

A Revised Pseudo-Second Order Kinetic Model for Adsorption, Sensitive to Changes in Sorbate and Sorbent Concentrations

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Much contemporary research considers the development of novel sorbents for the removal of toxic contaminants. Whilst these studies often include experimental adsorption kinetics, modelling is normally limited to application of the pseudo-second order (PSO) rate equation, which provides no sensitivity towards changes in experimental conditions and thus no predictive capability. We demonstrate a relatively simple modification of the PSO model, with the final form $dqt/dt = k'C_t(1-(q_t/q_e))^2$ where $k'=k_2^*(q_e^{*/2})/C_0^*$. We demonstrate that unlike the PSO model, this new rate equation provides first-order dependence upon initial sorbate concentration (observed experimentally as $\bar{x}=0.829\pm0.417$), whilst rate constant k' is significantly less sensitive to changes in C_0 and C_s than PSO rate constant k_2 . We demonstrate that this model improves predictive capacity towards changes in C_0 and C_s , particularly when q_e is calculated using the Langmuir or Freundlich adsorption isotherm. Finally, we explore how the new rate constant, k', responds to changes in sorbent morphology, identifying that particle radius is a better constraining parameter than surface area. In this new equation, the conditionality of the rate constant upon experimental conditions is significantly decreased, facilitating better comparison of new results with the literature.

File list (2)

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A revised pseudo-second order kinetic model for adsorption, sensitive to changes in sorbate and sorbent concentrations

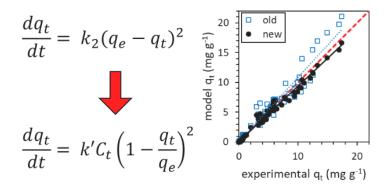
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Graphical Abstract



Keywords

Adsorption kinetics; kinetic model; Lagergren; Ho and McKay; intraparticle diffusion; particle size

Abstract

Much contemporary research considers the development of novel sorbents for the removal of toxic contaminants. Whilst these studies often include experimental adsorption kinetics, modelling is normally limited to application of the pseudo-second order (PSO) rate equation, which provides no sensitivity towards changes in experimental conditions and thus no predictive capability. We demonstrate a relatively simple modification of the PSO model, with the final form $dqt/dt = k'C_t(1-(q_t/q_e))^2$ where $k'=k_2*(q_e*^2)/C_0*$. We demonstrate that unlike the PSO model, this new rate equation provides first-order dependence upon initial sorbate concentration (observed experimentally as $\bar{x}=0.829\pm0.417$), whilst rate constant k' is significantly less sensitive to changes in C_0 and C_s than PSO rate constant k_2 . We demonstrate that this model improves predictive capacity towards changes in C_0 and C_s , particularly when c_0 is calculated using the Langmuir or Freundlich adsorption isotherm. Finally, we explore how the new rate constant, c_0 is a better constraining parameter than surface area. In this new equation, the conditionality of the rate constant upon experimental conditions is significantly decreased, facilitating better comparison of new results with the literature.

1. Introduction

There is a great wealth of recent literature concerning the development of novel sorbent materials for the remediation of contaminated water. This includes composite materials offering superior stability ¹, ease of separation from effluent ^{2 3}, or multifunctional capabilities such as photocatalytic activity ⁴ ⁵. Understanding adsorption kinetics is important for identifying the minimum duration needed for batch treatments and estimating maximum flow rates for column or continuous-flow treatments ⁶. Environments will be encountered with different concentrations of contaminants (with the initial concentration denoted as C₀), where different amounts of sorbent (with sorbent concentration denoted as C_s) are required to achieve desired filter device life-times. Predicting the range of experimental conditions under which novel sorbents will be deployed is challenging, and laboratory experiments can only hope to capture the approximate environments in which new sorbents will operate. It is thus desirable to have an element of predictive capability towards understanding adsorption kinetics.

There are two main classes of kinetic adsorption models, differing in their mechanistic description: diffusion-controlled and adsorption-controlled. A summary of the major kinetic models in each category is given in Table 1. For diffusion-controlled models, external diffusion is rarely considered rate limiting 7 . Instead, adsorption rates are controlled by intraparticle diffusion, with a diffusivity parameter reflecting the rate at which sorbate diffuses through the porous spaces within sorbent particles 7 . This category includes the Crank model (1956) 8 which approximates the sorbent as spherical particles and the concentration of sorbate at the surface is constant 7 . Adsorption kinetics can often be linearised using the Weber-Morris model (1963) 9 , where q_t is proportional to $t^{1/2}$ 10 .

Adsorption-controlled kinetic models consider formation of the surface complex to be the rate determining step. The original adsorption-controlled model was the Lagergren, or pseudo-first order model (1898) 11 , with a reaction rate that is first order with respect to (q_e - q_t), where q_e is the concentration of sorbate adsorbed at equilibrium, and q_t is the concentration of sorbate adsorbed at time t 10 . The Elovich equation (1934) 12 is another adsorption-control kinetic model, where the reaction rate decreases exponentially with increasing adsorption progress 10 .

At present, the pseudo-second order (PSO) rate equation, popularised by Ho and Mckay (1999) 13 , is probably the most popular model used to describe adsorption kinetics 14 , especially for new and novel sorbent materials. The PSO rate equation is the same as the earlier Lagergren model, save that the dependence of reaction rate upon (q_e - q_t) is second-order rather than first-order. The PSO rate equation takes the form:

$$\frac{dq_t}{dt} = k_2 (q_e - q_t)^2$$

Equation 1

where t is time (minutes), q_t is the amount of sorbate adsorbed at time t (mg g⁻¹), k_2 is the pseudo-second order rate constant (g mg⁻¹ min⁻¹), and q_e is the amount of sorbate adsorbed at equilibrium (mg g⁻¹) ¹³. The vital limitation of the PSO is that it does not explicitly include the variables C_0 and C_s (the initial concentration of sorbate and sorbent, respectively) and therefore lacks sensitivity to changes in C_0 and C_s . The PSO therefore does not allow us to identify appropriate sorbent

concentrations for water treatment design under different contaminant concentrations. Table 1 demonstrates that the lack of sensitivity towards C_t and C_s is common to most adsorption kinetic models – they are without predictive capacity, and their parameters are valid only for the particular experimental conditions under which they were calculated.

Furthermore, k_2 is only valid for the experimental conditions under which it was determined, and so comparison of literature studies with different experimental values of C_0 and C_s can lead to false conclusions ¹⁵. The ability to normalise rate constants to C_0 and C_s would thus be another major advantage, allowing improvements in sorbent engineering to be truly identified and validated.

The aim of this work was to develop a kinetic adsorption model that could be used to predict changes in adsorption kinetics as a function of sorbate and sorbent concentration. Furthermore, we explored how this model could provide normalised rate constants for better comparison between different sorbents. Our motivation towards both these aims was to help researchers consider the operating conditions under which their sorbents may be best used, e.g. comparative modelling of batch and continuous-flow systems ⁶.

We used the PSO model as a starting point, due to it being well-known and ubiquitous, and capable of describing most kinetic data, despite doubts over whether adsorption or diffusion-controlled models are more mechanistically appropriate 16 . To achieve these aims, we (1) collected a wide range of literature where the influence of C_0 and C_s on adsorption kinetics were investigated; (2) determined the relationship between C_0 and C_s on adsorption kinetics within this data set; (3) built the observed C_0 and C_s dependence into a modified form of the PSO model; (4) verified that the rate constants of this new model are less conditional than PSO rate constant k_2 ; (5) used the modified model to predict experimental data; (6) explored the influence of surface morphology on the remaining variance between rate constants in the literature.

Table 1: A selection of kinetic models widely used to describe adsorption. The table illustrates that in most of these kinetic models $\frac{dq_t}{dt}$ is sensitive to neither changes in C_0 nor changes in C_s , and are thus limited in predictive capability. C_0 is the initial concentration of aqueous sorbate, C_t is the concentration of aqueous sorbate at time t, C_e is the concentration of aqueous sorbate at equilibrium, q_t is the quantity of sorbate adsorbed at time t (e.g. $mg\ g^{-1}$), q_e is the quantity of sorbate adsorbed at equilibrium ($mg\ g^{-1}$), k is the rate constant, a is the desorption constant, a is the initial adsorption rate, D is molecular diffusion coefficient of the sorbate in solution, r is the radial coordinate, p is particle density, e is particle porosity, e is the local concentration of sorbate in the adsorbed phase (presumably equivalent to q_t), D_s is surface diffusivity (assumed constant), ϑ_t is surface coverage at time t, ϑ_e is surface coverage at equilibrium, u_e is the relative sorbate uptake at equilibrium, with $u_e = 1 - \frac{C_e}{C_0}$, k_a is a rate constant for adsorption. q_e^{*2} and C_0^{*2} denote the values of q_e and C_0 used to determine k_2 from a given experimental data set.

Model	Equation	Is the rate equation sensitive to changes in Ct?	Is the rate equation sensitive to changes in C _s ?	Reference					
Intraparticle diffusion-controlled models									
Crank model	$\frac{\partial C_t}{\partial t} = \frac{D}{T} \left(\frac{\partial^2 C_t}{\partial r^2} + 2 \frac{\partial C_t}{r \partial r} \right) - \frac{\rho}{\varepsilon_r} \frac{\partial N}{\partial t}$	Yes	No	17					
Simplified Crank model	$\frac{\partial q_t}{\partial t} = \frac{D_s}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial q_t}{\partial r} \right)$	No (model assumes constant surface concentration of sorbate)	No	7					
Web and Morris	$q_t = kt^{\frac{1}{2}}$	No	No	7					
	Adsor	ption-controlled models							
Pseudo-first order	$\frac{dq_t}{dt} = k(q_e - q_t)$	No	No (unless q _e is replaced with a variable term)	10					
Pseudo- second order	$\frac{dq_t}{dt} = k(q_e - q_t)^2$	No	No (unless q _e is replaced with a variable term)	10					
Elovich equation	$\frac{dq_t}{dt} = ae^{-\alpha q_t}$	No	No	10					
Second-order rate equation (aqueous phase)	$\frac{dC_t}{dt} = -k_2 C_t^2$	Yes (second order)	No	10					
Integrated kinetic Langmuir model	$\begin{split} \frac{d\theta_t}{dt} &= k_a C_0 [(1 - \theta_e u_e) \left(1 - \frac{q_t}{q_e}\right) \\ &+ \theta_e u_e \left(1 - \frac{q_t}{q_e}\right)^2] \end{split}$	Yes (but C_0 only and not C_t)	Yes $(\theta_e \text{ and } u_e \text{ will change} \\ \text{with } C_s)$	18					
This work	$rac{dq}{d_t}=\ k'C_t\left(1-rac{q_t}{q_e} ight)^2$ where $\mathrm{k'}=rac{k_2q_e*2}{C_0*}$	Yes (1 st order)	Yes (qe is calculated at each time interval using adsorption isotherms)	This work					

2. Experimental

2.1. Data sets

Literature sources that experimentally investigated the influence of C_0 , C_s or particle size upon adsorption kinetics were compiled (references are given in the Supplementary Information). Studies using oxyanions, metal cations and organic dyes as the sorbate were included, to ensure our findings would be general and non-sorbate-specific. Both mineral sorbents and organic sorbents (activated carbon and chitosan) were included, however zeolites and metal-organic frameworks (MOFs) were not since the sorption mechanism of sorbate trapping within cages might present inconsistent results. None of the literature sources found gave any mechanistic account or mathematical explanation for observed differences in adsorption kinetics due to varying C_0 , C_s or particle size. The majority of literature sources used the PSO equation to model adsorption kinetics.

In total 79 literature sources with around 200 kinetic experiments were collected. To investigate the influence of initial sorbate concentration, 11 literature sources with a combined 43 kinetic experiments were collected. For initial sorbent concentration, 11 literature sources with 43 experiments were collected. For particle size, 15 data sets from 6 literature sources, with a combined 47 experiments were collected. A data set was considered to be all kinetic experiments using the same sorbate-sorbent system within a single literature source.

2.2. Calculation of initial rates

To investigate the dependency of adsorption kinetics on C_0 , C_s and particle radius, the method of initial rates was used ¹⁹. Experimental kinetic data from the literature was tabulated and initial reaction rates were determined. Many data sets lacked good resolution for the initial stages of adsorption, e.g. in ~30% of all experiments reaction progress had already exceeded 50% when the first data point after mixing sorbent with sorbate was collected. Calculating the initial rate (the rate of reaction at t=0) using the slope of q_t versus time at the earliest data points would lead to a systematic under-calculation of the initial reaction rate, as early curvature in adsorption kinetics would be missed (Figure 1). This error would become more significant the later that initial adsorption data is collected. Determination of initial adsorption kinetics using the slope of q_t versus t would also give a random error associated with calculating rates from two data points only.

To provide a better estimation of the initial rate, the PSO model was fit to each data set and the rate calculated at t=0. PSO parameters (k_2 and q_e) were determined using the linearised form of the PSO rate equation, which is as follows:

$$\frac{t}{q_t} = \frac{1}{k_2 q_e^2} + \frac{t}{q_e}$$

Equation 2

Adsorption kinetic profiles were plotted as $\frac{t}{q_t}$ against t and the linear regression obtained using the LINEST function in Excel. q_e was obtained via the relationship $q_e=\frac{1}{m}$ where m is the gradient of the

linear regression, and k_2 via $k_2 = c \times q_e^2$ where c is the y-intercept. The initial rate of adsorption was calculated through simplifying Equation 1 as:

Initial rate =
$$\frac{dq_{t=0}}{dt} = k_2 q_e^2$$

Equation 3

Parameters were normalised to the same units for ease of comparison: $k_2 - g \ mg^{-1} \ min^{-1}$; $q_e - mg \ g^{-1}$; and initial rate $- mg \ g^{-1} \ min^{-1}$. The advantages in determining the initial rate of adsorption using the PSO model compared with the initial slope of q_t versus time is demonstrated in Figure 1. In the example, the initial rate was three times greater when calculated using the PSO model compared with the initial slope (4.4 versus 1.3 mg $g^{-1} \ min^{-1}$). In this experiment, the first data point collected after mixing gave a value of $q_t = 4 \ mg \ g^{-1}$. Given that $q_e = 5.4 \ mg \ g^{-1}$, adsorption progress had already reached 75% completion when the first data was collected, and it is therefore not surprising that the slope of q_t versus time gives a significantly lower initial rate compared with the PSO model: the PSO includes curvature within its interpolation of early adsorption kinetics (interpolating between t=0 and 3 minutes). Not only are systematic errors reduced, but the PSO method also reduces the random error due to uncertainties in determination of sorbate concentrations, since most kinetic experiments gave at least 5 data points with which to fit the PSO parameters, rather than just two data points used in the initial slope method.

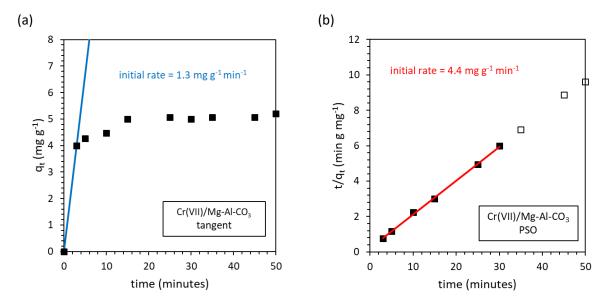


Figure 1: Illustration of how calculation of initial rates using the slope between the first two data points of q_t as a function of t gives a systematically lower result than when fitting the PSO model using the linearised plot of t/q_t as a function of t. The data presented is from the Cr(VII)/Mg-Al-CO₃ data set with 10 mg L⁻¹ sorbate and 1.5 g L⁻¹ sorbent ²⁰. The initial rate was calculated to be 8.5 mg g^{-1} min⁻¹ when using the PSO method with only the first two data points.

2.1. Determining the order of reaction

The order of reaction with respect to C₀, C_s or particle radius was determined by calculating the gradient of log(initial rate) versus log(independent variable) ¹⁹, where each data point represents a single kinetic experiment within a given literature study where the influence of either C₀ or C_s on adsorption kinetics was investigated.

order of reaction =
$$\frac{\Delta \log(initial\ rate)}{\Delta \log(C_0, C_s\ or\ r)}$$

Equation 4

For each data set, the order of reaction with respect to the independent variable was calculated using the LINEST formula in Microsoft Excel. The error of each data set was calculated as the standard error of the slope. A generalised order of reaction was calculated as the average (mean, \bar{x}) and median (\tilde{x}) order of reaction between all data sets collected, with errors reported as the standard deviation between all data sets. The dependencies of k_2 and k' upon C_0 , C_s and r were determined using the same method, substituting initial rates for k_2 or k'.

2.2. Modelling

As the modified rate equation was not easily linearised, this new model was simulated using Microsoft Excel. The quantity of sorbate adsorbed at the nth data point was calculated using the following formula:

$$q_n = q_{n-1} + \left((t_n - t_{n-1}) \times \left(k' C_{t(n-1)} \left(1 - \frac{q_{n-1}}{q_e} \right)^2 \right) \right)$$

Equation 5

With the concentration of aqueous sorbate remaining, C_t, decreasing according to the following equation:

$$C_{t(n)} = C_{t(n-1)} - C_s \times (q_n - q_{n-1})$$

Equation 6

The time interval between data points, $(t_n - t_{n-1})$ or Δt , was minimised until the magnitude of Δt had no significance on the observed kinetics.

3. Results and Discussion

3.1. Influence of C₀ and C_s upon adsorption rates

We first aimed to establish the influence that initial sorbent concentration, C_0 , and sorbent concentration, C_s , have on adsorption kinetics. We collected a range of experimental data, with oxyanions, metal cations and organic dyes represented as sorbates, as well as single component minerals, composite systems and activated carbon represented as sorbents. We calculated the initial rate of reaction (interpolated from the linearised PSO plot) and calculated the order of reaction with respect to C_0 and C_0 as the gradient in the slope of log(initial rate) as a function of log(C_0 or C_s). In total 11 data sets with a combined 43 kinetic experiments were collected to investigate the influence of C_0 . For C_s , 11 data sets with a combined 43 kinetic experiments were collected.

The data compiled from the literature generally shows a linear relationship between initial sorbate concentration and initial rate (normalised to mass, mg g⁻¹ min⁻¹). This is shown in Figure 2a where the average result for the gradient between log(initial rate) and log(C_0) was \bar{x} =0.829±0.417, \bar{x} =0.801. Despite large uncertainties, the average and median results were significantly closer to first-order than zero-order or second-order. This is intuitive for both diffusion and adsorption-controlled mechanisms, as twice as much sorbate should lead to twice as much sorbate flux from sorbent surface into pores, and collisions with the sorbent surface should be twice as frequent.

Similarly, a first order dependence of reaction rate on C_s was observed in the compiled data sets. This is shown in Figure 2b where the average gradient was $\bar{x}=1.18\pm0.61$, $\tilde{x}=1.15$. This is also intuitive for both diffusion and adsorption-controlled mechanisms, as when C_s is doubled, the total surface area is doubled, and the overall flux of sorbate entering sorbent pores is doubled, whilst the rate of collisions between sorbate and sorbent surface is also doubled. When normalised to mass (mg g⁻¹ min⁻¹) the initial rate was zero-order with respect to C_s , as expected. Again, despite large statistical errors, the relationship between initial rate and C_s was significantly closer to first-order (when rate is normalised to mg L⁻¹ min⁻¹) than both zero-order and second-order. For both C_0 and C_s , the median result was very close to the average, indicating that the analysis was not significantly skewed by any outlying data sets.

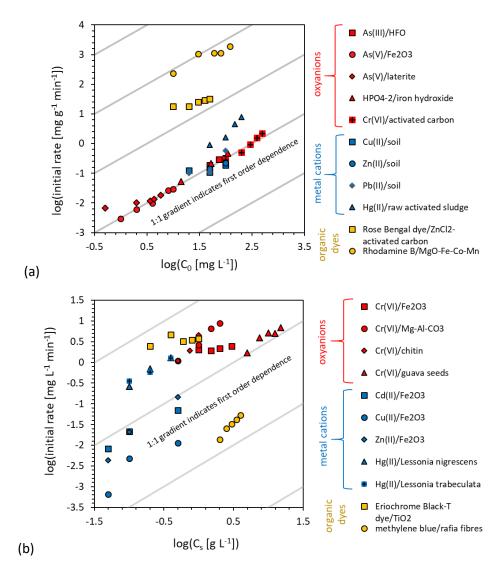


Figure 2: The initial rate of adsorption as a function of (a) initial sorbate concentration (C_0) and (b) initial sorbent concentration (C_s). Each data point represents a single kinetic experiment – each experiment was fitted with the pseudo-second order model and the rate at time t=0 calculated as discussed in the experimental section. Kinetic experiments are grouped into data sets, where all conditions except for (a) C_0 or (b) C_s were kept constant. The gradients of most data sets align with the 1:1 gradient (grey guidelines given for visual reference) indicating that the adsorption reaction is first order with respect to the independent variables C_0 and C_s .

3.2. Conditionality of k2 and limitations to predictive capabilities of the PSO

To predict the sensitivity of adsorption kinetics as a function of changes to C_0 and C_s , and to facilitate comparison of adsorption kinetics within the literature, it is necessary that the rate constant determined under one set of experimental conditions can used to model further experiments differing in C_0 and C_s . Having identified a first-order dependence of reaction rate on C_0 and C_s , we aimed to identify the influence that independent variables C_0 and C_s have on k_2 , using the same data sets.

Whilst section 3.1 demonstrated that adsorption kinetics are approximately first order with respect to C_0 (Figure 2a), the PSO rate constant, k_2 , is inversely proportional to C_0 , i.e. doubling initial sorbate concentration results in the rate constant k_2 decreasing by a factor of 2 (Figure 3a). The inverse relationship between k_2 and C_0 is explained by the second order dependence of PSO on the **absolute** concentration of available surface sites, (q_e-q_t) , and is demonstrated mathematically in the following discussion.

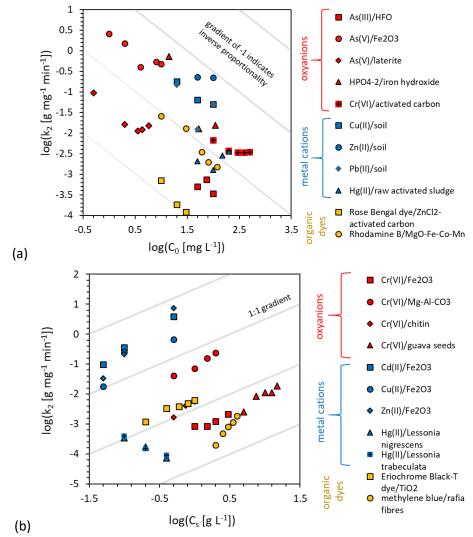


Figure 3: Pseudo-second order (PSO) rate constant k_2 as a function of (a) initial sorbate concentration (C_0) and (b) initial sorbent concentration (C_s). Grey lines indicate gradients and thus reaction orders of (a) -1 and (b) +1.

Many studies of adsorption kinetics operate under conditions where the sorbent remains unsaturated at equilibrium. This is natural as the authors wish to investigate the conditions under which their sorbent successfully removes contaminants. At low values of C_e , a linear relationship is observed between C_e and q_e , known as Henry's adsorption isotherm 21 . If we approximate the adsorption isotherm as a linear relationship between C_e and q_e , and let coefficient α represent the factor increase in C_0 between experiments (1) and (2), we get the following:

$$a = \frac{reaction\: rate\: (2)}{reaction\: rate\: (1)} = \frac{C_{0(2)}}{C_{0(1)}} \approx \frac{C_{e(2)}}{C_{e(1)}} \approx \frac{q_{e(2)}}{q_{e(1)}}$$

Equation 7

where coefficient a is equal not just to the quotient of $C_{0(2)}$ and $C_{0(1)}$, but also the quotient of reaction rates, given the first order dependence of reaction rate upon C_0 observed in section 3.1. $q_{e(2)}$ can thus be substituted for the product of coefficient a and the old $q_{e(1)}$:

$$q_{e(2)} \approx aq_{e(1)}$$

Equation 8

Rearrangement of Equation 1 gives the following:

$$k_2 = \frac{reaction\ rate}{(q_e - q_t)^2}$$

Equation 9

which at time t=0 reduces to:

$$k_2 = \frac{initial\ rate}{{q_e}^2}$$

Equation 10

Substitution of $q_{e(2)}$ with $aq_{e(1)}$ as per Equation 8 gives:

$$k_{2(2)} = \frac{initial\ rate\ (2)}{{q_{e(2)}}^2} \approx \frac{initial\ rate\ (2)}{(aq_{e(1)})^2}$$

Equation 11

Substituting initial rate (2) for the product of *a* and initial rate (2) as per Equation 7 gives:

$$k_{2(2)} \approx \frac{initial\ rate\ (2)}{\left(aq_{e(1)}\right)^2} = \frac{a \times initial\ rate\ (1)}{a^2q_{e(1)}^2} = \frac{a \times k_{2(1)}}{a^2} = \frac{k_{2(1)}}{a}$$

Equation 12

This mathematically demonstrates that k_2 is inversely proportional to C_0 , to a first approximation, when the sorbent is unsaturated and adsorption can be represented by the linear adsorption isotherm. This was observed experimentally, as whilst the reaction rate only doubles with a doubling of C_0 (Figure 2a), the PSO rate constant, k_2 , conditional to the value of C_0 used, decreased by a factor of two. The final results for the order of reaction with respect to C_0 were \bar{x} =-0.761 ±0.663, \tilde{x} =-0.765, N=11 data sets and 43 kinetic experiments. The standard deviation is large, nearly including x=0 for a zero-order relationship between k_2 and C_0 .

Most data sets showed first-order dependence of k_2 upon C_s (Figure 3b). This can also be explained by considering adsorption isotherms and the term $(q_e-q_t)^2$. If coefficient b indicates the factor increase in C_s between two experiments, then:

$$b = \frac{C_{s(2)}}{C_{s(1)}}$$

Equation 13

When the sorbent is saturated and $C_e >> (C_s * q_e)$, the adsorption isotherm approaches a plateau and increasing C_s has limited effect on q_e .

$$q_{e(2)} \approx q_{e(1)}$$

Equation 14

Since a zero-order dependence of rate (when normalised to mass, mg g⁻¹ min⁻¹) upon C₅ was identified in section 3.1, then:

initial rate (2)
$$\approx$$
 initial rate (1)

Equation 15

From Equation 10, we obtain:

$$k_{2(2)} = \frac{initial\ rate\ (2)}{{q_{e(2)}}^2} \approx \frac{initial\ rate\ (1)}{{q_{e(1)}}^2} \approx k_{2(1)}$$

Equation 16

 C_s does not enter the equations, and thus k_2 is predicted to have zero order dependence upon C_s at high values of C_e where the adsorption isotherm reaches a plateau.

We can also consider the opposite case, being the low C_e region of the adsorption isotherm. When $C_e << (C_s * q_e)$ and the majority of sorbate is removed by adsorption at equilibrium regardless of sorbent concentration, and increasing C_s causes a decrease in q_e as limited sorbate is divided across a larger surface area. In this case:

$$q_{e(2)} \approx \frac{q_{e(1)}}{b}$$

Equation 17

If Equation 15 based upon our observations from section 3.1 is still valid, then:

$$k_{2(2)} = \frac{initial\ rate\ (2)}{{q_{e(2)}}^2} \approx \frac{initial\ rate\ (1)}{\left(\frac{q_{e(1)}}{b}\right)^2} \approx \frac{b^2 \times initial\ rate\ (1)}{{q_{e(1)}}^2} \approx b^2 k_{2(1)}$$

Equation 18

Which gives a second-order dependence of k₂ upon C_s.

It therefore follows that the low and upper bounds of k_2 dependence upon C_s are zero and second-order respectively. These bounds fit the majority of data sets, as shown in Figure 3b. The final results for the order of reaction with respect to C_0 were \bar{x} =1.16 ±1.28, \tilde{x} =1.39, N=11 data sets and 43 kinetic experiments. Zero, first and second-order dependencies between k_2 and C_s are all included within the standard deviation. Out of 11 data sets, three showed a relationship between k_2 and C_s outside of the predicted constraints: methylene blue/raffia fibres demonstrated a dependency of 3.1 ±0.2, and the two Hg(II)/biosorbent data sets gave a dependency of -1.1 ±0.2.

Having demonstrated the conditionality of PSO rate constant k_2 with respect to C_0 and C_s , we investigated the implications of this conditionality with regards to predictive adsorption kinetic

modelling. For predictive modelling, a single rate constant should be able to model experiments differing in C_0 and C_s . We used a single value of k_2 , being the average value of k_2 for all kinetic experiments within a given data set, to model experiments with different initial conditions (C_0 and C_s). The unique value of q_e determined for each kinetic experiment was kept for modelling. All parameters are tabulated in the supplementary information. Figure 4 shows that the resulting PSO model cannot accurately describe data sets differing in C_0 or C_s when a fixed value for rate constant k_2 is used. The implication is that (a) the PSO cannot be used to predict changes in adsorption kinetics as a function of C_0 and C_s , and (b) comparison of the PSO rate constant, k_2 , between different literature studies, is only valid if PSO parameters were determined under the same experimental conditions (C_0 and C_s).

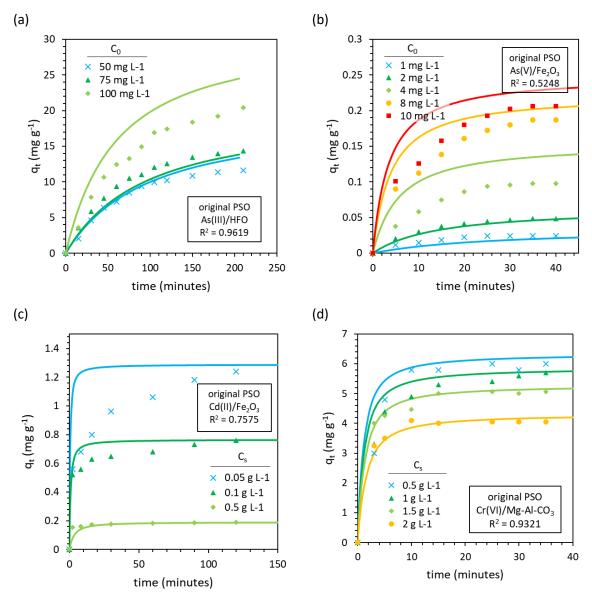


Figure 4: Demonstrating the lack of predictive capability in the PSO model due to the C_0 and C_s conditionality of k_2 . A selection of literature data sets were modelled using PSO kinetics with a single value of k_2 , calculated as the average value of PSO rate constant, k_2 , across all kinetic experiments within the given data set. (a) and (b) are data sets wherein C_0 is varied whilst (c) and (d) are data sets where C_s is varied. Experimental data is from C_s 22 23 24 20.

3.3. Development of a kinetic model sensitive to changes in C₀ and C_s

To provide sensitivity when predicting changes in adsorption kinetics due to changes in C_0 and C_s , we modified the PSO rate equation, giving a new pseudo-second order kinetic model, as follows. Firstly, to remove the inverse relationship between the rate constant upon C_0 (Figure 3a), the right-hand side of the equation must be made proportional to C_0 (Figure 2a) and not proportional to C_0^2 , as the unmodified PSO equation approximates to through the term $(q_e-q_t)^2$ (Equation 1). This problem is solved by replacing $(q_e-q_t)^2$, which provides second order dependence on the **absolute amount** of adsorption capacity remaining, with a new term providing second order dependence on the **relative amount** of adsorption capacity remaining. The term used herein is of the form $\left(1-\frac{q_t}{q_e}\right)^2$. This modification of Equation 1 gives the following:

$$\frac{dq_t}{d_t} = k' \left(1 - \frac{q_t}{q_e} \right)^2$$

Equation 19

where $k'=k_2q_e*^2$. Here, q_e^* denotes the equilibrium concentration of adsorbed sorbate in the particular kinetic experiment used to calculate k_2 . At time t=0, this new term, $\left(1-\frac{q_t}{q_e}\right)^2$, always returns a value of 1, independent of the sorbate concentration.

The first order dependence of rate upon the sorbate concentration can then be explicitly defined, giving the final equation:

$$\frac{dq_t}{dt} = k'C_t \left(1 - \frac{q_t}{q_e}\right)^2$$

Equation 20

where $k'=\frac{k_2q_e*2}{C_0*}$ with C_0^* denoting the initial sorbate concentration in the kinetic experiment used to calculate k_2 . Modification of the equation to this form, with second order dependence on the **relative** concentration of unused adsorption capacity rather than the **absolute** concentration, results in no change to adsorption kinetics when simulated under the same experimental conditions.

Figure 2b demonstrated a first-order dependence of initial rate upon C_s when normalised to volume (mg L⁻¹ min⁻¹) and zero order when normalised to sorbent mass (mg g⁻¹ min⁻¹). The current modified equation is dependent on the <u>relative</u> availability of adsorption capacity rather than the <u>absolute</u> availability. Any changes in q_e due to varying C_s do not affect the term $(1 - \frac{q_t}{q_e})^2$ at t=0, as it continues to reduce to unity. Therefore, when the rate equation is normalised to mg g⁻¹ min⁻¹ (i.e. $\frac{dq_t}{dt}$ rather than $\frac{dC_t}{dt}$) no modification of the rate equation for C_s is required. Our modified rate equation, Equation 20, is similar to the adsorption-only form of the kinetic Langmuir model (kLm), which at high surface coverage is first order with respect to C_t and second order to $(1 - \frac{q_t}{q_s})^{25 \ 18}$.

3.4. Confirming that k' is less conditional than k₂

A predictive adsorption kinetic model, sensitive to changes in C_0 and C_s , requires a rate constant that is unaffected by either independent variable. Since the ideal rate constant is therefore entirely unaffected by changes to C_0 and C_s , the slope of log(initial rate) as a function of log(C_0) or log(C_s) should return a gradient of zero, indicating zero-order dependence. We calculated the dependency of K' upon K_0 and K_0 identically as for K_0 in section 3.2. We then validated whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying whether K' is less conditional than K_0 by identifying K' in K' is less conditional than K_0 by identifying K' in K' is less conditional than K' in K' is less conditional than K' in K' in K' is less conditional than K' in K' is less conditional than K' in K' in K' in K' is less conditional than K' in K'

The visual comparison of k_2 and k' dependencies upon C_0 and C_s is given in Figure 5. k_2 displayed an inverse dependence on C_0 with \bar{x} =-0.761±0.663, \tilde{x} =-0.765, and an approximately first-order dependence on C_s with \bar{x} =1.16±1.28, \tilde{x} =1.39. The new rate constant k' was significantly closer to zero-order dependency than any other relationship with regards to both C_0 and C_s . For C_0 the results were \bar{x} =-0.28±0.53, \tilde{x} =-0.38 for C_0 dependency, and \bar{x} =0.04±0.61, \tilde{x} =-0.02 for C_s dependency. The standard deviation values of k' were also smaller than for k_2 , indicating that the relationships between independent variables and rate constants were most consistent across data sets for k' than k_2 . These results suggest that the new rate constant k' is less conditional than k_2 , and that the dependency of adsorption kinetics upon C_0 and C_s has been captured by the new rate equation, at least in part, if not fully.

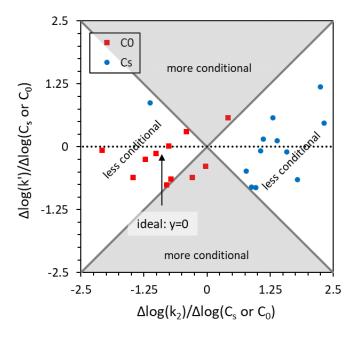


Figure 5: Mathematical demonstration of reduced sensitivity to C_0 and C_s in the new rate constant k' compared with k_2 . Where the rate constant is not influenced by changes in C_0 and C_s , a gradient of the log(rate constant) versus either $\log(C_0)$ or $\log(C_s)$ will return a value of zero (indicating that the rate constant does not change). A value of +1 indicates a first order relationship, -1 indicates an inverse relationship and so on. The white shaded area of the plot indicates experimental data sets where k' was less sensitive to changes in C_0 and C_s than C_s

3.5. Use of adsorption isotherms for modelling adsorption kinetics

The PSO model traditionally uses a unique, fixed value of q_e to model each adsorption kinetic experiment. However, q_e is sensitive to both C_0 and C_s , and so the single value of q_e obtained under

one set of experimental conditions cannot be used to predict adsorption kinetics once C_0 and C_s are changed. To provide appropriate sensitivity, the modified kinetic model must therefore have a q_e term that is sensitive to sorbate concentration. This can be achieved by replacing q_e with adsorption isotherms such as the Langmuir or Freundlich models 21 . Using adsorption isotherms, q_e can either be set as the concentration of adsorbed sorbate at equilibrium (as per convention), or q_e can be recalculated at each point in time, giving an out-of-equilibrium value of q_e which decreases throughout the kinetic experiment as adsorption progress increases. In this second case, the term C_e within the adsorption isotherm would be replaced with C_t . Huang et al. previously demonstrated that this second case actually gives better account of the true driving force of the reaction during the initial stages of adsorption 26 . Recalculating ' q_e ' using C_t in this way is also more appropriate when modelling column systems, where $C_t << C_e$ for the majority of the experiment. Langmuir and Freundlich adsorption isotherm data from the literature was incorporated into the modified rate equation (Equation 19), and a selection of data sets were then modelled using this approach, presented in section 3.6.

3.6. Validating improvements in the modified equation

In order to validate our modified model, we used Equation 19 to model the same selection of data sets previously modelled using the PSO model with an average value of k_2 (Figure 4). A single value of k' was used to model all kinetic experiments within each data set, taken as the average value of k' across the data set. All parameters are tabulated in the supplementary information. The results are given in Figure 6 and in all cases the goodness of fit (represented by R^2) was improved compared with when predicting adsorption kinetics using the PSO model with a value of k_2 taken as the average between all kinetic experiments within the data set. The results were R^2 =0.9735 vs. 0.9619 for As(III)/HFO (with C_0 varied), R^2 =0.8225 vs. 0.5248 for As(V)/Fe $_2$ O $_3$ (C_0 varied), R^2 =0.8833 vs. 0.7575 for Cd/Fe $_2$ O $_3$ (C_0 varied) and R^2 =0.9751 vs. 0.9321 for Cr(VI)/Mg-Al-CO $_3$ (C_0 varied). This is especially significant given that in section 3.2 a unique value of C_0 determined by experimental fitting was used for the PSO model, whilst for the modified kinetic model, C_0 was calculated using a single adsorption isotherm (Langmuir or Freundlich) for all kinetic experiments within the data set. The modified model thus not only provided a better fit than the unmodified pseudo-second order model, but provided a better fit whilst using fewer fitting parameters.

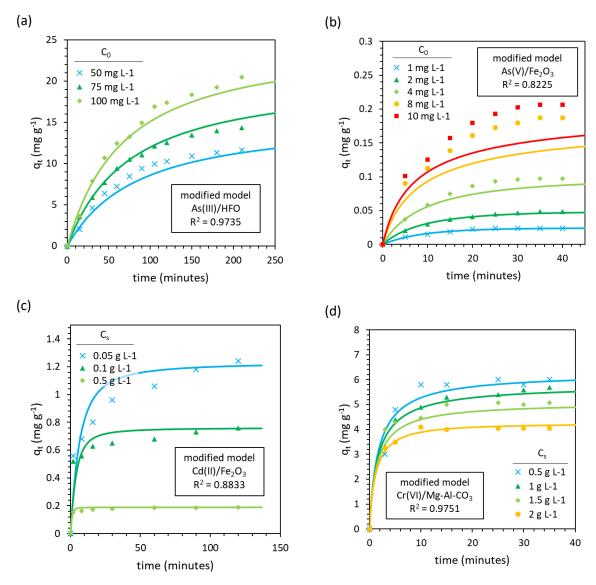


Figure 6: Modelling adsorption kinetics using the modified model. For each data set a single value of rate constant k' was used (chosen as the average value of k' between all kinetic experiments within the given data set) and q_e was determined for all points in time, within all kinetic experiments using the same adsorption isotherm parameters. The single kinetic model was used to model experiments differing in C_0 (a and b) and C_s (c and d). The Freundlich adsorption isotherm was used for all experiments in (a) and (d), and the Langmuir adsorption isotherm for all experiments in (c) and (d). Experimental data is from C_0 22 32 24 20

The q_t values predicted by the PSO and modified models were cross-calibrated against the values of q_t observed experimentally (Figure 7). Using the unmodified PSO with a single value of k_2 but unique experimentally-fitted values of q_e gave a cross-calibration slope of 1.0770 \pm 0.0186 (R²=0.9638), i.e. an error of 7.7% and uncertainty of 1.7% (Figure 7a). The cross-calibration between experiment and model was improved when using the modified kinetic model, with a slope of 0.9598 \pm 0.0061 (R² = 0.9949) – an error of 4.0% and an uncertainty of 0.6%, both lower than in the unmodified PSO model. In the logarithmic form (Figure 7b) the cross-calibration slope of the modified model versus experiment was closer to unity with a gradient of 1.0117 \pm 0.0063 and R² = 0.9957, indicating that the linear regression in the previous panel (Figure 7a) is skewed by the few data points at high q_t values. Again, there is a better goodness of fit in the modified model compared with the original PSO (slope=1.0104 \pm 0.0106 and R²=0.9880). The modified rate equation presented in this work therefore goes some way towards providing sensitivity towards changes in C₀ and C_s.

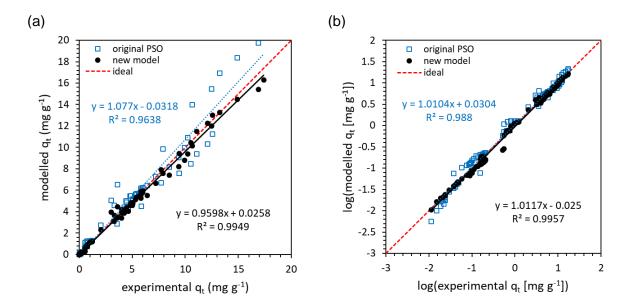


Figure 7: Calibration curves between the experimentally observed q_t and q_t predicted by the original PSO and modified kinetic models in (a) absolute form and (b) logarithmic form. Again, a single averaged value of k' was used to model all kinetic experiments in a single data set. N = 132 data points were included in the calibration plots. Shown are data points for the original PSO model (blue open squares) and the modified model (black filled circles) along with their linear regressions (blue dotted and black solid lines respectively). The red dashed line indicates the one-to-one slope for a perfect model.

Using the original PSO model, two unique parameters (k_2 and q_e) were required for each kinetic experiment within each data set, and no predictive capability was offered. In the modified model, one kinetic parameter, rate constant k', along with two adsorption isotherm parameters (K_L and q_{max} for Langmuir or K_F and n for Freundlich), can be used to model multiple kinetic experiments, offering some (though clearly imperfect) predictive capability.

3.7. Influence of sorbent morphology on rate constant k'

Whilst rate constants were successfully normalised to differences in experimental conditions C_0 and C_s by converting the PSO model with rate constant k_2 to the modified rate equation with rate constant k', values of k' calculated for given sorbate-sorbent combinations still showed significant variation across the literature (Supplementary Information). One important factor not yet discussed is sorbent morphology, i.e. particle size, porosity and surface area, which all vary depending on the preparation of the given sorbent sample. We thus investigated whether sorbent morphology accounts for the remaining variance in rate constants.

3.7.1. Surface area

The influence of surface area on the initial rate of reaction could not be probed as fully as the influence of C_0 and C_s was in section 3.1, due to a lack of literature investigating the effects of surface area on adsorption kinetics. However, an example of the influence of surface area on initial kinetics is given in Figure 8. Instead, different literature sources with the same sorbate-sorbent combinations were compared on the basis of surface area.

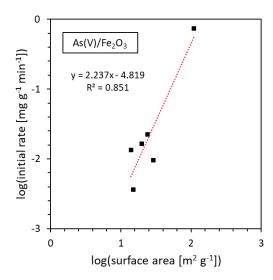


Figure 8: Example of the influence of surface area on initial kinetics. Experimental data from Di et al. 27.

A correlation between increasing surface area and k' was identified from a comparison of the compiled literature data (Figure 9). For arsenic oxyanions with mineral sorbents this correlation was weak: the average gradient of log(k') as a function of log(surface area) was just 0.404 ± 0.472 ($R^2=0.287$ only, N=5 sorbate-sorbent combinations). The correlation was more significant when considering all arsenic oxyanion-sorbent combinations as a single data set, giving a gradient of 0.564 ± 0.192 ($R^2=0.291$, N=23 data sets). Whilst the upper bound of log(k') values increased with increasing surface area, the lower bound changed little, and this generated the large uncertainty in the gradient and the source for low R^2 values.

Similar results were found for activated carbon/chitosan data sets: whilst the upper bound of log(k') increased with surface area, the lower bound changed little. The average gradient was 0.150 ± 0.167 (R^2 =0.043, N=4 sorbate-sorbent combinations), whilst the combination of all data into a single data set gave a gradient of 0.197 ± 0.535 (R^2 =0.292, N=20). The fluoride/ Al_2O_3 and methylene blue/ TiO_2 systems showed the strongest correlations. For fluoride/ Al_2O_3 the gradient was 4.36 ± 2.49 (R^2 =0.435, N=6), and for methylene blue/ TiO_2 the gradient was 0.97 ± 0.81 (R^2 =0.419, N=3).

High surface areas can be achieved both by reducing particle radius and by introducing greater porosity. The poor goodness of fit in the linear regression might be due to a convolution of particle radius and porosity effects within the measured surface area, meaning that surface area is an inappropriate choice for constraining variables. It is also worth noting that the PSO rate constant k_2 tended to decrease with increasing surface area (Supplementary Information). This is explained by the term $(q_e-q_t)^2$ within the PSO rate equation and the fact that q_e (mg g^{-1}) is roughly proportional to surface area.

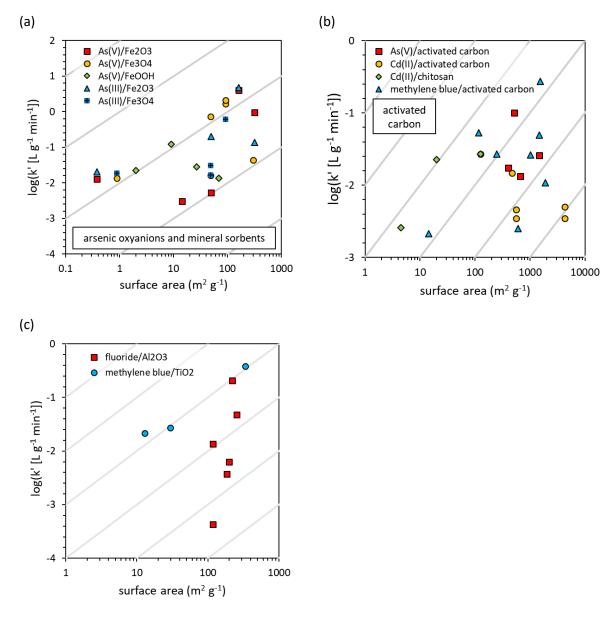


Figure 9: Greater values of rate constant k' were obtained with increasing surface area, demonstrated for (a) arsenic oxyanion and mineral sorbent systems, (b) activated carbon and chitosan systems, and (c) fluoride/ Al_2O_3 and methylene blue/ TiO_2 . Each data point within a given data set refers to a separate study, so despite sharing a chemical formula, the sorbents within a data set may differ in sorbent morphology. Grey lines indicate the 1:1 gradient between log(k') and log(surface area).

3.7.2. Particle size

In the intraparticle diffusion model, adsorption kinetics are controlled by the rate at which sorbate diffuses within sorbent pores ²⁸. One form of the intraparticle diffusion kinetic model is as follows:

$$\frac{\delta q_t}{\delta t} = \frac{D}{r^2} \frac{\delta}{\delta r} (r^2 \frac{\delta q_t}{\delta r})$$

Equation 21

where D is the effective diffusivity of the sorbate, and r is the radius of spherical sorbent particles 29 . In the intraparticle diffusion model the rate of adsorption is thus proportional to the inverse of r^2 .

The influence of particle size on adsorption kinetics has been investigated more thoroughly in the literature than the influence of surface area, so an analysis of initial rates was performed, with the dependence of initial rate, k' and k_2 upon particle radius illustrated in Figure 10 (N=14 data sets with 37 kinetic experiments). Due to the wide variety of techniques used to report particle size, values of r have a large degree of uncertainty: the literature reported particle size as the size fractionation achieved through mesh sieving (either upper bounds, lower bounds, or both); the particle size distribution of sorbent suspensions determined by dynamic light scattering (itself strongly dependent on pH); and from SEM and TEM images.

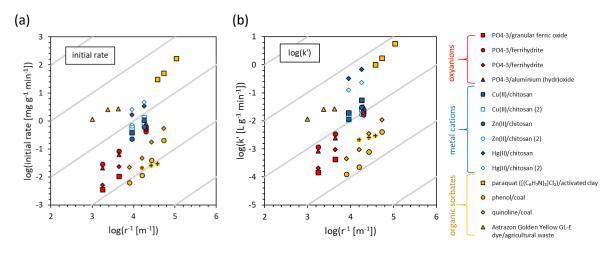


Figure 10: The relationship between (a) the initial rate and (b) k' and particle radius. Each data set is sourced from a separate study, i.e. all data points within a given data set uses the same sorbent, with the same morphology. Grey lines are a visual reference for the 1:1 gradient. The relationship between particle radius and k_2 is presented in the Supplementary Information.

Log(initial rate) as a function of $\log(\frac{1}{r})$ gave an average gradient of 1.25±0.57 (\tilde{x} =1.31). For $\log(k_2)$ the gradient was only 0.982±0.891 (\tilde{x} =1.08). The new rate constant, as $\log(k')$, gave a gradient of 1.12±0.63 (\tilde{x} =1.15), lying between those of log(initial rate) and $\log(k_2)$. This again suggests that the modified model gives a better description of kinetic sensitivities to experimental conditions, in this case, particle size. With a gradient of 1.25±0.57, the dependence of initial rate upon particle radius was thus determined to be closer to first-order than second-order, in contradiction to the $(\frac{1}{r^2})$ dependency of rate given by the intraparticle diffusion model (Equation 21).

Having determined the empirical relationship between initial rate, k' and particle radius using the method of initial rates, we then investigated whether adsorption kinetic data reported in the literature using the same sorbate-sorbent concentrations could be rationalised on the basis of particle radius. The relationship between particle radius and log(k') was weaker when incorporating multiple literature sources into a single sorbate-sorbent data set, compared with when using a single literature source where the influence of particle radius was explicitly investigated (Figure 11).

As discussed, using a single literature source for each sorbate-sorbent combination, an average gradient of 1.25±0.57 was obtained (median=1.31, N=14 data sets ad 37 kinetic experiments), indicating a first-order, or greater than first-order, dependence of k' upon 1/r. However, when comparing the same sorbate-sorbent combinations but between different literature sources, the dependency was only 0.405±0.341 (x=0.353, N=12 data sets and 64 sources) indicating a relationship that is weaker than first-order (Figure 11a-d). In the first case, literature studies investigating the influence of particle size upon adsorption kinetics primarily achieved a range of sorbent particle sizes

by size fractionation (sieving) which often maintained surface area and porosity ³⁰. In the second case where sorbate-sorbent combinations are compared between different literature sources, despite having the same chemical formula, sorbents were synthesised under different experimental conditions and the surface morphology and porosity cannot be assumed constant. Combining all data points, when not categorised into sets of the same sorbate-sorbent combination, a gradient of 0.329±0.095 (N=12 data sets and 64 sources) was obtained, not dissimilar to the gradient obtained when data was catalogued into same sorbate/sorbent combination sets.

With each sorbate-sorbent combination considered as a separate data set, the average order of reaction was 0.404 ± 0.472 with respect to surface area, and 0.405 ± 0.341 with respect to 1/r. Combining all data points (with different sorbate-sorbent combinations) into a single data set gave an average order of reaction of 0.564 ± 0.192 with respect to surface area, and 0.329 ± 0.095 with respect to 1/r. In both cases the uncertainties in the linear regression between log(k') and $log(r^{-1})$ were smaller than the regression between log(k') and log(surface area) both on an absolute and relative basis. Particle radius is thus a better constraining parameter for adsorption kinetics than surface area, and whilst rate constant k' could not be normalised across literature data into a single unifying value on the basis of particle radius alone (Figure 11a-d), particle size is clearly an important factor in any discussion or cross-comparison of adsorption kinetics and normalised rate constants such as k'.

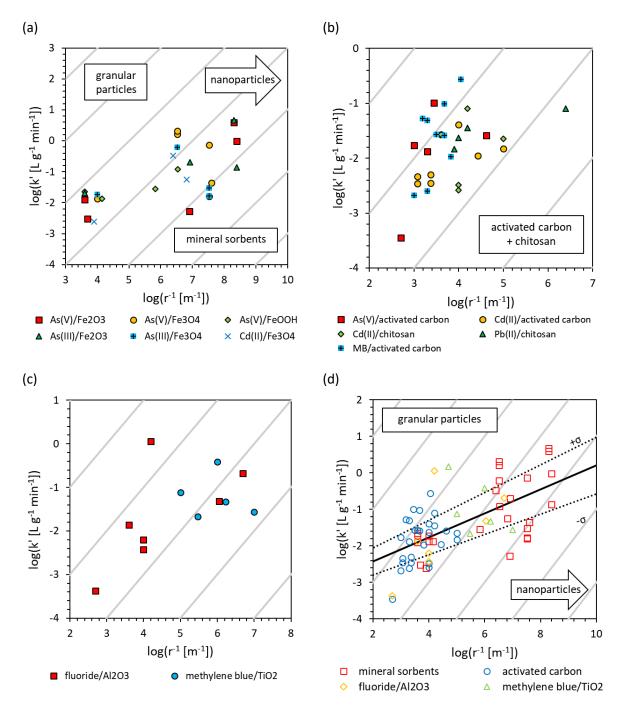


Figure 11: The relationship between k' and particle radius for (a) arsenic oxyanions/mineral sorbents, (b) activated carbon and chitosan sorbents, and (c) F'/Al_2O_3 and methylene blue/ TiO_2 systems. Each data point within a given data set is a separate study within the literature, i.e. despite sharing a chemical formula, data points within the set may differ in morphology. The solid and dashed black lines in figure (d) presents the average gradient of all data points combined into a single data set (0.329 ± 0.051) .

4. Conclusions

This work demonstrates that the PSO kinetic model can be made sensitive towards changes in initial sorbate concentration and sorbent concentration through modification into the form $\frac{dq_t}{dt} = k' C_t (1 - t)$ $\frac{q_t}{q_e}$)² where $\mathbf{k}' = \frac{k_2 q_e * 2}{C_0 *}$. This model provides a better fit to experimental data varying in C_0 and C_s than the unmodified PSO with fewer fitting parameters, and therefore provides a limited amount of predictive capability. The new rate equation is similar to an adsorption-only form of the kinetic Langmuir model (kLm), which at high surface coverage is first order with respect to Ct and second order to $(1-\frac{q_t}{q_e})^{25}$ 18, however with fewer parameters it is simpler, and may be more useful for the non-expert. Furthermore, normalising conditional rate constants k2 to our new k' is recommended to remove some of the error that arises when comparing literature adsorption kinetics data where experimental conditions are different 15. Finally, we explored the influence of sorbent morphology upon rate constant k' and demonstrated that particle size is a better constraining parameter than surface area. Particle radius was, however, insufficient to fully account for the variance remaining between experimentally determined values of rate constant, k'. This study indicates that the modified kinetic model and its rate constant are better equipped for comparing new sorbents with literature data than the original pseudo-second order model and k2, with k' being easily calculated from PSO parameters through the expression $k' = \frac{k_2 q_e^{*2}}{C_0^{*}}$.

5. Acknowledgements

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A revised pseudo-second order kinetic model for adsorption, sensitive to changes in sorbate and sorbent concentrations

Supplementary Information

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1. Data sets

Table 1: Data sets collected, and the experimental conditions varied across each set. C_0 is the initial sorbate concentrations $(mg\ L^{-1})$, C_s is the sorbent concentration $(g\ L^{-1})$ and r is particle radius $(nm, \mu m\ or\ mm)$.

Sorbate	Sorbent	Factor varied	Reference
As(V)	Fe ₂ O ₃	Initial sorbate concentration, C ₀	1
As(V)	Laterite	C ₀	2
As(III)	Hydrous ferric oxide	C ₀	3
HPO ₄ ²⁻	Iron hydroxide	C ₀	4
Cr(VI)	Activated carbon	C ₀	5
Cu(II)	Soil	C ₀	6
Zn(II)	Soil	C ₀	6
Pb(II)	Soil	C ₀	6
Hg(II)	Raw activated sludge	C ₀	7
Rose Bengal dye	ZnCl2-activated carbon	C ₀	8
Rhodamine B	MgO-Fe-Co-Mn	C ₀	9
Cr(VI)	Fe ₂ O ₃	sorbent concentration, C _s	10
Cr(VI)	Mg-Al-CO ₃	Cs	11
Cr(VI)	Chitin	Cs	12
PO ₄ 3-	Peat/amorphous iron(III) hydroxide	C _s	13
Cr(VI)	Guava seeds	Cs	14
Cd(II)	Fe ₂ O ₃	Cs	15
Cu(II)	Fe ₂ O ₃	Cs	15
Zn(II) Fe ₂ O ₃		C _s	15
Hg(II)	Hg(II) Lessonia nigrescens (kelp)		16
Hg(II)	Lessonia trabeculata (kelp)	C _s	16
Eriochrome Black-T dye	TiO ₂	C _s	17
Methylene blue	Rafia fibres	Cs	18
PO ₄ -3	PO ₄ -3 Granular ferric oxide		19
PO ₄ -3	Ferrihydrite	r	19
PO ₄ -3	Ferrihydrite	r	19
PO ₄ -3	Aluminium (hydr)oxide	r	19
Cu(II)	Chitosan	r	20

Sorbate	Sorbent	Factor varied	Reference		
Zn(II)	Chitosan	r	20		
Hg(II)	Chitosan	r	20		
paraquat, [(C ₆ H ₇ N) ₂]Cl ₂	Activated clay	r	21		
phenol	Coal	r	22		
Omega Chrome Red ME dye	Fly ash	r	23		
Astrazon Golden Yellow GL- E dye	Agricultural waste	r	24		
As(V)	Fe ₂ O ₃	-	1		
As(V)	Fe ₂ O ₃	-	25		
As(V)	Fe ₂ O ₃	-	26		
As(V)	Fe ₂ O ₃	-	27		
As(V)	Fe ₂ O ₃	-	28		
As(V)	Fe ₂ O ₃	-	29		
As(V)	Fe ₃ O ₄	-	27		
As(V)	Fe ₃ O ₄	-	30		
As(V)	Fe ₃ O ₄	-	31		
As(V)	Fe ₃ O ₄	-	32		
As(V)	Fe ₃ O ₄	-	33		
As(V)	Fe ₃ O ₄	-	30		
As(V)	FeOOH	-	34		
As(V) FeOOH		-	27		
As(V) FeOOH		-	35		
As(V) FeOOH		-	36		
As(V) FeOOH		-	37		
As(III)	Fe ₂ O ₃	-	26		
As(III)	Fe ₂ O ₃	-	27		
As(III)	Fe ₂ O ₃	-	28		
As(III)	Fe ₂ O ₃	-	29		
As(III)	Fe ₃ O ₄	-	27		
As(III)	Fe ₃ O ₄	-	30		
As(III)	Fe ₃ O ₄	-	32		
As(V) Activated carbon		-	38		

Sorbate	Sorbent	Factor varied	Reference		
As(V)	Activated carbon	-	39		
As(V)	Activated carbon	-	40		
As(V)	Activated carbon	-	41		
As(V)	Activated carbon	-	42		
Cd(II)	Activated carbon	-	43		
Cd(II)	Activated carbon	-	44		
Cd(II)	Activated carbon	-	45		
Cd(II)	Activated carbon	-	46		
Cd(II)	Activated carbon	-	47		
Cd(II)	Activated carbon	-	48		
Methylene blue	Activated carbon	-	49		
Methylene blue	Activated carbon	-	50		
Methylene blue	Activated carbon	-	51		
Methylene blue	Activated carbon	-	52		
Methylene blue	Activated carbon	-	53		
Methylene blue	Activated carbon	-	54		
Methylene blue	Activated carbon	-	55		
Methylene blue	Activated carbon	-	56		
Methylene blue	Activated carbon	-	57		
Methylene blue	Activated carbon	-	58		
Cd(II)	Chitosan	-	59		
Cd(II)	Chitosan	-	60		
Cd(II)	Chitosan	-	61		
Cd(II)	Chitosan	-	62		
Cd(II)	Chitosan	-	63		
Cd(II)	Chitosan	-	64		
Methylene blue	TiO ₂	-	65		
Methylene blue	TiO ₂	-	66		
Methylene blue	TiO ₂	-	67		
Methylene blue	TiO ₂	-	68		
Methylene blue	TiO ₂	-	69		
Methylene blue	TiO ₂	-	70		

Sorbate	Sorbent	Factor varied	Reference	
Methylene blue	TiO ₂	-	71	
Methylene blue	TiO ₂	-	72	
Fluoride	Al ₂ O ₃	-	73	
Fluoride	Al ₂ O ₃	-	74	
Fluoride	Al ₂ O ₃	-	75	
Fluoride	Al ₂ O ₃	-	76	
Fluoride	Al ₂ O ₃	-	77	
Fluoride	Al ₂ O ₃	-	78	
Fluoride	Al ₂ O ₃	-	79	

2. Adsorption isotherm fitting

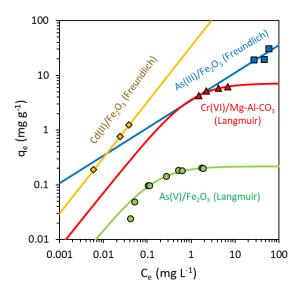


Figure 1: Adsorption isotherms were fit using experimental data and then used to recalculate q_e at each point in time when modelling kinetic experiments using our modified form of the pseudo-second order rate equation.

3. Kinetic parameters for predictive modelling

Table 2: Parameters used to model selected data sets using the PSO model and the modified model, using a single rate constant to model experimental data where C_0 and C_s are varied.

			PSO model		modified model						
system	C ₀ (mg L ⁻¹)		k ₂ (g mg ⁻¹ min ⁻¹)	Qe D2	k'	Langmuir parameters		Freundlich parameters			
,,,,,				(mg g ⁻¹)	R ²	(L g ⁻¹ min ⁻¹)	K _L (L mg ⁻¹)	Q _{max} (mg L ⁻¹)	K _F (mg g ⁻¹ (mg L ⁻¹) ^{-1/n})	n	R ²
	50	2		19.3	0.9619	0.0036					
As(III)/HFO	75	2	0.00053	19.8					3.5	1.98	0.9735
	100	2		31.1							
	1	40	1.10	0.034	0.5248	0.0029	7.21	0.22			
	2	40		0.063							
As(V)/Fe ₂ O ₃	4	40		0.15							0.7968
	8	40		0.22							
	10	40		0.25							
	0.1	0.05	1.46	1.29	0.7575	1.72					
Cd(II)/Fe ₂ O ₃	0.1	0.1		0.77					34.4	0.98	0.7617
	0.1	0.5		0.19							
	10	0.5		7.33	0.8325	0.34	0.99	7.20			
Cr(VI)/Mg- Al-CO₃	10	1	0.127	6.11							0.8732
	10	1.5	0.127	5.28							0.0732
	10	2		4.32							

4. Further results

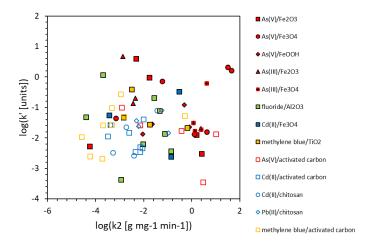


Figure 2: After normalising for C_0 and C_s , significant variation in k' remains between different literature sources. This is at least partially due to differences in sorbent morphology.

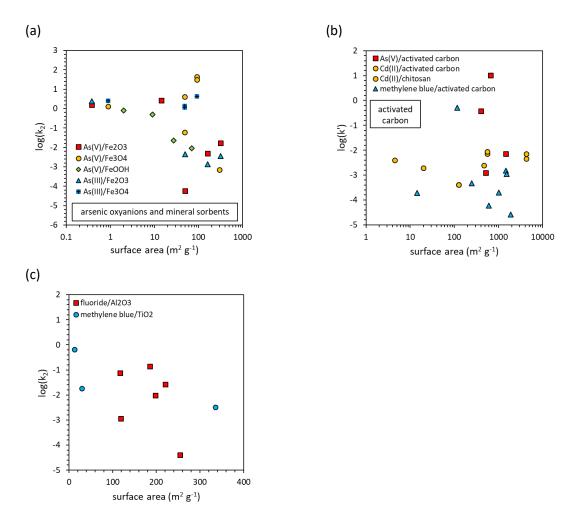


Figure 3: Influence of surface area on $log(k_2)$.

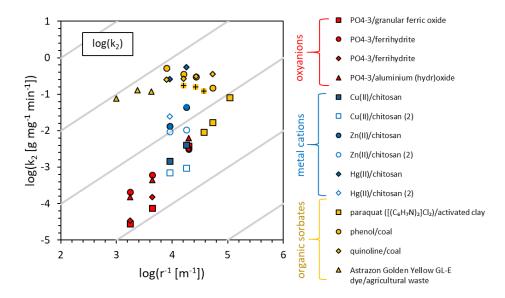


Figure 4: Influence of particle size on $log(k_2)$.

5. Final data table

Table 3: Initial sorbate concentration (C_0), sorbent concentration (C_s), surface area and particle size as well as the calculated values of PSO rate constant k_2 , new rate constant k_2 , new rate constant k_3 , new rate constant k_4 , and the initial rate (calculated via the PSO model), for adsorption kinetic experiments collected from the literature. In most cases, but not all, q_e and k_2 were calculated in this work. In a few other cases these parameters have been pulled directly from the literature.

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g⁻¹)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
			Varying initial s	orbate concentrati	on, C ₀				
	1	40	14.4	0.2	0.0341	2.58	0.003	0.003	
	2	40	14.4	0.2	0.0629	1.51	0.003	0.00597	
As(V)/Fe2O3	4	40	14.4	0.2	0.154	0.405	0.00241	0.0096	1
	8	40	14.4	0.2	0.222	0.532	0.00329	0.0262	
	10	40	14.4	0.2	0.249	0.480	0.00297	0.030	
	1	2	18.05	0.23-0.65	0.267	0.267	0.0136	0.019	
	2	2	18.05	0.23-0.65	0.789	0.789	0.005	0.491	
As(V)/laterite	4	2	18.05	0.23-0.65	1	1	0.00317	1	2
	8	2	18.05	0.23-0.65	1.05	1.05	0.00309	1.16	
	10	2	18.05	0.23-0.65	1.09	1.09	0.00309	1.3	
	50	2		0.14-0.29	19.3	0.000503	0.00376	0.187	
As(III)/HFO	75	2		0.14-0.29	19.8	0.000738	0.00384	0.289	3
	100	2		0.14-0.29	31.1	0.000335	0.00324	0.324	
	2.8	7.5		0.05-0.315	0.35	0.722	0.0316	0.0884	4
HPO4-2/iron hydroxide	14	7.5		0.05-0.315	2.03	0.0128	0.00374	0.0527	

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-1)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
	53	7.5		0.05-0.315	3.73	0.0156	0.0041	0.217	
	110	7.5		0.05-0.315	9.26	0.00551	0.0043	0.472	
	20	10	287	<1	0.84	0.177	0.00625	0.125	
Cu(II)/soil	50	10	287	<1	1.29	0.0636	0.00212	0.106	6
	100	10	287	<1	1.94	0.0505	0.00189	0.19	
	50	10	287	<1	0.797	0.227	0.00288	0.144	6
Zn(II)/soil	100	10	287	<1	0.995	0.223	0.00221	0.221	
	20	10	287	<1	0.831	0.146	0.00504	0.101	
Pb(II)/soil	50	10	287	<1	3.04	0.0123	0.00227	0.114	6
	100	10	287	<1	3.35	0.0507	0.00569	0.569	
	10	0.1	59.33		95.4	0.0259	23.6	236	
	30	0.1	59.33		285	0.013	35.2	1060	
Rhodamine B/MgO-Fe-Co-Mn	60	0.1	59.33		568	0.0035	18.8	1130	9
	80	0.1	59.33		752	0.002	14.1	1130	
	120	0.1	59.33		1120	0.0015	15.6	1880	
	50	2		0.125-0.212	20.7	0.0021	0.018	0.9	
	100	2		0.125-0.212	35.5	0.00126	0.016	1.59	7
Hg(II)/raw activated sludge	150	2		0.125-0.212	39.5	0.00287	0.0299	4.48	,
	200	2		0.125-0.212	46.9	0.00353	0.0389	7.76	
	100	0.5	38.7	0.15-0.42	6.41	0.00662	0.00272	0.272	5
Cr(VI)/activated carbon	200	0.5	38.7	0.15-0.42	11.6	0.00371	0.00252	0.499	

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-1)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
	300	0.5	38.7	0.15-0.42	16.5	0.00336	0.00303	0.915	
	400	0.5	38.7	0.15-0.42	21.2	0.00338	0.00382	1.52	
	500	0.5	38.7	0.15-0.42	25.1	0.0034	0.0043	2.14	
	10	0.05		0.073-0.15	160	0.000706	1.8	18.1	
	20	0.05		0.073-0.15	312	0.000185	0.903	18	
Rose Bengal dye/activated carbon	30	0.05		0.073-0.15	451	0.000123	0.83	25	8
	40	0.05		0.073-0.15	584	0.0000839	0.715	28.6	
	50	0.05		0.073-0.15	709	0.0000643	0.646	32.3	
l			Varying sor	bent concentration,	Cs				
	50	1			49	0.000838	0.0402	2.01	
	50	1.5			39.4	0.000826	0.0257	1.28	40
Cr(VI)/Fe2O3	50	2			29.9	0.00121	0.0217	1.08	10
	50	3			19.6	0.00213	0.0164	0.818	
	10	0.5	83.95	0.125-0.25	6.35	0.0812	0.328	3.27	
	10	1	83.95	0.125-0.25	5.86	0.0879	0.302	3.02	44
Cr(VI)/Mg-Al-CO3	10	1.5	83.95	0.125-0.25	5.28	0.157	0.438	4.38	11
	10	2	83.95	0.125-0.25	4.32	0.239	0.446	4.46	
	100	0.5		0.25-0.42	36.7	0.00164	0.0221	2.21	
	100	0.75		0.25-0.42	25.4	0.00402	0.026	2.59	12
Cr(VI)/chitin	100	1		0.25-0.42	29	0.00548	0.046	4.61	12
	100	1.5		0.25-0.42	8.14	0.0284	0.0188	1.88	

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-1)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
	100	2		0.25-0.42	7.98	0.013	0.00826	0.828	
	100	2.5		0.25-0.42	5.53	0.0205	0.00627	0.627	
PO4-3/peat/amorphous iron(III)	25	0.2	43.8		4.72	0.001	0.000891	0.0223	13
hydroxide	25	1	43.8		2.02	0.037	0.00604	0.151	20
	75	5		0.3-0.8	10	0.0101	0.0136	1.01	
	75	7.5		0.3-0.8	7.63	0.0136	0.0105	0.792	
Cr(VI)/guava seeds	75	10		0.3-0.8	6.63	0.0168	0.00988	0.738	14
	75	12.5		0.3-0.8	5.79	0.0169	0.00753	0.567	
	75	15		0.3-0.8	4.88	0.0266	0.00844	0.633	
	0.1	0.05	31.7		1.29	0.0986	1.63	0.164	
Cd(II)/Fe2O3	0.1	0.1	31.7		0.765	0.362	2.12	0.212	15
	0.1	0.5	31.7		0.189	3.9	1.4	0.139	
	0.1	0.05	31.7		0.852	0.0181	0.129	0.0131	
Cu(II)/Fe2O3	0.1	0.1	31.7		0.435	0.252	0.47	0.0477	15
	0.1	0.5	31.7		0.187	0.656	0.225	0.0229	
	0.1	0.05	31.7		1.61	0.0336	0.894	0.0871	
Zn(II)/Fe2O3	0.1	0.1	31.7		0.983	0.216	2.13	0.209	15
	0.1	0.5	31.7		0.197	7.47	2.97	0.29	
	50	0.4		0.125-0.25	97.4	0.00034	0.0645	3.23	
Hg(II)/Lessonia nigrescens	50	0.2		0.125-0.25	143	0.000171	0.0695	3.5	16
	50	0.1		0.125-0.25	190	0.000073	0.0526	2.64	

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-¹)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
	50	0.4		0.125-0.25	90.5	0.00039	0.0639	3.19	
Hg(II)/Lessonia trabeculata	50	0.2		0.125-0.25	140	0.000148	0.0579	2.9	16
	50	0.1		0.125-0.25	200	0.000089	0.0713	3.56	
	25	0.2			101	0.0012	0.489	12.2	
	25	0.4			58.8	0.00335	0.463	11.6	
Eriochrome Black-T dye/TiO2	25	0.6			37.4	0.0038	0.212	5.32	17
	25	0.8			30	0.00484	0.174	4.36	
	25	1			24.7	0.00613	0.15	3.74	
	11	2	1.08		5.91	0.000198	0.000629	0.00692	
	11	2.5	1.08		4.55	0.000491	0.000922	0.0102	
methylene blue/rafia fibres	11	3	1.08		3.66	0.000808	0.000985	0.0108	18
	11	3.5	1.08		3.21	0.00114	0.00107	0.0117	
	11	4	1.08		2.67	0.00188	0.00122	0.0134	
			Varyi	ng surface area					
	35.21	1	20		2.43	0.00295	0.000493	0.0174	
	35.21	1	29		2.02	0.00245	0.000284	0.01	
	35.21	1	14		2.1	0.0032	0.000402	0.0141	80
As(V)/Fe2O3	35.21	1	24		2.99	0.0027	0.000685	0.0241	35
	35.21	1	15		0.911	0.00454	0.000107	0.00377	
	35.21	1	109		75.8	0.000141	0.023	0.81	
	1		Varyin	g particle radius		<u> </u>			

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-¹)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
	25	2	244	1.13	11.3	0.000028	0.000143	0.00358	
PO4-3/granular ferric oxide	25	2	244	0.45	11.8	0.000075	0.000418	0.0104	19
	25	2	244	0.1	12.1	0.0038	0.0223	0.556	
	25	2	179	1.13	11.8	0.00021	0.00117	0.0292	
PO4-3/ferrihydrite	25	2	179	0.45	11.7	0.00062	0.00339	0.0849	19
	25	2	179	0.1	11.6	0.0031	0.0167	0.417	
	25	2	119	1.13	12.4	0.000034	0.000209	0.00523	
PO4-3/ferrihydrite	25	2	119	0.45	12.6	0.00015	0.000953	0.0238	19
	25	2	119	0.1	12.6	0.0031	0.0197	0.492	
	25	11	235	1.13	11.8	0.00015	0.000835	0.0209	
PO4-3/aluminium (hydr)oxide	25	12	235	0.45	11.9	0.00044	0.00249	0.0623	19
	25	13	235	0.1	11.3	0.0063	0.0322	0.804	
	30	0.25	266.91	0.053	57.8	0.0092	1.02	30.7	
paraquat ([(C ₆ H ₇ N) ₂]Cl ₂)/activated clay	30	0.25	266.71	0.037	55.6	0.0166	1.71	51.3	21
Clay	30	0.25	229.42	0.0185	45.5	0.082	5.65	170	
	50	40		0.037	1.16	0.15	0.00404	0.202	
	50	40		0.074	0.36	0.303	0.000785	0.0393	
phenol/coal	50	40		0.125	0.18	0.353	0.000229	0.0114	22
	50	40		0.25	0.11	0.52	0.000126	0.00629	
	50	40		0.037	1.24	0.353	0.0108	0.543	
quinoline/coal	50	40		0.074	0.76	0.308	0.00356	0.178	81

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g⁻¹)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
	50	40		0.125	0.42	0.27	0.000952	0.0476	
	20	0.125-0.5		0.11	16.3	0.00407	0.054	1.08	
Cu(II)/chitosan	20	0.125-0.5		0.22	16.1	0.00148	0.0191	0.384	20
	50	0.125-0.5		0.11	36.9	0.000958	0.0261	1.3	
Cu(II)/chitosan (2)	50	0.125-0.5		0.22	28.3	0.000707	0.0113	0.566	20
	20	0.125-0.5		0.11	3.75	0.0436	0.0307	0.613	20
Zn(II)/chitosan	20	0.125-0.5		0.22	4.13	0.0134	0.0114	0.229	20
	50	0.125-0.5		0.11	8.98	0.0105	0.0169	0.847	
Zn(II)/chitosan (2)	50	0.125-0.5		0.22	8.57	0.00939	0.0138	0.69	20
	20	0.125-0.5		0.11	2.5	0.546	0.683	3.41	20
Hg(II)/chitosan	20	0.125-0.5		0.22	2.5	0.263	0.329	1.64	20
	50	0.125-0.5		0.11	10.1	0.0457	0.232	4.66	
Hg(II)/chitosan (2)	50	0.125-0.5		0.22	10.1	0.0248	0.127	2.53	20
	10		5.77	0.053	0.496	0.121	0.00298	0.0298	
Omega Chrome Red ME dye/fly ash	10		5.77	0.075	0.406	0.156	0.00257	0.0257	23
	10		5.77	0.125	0.35	0.173	0.00212	0.0212	
		ı	Extra data for li	terature cross-compa	rison	1			
As(V)/Fe2O3	1.5	4	56	0.0001	7.13	0.116	3.94	5.9	82
As(V)/Fe2O3	0.2	0.01	50	6.25E-08	4.31	0.0000567	0.00527	0.00105	26
As(V)/Fe2O3	1.5	0.1	0.38	0.000125	0.109	1.58	0.0126	0.0188	27
As(V)/Fe2O3	0.01	0.005	162	2.5E-09	8.87	0.004	33.1	0.315	28

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-1)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
	0.01	0.007	162	2.5E-09	8.01	0.006	40.5	0.385	
	0.01	0.01	162	2.5E-09	7.12	0.019	101	0.963	
	0.01	0.02	162	2.5E-09	4.78	0.033	79.4	0.754	
As(V)/Fe2O3	10	0.4	317	0.000000002	23.7	0.017	0.955	9.55	29
	2000.36	2.5	202		176	1.29	20	40000	
	40.01	2.5	202		124	0.000859	0.328	13.2	83
As(V)/ferrihydrite	2000.36	2.5	202		19.7	0.458	0.0888	178	
	40.01	2.5	202		19.8	0.989	9.72	388	
As(V)/ferrihydrite	50	1	120		31.5	0.00066	0.0131	0.655	84
As(V)/Fe3O4	1.5	0.1	0.89	0.00005	0.125	1.29	0.0133	0.0202	27
	0.04	1	92	0.0000015	0.0364	44.6	1.64	0.0591	30
As(V)/Fe3O4	0.07	1	92	0.0000015	0.0663	31	2.07	0.136	
As(V)/Fe3O4	1	0.2	94.07	0.00000025	4.3	2.75	50.9	50.8	31
As(V)/Fe3O4/γ-Fe2O3	1	0.4	49	0.00000015	0.0619	4.15	0.0159	0.0159	32
As(V)/Fe3O4/γ-Fe2O3	2	0.4	49	0.00000015	4.97	0.059	0.729	1.46	32
As(V)/Fe3O4-HBC	0.1	0.2	0.03		0.474	0.0162	0.0365	0.00364	33
As(V)/Fe3O4	1	0.1	300	1.25E-08	7.89	0.000704	0.0438	0.0438	85
As(V)/FeOOH	8.99	1.98	27	0.0000075	3.34	0.0228	0.0282	0.254	34
As(V)/FeOOH	1.5	0.1	2.01	0.000125	0.205	0.797	0.0224	0.0335	27
As(V)/β-FeOOH	3.36	0.27	9	0.00000145	0.911	0.488	0.121	0.405	35
As(V)/FeOOH	5	1.8	69.62	0.00000145	2.73	0.00496	0.00739	0.037	36

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-1)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
As(V)/FeOOH	5	1.8	69.62	0.000035	2.73	0.009	0.0134	0.0671	36
As(V)/FeOOH	74.92	2.5	103		12.6	0.00984	0.0208	1.56	37
As(III)/Fe2O3	0.2	0.01	50	0.0000006	3.02	0.00443	0.202	0.0404	26
As(III)/Fe2O3	1.5	0.1	0.38	0.000125	0.112	2.4	0.0201	0.0301	27
	0.12	0.01	162	2.5E-09	8.92	0.002	1.38	0.159	
	0.12	0.02	162	2.5E-09	5.46	0.006	1.56	0.179	28
As(III)/Fe2O3	0.12	0.04	162	2.5E-09	2.86	0.027	1.92	0.221	
	0.12	0.06	162	2.5E-09	1.92	0.086	2.76	0.317	
As(III)/Fe2O3	10	0.4	317	0.000000002	19.5	0.0036	0.137	1.37	29
	2000.36	2.5	202		306	0.0059	0.277	552	
	40.01	2.5	202		378	0.00279	9.98	399	83
As(III)/ferrihydrite	2000.36	2.5	202		19.7	0.422	0.082	164	03
	40.01	2.5	202		20	2.3	22.9	920	
As(III)/ferrihydrite	50	1	120		46.8	0.000913	0.0399	2	84
As(III)/ferrihydrite	50	1	120		48.1	0.00114	0.0529	2.64	04
As(III)/Fe3O4	1.5	0.1	0.89	0.00005	0.106	2.49	0.0186	0.028	27
As(III)/Fe3O4	0.16	1	92	0.0000015	0.155	4.22	0.619	0.101	30
	1	0.4	49	0.00000015	0.161	1.17	0.0304	0.0303	32
As(III)/Fe3O4/γ-Fe2O3	2	0.4	49	0.00000015	0.154	1.39	0.0165	0.033	32
As(III)/FeOOH	74.92	2.5	103		17.2	0.0413	0.164	12.2	37
As(III)/FeOOH	1.5	0.1	2.01	0.000125	0.212	1.12	0.0334	0.0503	27

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-1)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
Cd(II)/Fe3O4	50	5	47.87	0.0000625	0.938	0.139	0.00245	0.122	86
	200	1		0.00000138	174	0.000365	0.0554	11.1	
	200	1		0.00000138	124	0.000559	0.0433	8.6	87
Cd(II)/Fe3O4	200	1		0.00000138	133	0.000216	0.0192	3.82	.,
	200	1		0.00000138	118	0.000469	0.0324	6.53	
Cd(II)/Fe3O4	100	5		0.0000002	10.4	0.303	0.328	32.8	88
Cd(II)/activated carbon	50	1	471.67	0.000005	17.3	0.00248	0.0148	0.742	43
Cd(II)/activated carbon	495.61	2.5		0.0000185	131	0.000323	0.0111	5.54	44
Cd(II)/activated carbon	60	0.4	560	0.000425	5.41	0.00709	0.00346	0.208	45
Cd(II)/activated carbon	500	20	4273	0.000213	18.7	0.00716	0.00499	2.5	46
Cd(II)/activated carbon	100	0.05		0.00005	20.4	0.0099	0.041	4.12	47
Cu(II)/activated carbon	60	0.4	560	0.000425	5.64	0.00872	0.00462	0.277	45
Pb(II)/activated carbon	500	20	4273	0.000213	19.6	0.00452	0.00349	1.74	46
Cd(II)/chitosan	100	1	125.24	0.000125	81.3	0.000405	0.0268	2.68	60
Cd(II)/chitosan	100	5		0.00005	25	0.000525	0.00328	0.328	61
Cd(II)/chitosan	100	0	4.45	0.00005	8.08	0.00398	0.00259	0.26	62
Cd(II)/chitosan	50	4	20.06	0.000005	24.5	0.00189	0.0227	1.13	63
Cd(II)/chitosan	50	1.72		0.0000315	10.5	0.0363	0.0805	4	64
Pb(II)/chitosan	100	5		0.00005	19.6	0.00618	0.0237	2.37	61
Pb(II)/chitosan	50	1.72		0.0000315	18.9	0.00504	0.0359	1.8	64
Pb(II)/Fe3O4	100	5		0.0000002	11.9	0.0564	0.0801	7.99	88

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-1)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
Cu(II)/Fe3O4	50	5	47.87	0.0000625	2.6	0.108	0.0146	0.73	86
methylene blue/activated carbon	4.48	4	116	0.000325	0.682	0.508	0.0528	0.236	49
methylene blue/activated carbon	200	1		0.000106	203	0.000468	0.0968	19.3	50
methylene blue/activated carbon	750	10	1483	0.00025	157	0.00151	0.0492	37.2	51
methylene blue/activated carbon	250	1	1531	0.000045	250	0.00109	0.273	68.1	52
methylene blue/activated carbon	500	1	1894	0.000075	455	0.000026	0.0107	5.38	53
methylene blue/activated carbon	1000	0.5	600	0.00025	205	0.0000593	0.00249	2.49	54
methylene blue/activated carbon	600	1.333	1029	0.000106	280	0.000201	0.0262	15.8	55
methylene blue/activated carbon	200	1.8	14.27	0.0005	47.2	0.00019	0.00212	0.423	56
methylene blue/activated carbon	500	1.6	248	0.000158	169	0.00047	0.027	13.4	58
methylene blue/TiO2	12.5	0.5	13	0.0000017	0.647	0.642	0.0215	0.269	65
methylene blue/TiO2	100	2	29.6	0.0000005	12.4	0.0179	0.0274	2.75	66
methylene blue/TiO2	200	0.8		0.0000003	80.3	0.00145	0.0467	9.35	67
methylene blue/TiO2	20	0.2	335	0.000005	49	0.00321	0.386	7.71	68
methylene blue/TiO2	10	2		0.000005	4.06	0.0462	0.0762	0.762	69
fluoride/Al2O3	10	1	221	0.0000001	8.8	0.0268	0.207	2.08	73
fluoride/Al2O3	10	6.667	118.6	0.001	1.93	0.00115	0.000426	0.00428	74
fluoride/Al2O3	5	1		0.0000315	169	0.0002	1.15	5.71	75
fluoride/Al2O3	304	0.5	254.1	0.0000045	600	0.0000404	0.0479	14.5	76
fluoride/Al2O3	10	10	185.6	0.00005	0.522	0.137	0.00372	0.0373	77
fluoride/Al2O3	5	4	117.4	0.000125	0.944	0.0762	0.0136	0.0679	78

System	C ₀ (mg L ⁻¹)	Cs (g L ⁻¹)	Surface area (m² g-¹)	Particle size (mm)	q _e (mg g ⁻¹)	k ₂ (g mg ⁻¹ min ⁻¹)	k' (L g ⁻¹ min ⁻¹)	Initial rate (mg g ⁻¹ min ⁻¹)	Reference
fluoride/Al2O3	5	2.2	198	0.00005	1.81	0.00949	0.00623	0.0311	79
As(V)/Fe2O3	0.1	0.005	162	2.5E-09	8.77	0.00487	3.95	0.375	28
As(III)/Fe2O3	0.1	0.005	162	2.5E-09	18	0.00138	4.71	0.447	28
As(V)/activated carbon	4.5	1		0.001	0.0231	2.99	0.000354	0.0016	38
As(V)/activated carbon	10	1	1470	0.000012	6.05	0.00707	0.0258	0.259	39
As(V)/activated carbon	2	1	405	0.0005	0.303	0.376	0.0172	0.0345	40
As(V)/activated carbon	0.1	0.03	520	0.00018	2.87	0.00122	0.101	0.01	41
As(V)/activated carbon	0.15	5	660	0.00025	0.014	10.2	0.0133	0.002	42
Pb(II)/Al2O3	414.4	13.3	100		0.000591	93.9	7.92E-08	0.0000328	89
Pb(II)/Al2O3	20	5	243	0.0000005	4.27	0.0109	0.00992	0.199	90
methylene blue/TiO2	10	0.5	51	0.00001	21.1	0.0333	1.49	14.8	71

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