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On the Accurate Description of Photoionization Dynamical Parameters

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Abstract

Calculation of dynamical parameters for photoionization requires an accurate description of both initial and final states of the system, as well as of the outgoing electron. We here show, that using a linear combination of atomic orbitals (LCAO) B-spline density functional (DFT) method to describe the outgoing electron, in combination with correlated equation-of-motion coupled cluster singles and double (EOM-CCSD) Dyson orbitals, gives good agreement with experiment and outperforms other simpler approaches, like plane and Coulomb waves, used to describe the photoelectron. Results are presented for cross sections, angular distributions and dichroic parameters in chiral molecules, as well as for photoionization from excited states. We also present a comparison with the results obtained using Hartree-Fock (HF) and density-functional theory molecular orbitals selected according to Koopmans' theorem for the bound states.

The last two decades have witnessed a remarkable increase in the study of sophisticated light-matter interactions, due to enormous advancements in synchrotron and laser sources. This is dictating high demands for reliable theoretical and computational methodologies.^{1,2} Photoelectron spectroscopy (PES) offers detailed information about the electronic structure of the target molecule and the properties of the ejected electrons. Photoionization has recently become important as an ultrafast probe in time-resolved experiments, both in the femto and atto second domains, as well as in the description of many strong-field phenomena.²⁻⁵ However, the interpretation of such spectroscopic signatures requires that relaxation and correlation effects in the description of bound and continuum states are taken into account. Even though, in general, the distinction between relaxation and correlation is not always well defined, we focus here on correlation effects, which are the dominant effects for valence-shell ionization. Correlation effects in the bound states and on the ionization energies have been studied extensively using several many-body approaches, like configuration interaction (CI),⁶⁻⁸ perturbation theory (PT),⁶ coupled cluster (CC)⁹⁻¹³ and Green's functions (GF).^{6,14-16} Additional correlation effects are hidden in the description of the continuum, the interaction between continuum channels or between continuum and bound states.

The calculation of the photoionization observables is based on the evaluation of the dipole transition moments,

$$\vec{D}^{if} = \langle \Psi_i^N | \vec{\mu} | \Phi_f \rangle \quad (1)$$

which connect the initial (Ψ_i^N) and final (Φ_f) wave functions. The $\vec{D}^{if}$ transition elements are linked to the differential cross section, given by (in a.u.)¹⁷

$$\frac{d\sigma}{d\vec{k}} = 4\pi^2 \alpha \omega |\vec{D}^{if}|^2 \quad (2)$$

where α is the fine structure constant, ω is the photon energy and $\vec{k}$ is the momentum of the photoelectron in the molecular frame.

Koopmans' theorem is the simplest way to elucidate PES. The basic assumption is a

single determinant (or single particle) description of both the initial and final states, without including orbital relaxation effects. This approximation gives a one-to-one correspondence between a molecular orbital and a spectral band, such that the ionization energy equals minus the corresponding orbital energy, $IE = -\epsilon_i$. The model fails for ionizations involving multielectron excitations, shake-up or satellite bands.^{7,16}

To overcome these limitations, correlation effects need to be taken into account. A key quantity in doing so is the Dyson orbital. Considering a single channel approximation of the final state^{11,18,19}

$$\Phi_f = \Psi_f^{N-1} \phi_\epsilon, \quad (3)$$

the photoelectron matrix element $\vec{D}^{if}$ can be rewritten as

$$\vec{D}^{if} = \langle \phi_{if}^d | \vec{\mu} | \phi_\epsilon \rangle \quad (4)$$

where N is the number of electrons in the initial state, $\vec{\mu}$ is the electric dipole moment operator, ϕ_ϵ is the wave function of the ejected electron and ϕ_{if}^d is the Dyson orbital. The latter is defined as the overlap of the normalized wave functions for the bound N and $(N-1)$ systems:^{14,20}

$$\phi_{if}^d = \sqrt{N} \int \Psi_i^N(x_1, x_2, \dots, x_N) \Psi_f^{N-1}(x_2, \dots, x_N) dx_2 \dots dx_N = \sum_p \gamma_p \phi_p. \quad (5)$$

As indicated by the second equality, the Dyson orbital can be shown to reduce to a linear combination of ground-state molecular orbitals (ϕ_p), with coefficients γ_p . We refer to the Supporting Information for details on the derivation of Eqs. (4) and (5).

Thus, within the single channel approach, correlation effects in the bound states (both initial and final), that are in many cases the dominant ones, can be effectively incorporated via the use of the Dyson orbital approach. The nature of the Dyson orbitals acts as a marker of the effect of correlation upon changing from neutral to ionic system. It can be thought

of as a correlated counterpart of a Hartree-Fock orbital describing the initial state of the ionized electron within Koopmans' theorem.

In this work we use a EOM-CCSD framework,¹¹ where the left (label L) and right (label R) Dyson orbitals are distinct.¹¹ For ionisation from the ground state, their expansion coefficients γ_p^L (defined as $\langle \Psi_L^N | a_p^\dagger | \Psi_R^{N-1} \rangle$) and γ_p^R (defined as $\langle \Psi_L^{N-1} | a_p | \Psi_R^N \rangle$) are given by:

$$\gamma_p^R = \langle \text{HF} | (1 + \hat{\Lambda}) \exp(-T_2 - T_1) a_p^\dagger R^{\text{IP}} \exp(T_1 + T_2) | \text{HF} \rangle \quad (6)$$

$$\gamma_p^L = \langle \text{HF} | L^{\text{IP}} \exp(-T_2 - T_1) a_p \exp(T_1 + T_2) | \text{HF} \rangle \quad (7)$$

whereas for ionisation from an excited state they are found as

$$\gamma_p^R = \langle \text{HF} | L^{\text{EE}} \exp(-T_2 - T_1) a_p^\dagger R^{\text{IP}} \exp(T_1 + T_2) | \text{HF} \rangle \quad (8)$$

$$\gamma_p^L = \langle \text{HF} | L^{\text{IP}} \exp(-T_2 - T_1) a_p R^{\text{EE}} \exp(T_1 + T_2) | \text{HF} \rangle \quad (9)$$

In Eqs. (6)–(9), we have used the standard EOM-CCSD notation,^{12,21,22}

$$\hat{\Lambda} = \sum_{\lambda_i=\lambda_1, \lambda_2} \bar{t}_{\lambda_i} \tau_{\lambda_i}^\dagger \quad (10)$$

$$R^{\text{IP}} = \sum_i r_i a_i + \frac{1}{2} \sum_{aij} r_{ij}^a a_a^\dagger a_j a_i \quad (11)$$

$$R^{\text{EE}} = r_0 + \sum_{ai} r_i^a a_a^\dagger a_i + \frac{1}{4} \sum_{abij} r_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i \quad (12)$$

Having taken care of correlation effects in the bound states via, e.g., the Dyson orbital, the next step is to choose a description of the single particle continuum. We remark here that the development of computational approaches to molecular photoionization has spanned several decades.^{23–25} Nonetheless, full close-coupling approaches, taking in principle account of interchannel interactions, have progressed slowly, and are still computationally intensive and limited to relatively small systems. In fact, most approaches for polyatomic molecules are still at the static-exchange level, which is the counterpart for the photoelectron of the

Koopmans description. Exceptions are the Stieltjes approach,^{23,26} and the equivalent Padé approach,²⁷ that completely avoid the explicit construction of continuum states, but reconstructs cross section from purely bound pseudo-spectra. However, due to the lack of proper asymptotic boundary conditions, the separation of individual channels becomes ambiguous, and angular distributions cannot be obtained. Also, large basis sets are required and the energy resolution is quite limited. Even then, these methods have proven quite successful in combination with highly correlated electronic structure methods such as linear response coupled cluster^{28–31} and algebraic diagrammatic construction (ADC).^{32–34}

The simplest approach to approximate the continuum is with plane waves (PW) or Coulomb waves (CW), possibly orthogonalized to the Dyson orbital.^{9,35,36} Although this approach becomes accurate at high photoelectron energies, at least several hundred eV, it is generally rather poor at low energies, especially for angular distributions, or description of single particle continuum structures, like shape resonances, Cooper minima, and so on. A more accurate compromise, based on the muffin-tin approximation, is the multiple scattering approach.²³

Another strategy is to explicitly solve the Schrödinger equation for the photoelectron wave function, which can be rather accurately done, for instance, at the static-exchange DFT level, as we do in this work. Actually, even a time-dependent TDDFT continuum can be employed, which is currently under development. An initial result will be presented here for the argon atom.

To obtain an accurate description of the continuum in large systems, a basis set approach capable of a multicentric (LCAO) description is needed. Again, several possibilities exist.^{24,37–40} A tricky aspect is that the description of the continuum requires basis sets which obey the particular boundary conditions of the continuum and, at the same time, reproduce the oscillating behaviour of the photoelectron wave function to infinity. Conventional Gaussian- or Slater-type orbitals are unsuitable for this purpose. B-spline functions, on the other hand, are quite useful in this regards. They are very flexible, they accurately

describe both the bound and continuum states with minimal numerical dependencies,⁴¹ and they are becoming popular for continuum calculations.^{38,40,42}

Here, we employ a LCAO B-spline approach where the primitive basis functions χ_{jlm} are expressed as product of radial, $B_j(r)$, and angular $Y_{lm}(\theta, \phi)$ functions,

$$\chi_{jlm} = \frac{1}{r} B_j(r) Y_{lm}(\theta, \phi) \quad (13)$$

A long-range, large angular momentum basis is centred on the origin, to describe the asymptotic part. Much smaller expansions are employed instead at the nuclear positions, to take care of the Coulomb cusps, and to avoid linear dependence. Converged results are easily obtained even for quite large systems and heavy atoms.

The continuum orbitals are computed by solving the Schrödinger equation in the angular momentum representation

$$h_{KS} \phi_{\epsilon, lm} = E \phi_{\epsilon, lm} \quad (14)$$

with real (K-matrix) boundary conditions. In the Hamiltonian operator

$$h_{KS} = -\frac{1}{2} \Delta + V_N + V_C + V_{XC} \quad (15)$$

the term V_N , V_C and V_{XC} are the nuclear attraction, Coulomb and exchange-correlation potentials, respectively.

Angular momentum analysis of the differential photoionization cross-section in Eq. (2) gives

$$\frac{d\sigma}{dk} = \frac{\sigma}{4\pi} \left[1 + \left(-\frac{1}{2} \right)^{|m_r|} \beta P_2(\cos \theta) + m_r D P_1(\cos \theta) \right] \quad (16)$$

This shows that the angular distribution of the photoelectron in the solid angle dk along the direction of propagation (k) can be completely defined by three parameters: the cross-section σ , the asymmetry parameter β and the dichroism parameter D . Here, P_i is a Legendre

polynomial of order i , whereas θ is the scattering angle between k and the laboratory frame defined by the polarization or the direction of propagation of the light. The parameter m_r specifies the light polarization, with three possible values, 0, +1 and -1, for linearly polarized (LP), left circularly polarized (LCP) and right circularly polarized (RCP) light, respectively. For LP, Eq. (16) assumes a very simple form,

$$\frac{d\sigma}{dk} = \frac{\sigma}{4\pi} [1 + \beta P_2(\cos \theta)] \quad (17)$$

The details of the incorporation of EOM-CCSD Dyson orbital coefficients to obtain the differential cross-section in Eq. (16) can be found in the Supporting Information.

In this letter, we present a combined EOM-CCSD Dyson-orbital/DFT continuum approach for the photoionization observables, and compare our results with both experiments and other computational approaches, including PW or CW results. In more detail, we compare the partial cross-sections and asymmetry parameters for atomic (Ar) and small molecular (CO, H₂O, CS and CH₂O) systems, for which reliable experimental or literature data are available, by treating the bound and continuum parts at different levels of theory—namely: HF molecular orbitals, DFT molecular orbitals and EOM-CCSD Dyson orbitals for the bound state and single-center and multi-center approaches for the continuum state. The dichroic parameter, as an example of photoionization of chiral molecules, is presented for methyloxirane. We also compute the photoionization dynamical parameters from the lowest lying excited states of furan, as accessible in pump-probe experiments,^{2,3} which can be taken up by experimentalists for further spectroscopic studies. Our study takes into account the individual pathways contributing towards the photoionization dynamical parameters. This gives a clear picture of the dominantly contributing ionizations of the system.

For most of the molecules, experimental geometries from the NIST database were used.⁴³ The geometry of (*S*)-methyloxirane was taken from previous work,⁴⁴ and so was the geometry used for furan.⁴⁵

The EOM-CCSD Dyson orbitals were computed using Q-Chem^{11,46} and the aug-cc-pVTZ basis set. PW and CW cross-sections and asymmetry parameters were computed using the ezDyson code.⁴⁷ As anticipated, we adapted the LCAO B-spline code⁴² in order to take into account the different left and right Dyson orbital coefficients. (See the discussion in the Supporting Information). To compute the continuum functions, ground state electronic densities were obtained from ADF using the LB94 exchange-correlation potential and the DZP basis from the ADF database.⁴⁸ The choice of the functional was guided by literature studies showing its good performance for the calculation of photoionization observables.^{49,50} All parameters used to calculate the observables are listed in Table S1 and S2. Notice that we present the results according to conventional symmetry labelling, even though the QChem calculations adopted non-Mulliken symmetry convention.⁴⁶

The performance of the various combinations of methods here employed to describe bound and continuum states was assessed for the three lowest-energy ionizations of CO, H₂O and CH₂O and for the two lowest of CS. Fig. 1 shows the results for CO and H₂O. The results for CS and CH₂O are documented in the Supporting Information (see Fig. S3-S6), due to lack of experimental data and to avoid repetition of results.

In general, the B-spline DFT method for the continuum in conjunction with the EOM-CCSD Dyson orbitals offers the best description of photoionization processes, amongst the various combinations here considered. For CO and H₂O, there is very little difference between the results obtained from the EOM-CCSD correlated Dyson orbitals and those from the Koopmans theorem-based HF and DFT MOs. This is quite reasonable, given that the major contributing MO in the Dyson orbital has a coefficient greater than 0.9 for the corresponding HF or DFT MO. A slight vertical deviation is seen for our method with respect to experiment for the third lowest-lying ionization (the ionization to B ²Σ⁺) of CO. Even though the PW and CW approximations often reproduce the spectral profile, it shows non-uniform large vertical shifts. The discrepancy is larger near the ionization threshold. This is justifiable as PW and CW are two extreme approximations and the actual charge on the ionized core is

Figure 1: Partial cross-section and asymmetry parameters of CO and H₂O relative to the three lowest energy ionizations. Experimental result taken from Ref. 51, 52, 53, 54, 55 and 56.

some value in the range $[0,1]$.⁹

The Cooper minimum is a spectral feature observed in the photoionization cross-section

of Ar at around 47 eV. It is a signature of the sign change in the photoelectron matrix element as the outgoing d-wave moves towards higher kinetic energy. Both the shape of the Dyson orbital and the description of the outgoing electron define the shape of the Cooper minimum, see Fig. 2. As previously ascertained,⁹ the PW and CW descriptions of the

Figure 2: Total and partial cross section and asymmetry parameter of Ar calculated with HF, DFT and EOM-CCSD methods. Multicenter B-spline DFT and TD-DFT functions are used to describe the continuum. The calculated results with DFT B-spline continuum are overlapping. Experimental results are from Refs. 57, 58, 59 and 60.

photoelectron wave function fail to reproduce the Cooper minimum (see also Fig. S1). The spectral profile of the total cross-section is reproduced using the B-spline DFT technique for the continuum, even though the minimum is shifted by about 15 eV. It is seen to be a feature

arising due to ionization from the 3p orbital of Ar. As the magnitude of the partial cross-section of ionization from 3p orbital is much larger than that of 3s, the total cross-section is very similar to the partial cross-section from 3p orbital (as shown in Fig. S1). Electron-electron interaction mixes the 3s and 3p channels. As the 3p cross-section is much larger, the Cooper minimum is also felt in the 3s ionization. However, the DFT B-spline continuum approach lacks correlation in the treatment of the continuum and thus does not yield the minimum in the partial cross-section of ionization from the 3s orbital. The time-dependent (TD)DFT description of the continuum overcomes this problem and provides a remarkable agreement with experiment, as it can be appreciated from Fig. 2. Notable improvement in the asymmetry parameter corresponding to ionization from the 3p orbital is also seen upon consideration of TD-DFT B-spline continuum. The minimum is obtained at the correct photon energy, whereas the dip in the spectral feature is overestimated by about 1 β unit.

The photoelectron dichroism parameter (D) is governed by the description of the nature of the ionized state. From now onwards, we do not report on the PW and CW descriptions of the continuum, having assessed their inadequacy to properly calculate photoionization dynamical variables.

Fig. 3 illustrates our approach on the D of (*S*)-methyloxirane. As immediately apparent, correlation effects must be accounted for while describing the bound state of the target. Koopmans’ theorem fails completely to reproduce the experimental observations, especially in the near-threshold region which usually acts as a fingerprint region. The HF-based D results (blue curves in Fig. 3) show large deviations from experiment or are completely opposite in sign. DFT molecular orbitals perform better. EOM-CCSD Dyson orbitals coupled with B-spline DFT give spectral features within the experimental error bar. Also, it is observed that the near threshold results are most sensitive to the bound state description.

Finally, we studied the ground and excited state dynamics of photoionization of furan, see Fig. 4. At the ground-state equilibrium geometry, the HOMO and HOMO–1 orbitals are $1a_2$ and $2b_1$ π -orbitals, respectively, which leads to two well resolved bands in PES. For

Figure 3: Photoelectron circular dichroism parameter D of (*S*)-methyloxirane (C_3H_6O) calculated with B-spline DFT functions to represent the outgoing electron and three different electronic structure methods for the bound states. Experimental results are taken from Ref. 61. The numerical label in each subfigure indicates the MO from which the ionization predominantly occurs.

the ground state ionizations, the partial cross-sections are steeply decreasing with no other significant feature. On the other hand, the experimental anisotropy parameters β increase rapidly near the threshold and become constant at about 50 eV. The same trend is roughly observed in the computed β . The two lowest-lying excited states of furan are the 1A_2 and 1B_2 states. Transition to 1A_2 state is dark and is of Rydberg ($\pi 3s$) character, whereas 1B_2 is a bright state. We considered the ionization from these two excited states to the lowest-energy

Figure 4: Partial cross-section and asymmetry parameter from ground and lowest lying excited states of furan ($\text{C}_4\text{H}_4\text{O}$) using ground state equilibrium geometry. EOM-CCSD-Dyson with multicenter B-Spline DFT continuum. Experimental results (shown by dots) are taken from Ref. 62.

ionized state ($^2\text{A}_2$ in both cases). The dark state is characterized by excitation from occupied $1a_2$ to virtual $10a_1$. However, the bright state is a combination of transition from $1a_2$ to $3b_1$ and $5b_1$ MO's. The square of the Dyson norm (here computed as $|\phi^L \cdot \phi^R|$) corresponding to ionization from both excited states is about 0.48. This value, close to 0.5, reflects ionization from a singly occupied orbital in the initial state.

For both ground and excited state ionization processes, β is more sensitive than partial σ and shows an initial decrease followed by gradual increase, finally being constant at around 1.5 β units. A steep descent in σ is observed in all cases, where the steepness is a measure of the diffuseness of the orbital from which ionization occurs.

To summarize, our methodology—using EOM-CCSD Dyson orbital coefficients to de-

scribe the bound initial and final states of the target system and B-spline DFT functions to describe the continuum—gives an accurate description of photoionization dynamic properties. On the other hand, simple PW or CW continuum results appear too erratic to be of much use at low electron kinetic energy.

The present approach is completely general, allowing the calculation of all photoionization observables, including, although not considered in this initial study, the photoionization from molecules aligned or fixed in space. This contrasts Stieltjes or Padé pure bound state approaches, which, although useful in limited contexts, lack sufficient generality. With the DFT B-spline continuum, only interchannel coupling effects, which are less widespread, lie outside our formulation. However, as our preliminary results illustrate in the case of argon, the use of the TD-DFT continuum may alleviate this deficiency. Further theoretical improvements include application to core ionizations by coupling the bound state calculation with the core-valence-separation (CVS) approach,^{63–65} and the description of multielectron excitations, i.e. shake-up states, which will require the inclusion of higher-order (beyond CCSD) correlation effects for an accurate description of the bound states.

Finally, the approach is computationally quite efficient and could be used for the description of nuclear motion problems, like vibrationally resolved spectra or imaging of nuclear trajectories, which require a large number of individual calculations. Besides being an always open channel if the photon energy is high enough, its ultrafast response and its high sensitivity to the nature of the orbital from which the electron is ionized make photoionization an ideal probe in time resolved experiments, as well as a source of coherent electronic wavepacket preparation in attosecond experiments.

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Supporting Information Available

The Supporting Information is available free of charge.

(1) Definitions; (2) Derivation of Eq. (4); (3) Photoelectron transition strength in the EOM-CC formalism; (4) Calculation of photoionization observables using the LCAO B-spline approach, (5) Parameters used in the LCAO-B-spline code and in the ezDyson code, (6) Ionization energies and Dyson norms; (7) Partial cross-sections and asymmetry parameters; (8) Excitation and Ionization energies of Furan; (9) Excited state cross-section and asymmetry parameter of furan at bent geometry; (10) Cartesian coordinates for all molecules considered.

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On the Accurate Description of Photoionization Dynamical Parameters.

Supporting Information

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Definitions

Our objective is to compute the photoelectron transition moments between an initial (label I) and a final (label F) state

$$D^{FI} = \langle \Psi_F | \hat{\mu} | \Psi_I \rangle \quad (1)$$

where $\hat{\mu}$ is the dipole moment operator.

Following the works of Arneberg et al.,¹ we write the final state (Ψ_F) for the photoionization process as an antisymmetrized product of the wavefunction of the photoelectron (the continuum electron), ϕ_ϵ , and a bound $(N - 1)$ -electron wavefunction Ψ^{N-1} :

$$\Psi_F \equiv \Psi_{F\epsilon} = \mathcal{A}(\Psi_F^{N-1} \phi_\epsilon) = a_\epsilon^\dagger \Psi_F^{N-1} \quad (2)$$

The (transition) dipole operator in second quantization is

$$\hat{\mu} = \sum_{pq} \mu_{pq} a_p^\dagger a_q \quad (3)$$

where $\mu_{pq} = \langle \phi_p | \vec{\mu} | \phi_q \rangle$. Then, starting from the reduction of the many particle transition moments and neglecting the conjugate Dyson, one can arrive at the following expression

$$D_x^{FI} = \sum_q \langle \phi_\epsilon | \mu_x | \phi_q \rangle \gamma_q \equiv \langle \phi_\epsilon | \mu_x | \phi^d \rangle, \quad \phi^d = \sum_q \gamma_q \phi_q \quad (4)$$

Derivation of Eq. (4)

The derivation of Eq. 4 follows Martin and Shirley² and Arneberg et al.¹ We consider an orthonormal molecular orbital (MO) basis $\{\phi_p\}$ (of the initial state) with associated creation $a_p^\dagger$ and annihilation a_p operators. We can expand the photoelectron function ϕ_ϵ on this basis

$$\phi_\epsilon = \sum_p \langle \phi_p | \phi_\epsilon \rangle \phi_p \quad (5)$$

Whereby the corresponding creation and annihilation operators become,

$$a_\epsilon^\dagger = \sum_p \langle \phi_p | \phi_\epsilon \rangle a_p^\dagger; \quad a_\epsilon = \sum_p \langle \phi_\epsilon | \phi_p \rangle a_p \quad (6)$$

Moreover

$$[a_p, a_q^\dagger]_+ = \delta_{pq}; \quad [a_\epsilon, a_q^\dagger]_+ = \langle \phi_\epsilon | \phi_q \rangle \quad (7)$$

From Eqs. (3) and (6), we can then write the transition dipole moment as

$$D^{FI} = \langle a_\epsilon^\dagger \Psi_F^{N-1} | \hat{\mu} | \Psi_I^N \rangle = \langle \Psi_F^{N-1} | a_\epsilon \hat{\mu} | \Psi_I^N \rangle = \sum_{pq} \mu_{pq} \langle \Psi_F^{N-1} | a_\epsilon a_p^\dagger a_q | \Psi_I^N \rangle \quad (8)$$

Using Eqs. (6) and (7) we rewrite

$$a_\epsilon a_p^\dagger a_q = \langle \phi_\epsilon | \phi_p \rangle a_q - a_p^\dagger a_\epsilon a_q = \langle \phi_\epsilon | \phi_p \rangle a_q + a_p^\dagger a_q a_\epsilon; \quad (9)$$

$$\begin{aligned} a_\epsilon \hat{\mu} &= \sum_{pq} \mu_{pq} a_\epsilon a_p^\dagger a_q \\ &= \sum_{pq} \mu_{pq} \langle \phi_\epsilon | \phi_p \rangle a_q - \sum_{pq} \mu_{pq} a_p^\dagger a_\epsilon a_q \\ &= \sum_q \sum_p \langle \phi_\epsilon | \phi_p \rangle \langle \phi_p | \vec{\mu} | \phi_q \rangle a_q + \left(\sum_{pq} \mu_{pq} a_p^\dagger a_q \right) a_\epsilon \\ &= \sum_q \langle \phi_\epsilon | \left(\sum_p |\phi_p\rangle \langle \phi_p| \right) \vec{\mu} | \phi_q \rangle a_q + \hat{\mu} a_\epsilon \\ &= \sum_q \langle \phi_\epsilon | \vec{\mu} | \phi_q \rangle a_q + \hat{\mu} a_\epsilon \\ &= \sum_q \langle \phi_\epsilon | \mu | \phi_q \rangle a_q + \hat{\mu} \sum_q \langle \phi_\epsilon | \phi_q \rangle a_q \end{aligned} \quad (10)$$

and finally arrive at

$$\begin{aligned}
D^{FI} &= \sum_q \langle \phi_\epsilon | \mu | \phi_q \rangle \langle \Psi_F^{N-1} | a_q | \Psi_I^N \rangle + \sum_q \langle \phi_\epsilon | \phi_q \rangle \langle \Psi_F^{N-1} | \hat{\mu} a_q | \Psi_I^N \rangle \\
&= \sum_q \langle \phi_\epsilon | \mu | \phi_q \rangle \gamma_q^F + \sum_q \langle \phi_\epsilon | \phi_q \rangle \eta_q^F
\end{aligned} \tag{11}$$

where we have introduced the definitions of the following amplitudes (expansion coefficients)

$$\gamma_q^F = \langle \Psi_F^{N-1} | a_q | \Psi_I^N \rangle; \quad \eta_q^F = \langle \Psi_F^{N-1} | \hat{\mu} a_q | \Psi_I^N \rangle \tag{12}$$

The first term in Eq. (11) is called the *direct* term, and the second one the *conjugate* term. According to Eq. (11), the full transition moment is given by 1-particle matrix elements with the continuum, multiplied by the γ_q^F and η_q^F amplitudes, which are matrix elements of the initial and final bound states. This is sufficient to get the cross section for a single channel calculation, with a product structure of the final wavefunction.

Introducing a resolution of identity on a set of $(N - 1)$ -electron wavefunctions into the conjugate term, we obtain

$$\langle \Psi_F^{N-1} | \hat{\mu} a_q | \Psi_I^N \rangle = \sum_J \langle \Psi_F^{N-1} | \mu | \Psi_J^{N-1} \rangle \langle \Psi_J^{N-1} | a_q | \Psi_I^N \rangle \tag{13}$$

$$\eta_q^F = \sum_J D_{N-1}^{FJ} \gamma_q^J \tag{14}$$

where D_{N-1}^{FJ} indicates the transition dipole moment between two electronic states of the $(N - 1)$ -electron ionic (bound) system. At high energy, or simply when the photoelectron wavefunction is considered orthogonal to all bound orbitals included in the expansion of the initial state, the conjugate term is negligible, and only the direct term survives.

Finally, introducing in the direct term the definition of the *Dyson orbital*, ϕ^d :

$$\phi^d = \sum_q \phi_q \gamma_q^F \tag{15}$$

we arrive at the expression for the transition moment already given in Eq. (4)

$$D_x^{FI} = \sum_q \langle \phi_\varepsilon | \mu_x | \phi_q \rangle \gamma_q^F \equiv \langle \phi_\varepsilon | \mu_x | \phi^d \rangle$$

At the independent particle level (unrelaxed Hartree-Fock configurations), we have that $\gamma_q = 1$ for the primary ionic states, and $\gamma_q = 0$ otherwise, i.e.

$$|\Psi_F^{N-1}\rangle = a_q |\text{HF}\rangle \quad (16)$$

Thus, those are the only final states that are allowed, and their corresponding Dyson orbitals are simply the canonical occupied Hartree-Fock (HF) orbitals ϕ_p , with unit norm.

The square norm of the Dyson orbital

$$R_F \equiv ||\phi^d||^2 = \sum_p |\gamma_p^F|^2 \quad (17)$$

is often called *spectral strength* or *pole strength* of the final bound state.

Correlation lowers the spectral strengths of the primary ionic states, and gives intensity to additional states (satellite or shake-up states) characterized by additional electronic excitations. The usual values for the outermost primary states (outer valence) are about 0.8–1.00. The effects become especially strong for the inner valence region (higher ionic states of the same symmetry) mainly because of the mixing with 2h-1p configurations relative to outer excitations, and the spectral strength can spread over many final states of low intensity. Another important effect is the mixing of different canonical MOs in the Dyson orbital, due to rotation of the occupied orbitals upon ionization.

Photoelectron transition strength in the EOM-CC formalism

In coupled cluster (CC) theory, a transition strength (matrix element) between two states I and F is written as

$$S_{xy}^{IF} = \frac{1}{2} \{ D_x^{IF} D_y^{FI} + (D_y^{IF} D_x^{FI})^* \} \quad (18)$$

where the left and right moments are,

$$D_x^{IF} = \langle \Psi_I | \mu_x | \Psi_F \rangle^{(L)}; \quad D_x^{FI} = \langle \Psi_F | \mu_x | \Psi_I \rangle^{(R)} \quad (19)$$

The labels (L) or (R) are used to highlight that, in EOM-CC, the above are not true scalar products, since they are not Hermitian, $D_x^{FI} \neq (D_x^{IF})^*$. We can then write

$$\begin{aligned} D_x^{IF} &= \langle (\phi_{fi}^d)^L | \mu_x | \phi_\epsilon \rangle; & \phi_{fi}^L &= \sum_p \gamma_p^L \phi_p \\ D_x^{FI} &= \langle \phi_\epsilon | \mu_x | (\phi_{fi}^d)^R \rangle & \phi_{fi}^R &= \sum_p \gamma_p^R \phi_p \end{aligned} \quad (20)$$

From now onwards, we will use ϕ_{fi}^R instead of $(\phi_{fi}^d)^R$ and similarly for the left one. The transition moments in Eq. (20) are now true scalar products (as computed in our program) between the continuum and the left and right Dyson orbitals. Using the above expressions, we then consider the photoelectron transition strength matrix

$$S_{xy}^{FI} \equiv \langle \Psi_F | \mu_x | \Psi_I \rangle \langle \Psi_I | \mu_y | \Psi_F \rangle \quad (21)$$

$$= \frac{1}{2} [(\langle \phi_\epsilon | \mu_x | \phi_{fi}^R \rangle \langle \phi_{fi}^L | \mu_y | \phi_\epsilon \rangle) + (\langle \phi_\epsilon | \mu_y | \phi_{fi}^R \rangle \langle \phi_{fi}^L | \mu_x | \phi_\epsilon \rangle)^*] \quad (22)$$

$$= \frac{1}{2} [\langle \phi_\epsilon | \mu_x | \phi_{fi}^R \rangle \langle \phi_\epsilon | \mu_y | \phi_{fi}^L \rangle^* + \langle \phi_\epsilon | \mu_x | \phi_{fi}^L \rangle \langle \phi_\epsilon | \mu_y | \phi_{fi}^R \rangle^*] \quad (23)$$

Thus, in practice, any expression (in the original code for variational wavefunction) that contains Hermitian products of two dipole matrix elements is transformed (in the non-variational

CC case) as

$$D_{\alpha}^{FI}(D_{\beta}^{FI})^* \rightarrow \frac{1}{2} \{ D_{R,\alpha}^{FI} D_{L,\beta}^{IF*} + D_{L,\alpha}^{IF} D_{R,\beta}^{FI*} \} \quad (24)$$

where α and β refer to different cartesian components x , y or z , which is the form employed in Eq (30).

Calculation of the photoionization observables using the LCAO B-spline approach

The differential cross section is given by,

$$\frac{d\sigma}{dk} = 4\pi^2 \alpha \omega |\langle \Psi_F | \vec{\varepsilon} \cdot \vec{\mu} | \Psi_i \rangle|^2 \quad (25)$$

The terms in Eq. (25) have usual meaning as defined in the main article.

To obtain the differential cross section, we first compute the continuum orbitals in the angular momentum representation

$$H\phi_{\epsilon,lm} = E\phi_{\epsilon,lm} \quad (26)$$

with real (K-matrix) boundary conditions, as well as the transition dipole moments with left and right Dyson orbitals ϕ^L and ϕ^R .

Then, $\phi_{\epsilon,lm}$ and the corresponding dipole moments are transformed to incoming-wave boundary conditions, $\phi_{\epsilon lm}^{(-)}$, and finally to $\phi_k^{(-)}$, where

$$\phi_k^{(-)} = \frac{1}{\sqrt{m}} \sum_{lm} i^l e^{-i\sigma_l} Y_{lm}^*(\hat{k}) \phi_{\epsilon lm}^{(-)} \quad (27)$$

Omitting, for ease of notation, the superscript FI , the left and right photoelectron dipole matrix elements (in Eq. (20)) are obtained as

$$D_{\epsilon lm, \gamma}^R = \langle \phi_{\epsilon lm}^{(-)} | \mu_\gamma | \phi_{fi}^R \rangle \quad (28)$$

and similarly for $D_{\epsilon lm, \gamma}^L$. Following standard manipulations as adopted by Chandra,³ one obtains, for randomly oriented molecules,

$$\frac{d\sigma}{d\vec{k}} = \pi\alpha\omega(-1)^{m_r} \sum_L A_L(k) P_L(\cos\theta) \quad (29)$$

where m_r specifies the polarization of light [$m_r = 0, +1, -1$ for linearly polarized (LP), left circularly polarized (LCP) and right circularly polarized (RCP) light, respectively]; θ is the angle between the momentum of the electron and either the polarization vector (for LP) or the direction of propagation (for CP) of the light beam. The coefficients $A_L(k)$ are given by

$$\begin{aligned} A_L(k) = (2L+1) & \begin{pmatrix} 1 & 1 & L \\ m_r & -m_r & 0 \end{pmatrix} \sum_{\substack{lm\gamma \\ l'm'\gamma'}} (-1)^{m+\gamma} \sqrt{(2l+1)(2l'+1)} \\ & \times \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & l' & L \\ -m & m' & (m-m') \end{pmatrix} \begin{pmatrix} 1 & 1 & L \\ \gamma' & -\gamma & (\gamma-\gamma') \end{pmatrix} \\ & \times \frac{1}{2} (D_{\epsilon lm, \gamma}^R D_{\epsilon l' m', \gamma'}^{L*} + D_{\epsilon lm, \gamma}^L D_{\epsilon l' m', \gamma'}^{R*}). \quad (30) \end{aligned}$$

Establishing the correspondences

$$\frac{\sigma}{4\pi} = \pi\alpha\omega(-1)^{m_r} A_0 \quad (31)$$

$$D = \frac{A_1(m_r = 1)}{A_0}; \quad \beta = \frac{A_2(m_r = 0)}{A_0} \quad (32)$$

we finally obtain the usual expression for the differential cross section,

$$\frac{d\sigma}{dk} = \frac{\sigma}{4\pi} [1 + m_r D \cos \theta + \left(-\frac{1}{2}\right)^{m_r} \beta P_2(\cos \theta)] \quad (33)$$

or simply, for linear polarization ($m_r = 0$),

$$\frac{d\sigma}{dk} = \frac{\sigma}{4\pi} [1 + \beta P_2(\cos \theta)] \quad (34)$$

Parameters used in the LCAO B-spline code and in the ezDyson code

Table S1: Parameters used in the LCAO B-spline code. $L_{\max}$ and $l_{\max}$ are the maximum angular momentum employed in the one center expansion at origin and on each atom (off-center), respectively. $R_{\max}$ and $r_{\max}$ are the maximum radial grid length from origin and off-center atoms, respectively.

Molecule	# grid points	$L_{\max}$	$R_{\max}$ (a.u.)	LCAO descriptor	
				$r_{\max}$ (a.u.)	$l_{\max}$
Ar	125	2	25	-	-
CO	125	20	25	0.4(C,O)	2
CS	125	20	25	0.4(C,S)	2
H ₂ O	125	10	25	1.0(H)	2
CH ₂ O	100	12	25	0.2 (C,O,H)	2
C ₄ H ₄ O (planar)	100	15	25	1.0 (C,O,H)	2
C ₄ H ₄ O (bent)	100	15	25	1.0 (C,O,H)	2
(S)-C ₃ H ₆ O	50	12	25	0.6(O), 1.0(C,H)	2

Table S2: Parameters for calculations of cross-sections and asymmetry parameter using ezDyson code. $L_{\max}$ is the maximum angular momentum number for which spherical waves are generated and the dipole matrix elements are calculated. $E_k^{\min}$ and $E_k^{\max}$ are the minimum and maximum value, respectively, of the kinetic energy for which the cross-section and asymmetry parameter are computed. $\langle R^2 \rangle$ is the average square radial distance, used for analyzing the size and shape of the left Dyson orbital.

Molecule	$L_{\max}$	# grid points	$E_k^{\min}$ (eV)	$E_k^{\max}$ (eV)	$\langle R^2 \rangle$ (Å)
Ar	2	101	0.1	100.1	0.608698
CO	5	101	0.1	100.1	1.052489
CS	5	101	0.1	100.1	1.697160
H ₂ O	5	50	0.5	50.5	0.763224
CH ₂ O	5	101	0.1	100.1	1.121095

Ionization energies (IE) and Dyson norms

Table S3: IE (in eV) and EOM-CCSD Dyson square norms of the ground state using aug-cc-pVTZ basis set. Experimental results are taken from Ref. 4 and 5. Mulliken’s symmetry convention is assumed.

System	Irrep	KT IE (eV)	EOM-CCSD IE (eV)	Dyson sq. norm ($ \phi^L \cdot \phi^R ^2$)	Exp. IE (eV)
Ar	3s	34.83	29.91	0.859	29.2
	3p	16.05	15.66	0.965	15.8
CO	5 σ	15.10	14.20	0.952	14.0
	4 σ	21.91	19.82	0.934	19.7
	1 π	17.41	17.10	0.949	16.9
CS	7 σ	12.82	11.52	0.931	11.3
	2 π	12.60	13.06	0.953	12.8
H ₂ O	3a ₁	15.92	14.83	0.957	14.7
	1b ₂	19.51	19.00	0.962	18.5
	1b ₁	13.88	12.62	0.956	12.6
CH ₂ O	5a ₁	17.77	16.15	0.944	16.0
	2b ₂	12.05	10.86	0.951	10.8
	1b ₁	14.64	14.66	0.944	14.5
C ₄ H ₄ O (planar)	1a ₂	8.68	9.04	0.950	8.8
	2b ₁	10.80	10.40	0.946	10.4
(S)- methyl- oxirane	16a	11.78	10.37	0.944	10.4
	15a	12.17	11.21	0.951	11.3
	14a	14.01	13.23	0.952	13.0
	13a	14.37	13.44	0.952	13.6
	12a	15.26	14.40	0.953	14.5
	11a	15.97	15.12	0.952	15.1

Partial cross-sections and asymmetry parameters

Argon

Figure S1: Partial cross-section of Ar relative to 3s (left) and 3p (right) states calculated with plane wave, coulomb wave and DFT-B-spline functions to represent the outgoing electron coupled with EOM-CCSD/aug-cc-pVTZ Dyson orbitals.

Figure S2: Comparative study of asymmetry parameter of Ar relative to ionization from 3p orbital. EOM-CCSD Dyson orbitals are used throughout. Experimental result taken from Ref. 6.

Carbon sulphide, CS

Figure S3: Partial cross-section and asymmetry parameters of CS relative to 7σ (top) and 2π (bottom) states calculated with HF, DFT and EOM-CCSD methods. Multicenter B-spline functions are used to describe the outgoing electron.

Figure S4: Partial cross-section and asymmetry parameters of CS relative to 7σ (top) and 2π (bottom) states calculated with plane wave, coulomb wave and B-spline functions to represent the outgoing electron coupled with EOM-CCSD/aug-cc-pVTZ Dyson orbitals.

Formaldehyde

Figure S5: Partial cross-section and asymmetry parameters of CH₂O relative to 5a₁ (top) and 2b₂ (middle) and 1b₁ (bottom) states calculated with HF, DFT and EOM-CCSD methods. Multicenter B-spline functions are used to describe the outgoing electron. Experimental results are taken from Ref. 7 and 8.

Figure S6: Partial cross-section and asymmetry parameters of CH_2O relative to $5a_1$ (top) and $2b_2$ (middle) and $1b_1$ (bottom) states calculated with plane wave, coulomb wave and B-spline functions to represent the outgoing electron coupled with EOM-CCSD/aug-cc-pVTZ Dyson orbitals. Experimental results are taken from Ref. 7 and 8.

Excitation and ionization energies of furan

Table S4: Excitation energies of furan in eV. Experimental result are from Ref. 5. CASSCF, NEVPT2, TDDFT and ADC(2) results are from Ref. 9.

Geometry	State	EOM-CCSD	CAS-[6,9]	NEVPT2	TDDFT	ADC(2)	Exp.
Planar	1A_2 (S_1)	6.15	5.71	5.98	5.51	5.91	6.04
	1B_2 (S_2)	6.47	6.91	6.63	5.98	6.44	6.49
Bent	1A	2.11	-	-	-	-	-
	1A	3.85	-	-	-	-	-

Table S5: Ionization energy from the excited state of furan in eV. CASSCF results are from Ref. 9

Geometry	Excited state	Ionized state of cation	EOM-IP-CCSD	CAS-[6,9]
Planar	1A_2 (S_1)	2A_2	2.89	2.43
	1B_2 (S_2)	2A_2	2.57	1.23
Bent	1A (S_1)	2A	5.23	4.22
	1A (S_2)	2A	3.48	2.96

Excited state cross-section and asymmetry parameter of furan at bent geometry

Figure S7: Computed photoionization partial cross section (left) and photoelectron asymmetry parameter (right) for transition from S_1 (top) and S_2 (bottom) states of furan at a puckered geometry.

Cartesian coordinates of all molecules

CO (Aangs)

O	0.00000000	0.00000000	0.48356580
C	0.00000000	0.00000000	-0.64475440

H₂O (Aangs)

O	-0.00000000	-0.00000000	0.11730000
H	-0.75720000	-0.00000000	-0.46920000
H	0.75720000	0.00000000	-0.46920000

CS (Aangs)

S	0.00000000	0.00000000	0.41860909
C	0.00000000	0.00000000	-1.11629091

CH₂O (Aangs)

O	0.00000000	0.00000000	0.67595000
C	0.00000000	0.00000000	-0.52905000
H	0.94290000	0.00000000	-1.11665000
H	-0.94290000	-0.00000000	-1.11665000

C₄H₄O (Aangs)-Planar

O	0.00000000	0.00000000	1.15570089
C	-0.71808000	0.00000000	-0.96094011
C	0.71808000	0.00000000	-0.96094011
C	-1.09300200	0.00000000	0.35220389
C	1.09300200	0.00000000	0.35220389

H	2.05718500	0.00000000	0.85420589
H	-2.05718500	0.00000000	0.85420589
H	1.37995400	0.00000000	-1.82459211
H	-1.37995400	0.00000000	-1.82459211

C₄H₄O (Angs)- Bent

O	1.09645392	-0.24983976	-0.34616477
C	0.64775401	0.90476604	0.14769665
C	-0.87836149	0.84558708	-0.05929252
C	-1.07423176	-0.47773249	-0.13303448
C	0.19506068	-1.05871585	0.15814657
H	1.19377197	1.61166861	0.79100107
H	-1.51941099	1.72028223	0.01907603
H	-1.97177413	-1.14269353	0.02025521
H	0.18445315	-1.47396792	1.25788853

C₃H₆O (au)

C	-0.28956361	0.08176958	0.92701577
C	1.93885200	-1.15600766	-0.10800484
C	-2.79852375	-0.19094696	-0.27656907
O	1.53613696	1.47730615	-0.46998636
H	1.71860882	-2.32670888	-1.80795335
H	3.52273536	-1.64526642	1.13875511
H	-0.27744501	0.50560530	2.96417187
H	-2.56905279	-0.59002862	-2.30639911
H	-3.89960425	1.56511541	-0.07933409
H	-3.88892566	-1.73605572	0.59599929

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