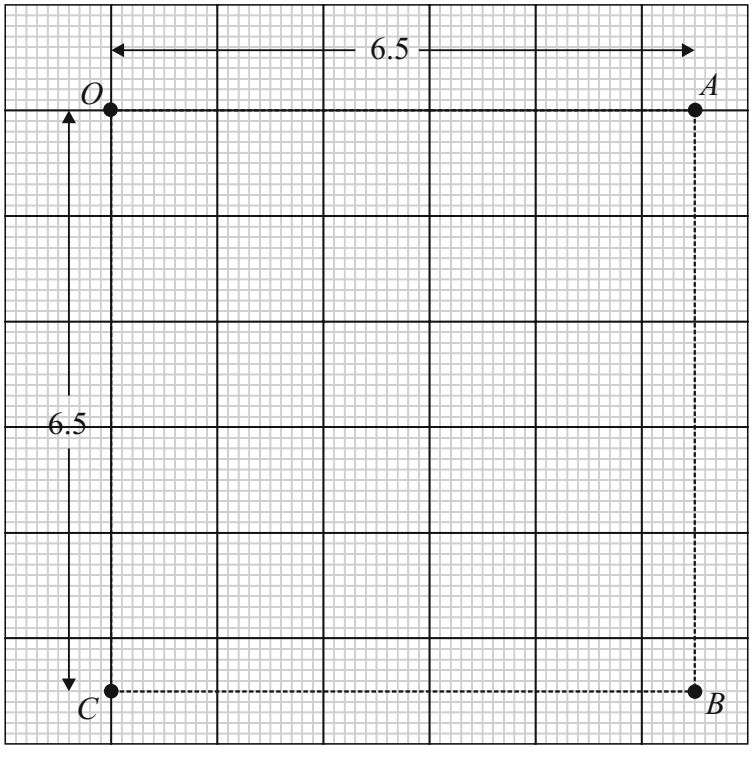
Question 5 (c)

The likelihood that the last roll is a six is $\frac{1}{6}$ as all events are independent of each other.

Die 1 \ Die 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Question 6 (25 marks)

Question 6 (a)



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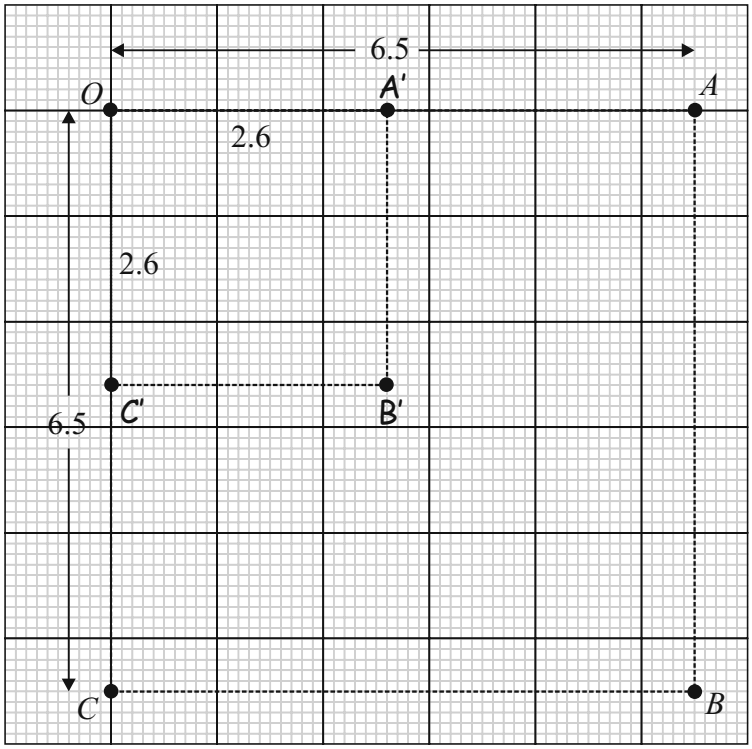
Question 6 (b)

$k = 0.4$

Length of side $= 6.5$

Scale factor $k = \dfrac{|\text{Image length}|}{|\text{Object length}|}$

$$k = \frac{|\text{Image length}|}{6.5} \Rightarrow |\text{Image length}| = 0.4 \times 6.5 = 2.6 \text{ units}$$



Question 6 (c)

Area $= 2.6 \times 2.6 = 6.76$ square units

Question 6 (d)

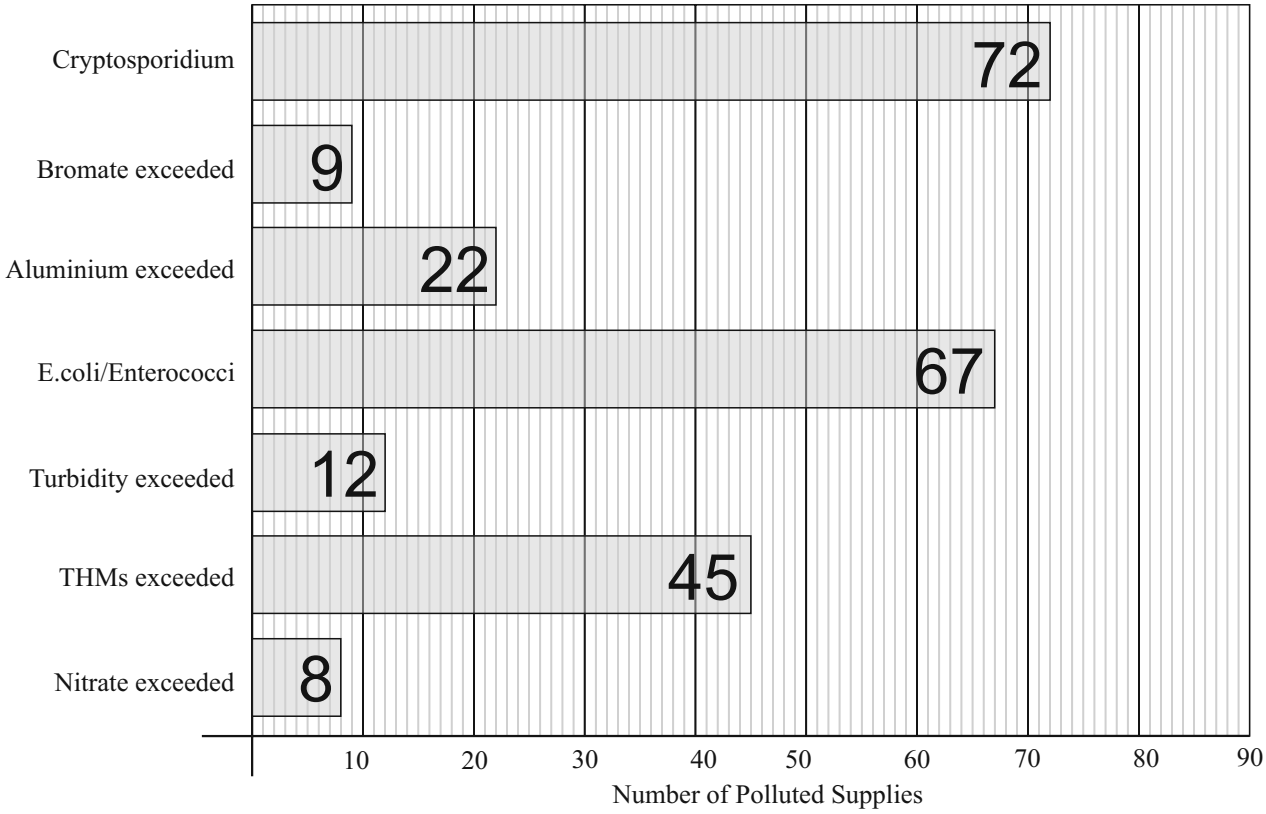
$$k^2 = \frac{95.0625}{6.5 \times 6.5} = \frac{9}{4}$$

$$\therefore k = \sqrt{\frac{9}{4}} = \frac{3}{2} = 1.5$$

$k^2 = \dfrac{|\text{Image area}|}{|\text{Object area}|}$

Question 7 (45 marks)

Question 7 (a)



Maximum number of supplies = 72 + 9 + 22 + 67 + 12 + 45 + 8 = 235

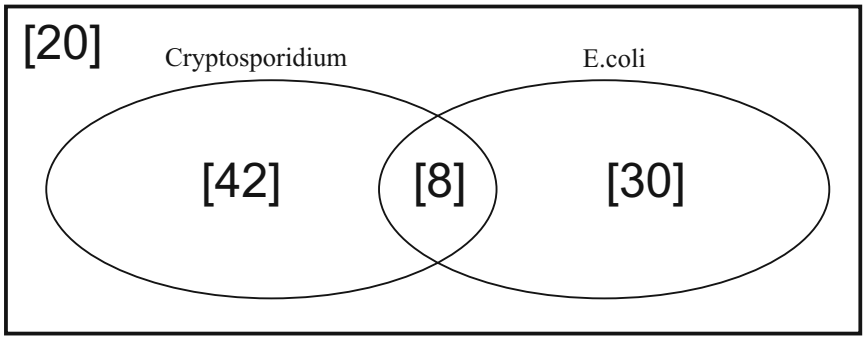
Question 7 (b)

$\frac{72}{300} \times 100\% = 24\%$ [72 water supplies had Cryptosporidium. They may have had other pollutants but they had Cryptosporidium at least.]

Question 7 (c)

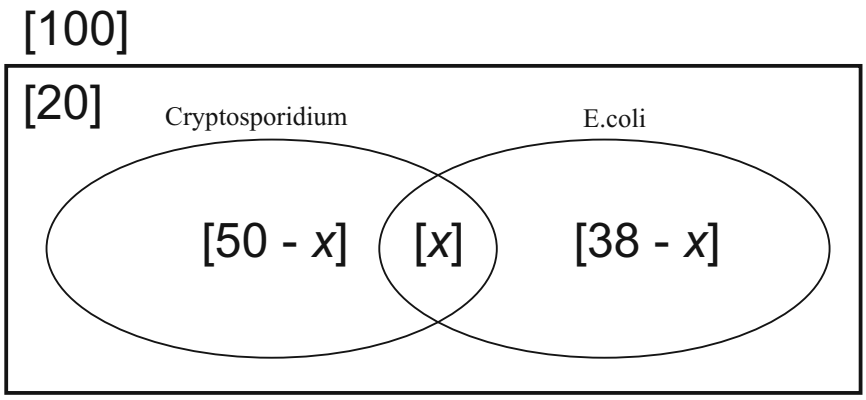
The name of the pollutants are too large to put on the horizontal axis, making them much harder to read.

Question 7 (d)



Total number of supplies = 100
Number of unpolluted supplies = 20
Number of polluted supplies = 80
Number of supplies polluted with Cryptosporidium = 50
Number of supplies polluted with E.coli = 38
Number of supplies polluted with Cryptosporidium **or** E.coli = 50 + 38 = 88
Number of supplies polluted with Cryptosporidium **and** E.coli = 88 – 80 = 8

OR



$$50 - x + x + 38 - x + 20 = 100$$
$$108 - x = 100$$
$$108 - 100 = x$$
$$\therefore x = 8$$

Question 7 (e) (i)

$$P(\text{Supply is polluted with Crytosporidium and E.coli}) = \frac{8}{100} = 0.08 = 8\%$$

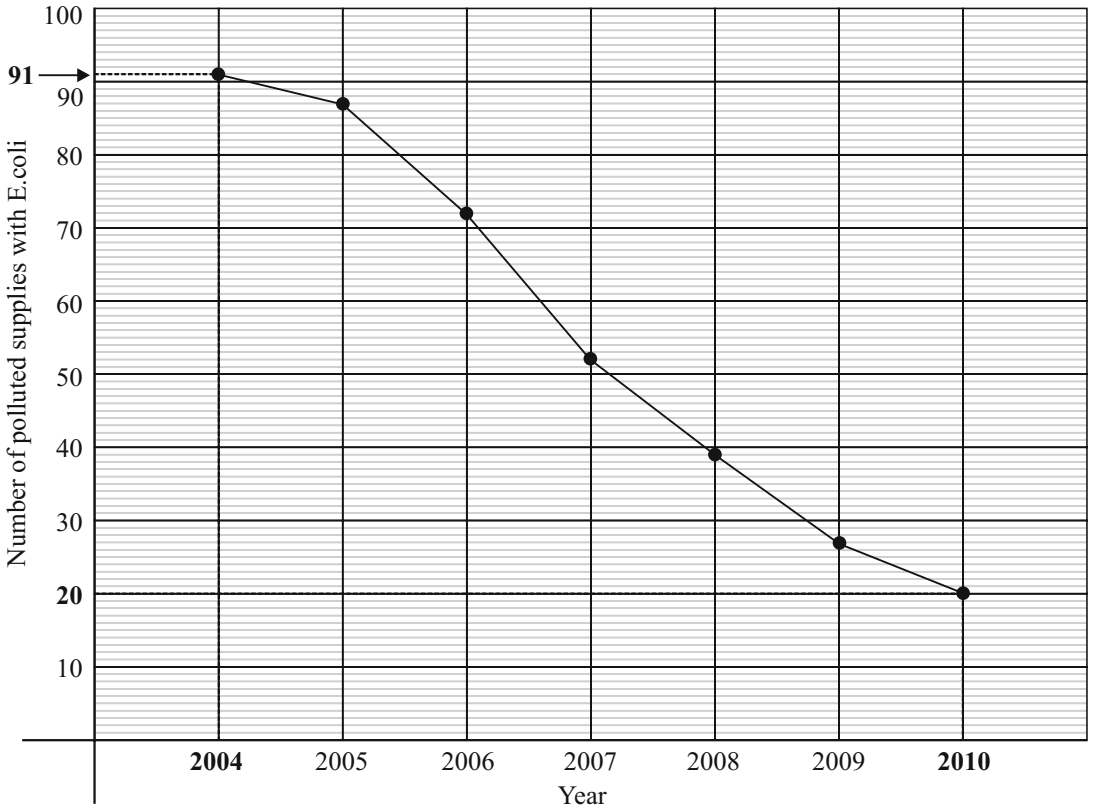
Question 7 (e) (ii)

$$P(\text{Supply is polluted with E.coli only}) = \frac{30}{100} = 0.3 = 30\%$$

Question 7 (f)

$$P(\text{3 supplies are polluted with Crytosporidium and E.coli}) = 0.08 \times 0.08 \times 0.08 = \frac{8}{15625}$$

Question 7 (g) (i)



$$\%\text{ of supplies in 2010 which had E.coli} = \frac{20}{945} \times 100\% = 2.12\%$$

Question 7 (g) (ii)

$$\% \text{ decrease in polluted supplies from 2004 to 2010} = \frac{91-20}{945} \times 100\% = 7.51\%$$

Question 7 (g) (iii)

Private supplies are inferior in terms of pollution.

Question 8 (30 marks)

Question 8 (a)

A simple random sample is one where every member of the population has an equal chance of being included in the sample.

Question 8 (b)

This would not give you a simple random sample because people without a telephone or who are ex-directory have no chance of being included in the sample.

Question 8 (c)

The null hypothesis H_o states that the Government does not call an election. $P \leq 50\%$

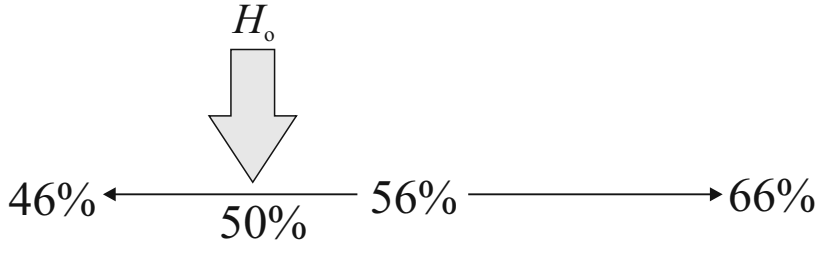
Question 8 (d)

$n = 100$

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}} = \pm\frac{1}{\sqrt{100}} = \pm 0.1 = \pm 10\%$$

Question 8 (e)

Proportion of those in the sample in favour of the Government = 56%
Test (using the sample proportion): 46% ← 56% → 66%
Since 50% is inside this interval, accept the null hypothesis which means the Government should not call an election.



Question 8 (f)

$n = ?$

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}} = \pm 5\% = \pm 0.05$$

$$\frac{1}{n} = (\pm 0.05)^2 = 0.0025$$

$$\therefore n = \frac{1}{0.0025} = 400$$

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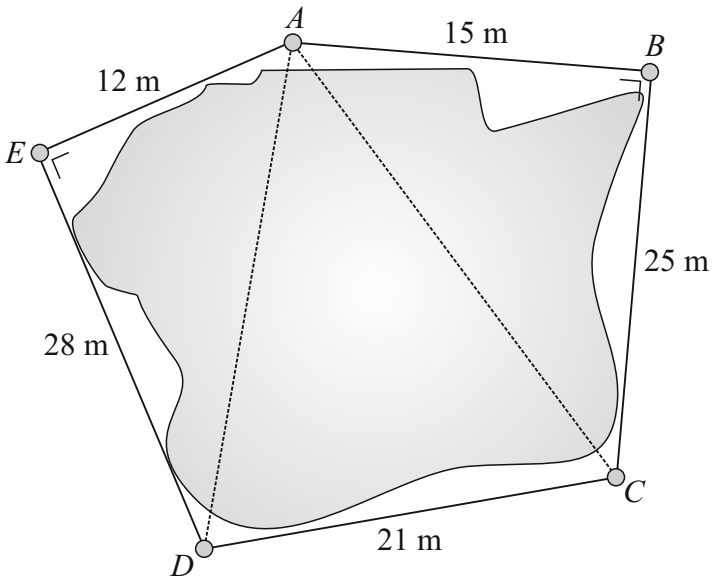
Question 9 (75 marks)



Question 9 (a) (i)

No, a regular pentagon has five **equal** sides, whereas a pentagon is a 5-sided figure.

Question 9 (a) (ii)

Perimeter = 15 + 25 + 21 + 28 + 12 = 101 m

Question 9 (b) (i)

The pedometer needs to know the length of Joan's stride.
Distance walked = Number of clicks × length of stride

Question 9 (b) (ii)

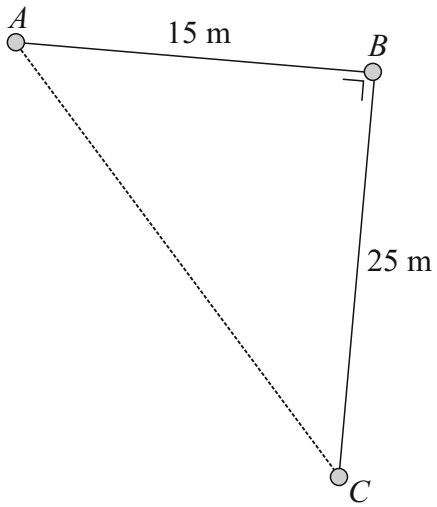
She will find the sum of the areas of triangles *AED*, *ADC* and *ABC*.

Question 9 (c) (i)

Name: Hypotenuse

Question 9 (c) (ii)

$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}(15)(25) = 187.5 \text{ m}^2$



Question 9 (c) (iii)

$|AC|^2 = 15^2 + 25^2$

$\therefore |AC| = \sqrt{15^2 + 25^2} = 29.2 \text{ m}$

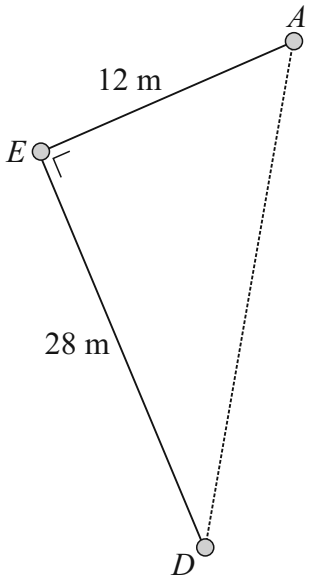
Question 9 (c) (iv)

$\tan(|\angle BAC|) = \frac{25}{15} \Rightarrow |\angle BAC| = \tan^{-1}(\frac{5}{3}) = 59^\circ$

$|\angle BCA| = 180^\circ - 90^\circ - 59^\circ = 31^\circ$

Question 9 (d) (i)

$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}(12)(28) = 168 \text{ m}^2$



Question 9 (d) (ii)

$|AD|^2 = 12^2 + 28^2$

$\therefore |AC| = \sqrt{12^2 + 28^2} = 30.5 \text{ m}$

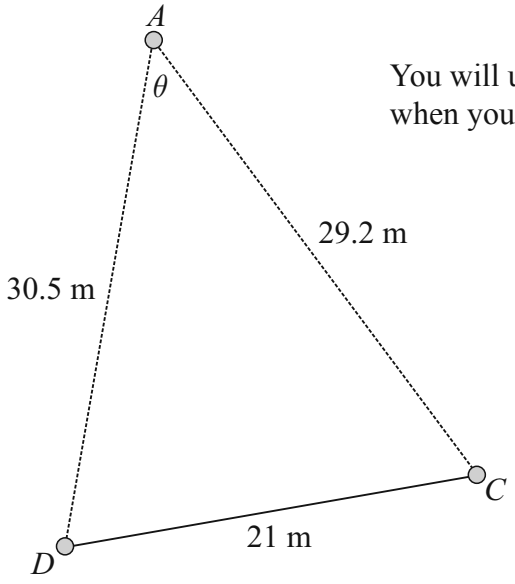
Question 9 (d) (iii)

No, for a triangle, base times height does not depend on the choice of base. ($A = \frac{1}{2}bh = \frac{1}{2}hb$)

Question 9 (e) (i)

Type of triangle: Scalene

Question 9 (e) (ii)



You will use the Cosine rule. This rule is used when you are given 3 sides and no angle.

Question 9 (e) (iii)

$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

$21^2 = 30.5^2 + 29.2^2 - 2(30.5)(29.2)\cos\theta$

$2(30.5)(29.2)\cos\theta = 30.5^2 + 29.2^2 - 21^2$

$1781.2\cos\theta = 1341.89$

$\theta = \cos^{-1}\left(\frac{1341.89}{1781.2}\right) = 41.1^\circ$

Question 9 (e) (iv)

$\boxed{\text{Area} = \frac{1}{2}ab\sin C}$

$\text{Area} = \frac{1}{2}(30.5)(29.2)\sin 41.1^\circ = 292.7 \text{ m}^2$

Question 9 (f) (i)

$\text{Area } |\Delta ABC| + \text{Area } |\Delta DEA| + \text{Area } |\Delta ADC|$

$= 187.5 + 168 + 292.7 = 648.2 \text{ m}^2 \approx 648 \text{ m}^2$

Question 9 (f) (ii)

$\%\text{ Error} = \frac{648 - 610}{610} = 6\%$

Sample Paper 3: Paper 1

Question 1 (25 marks)

Question 1 (a)

$123\% = €738$

$1\% = \dfrac{€738}{123} = €6$

$100\% = €6 \times 100 = €600$

Cost price = €600
VAT paid = €738 – €600 = €138

Question 1 (b)

$\$1 = \dfrac{1}{1 \cdot 32}€$

$\$500 = €\left(\dfrac{1}{1 \cdot 32}\right) \times 500 = €378 \cdot 79$

$\text{Loss} = €738 - €378 \cdot 79 = €359 \cdot 21$

$\%\ \text{Loss} = \dfrac{359 \cdot 21}{738} \times 100\% = 48 \cdot 7\%$

Question 1 (c) (i)

$€36\ 000 \times (0 \cdot 8)^3 = €18\ 432$

Question 1 (c) (ii)

$€36\ 000 \times (0 \cdot 4) = €14\ 400$

Question 1 (c) (iii)

Sells for the depreciation value:

$\text{Loss} = €36\ 000 - €18\ 432 = €17\ 568$

$\%\ \text{Loss} = \dfrac{€17\ 568}{€36\ 000} \times 100\% = 48 \cdot 8\%$

Sells for the scrap value:

$\text{Loss} = €36\ 000 - €14\ 400 = €21\ 600$

$\%\ \text{Loss} = \dfrac{€21\ 600}{€36\ 000} \times 100\% = 60\%$

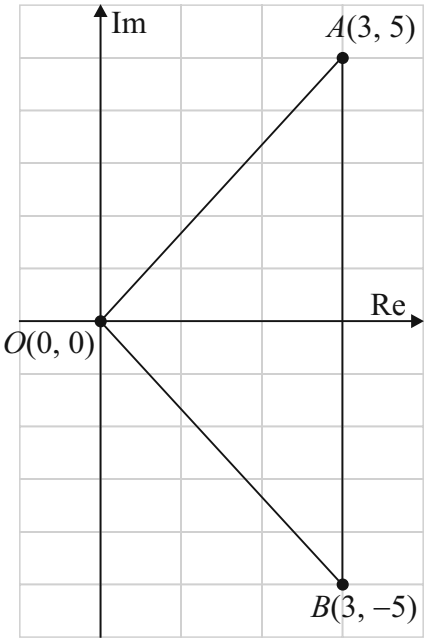
Question 2 (25 marks)

Question 2 (a)

$z = 3 + 5i : A(3, 5)$

$\overline{z} = 3 - 5i : B(3, -5)$

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$O(0, 0),\ A(3, 5)$

$|OA| = \sqrt{(3-0)^2 + (5-0)^2} = \sqrt{9+25} = \sqrt{34}$

$O(0, 0),\ B(3, -5)$

$|OB| = \sqrt{(3-0)^2 + (-5-0)^2} = \sqrt{9+25} = \sqrt{34}$

$\therefore |OA| = |OB|$ (Isosceles triangle)

Area $= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}(10)(3) = 15$

Question 2 (b)

$z = 4 - i$

$z + 4i = 4 - i + 4i = 4 + 3i$

$|z + 4i| = |4 + 3i| = \sqrt{16+9} = \sqrt{25} = 5$ $\quad |a + ib| = \sqrt{a^2 + b^2}$

$w = \dfrac{1}{z+4i} = \dfrac{1}{4+3i}$ [Multiply above and below by the conjugate of the denominator.]

$= \dfrac{1}{4+3i} \times \dfrac{4-3i}{4-3i} = \dfrac{4-3i}{16-12i+12i-9i^2}$ $\quad z = a + ib \Rightarrow \overline{z} = a - ib$

$= \dfrac{4-3i}{16+9} = \dfrac{4-3i}{25}$

$= \dfrac{4}{25} - \dfrac{3}{25}i$

LHS	*RHS*
$25w$	$\overline{z+4i}$
$25\left(\dfrac{4}{25} - \dfrac{3}{25}i\right)$	$\overline{4+3i}$
$4 - 3i$	$4 - 3i$

Question 3 (25 marks)

Question 3 (a)

$\frac{1}{x} = x + 1$ [Multiply across by x.]

$1 = x^2 + x$

$0 = x^2 + x - 1$

$a = 1,\ b = 1,\ c = -1$ $\quad \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

Question 3 (b)

$x = y + \frac{1}{2}$.....(**1**)(×2)

$2x = 3y - 2$...(**2**)

$2x - 2y = 1$

$2x - 3y = -2$

$y = 3$ [(**1**) − (**2**)]

$\therefore x = y + \frac{1}{2} = 3 + \frac{1}{2} = \frac{7}{2}$

Question 4 (25 marks)

Question 4 (a)

$\frac{125}{\sqrt{5}} = \frac{5^3}{5^{\frac{1}{2}}} = 5^{3-\frac{1}{2}} = 5^{\frac{5}{2}}$

$5^{3x-1} = \frac{125}{\sqrt{5}}$

$5^{3x-1} = 5^{\frac{5}{2}}$

$\therefore 3x - 1 = \frac{5}{2}$

$3x = \frac{7}{2}$

$\therefore x = \frac{7}{6}$

Formulae and Tables Book
Indices and logs [page 21]

$a^p \times a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

$a^{\frac{1}{2}} = \sqrt{a}$

Question 4 (b)

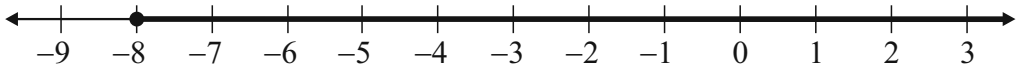
$x - 10 \le 2(x - 1)$

$x - 10 \le 2x - 2$

$-10 + 2 \le 2x - x$

$-8 \le x$

$\therefore x \ge -8$



Question 4 (c)

$\frac{1}{x-1} - \frac{1}{x} = \frac{1}{2}$ [Multiply across by the denominators.]

$\frac{2x(x-1)1}{x-1} - \frac{2x(x-1)1}{x} = \frac{2x(x-1)1}{2}$

$2x - 2(x-1) = x(x-1)$

$2x - 2x + 2 = x^2 - x$

$0 = x^2 - x - 2$

$0 = (x+1)(x-2)$

$\therefore x = -1,\ 2$

Question 5 (25 marks)

Question 5 (a) (i)

$y = 2x^3 - 7x^2 + 5x - 3$

$\frac{dy}{dx} = 6x^2 - 14x + 5$

Question 5 (a) (ii)

$y = (2x-1)(2x+1)$

$= 4x^2 - 1$

$\frac{dy}{dx} = 8x$

Question 5 (b) (i)

$v = 6 : v = 8t - 2t^2 = 6$

$2t^2 - 8t + 6 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 1 \text{ s}, 3 \text{ s}$

Question 5 (b) (ii)

$a = \frac{dv}{dt} = 8 - 4t$

$t = 0 \text{ s}: a = 8 - 4(0) = 8 \text{ m/s}^2$

Question 5 (b) (iii)

$a = 0 : a = 8 - 4t = 0$

$8 = 4t$

$t = 2 \text{ s}$

$t = 2 : v = 8t - 2t^2$

$= 8(2) - 2(2)^2$

$= 16 - 8 = 8 \text{ m/s}$

Question 6 (25 marks)

Question 6 (a)

$y = f(x) = 9 - x^2$

$x = -3 : f(-3) = 9 - (-3)^2 = 9 - 9 = 0$

$x = -2 : f(-2) = 9 - (-2)^2 = 9 - 4 = 5$

$x = -1 : f(-1) = 9 - (-1)^2 = 9 - 1 = 8$

$x = 0 : f(0) = 9 - (0)^2 = 9 - 0 = 9$

$x = 1 : f(1) = 9 - (1)^2 = 9 - 1 = 8$

$x = 2 : f(2) = 9 - (2)^2 = 9 - 4 = 5$

$x = 3 : f(3) = 9 - (3)^2 = 9 - 9 = 0$

x	-3	-2	-1	0	1	2	3
$f(x)$	0	5	8	9	8	5	0

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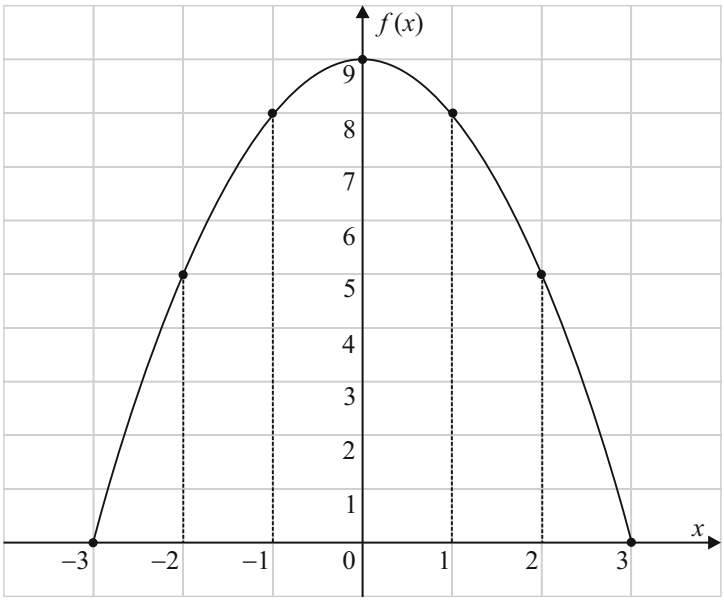
Question 6 (b)



Question 6 (c)

$A = \frac{1}{2}\{0+0+2(5+8+9+8+5)\} = 35$

QUESTION 7 (50 MARKS)

Question 7 (a)

MAY Bealtaine 2012

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6 ○	7	8	9	10	11	12 ◑
13	14	15	16	17	18	19
20	21 ●	22	23	24	25	26
27	28 ◐	29	30	31		

(i) The difference between corresponding numbers in consecutive rows is 15 – 8 = 7.

(ii) The difference between corresponding numbers in consecutive columns is 9 – 8 = 1.

Question 7 (b)

MAY Bealtaine 2012

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6 ○	7	8	9	10	11	12 ◑
13	14	15	16	17	18	19
20	21 ●	22	23	24	25	26
27	28 ◐	29	30	31		

(i) The difference between corresponding numbers in consecutive rows is $27 - 20 = 7$.

(ii) The difference between corresponding numbers in consecutive columns is $21 - 20 = 1$.

Question 7 (c)

Calendar Magic Number $= BL \times TR - BR \times TL$
$= 22 \times 10 - 24 \times 8 = 220 - 192 = 28$

Top Left (*TL*) Top Right (*TR*)

8	9	10
15	16	17
22	23	24

Bottom Left (*BL*) Bottom Right (*BR*)

Question 7 (d) (i)

Top Left (*TL*) Top Right (*TR*)

9	10	11
16	17	18
23	24	25

Bottom Left (*BL*) Bottom Right (*BR*)

Calendar Magic Number $= BL \times TR - BR \times TL$
$= 23 \times 11 - 25 \times 9 = 253 - 225 = 28$

Question 7 (d) (ii)

Top Left (*TL*) Top Right (*TR*)

1	2	3
8	9	10
15	16	17

Bottom Left (*BL*) Bottom Right (*BR*)

Calendar Magic Number $= BL \times TR - BR \times TL$
$= 15 \times 3 - 17 \times 1 = 45 - 17 = 28$

Question 7 (e) (i)

Top Left (*TL*) Top Right (*TR*)

a	$a+1$	$a+2$
$a+7$	$a+8$	$a+9$
$a+14$	$a+15$	$a+16$

Bottom Left (*BL*) Bottom Right (*BR*)

Question 7 (e) (ii)

Calendar Magic Number $= BL \times TR - BR \times TL$

$= (a+14) \times (a+2) - (a+16) \times a$

$= a^2 + 2a + 14a + 28 - a^2 - 16a$

$= 28$

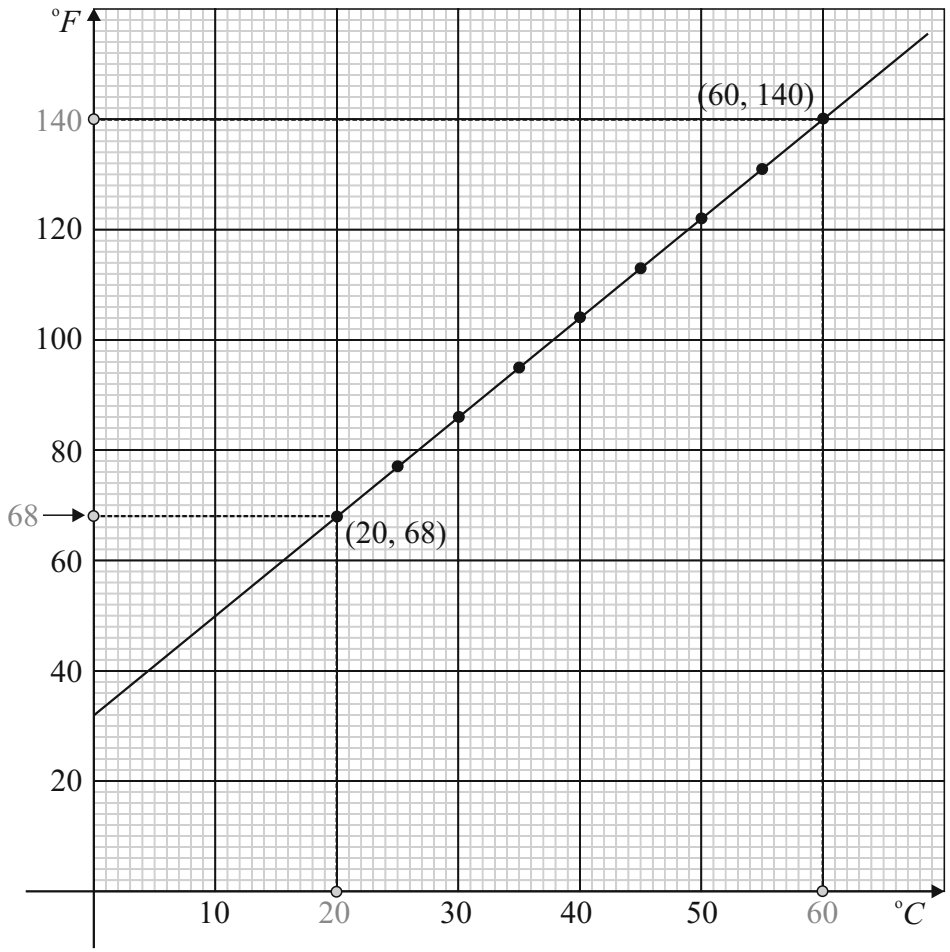
Question 7 (e) (iii)

All of the magic numbers are the same. The situation was generalised by allowing any number a to be the top left number. When the formula was applied in this general situation, 28 was produced. So it makes no difference what part of the page the 3 by 3 rectangle moves to.

Question 8 (50 marks)

Question 8 (a)

°C	20	25	30	35	40	45	50	55	60
°F	68	77	86	95	104	113	122	131	140



Question 8 (b)

Choose two points on the graph: (20, 68) and (60, 140)

$$m = \frac{140-68}{60-20} = \frac{9}{5} \qquad \boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Equation of line: Point(20, 68), $m = \frac{9}{5}$

$y - 68 = \frac{9}{5}(x - 20)$ $\quad \boxed{y - y_1 = m(x - x_1)}$

$F - 68 = \frac{9}{5}(C - 20)$

$F = \frac{9}{5}C - \frac{9}{5}(20) + 68$

$F = \frac{9}{5}C - 36 + 68$

$\therefore F = \frac{9}{5}C + 32$

Question 8 (c)

$F = \frac{9}{5}C + 32$

$F - 32 = \frac{9}{5}C$

$\therefore C = \frac{5}{9}(F - 32)$

Question 8 (d)

(i) $F = \frac{9}{5}C + 32$

$F = \frac{9}{5}(0) + 32 = 32^\circ\text{F}$

(ii) $F = \frac{9}{5}C + 32$

$F = \frac{9}{5}(100) + 32 = 180 + 32 = 212^\circ\text{F}$

(iii) $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(100 - 32) = \frac{5}{9}(68) \approx 38^\circ\text{C}$

(iv) $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(-22 - 32) = \frac{5}{9}(-54) = -30^\circ\text{C}$

Question 8 (e)

$F = \frac{9}{5}C + 32$

$C = \frac{5}{9}(F - 32)$

$F = C \Rightarrow \frac{9}{5}C + 32 = \frac{5}{9}(C - 32)$

$\frac{9}{5}C + 32 = \frac{5}{9}C - \frac{160}{9}$

$\frac{9}{5}C - \frac{5}{9}C = -\frac{160}{9} - 32$

$\frac{56}{45}C = -\frac{448}{9}$

$\therefore C = \frac{-448 \times 45}{9 \times 56} = -40^\circ\text{C}$

$\therefore -40^\circ\text{C} = -40^\circ\text{F}$

Question 8 (f) (i)

$h_1 = 3$ cm, $h_2 = 23$ cm, $h_3 = 17$ cm

$$C = 100\left(\frac{h_3 - h_1}{h_2 - h_1}\right) = 100\left(\frac{17-3}{23-3}\right) = 100\left(\frac{14}{20}\right) = 70^\circ\text{C}$$

Question 8 (f) (ii)

$C = \frac{5}{9}(F - 32)$

$F = 161^{\circ}\text{F}$: $C = \frac{5}{9}(161 - 32) = \frac{215}{3}^{\circ}\text{C}$

Error $= \frac{215}{3}^{\circ}\text{C} - 70^{\circ}\text{C} = \frac{5}{3}^{\circ}\text{C}$

$\%$ error $= \frac{\frac{5}{3}}{\frac{215}{3}} \times 100\% = 2.33\%$

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

$A = 12t^2 - \frac{2}{3}t^3$

$\frac{dA}{dt} = 24t - 2t^2$

$\left(\frac{dA}{dt}\right)_{t=1} = 24(1) - 2(1)^2$

$= 24 - 2 = 22 \text{ m}^2/\text{month}$

After one month, the area of the algae is increasing.

Question 9 (a) (ii)

$\left(\frac{dA}{dt}\right)_{t=14} = 24(14) - 2(14)^2$

$= 24 - 2 = -56 \text{ m}^2/\text{month}$

After 14 months, the area of the algae is decreasing.

Question 9 (b)

$\frac{dA}{dt} = 0 \Rightarrow 24t - 2t^2 = 0$

$12t - t^2 = 0$

$t(12 - t) = 0$

$\therefore t = 12$ months

Question 9 (c)

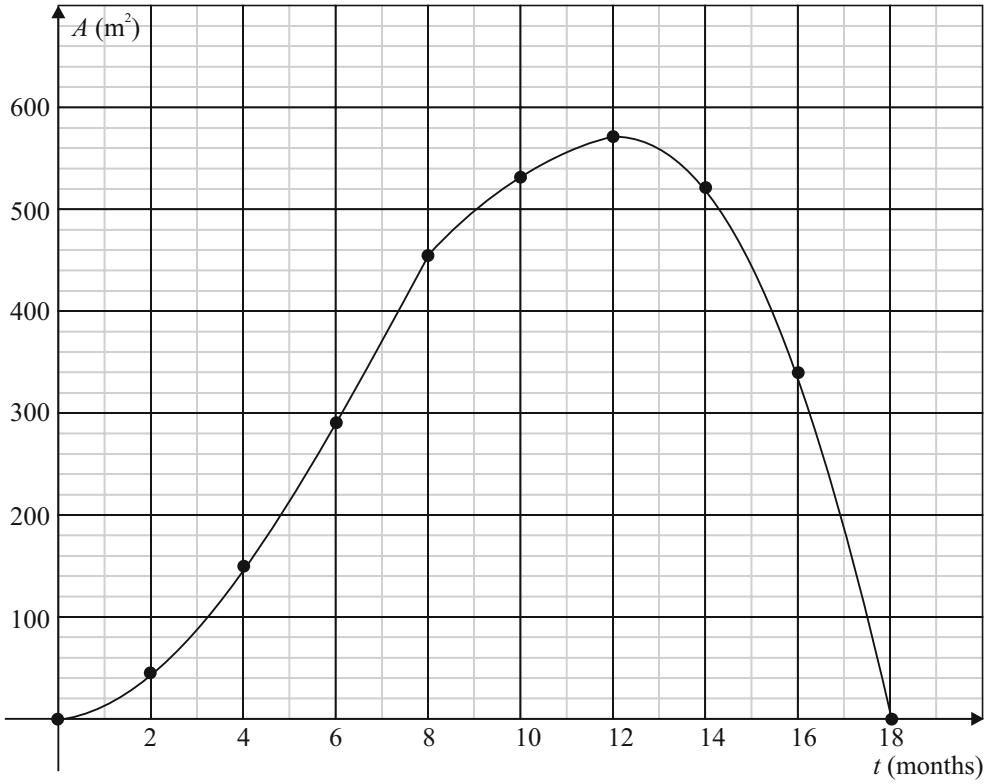
$A = 12t^2 - \frac{2}{3}t^3$

$t = 12$ months: $A_{\text{Max.}} = 12(12)^2 - \frac{2}{3}(12)^3 = 576 \text{ m}^2$

Question 9 (d)

t (Months)	0	2	4	6	8	10	12	14	16	18
A (m^2)	0	43	149	288	427	533	576	523	341	0

Question 9 (e)



Question 9 (f)

The algae grows slowly to a maximum area of 576 m^2 over the first 12 months, and then rapidly decreases to zero over the last six months.

Question 9 (g)

$A = 12t^2 - \frac{2}{3}t^3$

$t = 15$ months: $A = 12(15)^2 - \frac{2}{3}(15)^3 = 450$ m^2

$A = \pi r^2 \Rightarrow r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{450}{\pi}} \approx 12$ m

Sample Paper 3: Paper 2

Question 1 (25 marks)

Question 1 (a)

58	3	4	8						
59	0	0	2	3	7	8	8		
60	0	2	4	5	**7**	9			
61	1	3	3	5	6	6	8	8	9
62	0	1	3	4					

Key: 61|1 means 61.1 kg

There are 29 students. Arrange the weights in order. The median weight is now the weight of the middle student. This is the weight of the 15th. student.
Median weight = 60.7 kg

Question 1 (b)

Range $R = 62.4 - 58.3 = 4.1$ kg

58	**3**	4	8						
59	0	0	2	3	7	8	8		
60	0	2	4	5	7	9			
61	1	3	3	5	6	6	8	8	9
62	0	1	3	**4**					

Question 1 (c)

58	3	4	8						
59	0	0	2	**3**	**7**	8	8		
60	0	2	4	5	7	9			
61	1	3	3	5	6	**6**	**8**	8	9
62	0	1	3	4					

First quartile $Q_1 = \dfrac{59.3 + 59.7}{2} = 59.5$ kg

Third quartile $Q_3 = \dfrac{61.6 + 61.8}{2} = 61.7$ kg

Interquartile range = $Q_3 - Q_1 = 61.7$ kg $- 59.5$ kg $= 2.2$ kg

Question 1 (d)

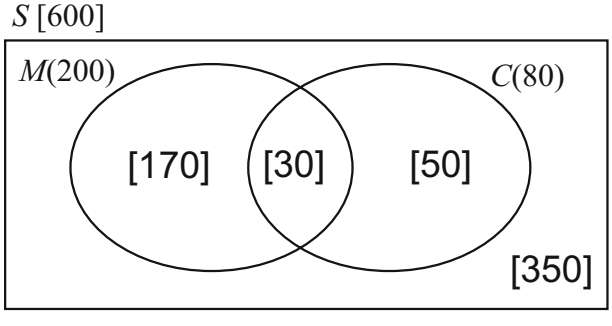
(i) The data is univariate as the only variable is weight.
(ii) The data is continuous as weight can be measured to any decimal place.

Question 2 (25 marks)

Question 2 (a)

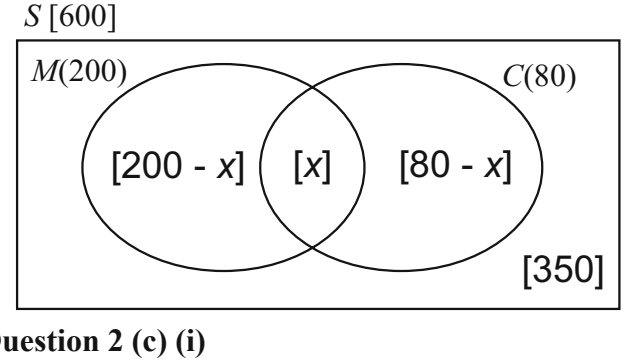
Number of students who neither sing nor play an instrument $= 600 \times \frac{7}{12} = 350$

Question 2 (b)



Total number of students = 600
Number of students who neither sing nor play an instrument = 350
Number of students who sing **or** play an instrument = 600 – 350 = 250
Number of students who play a musical instrument = 200
Number of students who sing in the choir = 80
Number of students who sing **and** play an instrument = 280 – 250 = 30

OR



$200 - x + x + 80 - x + 350 = 600$
$630 - x = 600$
$630 - 600 = x$
$\therefore x = 30$

Question 2 (c) (i)

P(Student sings in choir and plays an instrument)

$$= \frac{\text{Number of students who sing in choir and play an instrument}}{\text{Number of students}} = \frac{30}{600} = \frac{1}{20}$$

Question 2 (c) (ii)

P(Student does not sing)

$$= \frac{\text{Number of students who do not sing}}{\text{Number of students}} = \frac{600 - 80}{600} = \frac{520}{600} = \frac{13}{15}$$

Question 2 (c) (iii)

P(Student plays a musical instrument only)

$$= \frac{\text{Number of students who play an instrument only}}{\text{Number of students}} = \frac{170}{600} = \frac{17}{60}$$

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Question 3 (25 marks)

Question 3 (a) (i)

$A(-1, -1)$, $B(7, 3)$

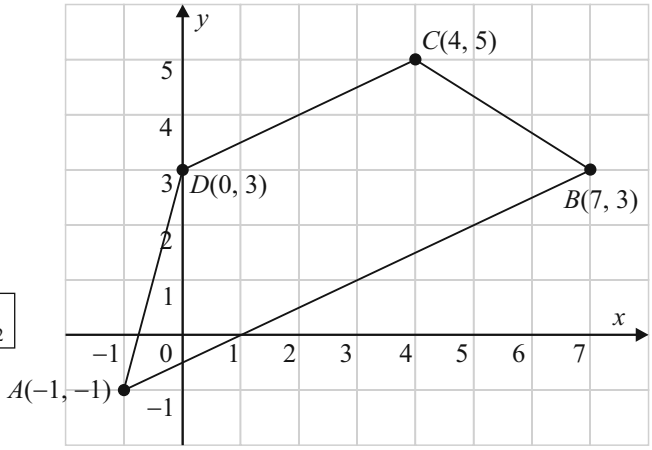
$$m_1 = \frac{3-(-1)}{7-(-1)} = \frac{4}{8} = \frac{1}{2}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$D(0, 3)$, $C(4, 5)$

$$m_2 = \frac{5-3}{4-0} = \frac{2}{4} = \frac{1}{2}$$

$m_1 = m_2 \Rightarrow AB \parallel DC$ Parallel lines: $m_1 = m_2$



Question 3 (a) (ii)

$A(-1, -1)$, $D(0, 3)$

$$|AD| = \sqrt{(0-(-1))^2 + (3-(-1))^2} = \sqrt{1+16} = \sqrt{17}$$

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$B(7, 3)$, $C(4, 5)$

$$|BC| = \sqrt{(4-7)^2 + (5-3)^2} = \sqrt{9+4} = \sqrt{13}$$

$$\therefore |AD| \neq |BC|$$

Question 3 (b)

Equation of line DB: $y = 3$

Equation of line AC:

Find slope of AC

$A(-1, -1)$, $C(4, 5)$ $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{5-(-1)}{4-(-1)} = \frac{6}{5}$$

$y - 5 = \frac{6}{5}(x-4)$ $\quad y - y_1 = m(x - x_1)$

$5y - 25 = 6x - 24$

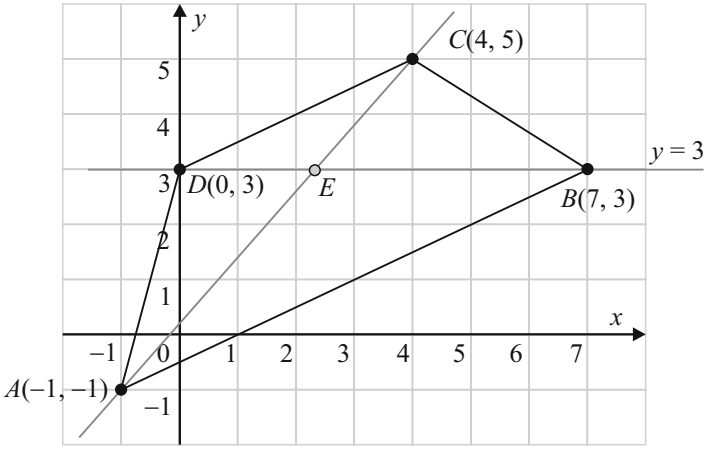
$0 = 6x - 5y + 1$

$AC \cap DB : 6x - 5y + 1 = 0 \cap y = 3$

$6x - 5(3) + 1 = 0$

$6x = 14 \Rightarrow x = \frac{7}{3}$

$\therefore E(\frac{7}{3}, 3)$



Sample 3 Paper 2

Question 3 (c)

Midpoint of DB:

$D(0, 3)$, $B(7, 3)$

$$\text{Midpoint} = \left(\frac{0+7}{2}, \frac{3+3}{2}\right) = (\tfrac{7}{2}, 3)$$

$$\boxed{\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Midpoint of AC:

$A(-1, -1)$, $C(4, 5)$

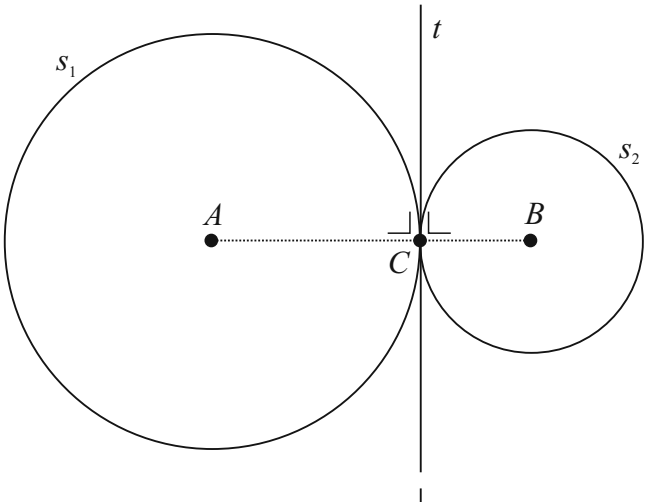
$$\text{Midpoint} = \left(\frac{-1+4}{2}, \frac{-1+5}{2}\right) = (\tfrac{3}{2}, 2)$$

The midpoint of each diagonal is not equal to E, the point where the diagonals intersect. Therefore, neither diagonal bisects the other.

QUESTION 4 (25 MARKS)

Question 4 (a)

The tangent t is perpendicular to the lines joining the point of contact C to the centres, A and B.
Therefore, ACB is a straight line and points A, B and C are collinear.

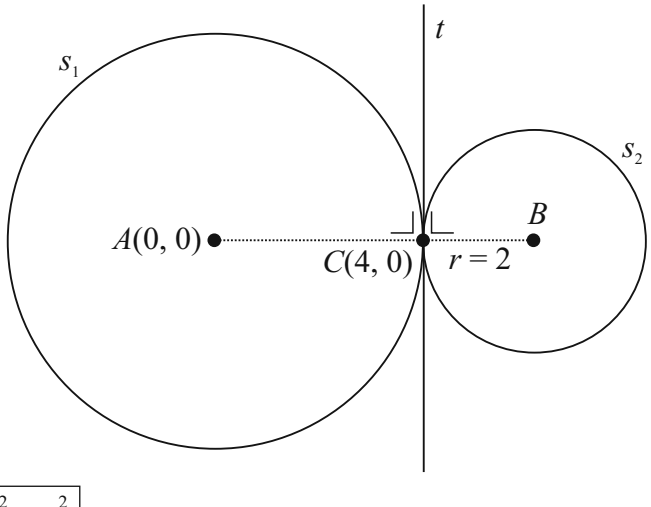


Question 4 (b) (i)

$$\text{Radius of } s_1 = |AC|$$
$$= \sqrt{(4-0)^2 + (0-0)^2}$$
$$= \sqrt{16} = 4$$

Or

The line AC lies along a horizontal line.
Radius of $s_1 = 4$



Question 4 (b) (ii)

Centre $B(6, 0)$

Question 4 (c)

Equation of s_1:
Centre $A(0, 0)$, $r = 4$

$$(x-0)^2 + (y-0)^2 = 4^2$$

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

$$s_1 : x^2 + y^2 = 16$$

Equation of s_2:
Centre $B(6, 0)$, $r = 2$

$$(x-6)^2 + (y-0)^2 = 2^2$$

$$s_2 : (x-6)^2 + y^2 = 4$$

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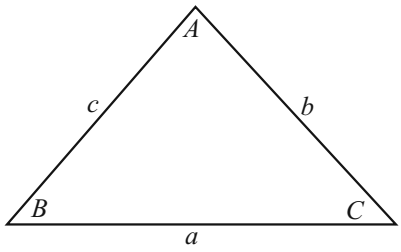
Question 5 (25 marks)

Question 5 (a)

Formula: $a^2 = b^2 + c^2 - 2bc\cos A$



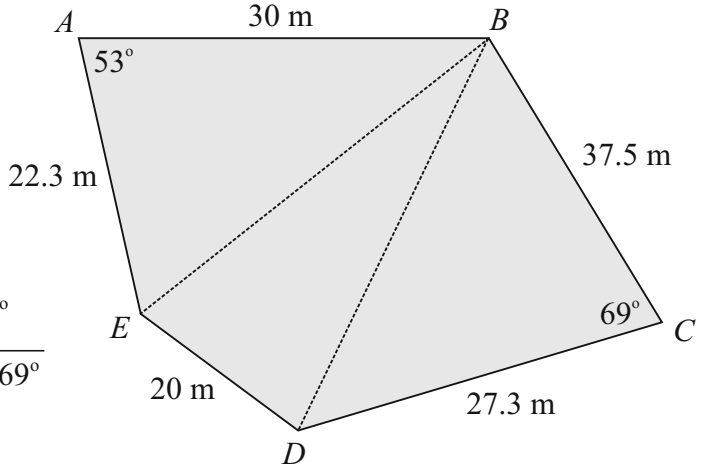
Question 5 (b) (i)

$a^2 = b^2 + c^2 - 2bc\cos A$

$|BE|^2 = 30^2 + 22.3^2 - 2(30)(22.3)\cos 53^\circ$

$\therefore |BE| = \sqrt{30^2 + 22.3^2 - 2(30)(22.3)\cos 53^\circ}$

$= 24.3 \text{ m}$



Question 5 (b) (ii)

$a^2 = b^2 + c^2 - 2bc\cos A$

$|BD|^2 = 37.5^2 + 27.3^2 - 2(37.5)(27.3)\cos 69^\circ$

$\therefore |BE| = \sqrt{37.5^2 + 27.3^2 - 2(37.5)(27.3)\cos 69^\circ}$

$= 37.7 \text{ m}$

Question 5 (b) (iii)

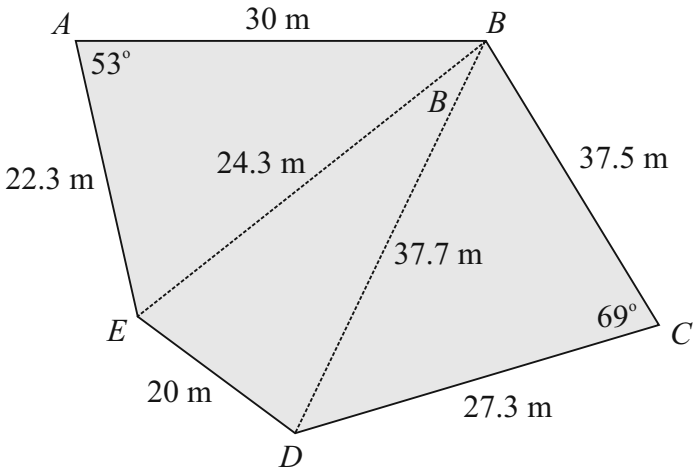
$a^2 = b^2 + c^2 - 2bc\cos A$

$20^2 = 24.3^2 + 37.7^2 - 2(24.3)(37.7)\cos B$

$2(24.3)(37.7)\cos B = 24.3^2 + 37.7^2 - 20^2$

$$\cos B = \frac{24.3^2 + 37.7^2 - 20^2}{2(24.3)(37.7)}$$

$$B = \cos^{-1}\left(\frac{24.3^2 + 37.7^2 - 20^2}{2(24.3)(37.7)}\right) = 28.4^\circ$$



Question 5 (b) (iv)

$$\boxed{\text{Area} = \tfrac{1}{2}ab\sin C}$$

Area of lake = Area of $|\Delta ABE|$ + Area of $|\Delta BED|$ + Area of $|\Delta BDC|$

$= \tfrac{1}{2}(30)(22.3)\sin 53^\circ + \tfrac{1}{2}(24.3)(37.7)\sin 28.4^\circ + \tfrac{1}{2}(27.3)(37.5)\sin 69^\circ$

$= 963 \text{ m}^2$

Question 6 (25 marks)

Question 6 (a)

If a theorem states that p implies q, then the converse of the theorem states that q implies p.
Converse: $q \Rightarrow p$

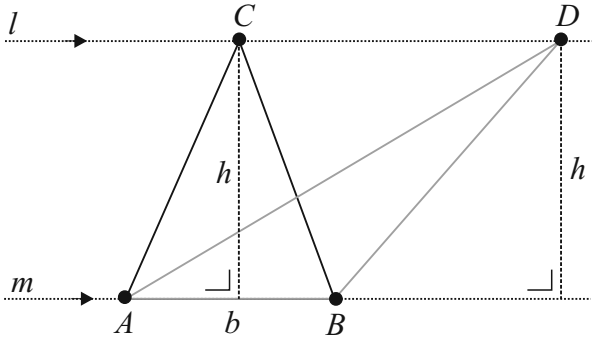
Question 6 (b)

Converse: 'If a number is divisible by 3, then it is also divisible by 6.'
The converse is false.
Example: 27 is divisible by 3 but not by 6.

Question 6 (c)

Converse: 'Triangles with equal areas are congruent.'
This statement is false.

Explain:



Draw two parallel lines l and m.
Draw a line $[AB]$ which forms the base of two triangles, ABC and ABD.
Each triangle drawn has the same base b and height h, and therefore, each triangle has the same area.
However, it is obvious that these triangles are **not** congruent.

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Question 7 (75 marks)

Question 7 (a)

Boys

Age	Number	Overweight (%)	Obese (%)
4	650	19	7
5	800	17	5
6	700	13	5
7	820	13	7
8	950	17	7
9	800	13	9
10	950	14	10
11	750	18	6
12	800	19	6
	7220		

Number of boys = 7220

Girls

Age	Number	Overweight (%)	Obese (%)
4	600	22	7
5	800	22	7
6	900	22	7
7	650	19	11
8	730	22	8
9	**900**	**22**	**9**
10	800	20	8
11	940	19	6
12	760	19	6
	7080		

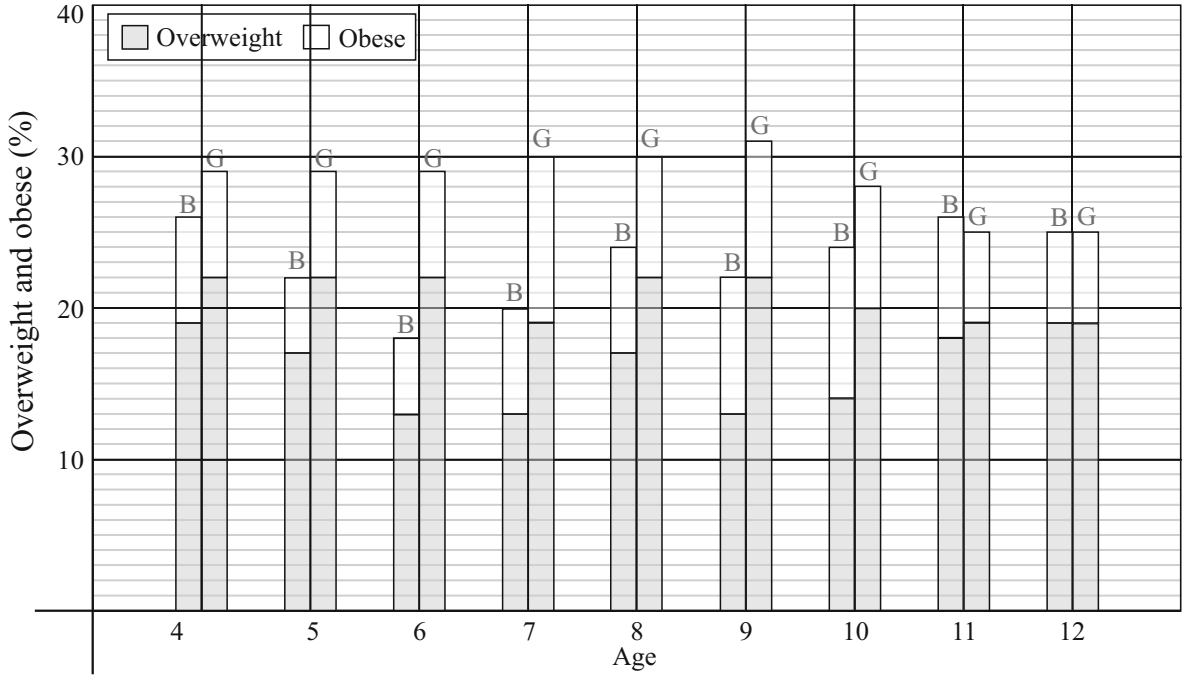
Number of girls = 7080

Number of children = 7220 + 7080 = 14 300

Question 7 (b)

Girls aged nine years have the highest percentage of being overweight and obese.
Percentage of being obese and overweight = 22% + 9% = 31%

Question 7 (c)



Girls on average are more obese than boys. Girls seem to get less overweight as they get older.

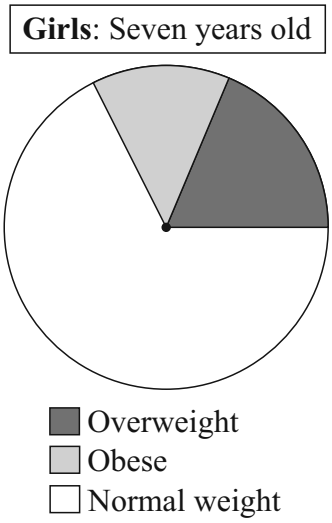
Question 7 (d) (i)

Seven-year-old girls:

Overweight 19% $\rightarrow \frac{19}{100} \times 360^{\circ} \approx 68^{\circ}$

Obese 11% $\rightarrow \frac{11}{100} \times 360^{\circ} \approx 40^{\circ}$

Normal weight 70% $\rightarrow \frac{70}{100} \times 360^{\circ} = 252^{\circ}$



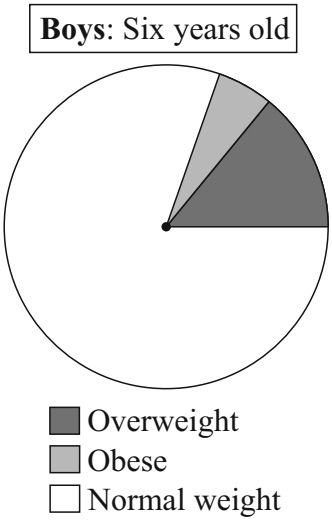
Question 7 (d) (ii)

Six-year-old boys:

Overweight 13% $\rightarrow \frac{13}{100} \times 360^{\circ} \approx 47^{\circ}$

Obese 5% $\rightarrow \frac{5}{100} \times 360^{\circ} = 18^{\circ}$

Normal weight 82% $\rightarrow \frac{82}{100} \times 360^{\circ} \approx 295^{\circ}$



Question 7 (e) (i)

Number of boys who are overweight or obese:

$650 \times 0.26 + 800 \times 0.22 + 7000 \times 0.18 + 820 \times 0.2 + 950 \times 0.24$
$+ 800 \times 0.22 + 950 \times 0.24 + 750 \times 0.24 + 800 \times 0.25$
$= 1647$

Question 7 (e) (ii)

Number who will be overweight as an adult $= 1647 \times 0.3 = 494.1$

Percentage of boys who will be overweight as adults $= \frac{494.1}{7220} \times 100\% = 6.8\%$

Question 7 (f)

You can work this out by counting how many times you need to add 10,000 on to 300,000 to reach 500,000 and you get 20 years.

Or

$$\text{Number of years} = \frac{500\,000 - 300\,000}{10\,000} = \frac{200\,000}{10\,000} = 20$$

Question 7 (g) (i)

$\text{Height} = 150 \text{ cm} = 1.5 \text{ m}$

$\text{Weight} = 37 \text{ kg}$

$\text{BMI} = \frac{37}{1.5^2} = 16.4 \text{ kg/m}^2$

Question 7 (g) (ii)

$\text{Height} = 1.78 \text{ m}$

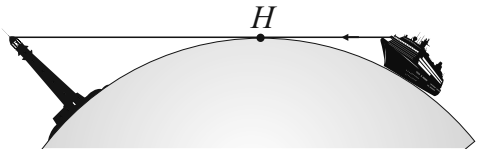
$\text{BMI} = 35 \text{ kg/m}^2$

$35 = \frac{\text{Weight}}{1.78^2} \Rightarrow \text{Weight} = 35 \times 1.78^2 = 110.9 \text{ kg}$

His weight must be less that 111 kg.

Question 8 (75 marks)

Question 8 (a) (i)



Question 8 (a) (ii)

The first part of the lighthouse that Julie sees is the top of the lighthouse, because the other points are below the horizon.

Question 8 (b) (i)

$d = 3.57\sqrt{h}$

$\therefore d = 3.57\sqrt{1.7} = 4.65 \text{ km}$

Question 8 (b) (ii)

$d = 3.57\sqrt{h}$

$\therefore d = 3.57\sqrt{20} = 15.97 \text{ km}$

Question 8 (b) (iii)

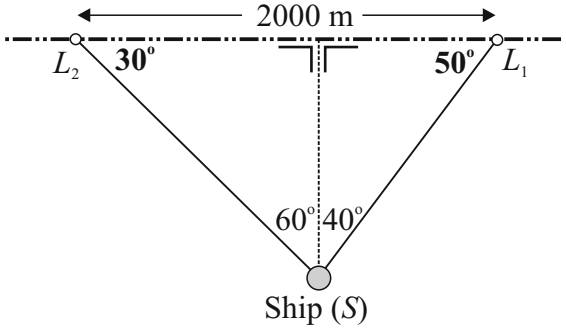
$d = 3.57\sqrt{h}$

$\therefore d = 3.57\sqrt{35} = 21.12 \text{ km}$

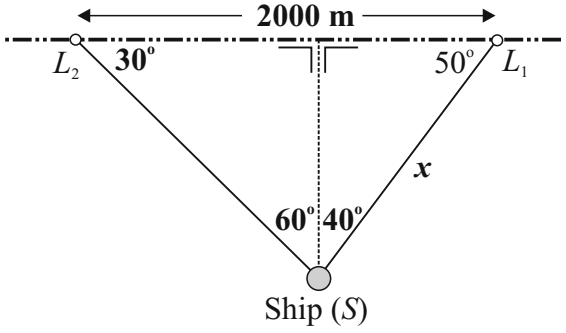
Question 8 (b) (iv)

Distance of ship to Paradise Island
$= 15.97 \text{ km} + 21.12 \text{ km} = 37.09 \text{ km} \approx 37 \text{ km}$

Question 8 (c) (i)



Question 8 (c) (ii)

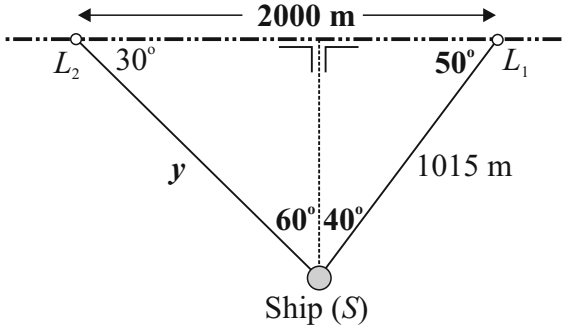


$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B}}$$

$$\frac{x}{\sin 30^\circ} = \frac{2000}{\sin 100^\circ}$$

$$x = \frac{2000 \sin 30^\circ}{\sin 100^\circ} \approx 1015 \text{ m}$$

Question 8 (c) (iii)

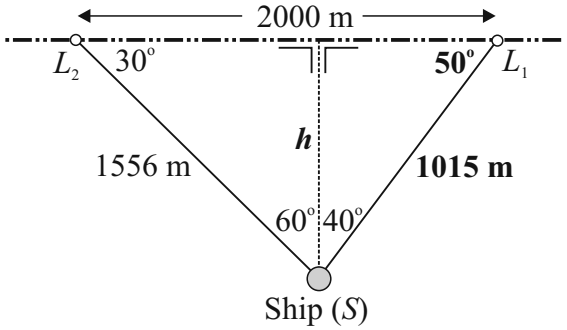


$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B}}$$

$$\frac{y}{\sin 50^\circ} = \frac{2000}{\sin 100^\circ}$$

$$y = \frac{2000 \sin 50^\circ}{\sin 100^\circ} \approx 1556 \text{ m}$$

Question 8 (c) (iv)



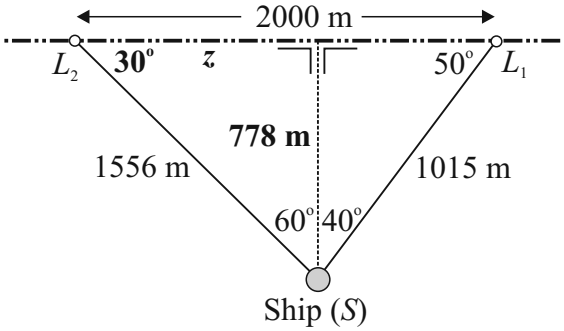
$$\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin 50^\circ = \frac{h}{1015}$$

$$\therefore h = 1015 \sin 50^\circ \approx 778 \text{ m}$$

Yes, it is further than 500 m from the shore.

Question 8 (d) (i)



$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan 30^\circ = \frac{778}{z}$$

$$\therefore z = \frac{778}{\tan 30^\circ} = 1347.5 \text{ m} = 1.3475 \text{ km}$$

Speed = 20 km/h

Distance = 1.3475 km

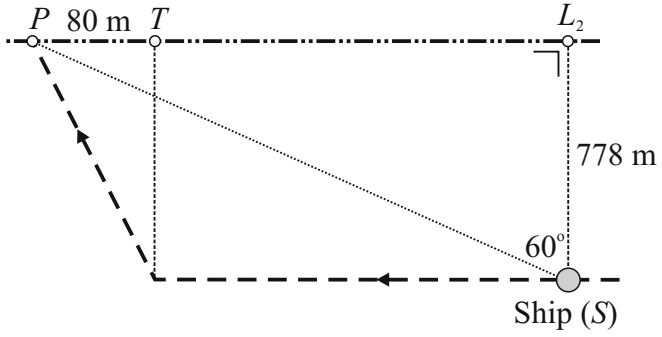
Time = t = ?

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$20 = \frac{1.3475}{t} \Rightarrow t = \frac{1.3475}{20} \times 60 \approx 4 \text{ minutes}$$

The ship is directly opposite the second lighthouse at 12.04 pm.

Question 8 (d) (ii)

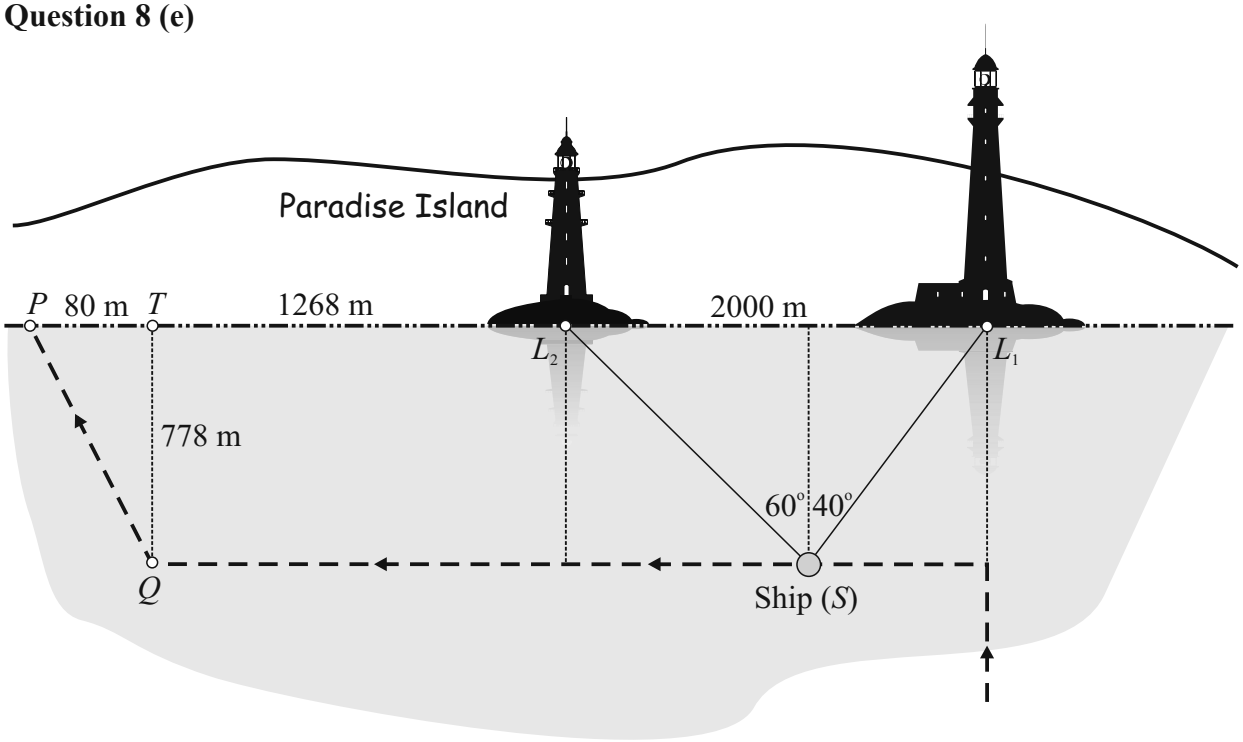


$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan 60^\circ = \frac{|PL_2|}{778}$$

$$\therefore |PL_2| = 778 \tan 60^\circ \approx 1348 \text{ m}$$

Question 8 (e)



$|TL_2| = 1348 - 80 = 1268$ m

$|PQ|^2 = 80^2 + 778^2$

$\therefore |PQ| = \sqrt{80^2 + 778^2} \approx 782$ m

Total distance travelled by ship parallel to coast and into the port = 2000 + 1268 + 782 = 4050 m

Sample Paper 4: Paper 1

Question 1 (25 marks)

Question 1 (a) (i)

$p = -3,\ q = -5$

$p + q = -3 - 5 = -8$

$pq = (-3)(-5) = 15$

Question 1 (a) (ii)

Roots: $x = p,\ x = q$

$\therefore (x - p)(x - q) = 0$

$x^2 - px - qx + pq = 0$

$x^2 - (p + q)x + pq = 0$

$x^2 - (-8)x + 15 = 0$

$\therefore x^2 + 8x + 15 = 0$

Question 1 (b)

$l : 3x + y = 25 \Rightarrow y = 25 - 3x$

$s : x^2 + y^2 = 65$

$\therefore x^2 + (25 - 3x)^2 = 65$

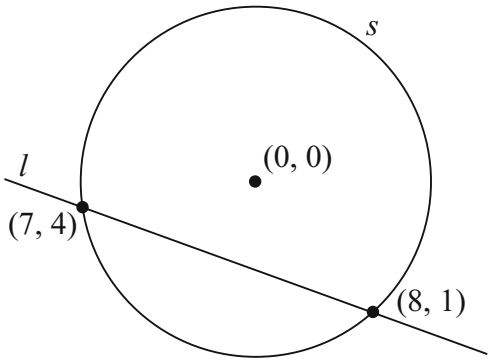
$x^2 + 625 - 150x + 9x^2 - 65 = 0$

$10x^2 - 150x + 560 = 0$

$x^2 - 15x + 56 = 0$

$(x - 7)(x - 8) = 0$

$\therefore x = 7,\ 8$



$x = 7 : y = 25 - 3(7) = 25 - 21 = 4$

$x = 8 : y = 25 - 3(8) = 25 - 24 = 1$

$\therefore (7,\ 4)$ and $(8,\ 1)$ are the points of intersection.

The line cuts the circle at two points whose centre is (0, 0). There the line l is a chord.

Question 2 (25 marks)

Question 2 (a)

$$\begin{aligned} & 3+2i(5+8i)-11i \\ &= 3+10i+16i^2-11i \\ &= 3+10i-16-11i \\ &= -13-i \end{aligned}$$

Question 2 (b) (i)

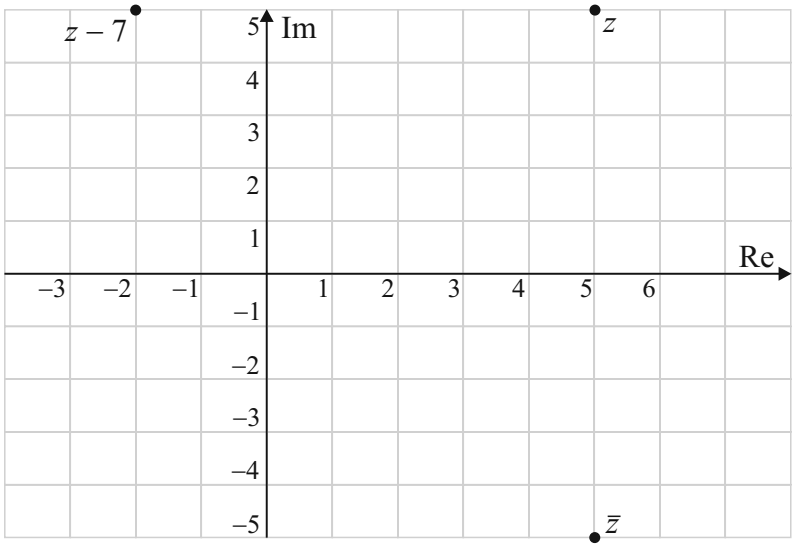
$$\begin{aligned} z &= (1+3i)(-i+2) \\ &= -i+2-3i^2+6i \\ &= -i+2-3(-1)+6i \\ &= -i+2+3+6i \\ &= 5+5i \end{aligned}$$

Question 2 (b) (ii)

$$\begin{aligned} z &= 5+5i \\ z-7 &= 5+5i-7 \\ &= -2+5i \end{aligned}$$

Question 2 (b) (iii)

$z = 5+5i \Rightarrow \overline{z} = 5-5i$



Question 2 (c)

$z = \dfrac{2+4i}{1-i}$ [Multiply above and below by the conjugate of the denominator.]

$z = a+ib \Rightarrow \overline{z} = a-ib$

$$\begin{aligned} &= \frac{(2+4i)}{(1-i)} \times \frac{(1+i)}{(1+i)} = \frac{2+2i+4i+4i^2}{1+i-i-i^2} \\ &= \frac{2+2i+4i-4}{1+i-i+1} = \frac{-2+6i}{2} \\ &= -1+3i \end{aligned}$$

$z = -1+3i$

$|z| = \sqrt{(-1)^2+3^3} = \sqrt{10}$

$\overline{z} = -1-3i$

$|a+ib| = \sqrt{a^2+b^2}$

$$\begin{aligned} z\overline{z} &= |z|^2 \\ (-1+3i)(-1-3i) &= (\sqrt{10})^2 \\ 1+3i-3i-9i^2 &= 10 \\ 1+9 &= 10 \end{aligned}$$

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Question 3 (25 marks)

Question 3 (a)

Arithmetic sequence: $-11,\ x,\ 5$

$\therefore x-(-11)=5-x$

$x+11=5-x$

$2x=-6$

$\therefore x=-3$

$-11,\ -3,\ 5$

$a=-11,\ d=-3-(-11)=8$ $\quad \boxed{T_n=a+(n-1)d}$

$T_n=-11+(n-1)8$

$=-11+8n-8$

$=8n-19$

Question 3 (b)

$2,\ 4,\ 6,\ 8,\ 10,\ 12,.....100$

$a=2,\ d=2$

$T_n=100=2+(n-1)2$

$\therefore 100=2+2n-2$

$100=2n$

$\therefore n=50$

Question 3 (c)

$2,\ 4,\ 6,\ 8,\ 10,\ 12,....$ $\quad \boxed{S_n=\frac{n}{2}[2a+(n-1)d]}$

$a=2,\ d=2,\ n=100$

$S_{100}=\frac{100}{2}[2(2)+(100-1)2]$

$=50[4+99(2)]$

$=50[202]$

$=10\ 100$

Question 4 (25 marks)

Question 4 (a)

Gross amount at the end of 5 years = €10 000 × 1.01^5 = €10 510.10
Interest earned = €510.10
Tax is payable on the interest at 30%, which means you get to keep 70% of the interest.
Amount of interest retained = €510.10 × 0.7 = €357.07
Amount payable after 5 years = €10 357.07

Question 4 (b)

Gross amount at the end of 5 years = €10 000 × 1.01^5 = €10 510.10

Bonus payment = €10 510.10 × 0.1 = €1051.01

Total interest earned = €510.10 + €1051.01 = €1561.11

Amount of interest retained = €1561·11×0·7 = €1092·78

Amount payable after 5 years = €11 092.78

Question 5 (25 marks)

Question 5 (a)

$f(x) = 3x^2 - 7x$

$f'(x) = 6x - 7$

$f'(3) = 6(3) - 7 = 18 - 7 = 11$

Question 5 (b) (i)

$y = ax^2 + bx$

$(3,\ 4) \in y = ax^2 + bx \Rightarrow 4 = a(3)^2 + b(3)$

$4 = 9a + 3b....(\mathbf{1})$

$y = ax^2 + bx$

$\frac{dy}{dx} = 2ax + b$

$\left(\frac{dy}{dx}\right)_{x=1} = 0 \Rightarrow 2a(1) + b = 0$

$2a + b = 0....(\mathbf{2})$

Solve equations (**1**) and (**2**) simultaneously to find a and b (see right).

$a = \frac{4}{3},\ b = -\frac{8}{3}$

$y = \frac{4}{3}x^2 - \frac{8}{3}x$

$9a + 3b = 4....(\mathbf{1})$

$2a + b = 0.....(\mathbf{2})(\times -3)$

$$\begin{array}{r} 9a + 3b = 4 \\ -6a - 3b = 0 \\ \hline 3a \qquad = 4 \end{array} \Rightarrow a = \frac{4}{3}$$

$2a + b = 0.....(\mathbf{2})$

$2(\frac{4}{3}) + b = 0$

$\frac{8}{3} + b = 0$

$\therefore b = -\frac{8}{3}$

Question 5 (b) (ii)

As the tangent is parallel to the x-axis, its slope is zero. Turning points are points whose tangents have zero slope.

Local minimum at $x = 1$

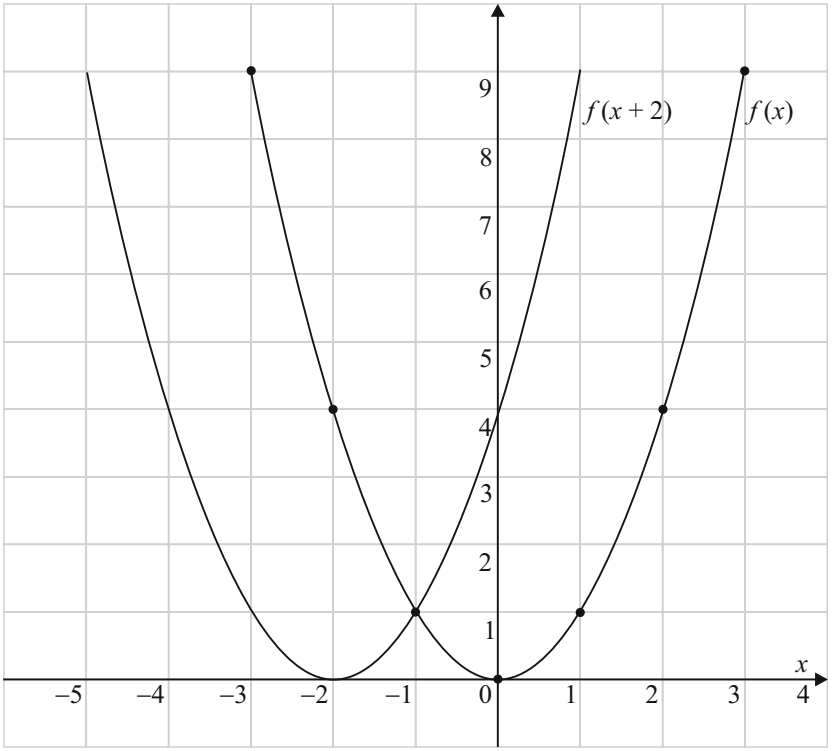
$y = \frac{4}{3}x^2 - \frac{8}{3}x = \frac{4}{3}(1)^2 - \frac{8}{3}(1) = \frac{4}{3} - \frac{8}{3} = -\frac{4}{3}$

Local minimum: $(1, -\frac{4}{3})$

Question 6 (25 marks)

Question 6 (a)

To draw the curve $f(x+2)$ given the curve $f(x)$, move the $f(x)$ curve 2 units to the left along the x-axis.



Question 6 (b)

$$f(x) = f(x+2)$$
$$x^2 = (x+2)^2$$
$$x^2 = x^2 + 4x + 4$$
$$0 = 4x + 4$$
$$-4 = 4x$$
$$\therefore x = -1$$

Question 6 (c)

$$y = (x+2)^2 = x^2 + 4x + 4 \qquad y = x^2$$
$$\frac{dy}{dx} = 2x + 4 \qquad \frac{dy}{dx} = 2x$$
$$\left(\frac{dy}{dx}\right)_{x=-1} = 2(-1) + 4 = 2 = m_1 \qquad \left(\frac{dy}{dx}\right)_{x=-1} = 2(-1) = -2 = m_2$$
$$m_1 + m_2 = 2 - 2 = 0$$

Question 7 (50 marks)

Question 7 (a)

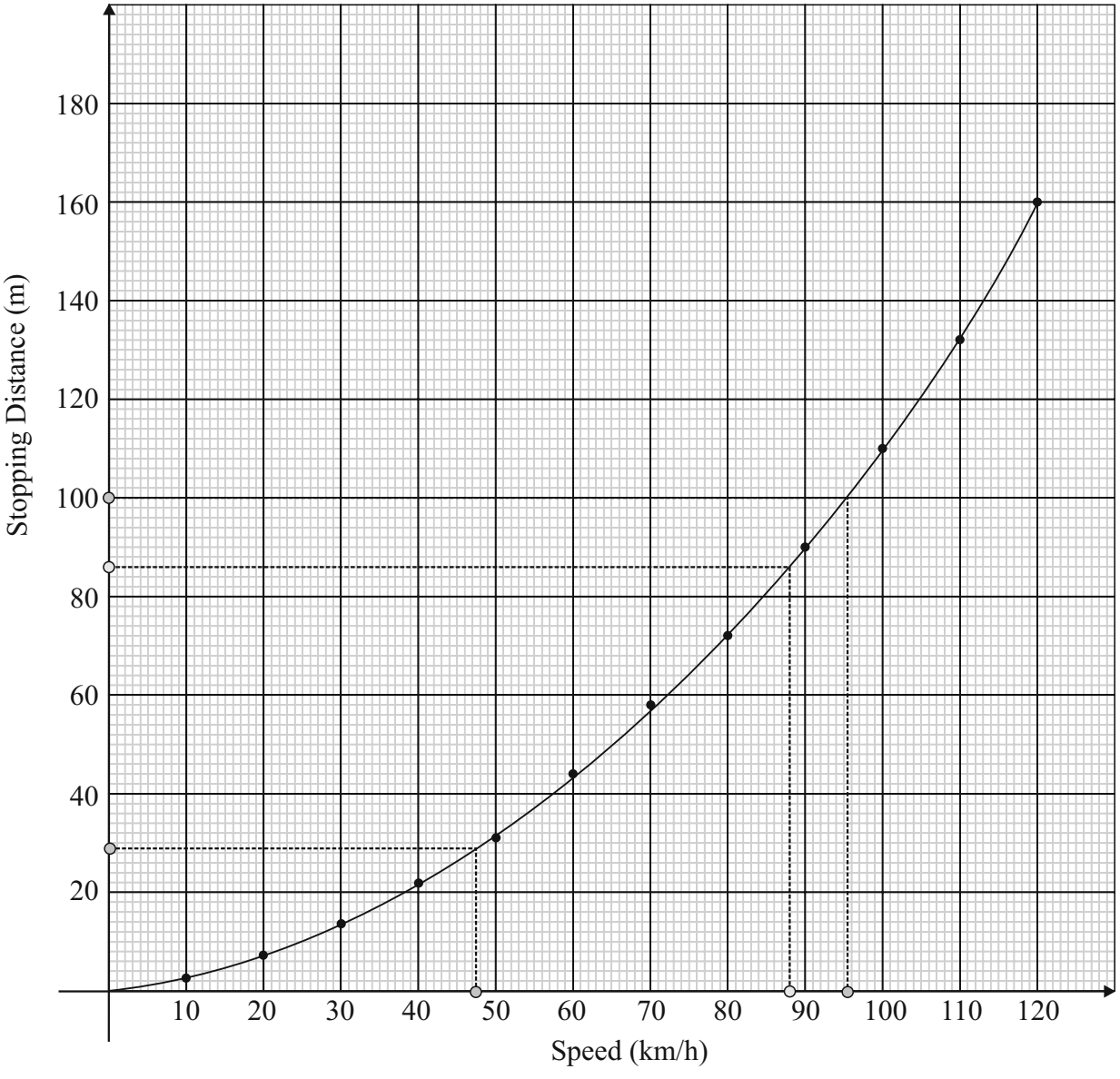
1 mile = 1.6 km

1 mph = 1.6 km/h

55 mph = 55×1.6 km/h = 88 km/h

Go to 88 km/h on the horizontal axis. Draw a line straight up to the graph and then straight across to the vertical axis. Read off the distance.

Answer: 86 m



Question 7 (b)

Go to 100 m on the vertical axis. Its corresponding speed is 95 km/h.
Half of this speed is 47.5 km/h.
Go to 47.5 km/h on the horizontal axis. Its corresponding distance is 29 m.

Question 7 (c)

$y = 0.014x^2 + 0.065x$

$x = 20 : y = 0.014(20)^2 + 0.065(20) = 6.9 \approx 7$ m

$x = 30 : y = 0.014(30)^2 + 0.065(30) = 14.55 \approx 15$ m

$x = 50 : y = 0.014(50)^2 + 0.065(50) = 38.25 \approx 38$ m

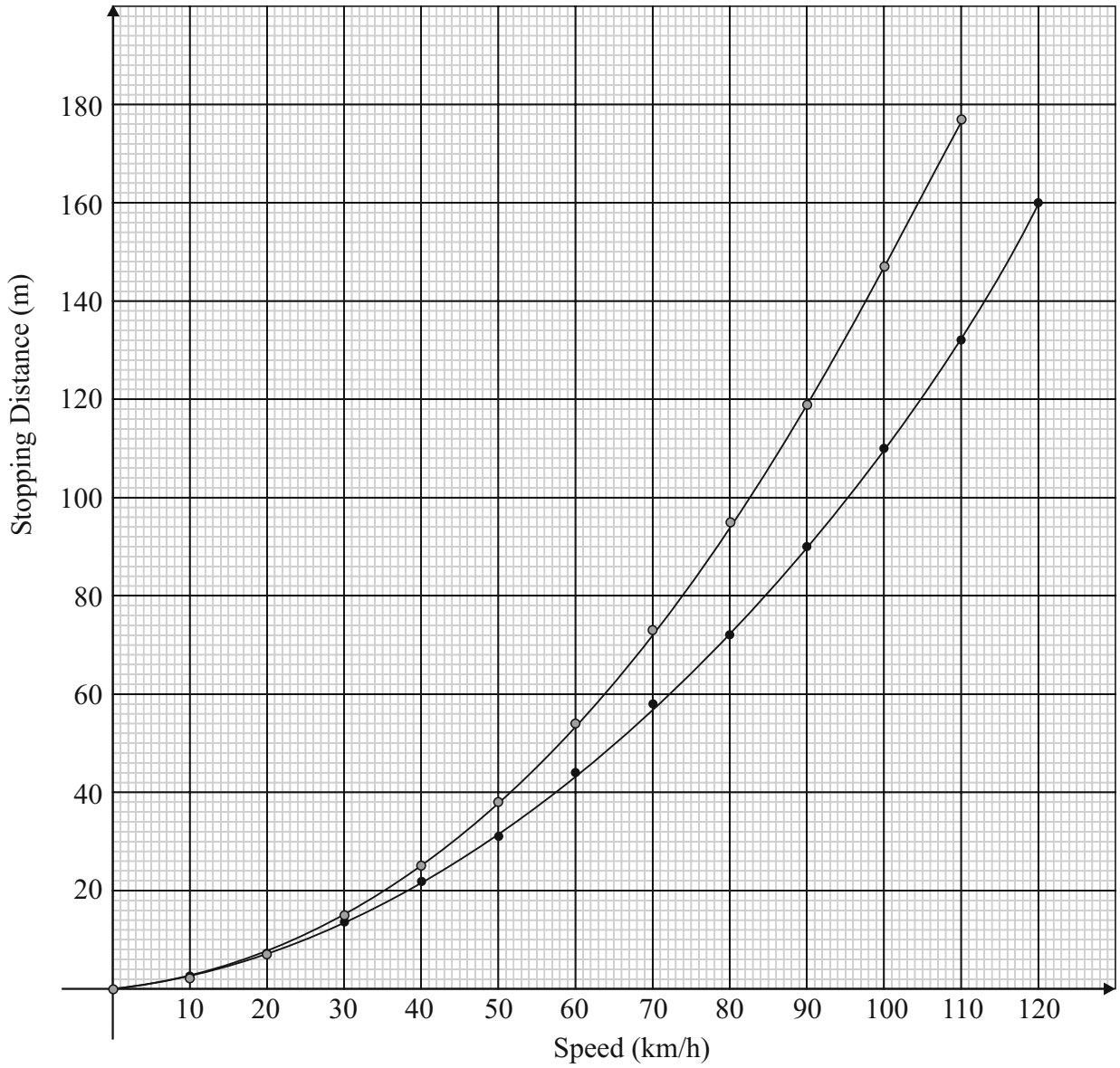
$x = 60 : y = 0.014(60)^2 + 0.065(60) = 54.3 \approx 54$ m

$x = 70 : y = 0.014(70)^2 + 0.065(70) = 73.15 \approx 73$ m

$x = 100 : y = 0.014(100)^2 + 0.065(100) = 146.5 \approx 147$ m

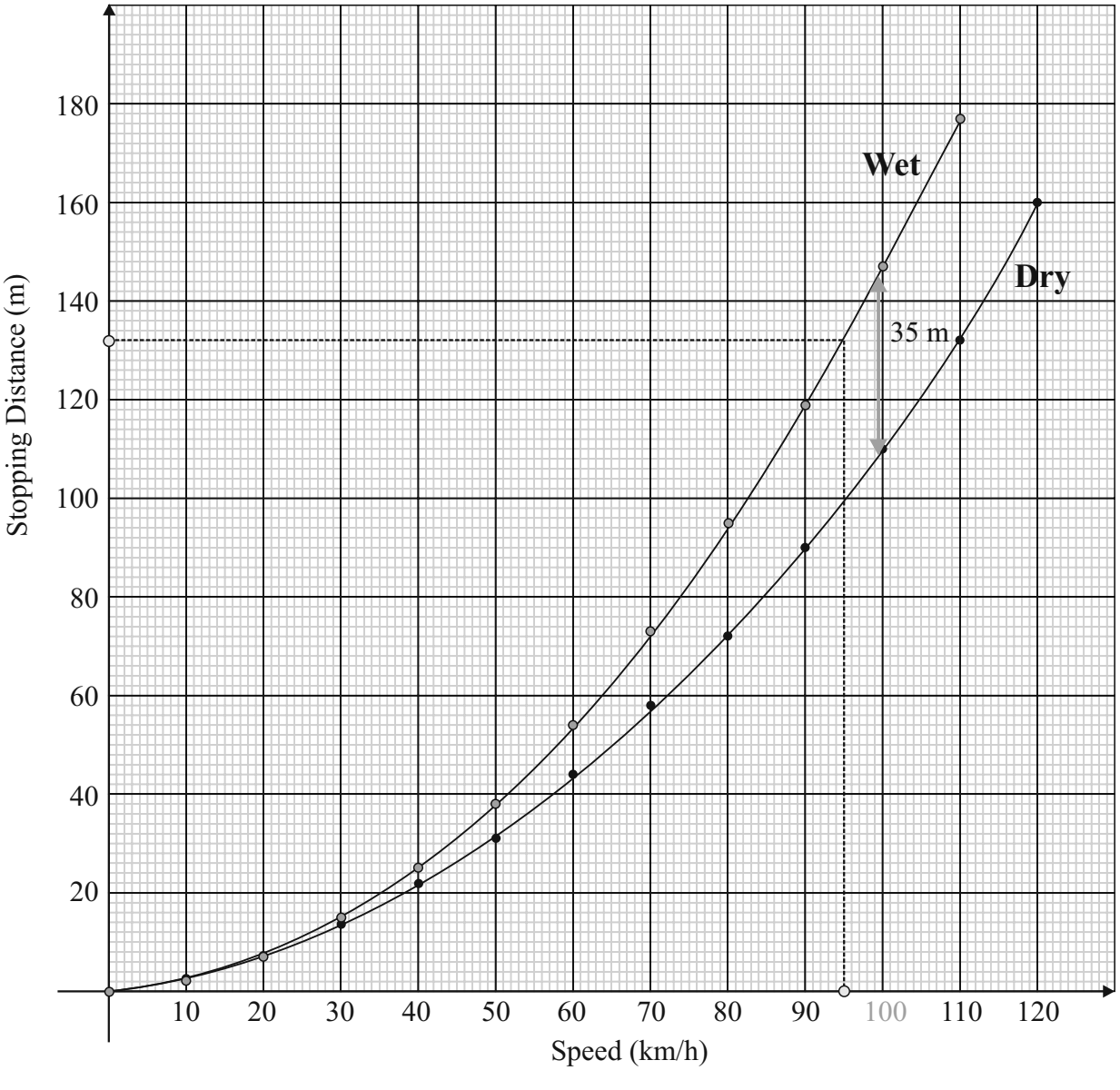
x	0	10	20	30	40	50	60	70	80	90	100	110
y	0	2	7	15	25	38	54	73	95	119	147	177

Question 7 (d)



Question 7 (e)

Go to 132 m on the vertical axis. Draw a line straight across until this line meets the wet graph. The corresponding value on the horizontal axis is 95 km/h.



Question 7 (f)

Find out where there is a vertical gap of 35 m between the wet road and dry road graphs. This occurs approximately at a speed of 100 km/h.

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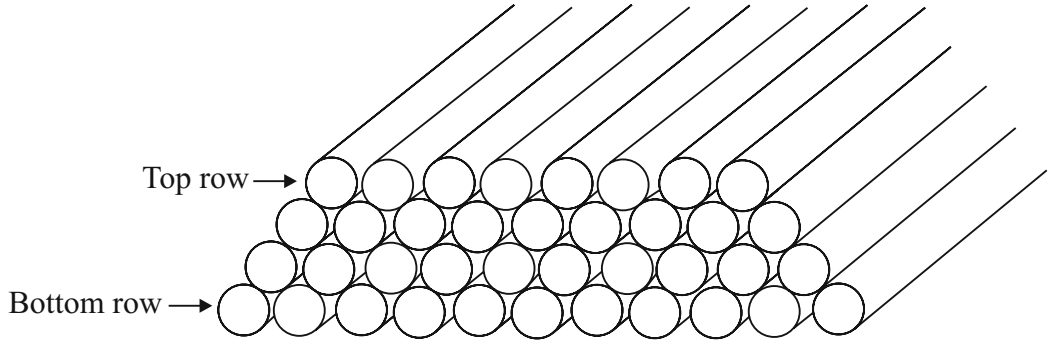
QUESTION 8 (50 MARKS)

Question 8 (a)



$$N = \frac{(b+t)(b-t+1)}{2}$$

$b = 20,\ t = 12$

$$\therefore N = \frac{(20+12)(20-12+1)}{2} = \frac{(32)(9)}{2} = 144$$

Question 8 (b)

$$N = \frac{(b+t)(b-t+1)}{2}$$

$b = 37,\ N = 598,\ t = ?$

$$598 = \frac{(37+t)(37-t+1)}{2}$$

$$1196 = (37+t)(38-t)$$

$$1196 = 1406 - 37t + 38t - t^2$$

$$t^2 - t - 210 = 0$$

$$(t-15)(t+14) = 0$$

$$\therefore t = 15$$

3	210
7	70
5	10
2	2
	1

Percentage of pipes on the top $= \frac{15}{598} \times 100\% \approx 2.5\%$

Question 8 (c) (i)

Cylinder: $V = \pi r^2 h$

$r = 3\,\text{cm},\ h = 30\ \text{cm},\ t = 15$

$V = 15 \times \pi \times 3^2 \times 30 = 4050\pi\ \text{cm}^3$

Question 8 (c) (ii)

$r = 3\,\text{cm},\ h = 30\ \text{cm},\ b = 37$

$V = 37 \times \pi \times 3^2 \times 30 = 9990\pi\ \text{cm}^3$

Question 8 (c) (iii)

Number of pipes in other rows $n = N - t - b = 598 - 15 - 37 = 546$

$r = 3\,\text{cm},\ h = 30\ \text{cm},\ n = 546$

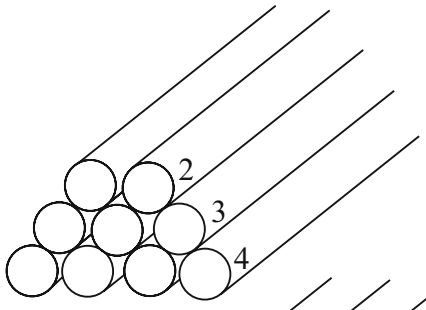
$V = 546 \times \pi \times 3^2 \times 30 = 147\ 420\pi\ \text{cm}^3$

Question 8 (d) (i)

$$N = \frac{(b+t)(b-t+1)}{2}$$

$b = 2t$

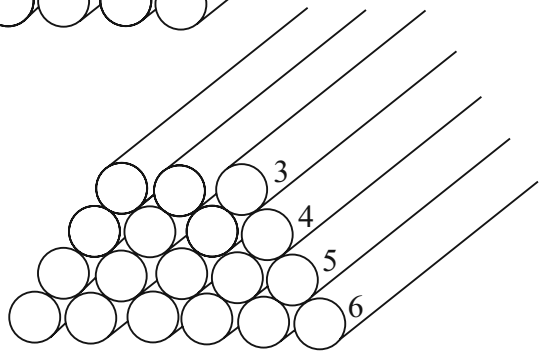
$$N = \frac{(2t+t)(2t-t+1)}{2} = \frac{(3t)(t+1)}{2}$$



Question 8 (d) (ii)

$t = 2$

$$N = \frac{(3t)(t+1)}{2} = \frac{(3(2))(2+1)}{2} = \frac{(6)(3)}{2} = 9$$



Question 8 (d) (iii)

$t = 3$

$$N = \frac{(3t)(t+1)}{2} = \frac{(3(3))(3+1)}{2} = \frac{(9)(4)}{2} = 18$$

Question 8 (e)

$t = 10,\ b = 20$

Total number of pipes $= 10+11+12+13+14+15+16+17+18+19+20$

$a = 10,\ d = 1,\ n = 11$

$S_{11} = \frac{11}{2}[2(10)+(11-1)1] = \frac{11}{2}[20+10] = \frac{11}{2}[30] = 165$ $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$

Question 8 (f)

Use the result in **8 (d) (i)**:

$$b = 2t \Rightarrow N = \frac{(3t)(t+1)}{2}$$

$$\therefore 513 = \frac{(3t)(t+1)}{2}$$

$1026 = 3t^2 + 3t$

$0 = t^2 + t - 342$

$0 = (t+19)(t-18)$

$\therefore t = 18$

2	342
3	171
3	57
19	19
	1

Yes, this is possible. There are 18 pipes in the top row and 36 in the bottom row. The total number of pipes will be 513.

Question 9 (50 marks)

Question 9 (a)

$S = 2200 - 2p \geq 0$ $\qquad$ $D = 4p - 320 \geq 0$

$2200 \geq 2p$ $\qquad$ $4p \geq 320$

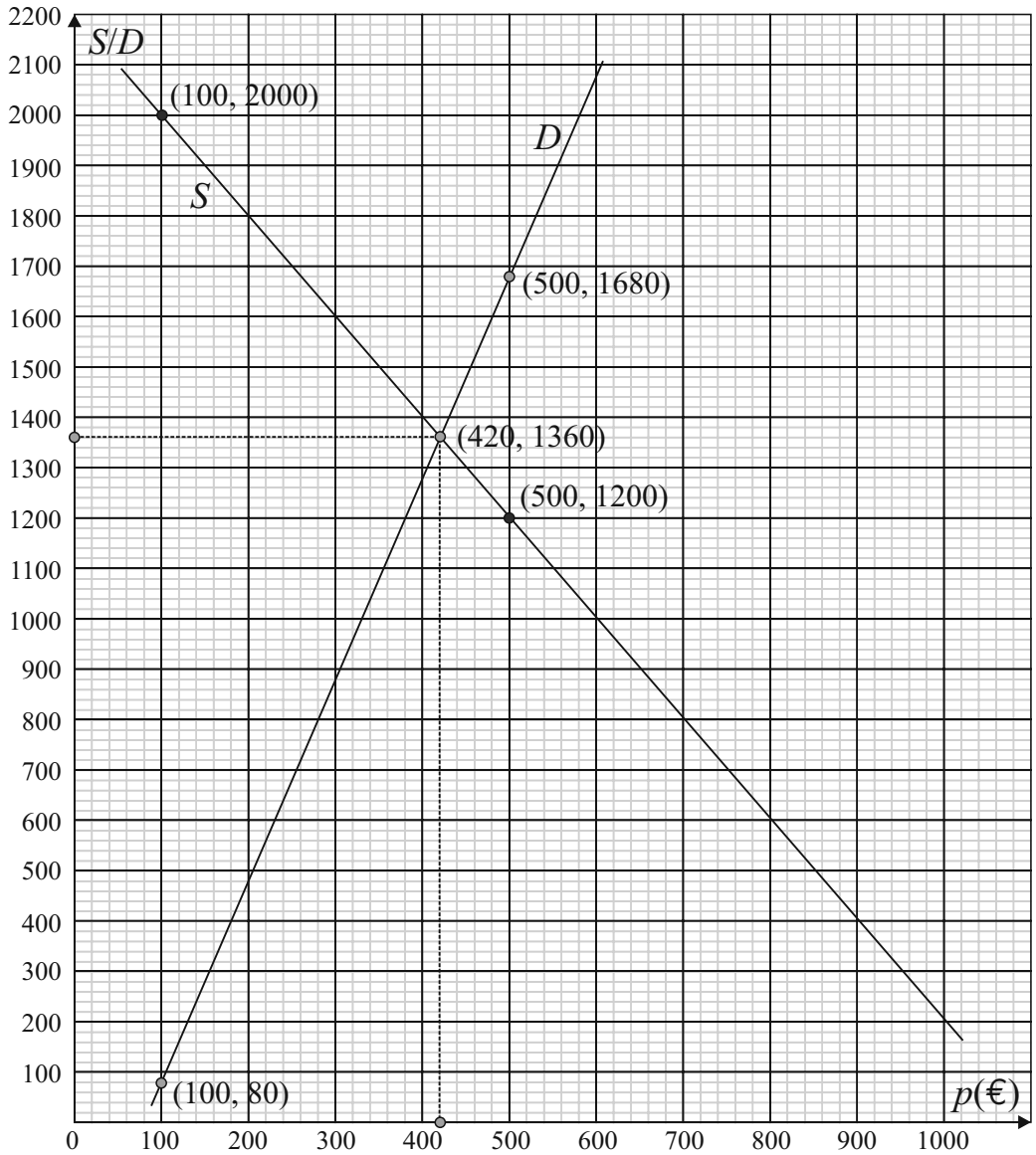
$1100 \geq p$ $\qquad$ $p \geq 80$

$\therefore p \leq 1100$

Price range: €$80 \leq p \leq$ €1100

Question 9 (b)

p (€)	100	500
S	2000	1200
D	80	1680



Question 9 (c)

ALGEBRA

$S = D$

$2200 - 2p = 4p - 320$

$2200 + 320 = 6p$

$2520 = 6p$

$\therefore p = €420$

GRAPH

The supply and demand lines on the graph intersect at $p = €420$. This is the equilibrium point.

Question 9 (d)

ALGEBRA

$p = €420 : S = 2200 - 2p = 2200 - 2(420) = 1360$

$p = €420 : D = 4p - 320 = 4(420) - 320 = 1360$

GRAPH

According to the graphs, the number of items supplied and demanded at $p = €420$ is 1,360 items.

Question 9 (e)

$S > D$: Manufacturers will have warehouses filled with items that consumers won't buy.
$S < D$: Stores will have empty shelves and unhappy customers asking for the product.

Question 9 (f)

$S = 40 - 2p^2$

$D = 8p - 2$

$S = D \Rightarrow 40 - 2p^2 = 8p - 2$

$0 = 2p^2 + 8p - 42$

$0 = p^2 + 4p - 21$

$0 = (p - 3)(p + 7)$

$\therefore p = €3$

Sample Paper 4: Paper 2

Question 1 (25 marks)

Question 1 (a)

Centre $(1, 2)$, $r = 5$ $\boxed{(x-h)^2+(y-k)^2=r^2}$

$(x-1)^2+(y-2)^2=5^2$

$(x-1)^2+(y-2)^2=25$

Question 1 (b)

$y=0: (x-1)^2+(0-2)^2=25$ Cuts the x-axis: Put $y=0$

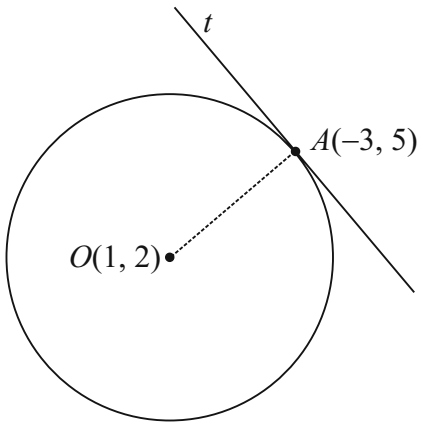
$(x-1)^2+4=25$

$(x-1)^2=21$

$(x-1)=\pm\sqrt{21}$

$x=1\pm\sqrt{21}$

$\therefore P(1+\sqrt{21}, 0), Q(1-\sqrt{21}, 0)$



Question 1 (c)

Is $A(-3, 5) \in (x-1)^2+(0-2)^2=25$?

$(-3-1)^2+(5-2)^2=(-4)^2+(3)^2=16+9=25$

Find the slope m_1 of OA:

$m_1=\dfrac{5-2}{-3-1}=\dfrac{3}{-4}=-\dfrac{3}{4}$ $\boxed{m=\dfrac{y_2-y_1}{x_2-x_1}}$

Slope of tangent t: $m_2=\dfrac{4}{3}$ Perpendicular lines: $m_1 \times m_2 = -1$

$y-5=\frac{4}{3}(x-(-3))$ $\boxed{y-y_1=m(x-x_1)}$

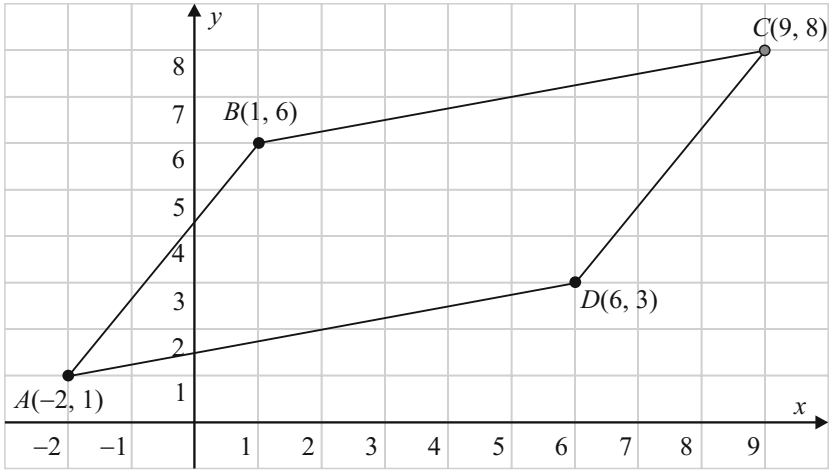
$3(y-5)=4(x+3)$

$3y-15=4x+12$

$0=4x-3y+27$

QUESTION 2 (25 MARKS)

Question 2 (a)



$A(-2, 1) \rightarrow B(1, 6)$ [Add 3, Add 5]

$D(6, 3) \rightarrow C(9, 8)$

Question 2 (b)

$A(-2, 1)$, $D(6, 3)$

$m = \dfrac{3-1}{6-(-2)} = \dfrac{2}{8} = \dfrac{1}{4}$ $\quad \boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

Slope of BE: $m = -4$ $\quad \boxed{\text{Perpendicular lines: } m_1 \times m_2 = -1}$

Equation of BE: $B(1, 6)$, $m = -4$ $\quad \boxed{y - y_1 = m(x - x_1)}$

$y - 6 = -4(x - 1)$

$y - 6 = -4x + 4$

$4x + y - 10 = 0$

Question 2 (c)

Equation of AD: $A(-2, 1)$, $m = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x - (-2))$

$4(y - 1) = 1(x + 2)$

$4y - 4 = x + 2$

$0 = x - 4y + 6$

Solve simultaneously the equations for AD and BE.

$4x + y = 10 \ldots\ldots (\mathbf{1})(\times 4)$

$x - 4y = -6 \ldots\ldots (\mathbf{2})$

$16x + 4y = 40$

$x - 4y = -6$

$17x \quad = 34 \Rightarrow x = 2$

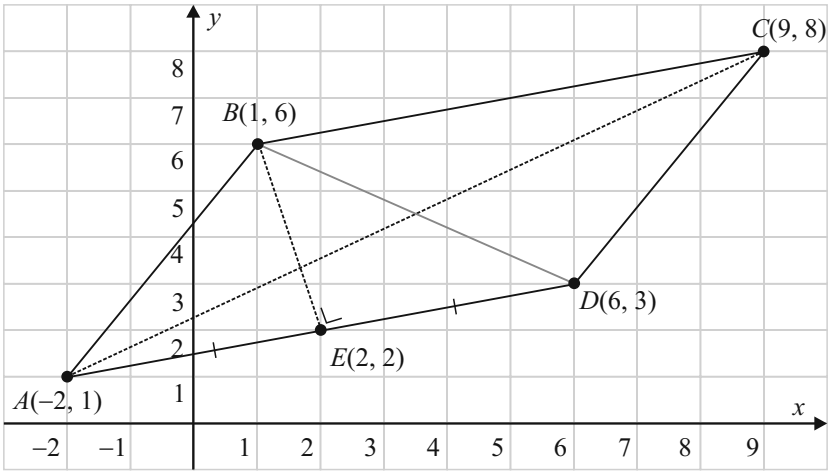
$x = 2 : 4(2) + y = 10 \Rightarrow y = 2$

Point of intersection of BE and AD: $E(2, 2)$

Midpoint of $[AD]$: $A(-2, 1)$, $D(6, 3)$

$$\text{MP} = \left(\frac{-2+6}{2}, \frac{1+3}{2}\right) = (2, 2) \quad \boxed{\text{MP} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Question 2 (d)



Method 1:

Area of a parallelogram: $\boxed{\text{Area} = b \times h}$

$\text{Area} = b \times h = |AD| \times |BE|$

$|AD| = \sqrt{(6-(-2))^2 + (3-1)^2} = \sqrt{8^2 + 2^2} = \sqrt{64+4} = \sqrt{68} = \sqrt{4 \times 17} = 2\sqrt{17}$

$|BE| = \sqrt{(1-2)^2 + (6-2)^2} = \sqrt{(-1)^2 + 4^2} = \sqrt{1+16} = \sqrt{17}$

$\therefore \text{Area} = |AD| \times |BE| = 2\sqrt{17}\sqrt{17} = 2(17) = 34$

Method 2:

The diagonal $[BD]$ bisects the area of the parallelogram.

$\text{Area} = 2 \times |\Delta ABD|$

$A(-2, 1) \rightarrow (0, 0)$ [Add 2, Subtract 1] $\boxed{A = \frac{1}{2}|x_1 y_2 - x_2 y_1|}$

$B(1, 6) \rightarrow (3, 5)$

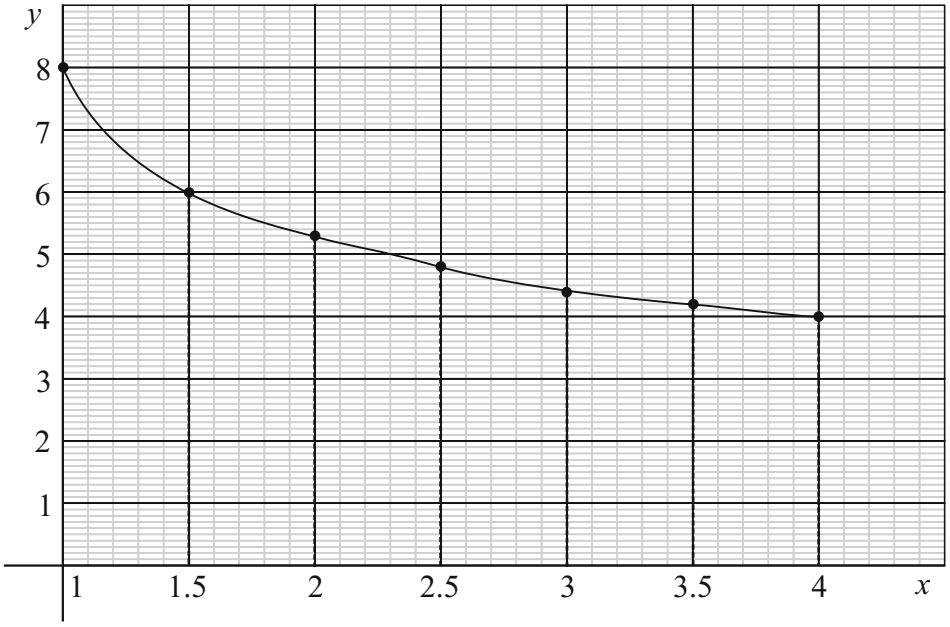
$D(6, 3) \rightarrow (8, 2)$

$\text{Area} = 2 \times \frac{1}{2}|(3)(2) - (5)(8)| = |6 - 40| = |-34| = 34$ square units

QUESTION 3 (25 MARKS)

Question 3 (a) & (b)

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	8.0	6.0	5.3	4.8	4.4	4.2	4.0



Question 3 (c)

$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + ... + y_{n-1})]$$

$h = 0.5$

$A \approx \frac{0.5}{2}[8 + 4 + 2(6 + 5.3 + 4.8 + 4.4 + 4.2)] = 15.35$ square units

EPL 104

Question 4 (25 marks)

Question 4 (a) (i)

If there are a ways of doing one operation, and b ways of doing another operation, then there are ab ways of doing both operations.

Question 4 (a) (ii)

$5 \times 6 \times 4 = 120$

There are five ways to fill the first box.
Once this box has been filled, there are four ways to fill the second box. Once the first two boxes have been filled, there are three ways to fill the third box.

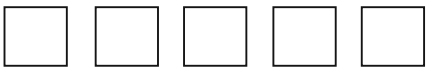
Number of possible three-course meals = 120

Question 4 (b)

Digits can be repeated:

There are nine ways to fill the first box (you cannot have a zero here as it would not be a five-digit number).
There are ten ways to fill the second box (you can use all digits).
There are ten ways to fill the third box, and so on.

$9 \times 10 \times 10 \times 10 \times 10 = 90\,000$

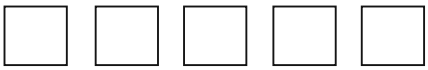


Number of five-digit numbers = 90 000

Digits cannot be repeated:

There are nine ways to fill the first box (you cannot have a zero here as it would not be a five-digit number).
There are nine ways to fill the second box (you cannot use what is in the first box but you can use a zero).
There are eight ways to fill the third box (you cannot have a number that is in the first two boxes).

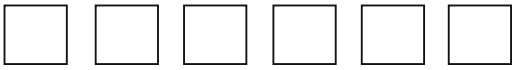
$9 \times 9 \times 8 \times 7 \times 6 = 27\,216$



Number of five-digit numbers with no digit repeated = 27 216

Question 4 (c)

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$



Number of arrangements of six people = 720

Glue the boys together and glue the girls together. There are two ways to arrange the groups. There are six ways to arrange the three boys in their group and there are six ways to arrange the three girls in their group.

$3 \times 2 \times 1 \quad 3 \times 2 \times 1$

Number of arrangements where boys and girls sit together = $2(3 \times 2 \times 1 \times 3 \times 2 \times 1) = 72$

Question 5 (25 marks)

Question 5 (a)

Gain in cent per dozen	20	12	8	4	0	–2
Probability	0.26	x	0.35	0.04	0.03	0.02

$P(x) = 0.26 + x + 0.35 + 0.04 + 0.03 + 0.02 = 1$
$\therefore x = 0.3$

Sum of the probabilities = 1

Question 5 (b)

Gain in cent per dozen	20	12	8	4	0	–2
Probability	0.26	0.3	0.35	0.04	0.03	0.02

Expected gain: $E(x) = 0 \times 0.26 + 12 \times 0.3 + 8 \times 0.35 + 4 \times 0.04 + 0 \times 0.3 + (-2) \times 0.02 = 11.72$ c

Question 5 (c)

Total number of eggs = 500 000

Number of dozens of eggs $= \dfrac{6\,000\,000}{12} = 500\,000$

Expected gain on his total production $= 500\ 000 \times 11 \cdot 72\,\text{c} = 5860\ 000\ \text{c} = €58\ 600$

Question 5 (d)

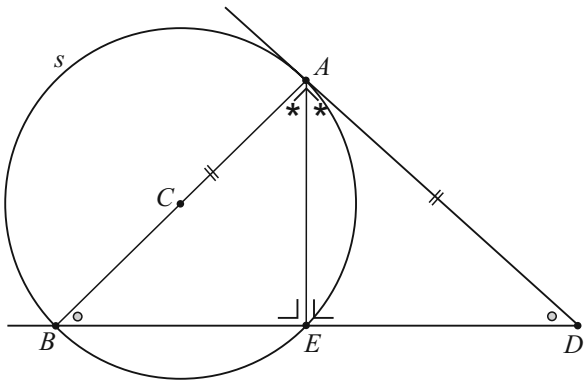
Gain in cent per dozen	20	12	8	4	0	–2
Probability	0.26	0.3	y	0.04	0.03	0.02

$E(x) = 12$ c
$\therefore 20 \times 0.26 + 12 \times 0.3 + 8y + 4 \times 0.04 + 0 \times 0.3 + (-2) \times 0.02 = 12$
$8y + 8.92 = 12$
$8y = 12 - 8.92 = 3.08$
$y = 0.385$

EPL 104

Question 6 (25 marks)

Question 6 (a)



Consider triangles ABE and AED.

$|AB| = |AD|$ (Given)

$|\angle ABD| = |\angle ADB|$ (Isosceles triangle)

$|AE| = |AE|$ (Common side)

$|\angle AEB| = 90^{\circ}$ (Angle standing on diameter is a right angle)

$\therefore |\angle AED| = 90^{\circ}$ (Straight angle)

Therefore, triangles ABE and AED are congruent.

$\therefore |BE| = |ED|$

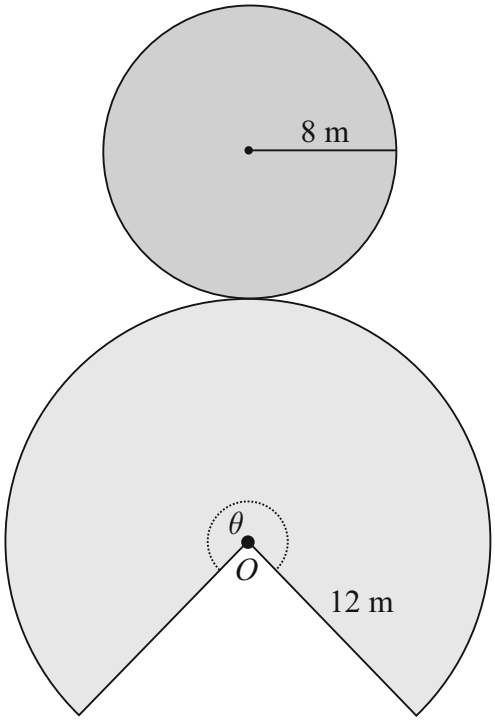
Question 6 (b)

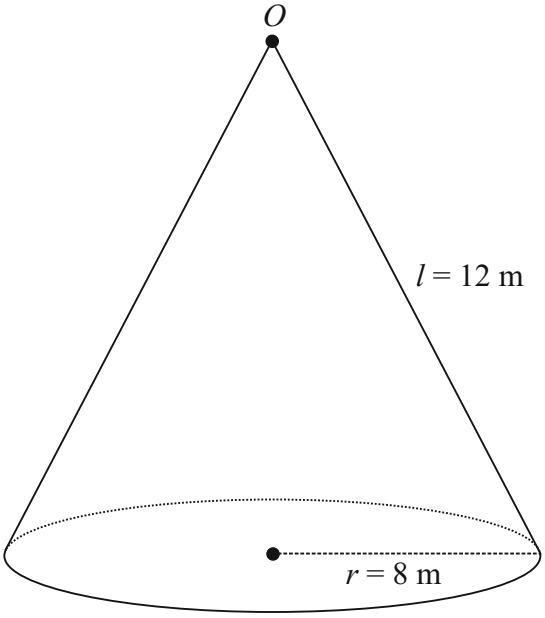
$|\angle BAD| = 90^{\circ}$ (Tangent is perpendicular to the point of contact.)

$|\angle EBA| = |\angle EDA| = 45^{\circ}$ (Isosceles triangle)

Question 7 (50 marks)

Question 7 (a)



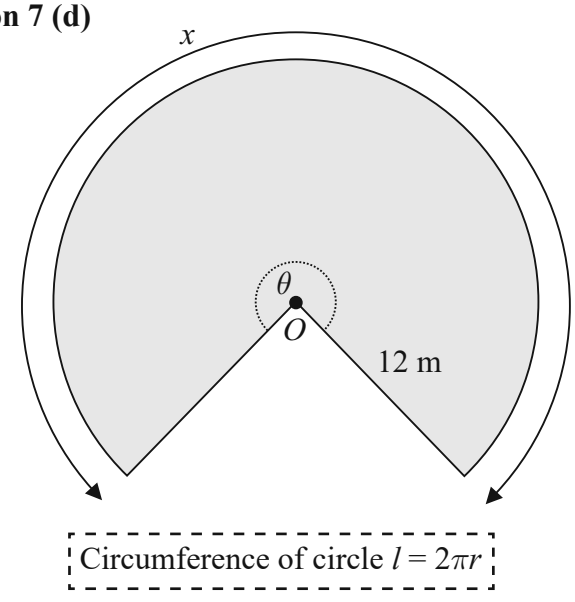


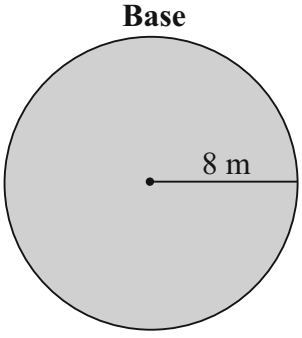
Question 7 (b&c)

Radius of base of cone $r = 8$ m

Slant height of the cone $l = 12$ m

Question 7 (d)





Circumference of circle $l = 2\pi r$

The length x is equal to the circumference of the circular base.

Length of circumference of base $= 2\pi(8) = 16\pi$ m $= x$

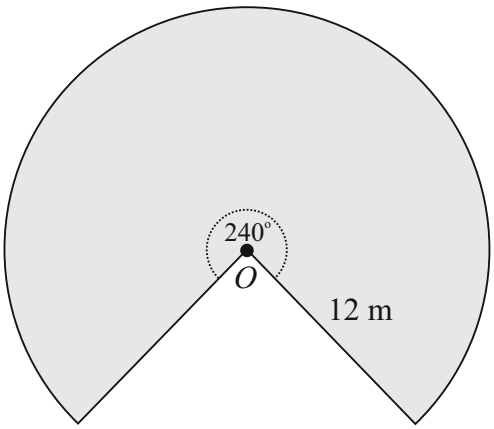
Length of circumference of big circle $= 2\pi(12) = 24\pi$ m

$$\frac{\text{Circumference of base}}{\text{Circumference of big circle}} = \frac{16\pi}{24\pi} = \frac{2}{3}$$

$$\theta = \tfrac{2}{3} \times 360^{\circ} = 240^{\circ}$$

EPL 104

Question 7 (e)



Area of circle $A = \pi r^2$

Curved surface area

$$= \frac{240^\circ}{360^\circ} \times \pi(12)^2$$

$$= 96\pi = 301{\cdot}6 \text{ m}^2$$

Question 7 (f)

Area of base

$$= \pi(8)^2$$

$$= 64\pi = 201{\cdot}1 \text{ m}^2$$

Total surface area = Curved surface area + Area of base
= 301.6 + 201.1 = 502.7 m^2

Question 7 (g)

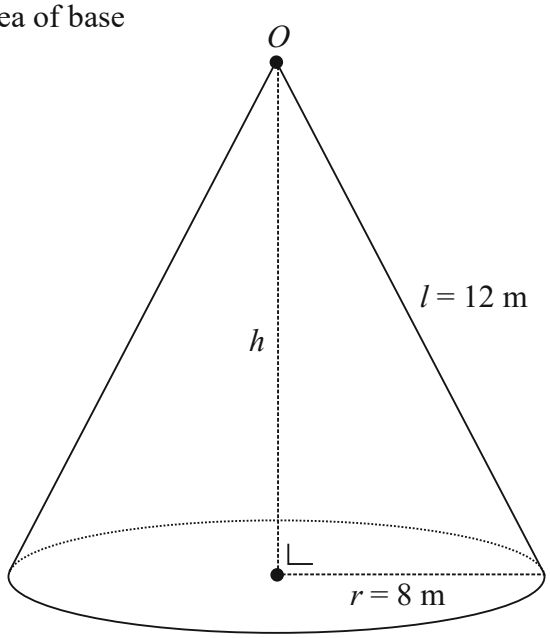
Volume of a cone $V = \frac{1}{3}\pi r^2 h$

$$r^2 + h^2 = l^2$$

$$h^2 = l^2 - r^2$$

$$h = \sqrt{l^2 - r^2} = \sqrt{12^2 - 8^2} = 4\sqrt{5} \text{ m}$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(4\sqrt{5}) \approx 600 \text{ m}^3$$



Question 8 (35 marks)

Question 8 (a)

$$n = 250\text{: Margin of error} = \pm\frac{1}{\sqrt{n}} = \pm\frac{1}{\sqrt{250}} = \pm 0\cdot 063 = \pm 6\cdot 3\%$$

Question 8 (b)

$$n = 500\text{: Margin of error} = \pm\frac{1}{\sqrt{n}} = \pm\frac{1}{\sqrt{500}} = \pm 0\cdot 045 = \pm 4\cdot 5\%$$

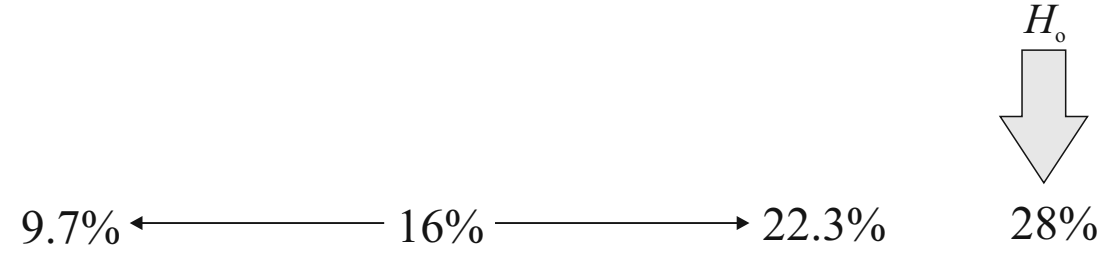
As can be seen from the calculation, doubling the sample size to 500 does not cause the margin of error to be halved.

Question 8 (c)

Claim (H_o): 28% of second level students in Ireland are 180 cm or taller.

Proportion of those in the sample who are 178 cm or taller $= \frac{40}{250} = 0\cdot 16 = 16\%$

Test (using the sample proportion): 9.7% ← 16% → 22.3%
Since 28% is outside this interval, reject the null hypothesis where the teacher claims that 28% of second level students in Ireland are 180 cm or taller.

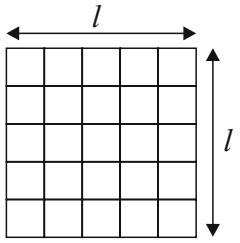


Question 8 (d)

The population is the entire group under investigation.
The sample is a group chosen to test the claim.
Population: All second level students in Ireland.
Sample: 250 students from our school.

EPL 104

Question 9 (65 marks)



Question 9 (a)

Length $l = 5$ units $= 5 \times 20$ m $= 100$ m

Area $A = 5 \times 5 = 25$ units squared $= 100$ m $\times$ 100 m $= 10\,000$ m^2

When converting units of length to metres, multiply by a factor of 20. (100 divided by 5)

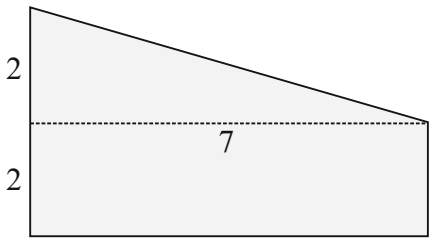
When converting units squared of area to m^2, multiply by a factor of 400. (10 000 divided by 25)

Question 9 (b)

Administration block

Area = Area of rectangle + Area of triangle

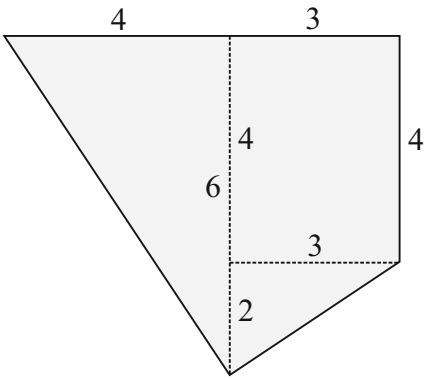
$$= 7 \times 2 + \tfrac{1}{2} \times 7 \times 2$$
$$= 14 + 7$$
$$= 21 \text{ units squared}$$
$$= 21 \times 400 \text{ m}^2$$
$$= 8400 \text{ m}^2$$



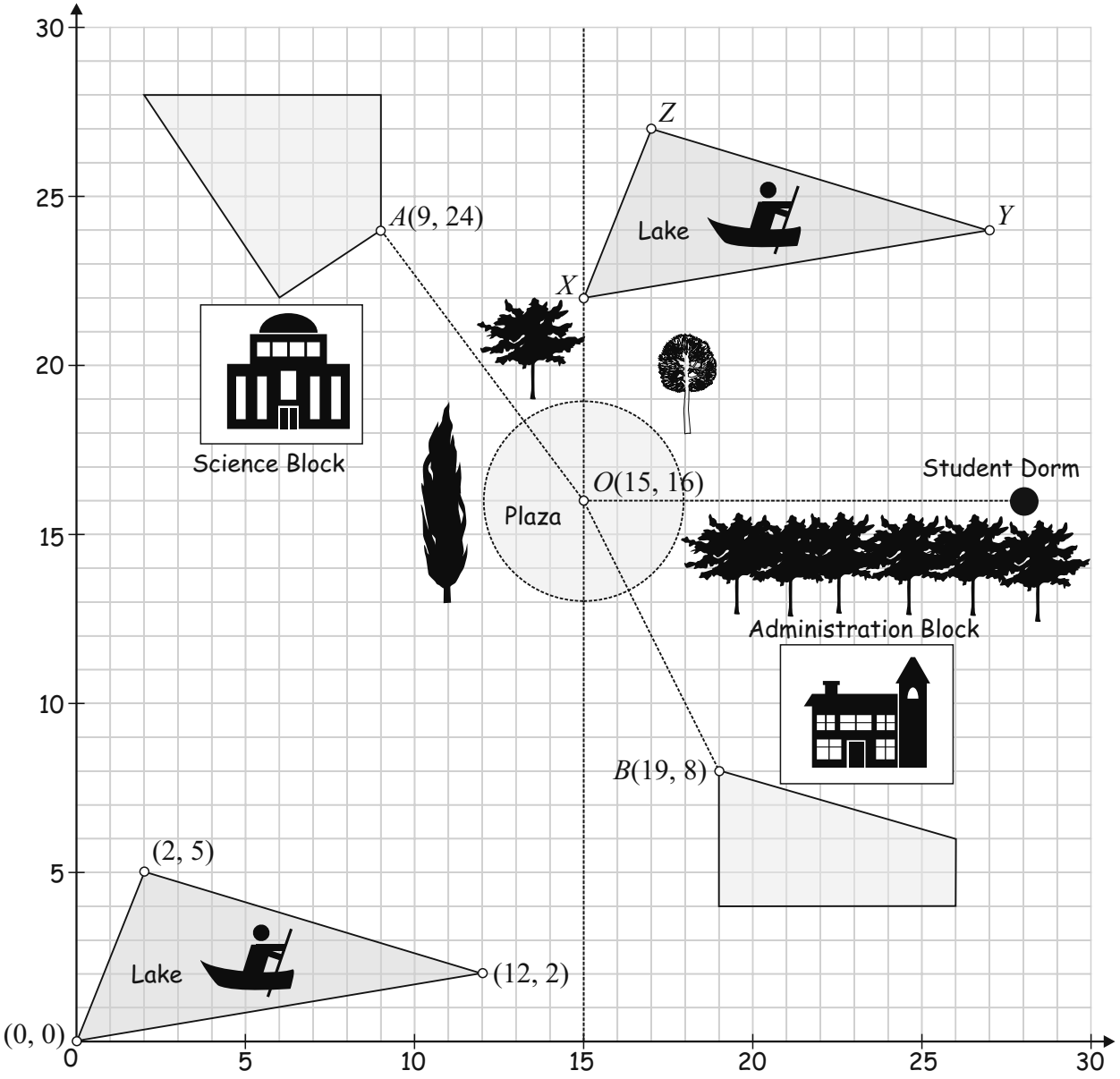
Science block

Area = Area of rectangle + Area of bigger triangle + Area of smaller triangle

$$= 4 \times 3 + \tfrac{1}{2} \times 4 \times 6 + \tfrac{1}{2} \times 3 \times 2$$
$$= 12 + 12 + 3$$
$$= 27 \text{ units squared}$$
$$= 27 \times 400 \text{ m}^2$$
$$= 10\,800 \text{ m}^2$$



Question 9 (c) (i)



Question 9 (c) (ii)

$(x_1, y_1) = (2, 5)$

$(x_2, y_2) = (12, 2)$

$A = \frac{1}{2}|x_1y_2 - x_2y_1| = \frac{1}{2}|(2)(2) - (12)(5)|$

$= \frac{1}{2}|4 - 60|$

$= \frac{1}{2}|-56|$

$= 28$ units squared

$= 28 \times 400$ m^2

$= 11200$ m^2

Question 9 (d) (i)

$A = (9, 24)$, $O(15, 16)$, $B(19, 8)$

Question 9 (d) (ii)

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$A(9, 24)$, $O(15, 16)$

$|AO| = \sqrt{(15-9)^2 + (16-24)^2} = 10$ units

$O(15, 16)$, $B(19, 8)$

$|OB| = \sqrt{(19-15)^2 + (8-16)^2} = 8.94$ units

$|AO| + |OB| = 10 + 8.94 = 18.94$ units

$= 18.94 \times 20 \approx 379$ m

Question 9 (e) (i)

Radius of the circle $r = 3$ units = 60 m

Length of circumference: $l = 2\pi r = 2\pi(60) = 377$ m

Question 9 (e) (ii)

Distance walked $= 13 \times 20 + \frac{1}{4}(377) = 354.3$ m

Question 9 (e) (iii)

Distance $= 354.3$ m $= 0.3543$ km

Speed $= 2.5$ km/h

Time $= ?$

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{0.3543 \text{ km}}{2.5 \text{ km/h}} = 0.14172 \text{ hr} = 0.14172 \times 60\text{s} \approx 9 \text{ mins}$$

Time of arrival: 10:09 am

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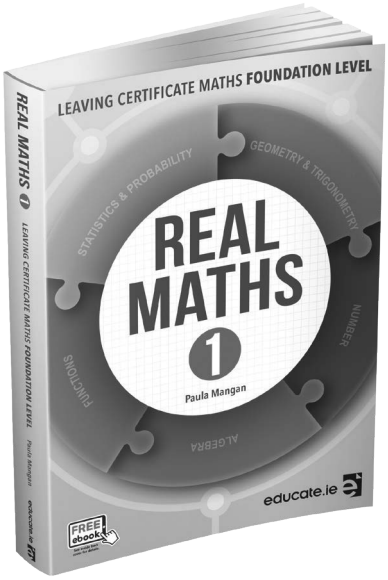

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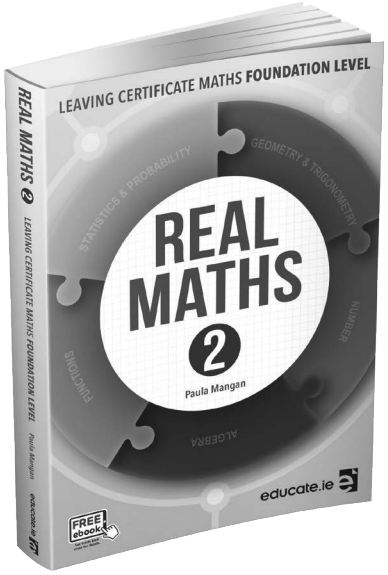

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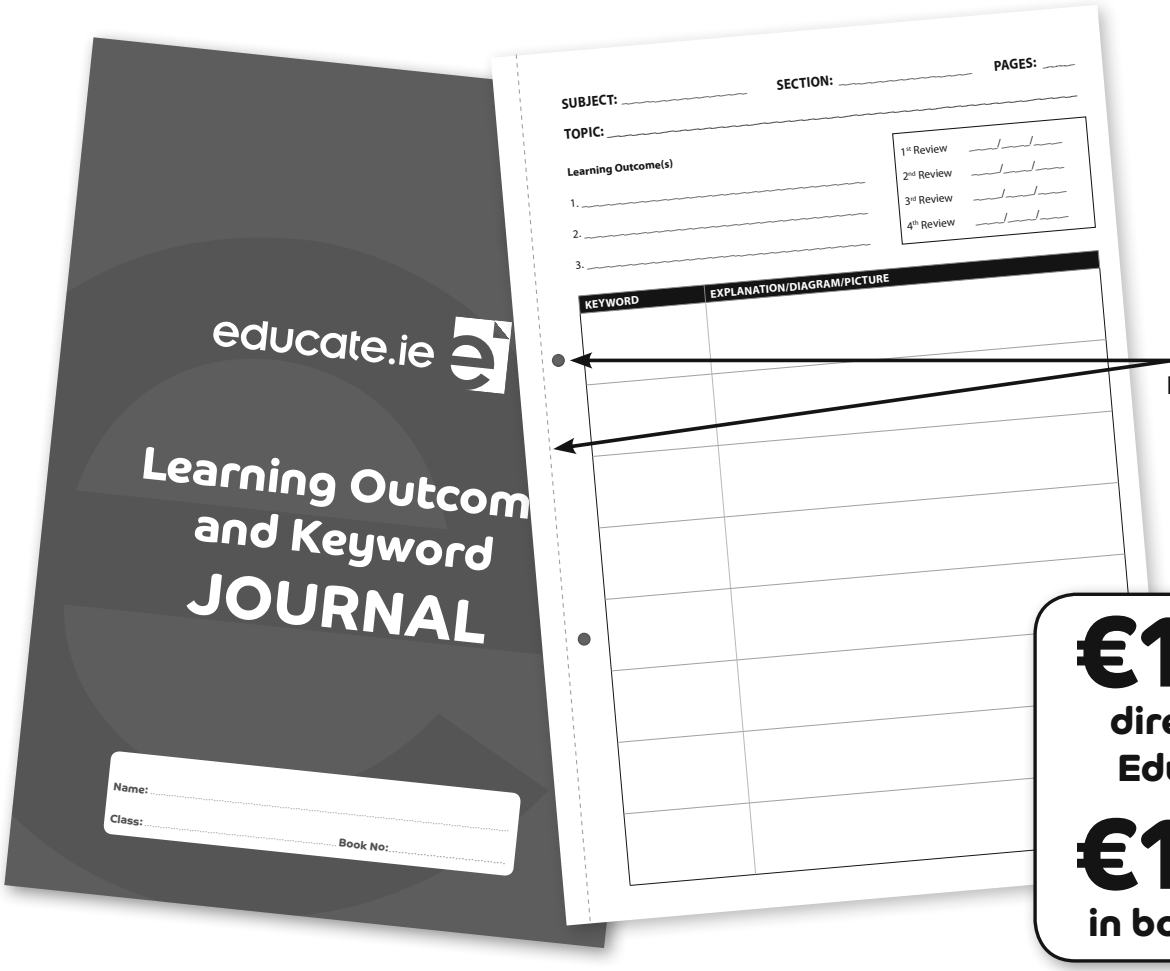

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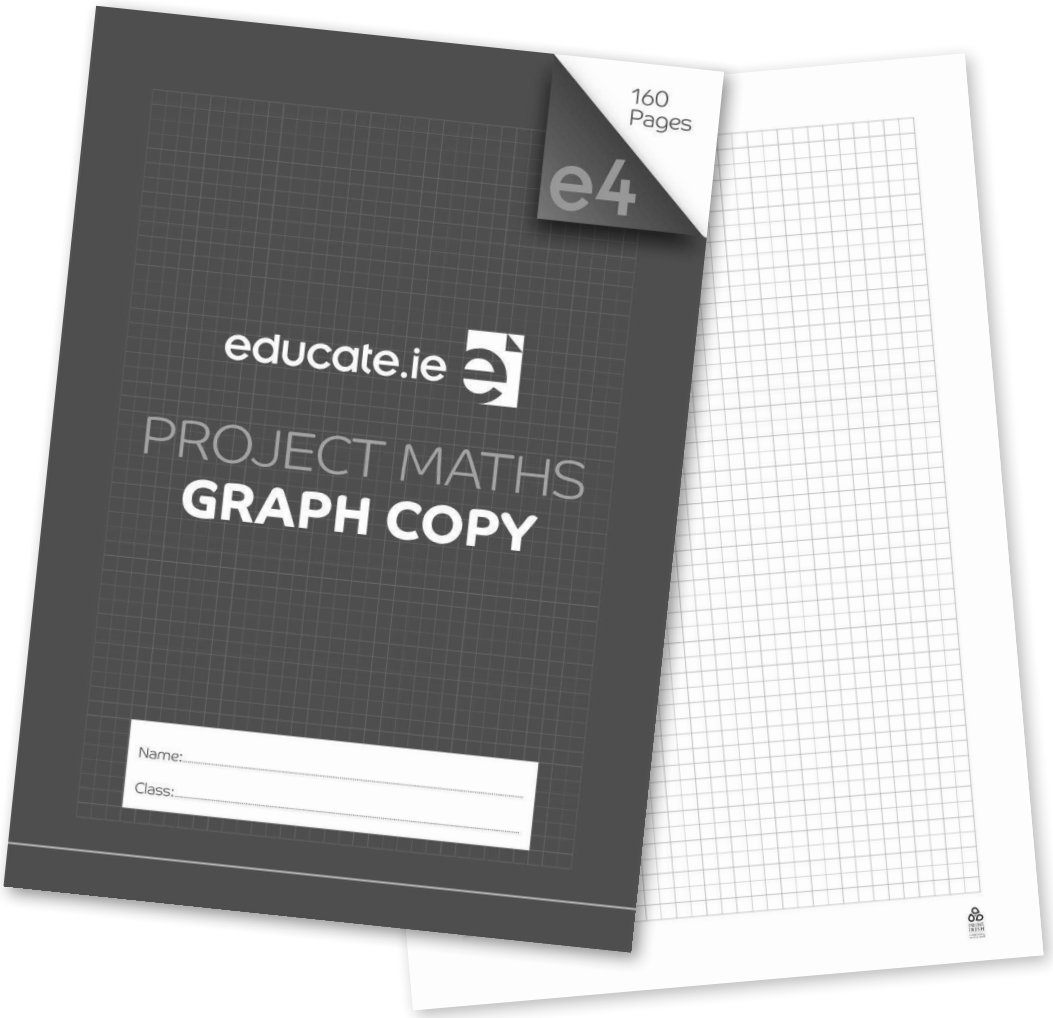

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