

Progression towards a standard written method of calculation

INTRODUCTION

The National Numeracy Strategy/Framework provides a structured and systematic approach to teaching number and there is a considerable emphasis on teaching mental calculation strategies. Up to the age of 9 (Year 4) informal written recording should take place regularly and is an important part of learning and understanding. More formal written methods should follow only when the child is able to use a wide range of mental calculation strategies.

REASONS FOR USING WRITTEN METHODS

- To aid calculation, particularly with larger numbers or decimals or when the problem is too difficult to be done mentally.
- To help communicate methods and solutions.
- To allow errors to be unpicked.
- To aid mental calculation by writing down some of numbers and answers involved.
- To provide a record of work carried out.

The lists below are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to more formal 'pencil and paper' methods of calculation.

TO CONSIDER:

Addition and subtraction

- Do they know how to add and subtract units/ones, tens or hundreds to numbers?
- Do they know addition and subtraction facts for all numbers to 20?
- Do they understand place value and can they partition and recombine numbers?
- Can they add three or more single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally, bridging through tens and hundreds when appropriate?
- Can they use a number line to add and find the difference between pairs of numbers?
- Can they explain their mental strategies orally and record them using informal jottings, such as the empty number line?

Multiplication and division

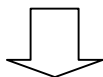
- Do they know (at least) the 2, 5, 10, 3 and 4 times tables by heart?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a place holder?
- Can they multiply two or three digit numbers by 10 and 100?
- Can they double and halve numbers mentally or with jottings?
- Do they understand place value and can partition and recombine numbers?
- Can they explain their mental strategies orally and record them using informal jottings?
- Can they draw and use arrays and groups of objects / jumps on a number line to describe their multiplication / division sentences?

A Whole School Approach

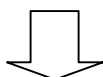
We believe it is vital that children have a wide repertoire of mental strategies to support them when first solving calculation. We will ensure children are taught to choose the most efficient method of calculation when appropriate.

Children will be taught to:

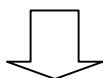
Attempt the calculation in their heads using an appropriate mental method



Use jottings to support their mental calculations when needed



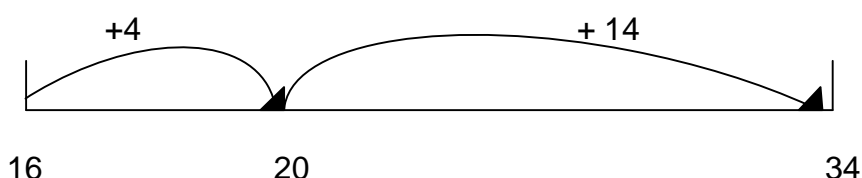
Use a pencil and paper procedure (from Y4 onwards)



Know when and how to use a calculator (Y5 and Y6)

- A wide range of mental methods will be established, refined and continue to be taught throughout the school, based on a **good understanding of place value in numbers**.
- **Place value** is an important concept which will be taught mentally, first in Reception where number tracks are used, progressing to numbers lines (to 10 or 20 or 100 as appropriate) in Year 1 and Year 2.
- The **empty number line** will be introduced and used to aid mental calculations in all four operations throughout key stages one and two.
- Pupils will be taught the importance of locating on a marked and unmarked number line to aid and calculation.
- Mental methods for subtraction will be taught mainly through the *difference* model – counting on forwards on the number line, although special cases need to be recognised, when counting backwards is more efficient. For example:

34 – 16 - count on **forwards**, using an empty number line where appropriate



- count back perhaps partitioning the *second* number only e.g. 256 – 32 or 256 – 30 – 2

- In all cases we will make sure that the children always look for special cases that can still be done entirely mentally e.g. 11 + 9, 15 + 5 (using number facts to 20)

- Children will be taught to estimate / approximate first

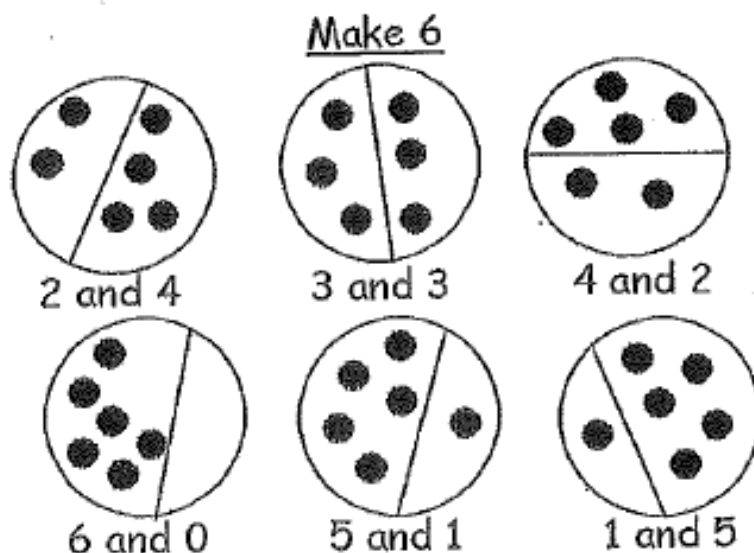
e.g. 29×3 (round up to the nearest 10, the answer will be near to 90).

- We will develop ways of checking answers and considering the reasonableness of solutions that have been found.

Written addition strategies

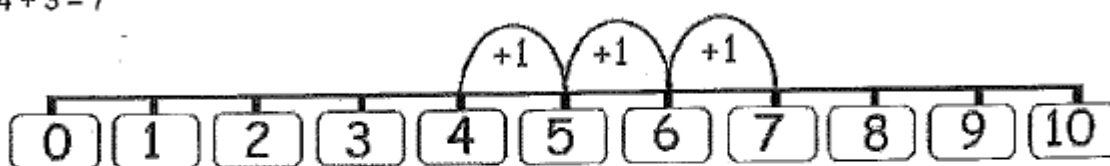
Reception and Yr 1

Children develop ways of recording calculations using pictures and apparatus for support.



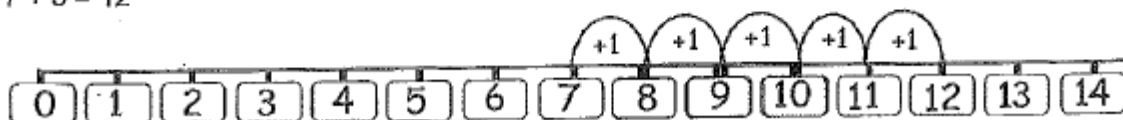
Children use numberlines and other practical resources (bead bars, counters etc) to support their addition calculations. Teachers demonstrate the use of a numberline.

$$4 + 3 = 7$$



Children begin to use landmarked numberlines to support their own calculations, counting on in ones including bridging through ten.

$$7 + 5 = 12$$



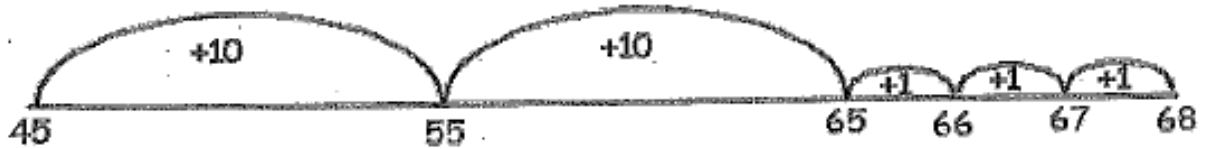
Teachers demonstrate starting with the larger number to count on. Teacher models that adding can be done in any order.

Children continue to use landmarked numberlines counting on in tens and ones.

Yr 2

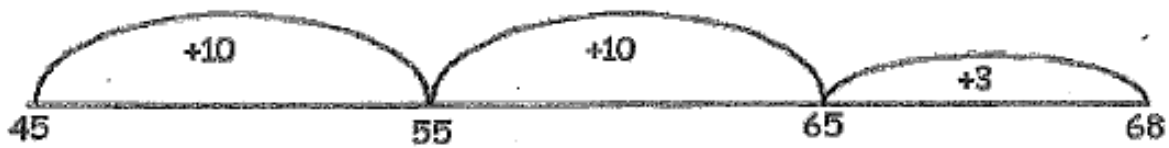
Children begin to use empty numberlines, counting on in tens and ones

$$45 + 23 = 68$$



This strategy becomes more efficient by counting on in tens and adding the units in one jump (using known addition facts i.e. $5 + 3 = 8$)

$$45 + 23 = 68$$



Adding the tens in one jump and the units in another jump.

$$45 + 23 = 68$$



Also bridging through ten

$$57 + 26 = 83$$



Lower Key Stage 2

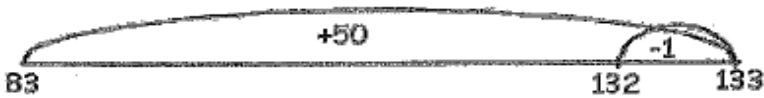
Children continue to use empty numberlines with increasingly large numbers, starting from the largest number.

$$57 + 85 = 142$$



Compensation should be demonstrated and practised on a numberline when adding 'near multiples'

$$49 + 83 = 132$$



Children should use jottings to support, record and explain their mental addition methods. Presentation of numbers needs to be clear with one digit per square.

When moving to vertical/column addition children should add the least significant digits first in preparation for carrying:

Example 1:

	T	U	
	6	7	
+	2	4	
	1	1	(7 + 4)
	8	0	(60 + 20)
	9	1	

Example 2:

	H	T	U	
		2	6	7
+		8	5	
		1	2	(7 + 5)
	1	4	0	(60 + 80)
	2	0	0	(200 + 0)
	3	5	2	

Upper Key Stage 2

Progressing from this 'expanded' method, children will begin to carry below the line.

The calculation should be read as 5 Units + 8 Units = 13 put down the 3 Units and carry the ten over to the tens column. (= 1 Ten)

2 Tens + 4 Tens = 6 Tens = 60

Plus the Ten carried = 7 tens = 70

$$\begin{array}{r}
 \text{H T U} \\
 625 \\
 + \quad 48 \\
 \hline
 673 \\
 \hline
 1
 \end{array}$$

Children extend the carrying method to numbers with four digits:

$$\begin{array}{r}
 \text{Th H T U} \\
 3587 \\
 + \quad 675 \\
 \hline
 4262 \\
 \hline
 111
 \end{array}$$

The calculation should be read as 7 Units + 5 Units = 12 put down the 2 Units and carry over the 10 to the Tens column. (1 Ten)

8 Tens + 7 Tens = 15 Tens = 150 plus the Ten carried = 16 Tens = 160 put down the 6 Tens and carry the Hundred over to the Hundreds column.

5 Hundreds + 6 Hundreds = 11 Hundreds = 1100

Plus the Hundred carried = 12 Hundreds = 1200

Put down the 2 Hundreds and carry the 1000 to the Thousands column.

3000 plus the Thousand carried = 4000

An informal mental calculation for this is to start with the largest digits first e.g. 3000, 500 + 600 = 1100, 80 + 70 = 150, 7 + 5 = 12 then add the answers together. (known as partitioning)

Children extend the carrying method to adding numbers with any number of digits:

	Th	H	T	U
	3	5	8	7
		6	7	5
	2	9	0	5
+			3	6
	7	2	0	3
	2	2	2	

Children extend the carrying method to adding numbers with decimal fractions:

$$26.4 + 37.351 = 63.751$$

	2	6	.	4
+	3	7	.	3 5 1
	6	3	.	7 5 1
	1			

	T	U	.	Tths	Hths	Thths
	2	6	.	4	0	0
+	3	7	.	3	5	1
	6	3	.	7	5	1
	1					

Useful tips:

1. Before calculating bring the decimal point into the answer line.
2. Ensure that there are the same number of digits after the decimal point by putting in the zeros. (This makes it easy to compare numbers)

The calculation should be read as: 0 Tths + 1 Tths = 1 Tths (Thousandths)

0 Hths + 5 Hths = 5 Hths (Hundredths)

4 Tths + 3 Tths = 7 Tths (Tenths)

And then 6 Units + 7 Units = 13

Put down the 3 Units and carry the Ten over to the Tens column.

When using a **calculator** children need to key in an appropriate number sentence e.g. $26.4 + 37.351 = 63.751$

Using similar methods children will:

- ✓ Add several numbers with different numbers of digits
- ✓ Add amounts of money
- ✓ Know that the decimal points line up under each other (be clear that the decimal point never moves the digits shift depending on their value)
- ✓ Add two or more decimal fractions
- ✓ Convert number sentences into written methods

When using written methods children should only move onto the next stage:

- When they are ready
- When they feel confident

Children should be encouraged to approximate their answers before calculating and check their answers after calculation using an appropriate strategy.

Children should be encouraged (and teacher should model) if a mental calculation would be more appropriate than using written methods. Partitioning mentally or informally e.g. $625 + 48 = 600 + 60 + 13 = 673$

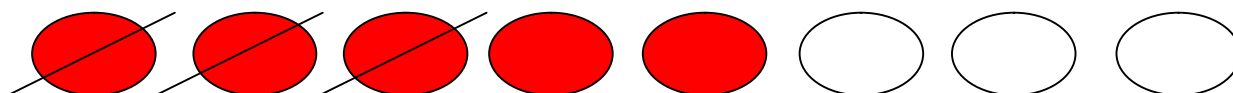
Written subtraction strategies

Reception and Yr 1

Children use pictorial methods to develop an understanding of subtraction eg. Counters, number songs.

They develop ways of recording calculations using pictures.

8 take away 3

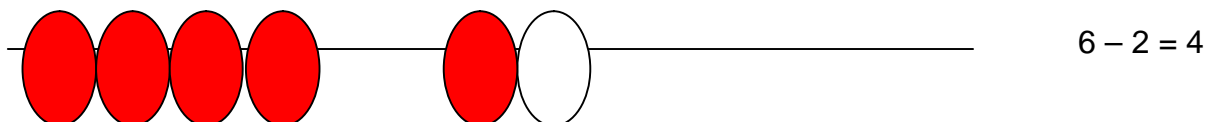


Practical work/games with counting apparatus.

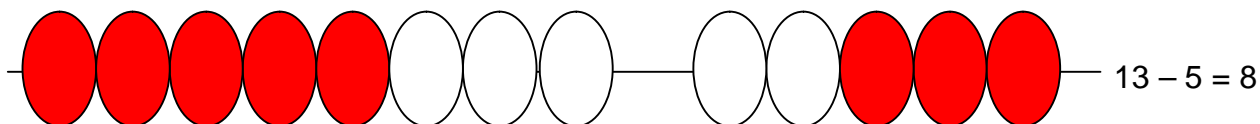
Songs and games.

Children are encouraged to develop a mental picture of the number system in their heads to use in calculation. They develop ways of recording calculations using pictures etc.

Bead strings or bead bars can be used to illustrate subtraction including bridging through 10 by counting back 3 then counting back 2.



They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of a number line.



Children then begin to use numbered lines to support their own calculations – using a numbered line to count back in ones then move onto tens and ones.

The numberline should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.

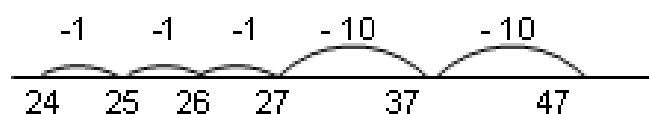
Yr 2

Children will begin to use empty number lines to support calculations.

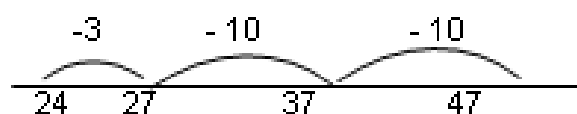
COUNTING BACK:

1. First counting back in tens and ones

$$47 - 23 = 24$$

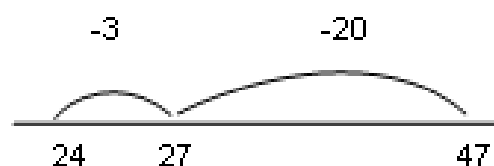


2. Then helping children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$)



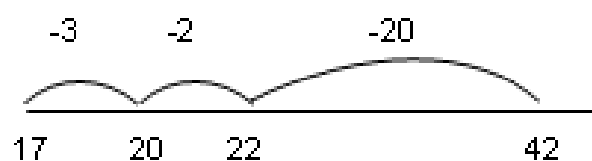
3. Subtracting the tens in one jump and the units in one jump

$$47 - 23 = 24$$



4. Bridging through ten can help children to become more efficient.

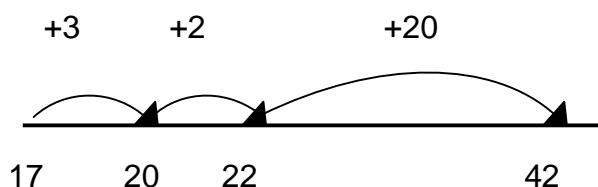
$$42 - 25 = 17$$



COUNTING ON:

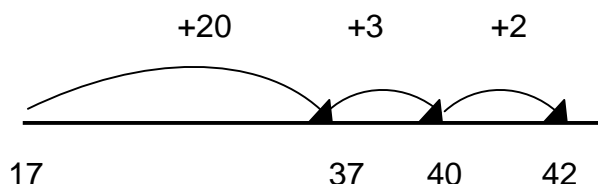
A counting on method (the difference model using a number line will continue throughout KS1 and KS2 to aid mental calculation when appropriate.)

$$42 - 17 = 25$$



Children might also use the following example if their mental mathematics is able to deal with larger jumps

$$42 - 17 = 25$$

**Lower Key stage 2**

A 'counting on' method using a number line will continue throughout KS2 to aid mental calculation when appropriate. This includes with larger numbers.

Underpinning skills must be in place throughout KS1 and Y3 before these stages are introduced, using both take away models and the difference/ 'Counting on'. The vocabulary of this model must also be reinforced. Children will begin using informal pencil and paper methods (jottings)

We have chosen to use decomposition as the main *written* method for subtraction in KS2.

Partitioning and decomposition (to break down)

Children are introduced to formal vertical methods of subtraction and to the concepts of partitioning and exchanging numbers. Partitioning means to separate the number into hundreds, tens and units or ones.

- Partitioning – demonstrated using arrow cards, 100 bead bar etc

$$\begin{array}{r} \text{TU} \\ 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 + 9 \\ 50 + 7 \\ \hline \end{array}$$

$$30 + 2 = 32$$

This calculation should be read as partition 89 to 80 + 9 and 57 to 50 + 7

9 Units – 7 Units = 2 Units

80 – 50 = 30

Then add 30 + 2 = 32

- Begin to exchange

$$\begin{array}{r} \mathbf{TU} \\ 71 \\ - 46 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Step 1 } 70 + 1 \\ - 40 + 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Step 2 } 60 + 11 \\ - 40 + 6 \\ \hline \end{array}$$

$$20 + 5 = 25$$

This should be read as take 6 Units from 1 Unit, this can't be done so take a Ten from the Tens column (this now leaves 60) and break it down into 10 units there are now 11.

This would be recorded by the children as:

$$\begin{array}{r} \overset{60}{\cancel{70}} + \overset{1}{1} \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

Moving to hundreds, tens and units

$$\begin{array}{r} \mathbf{HTU} \\ 754 = \\ - 86 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Step 1 } 700 + 50 + 4 \\ - \quad \quad 80 + 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Step 2 } 700 + 40 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{Step 3 } 600 + 140 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array}$$

$$600 + 60 + 8 = 668$$

This would be recorded by the children as

$$\begin{array}{r} \overset{600}{\cancel{700}} + \overset{1}{1} \overset{40}{\cancel{50}} + \overset{1}{1} 4 \\ \quad \quad \quad \overset{40}{\cancel{80}} + 6 \\ \hline \end{array}$$

$$600 + 60 + 8 = 668$$

This should be read as partition 754 and 86.

Take 6 Units from 4 Units, this can't be done so take a Ten from the Tens column (this leaves 40) break the 10 down there are now 14.

$$14 - 6 = 8$$

Take 80 from 40, this can't be done so take a Hundred from the Hundreds column break it down into 10 tens. You now have 140.

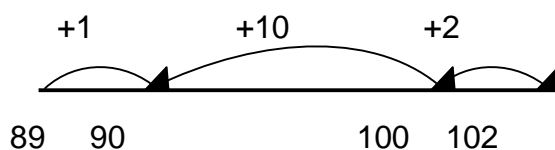
$$140 - 80 = 60$$

$$600 - 0 = 600$$

$$600 + 60 + 8 = 668$$

Where the numbers involved in the calculation are close together or near to multiples of 10, 100, 1000 counting on using a number line should be used.

$$102 - 89 = 13$$



Moving to a more contracted method

$$569 - 278$$

$$\begin{array}{r} 4 \ 1 \\ - 569 \\ \underline{278} \\ 291 \end{array}$$

This should be read as take 8 Units from 9 Units = 1 Unit. Take 7 Tens from 6 Tens, this can't be done so take a Hundred from the Hundreds column and break it down into 10 tens. You now have 16 Tens. 7 Tens from 16 Tens = 9 tens
Take 2 Hundreds from 4 Hundreds = 2 Hundreds.

An example of where you can't go to the next digit would be e.g.

$$500 - 268 = 232$$

$$\begin{array}{r} \mathbf{H \ T \ U} \\ 4 \ \ 9 \\ \cancel{5} \ \cancel{0} \ 10 \\ - 2 \ 6 \ 8 \\ \hline 2 \ 3 \ 2 \end{array}$$

This calculation should be read as take 8 Units from 0 Units, this can't be done so go to the Tens column there are no Tens to be taken so go to the Hundreds column and take 100 and break it down into 10 Tens. There are now 10 Tens in the Tens column. Take a Ten from the Tens column this leave 9 Tens. Now there are 10 units take away 8 Units from 10 = 2 Units
6 Tens from 9 Tens = 3 Tens
2 Hundreds from 4 Hundreds = 2 Hundreds

Upper Key Stage 2

Larger numbers and decimals and keying in subtraction calculations into the calculator.

Using similar methods children will:

- ✓ Subtract amounts of money
- ✓ Know that decimal points line up under each other and that place holder 0's should be positioned in the calculation.
- ✓ Know that the bigger number goes on top
- ✓ Convert number sentences into written methods

NB It is important for children to continue to make decisions about the most appropriate method of solving calculations. Children should always estimate and then decide whether they can find out the answer using a mental method first, whether to use jottings, such as the use of the empty number line, or whether to use a more formal written method. All calculations will be presented horizontally.

Written multiplication strategies

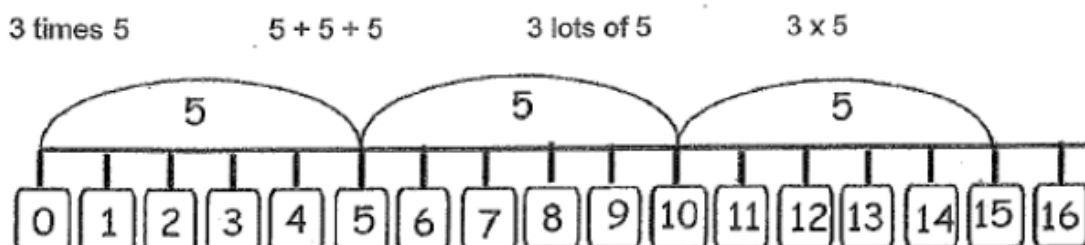
Reception and Yr 1

Children will experience equal groups of objects and will start to count in 2s, 5s and 10s. They will work on practical activities involving equal sets/ lots of/ groups.

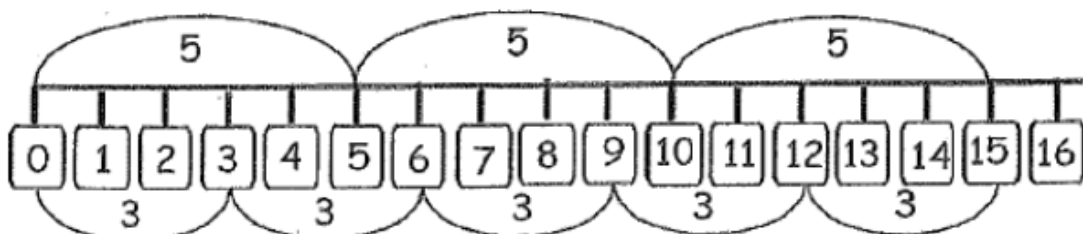


Yr 2

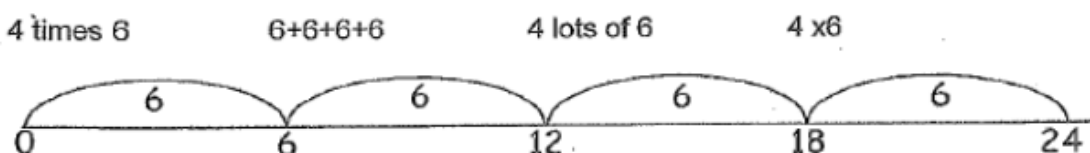
Children use landmarked number lines to develop their understanding of multiplication as repeated addition.



Children should have opportunities to experience that 3×5 has the same answer as 5×3 . This can be shown on the numberline.

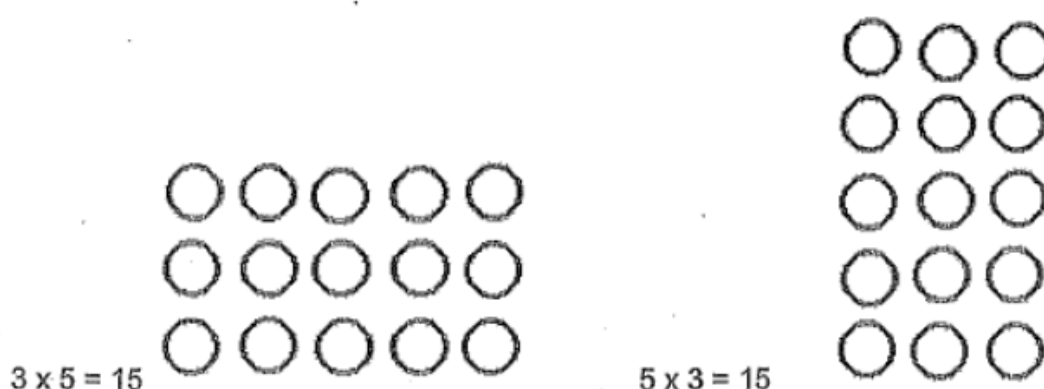


Children will continue to use empty numberlines to support their understanding of multiplication as repeated addition.



Lower Key Stage 2

Children begin to model a multiplication calculation using an array (this will support with the development of the grid method):



Children begin to use the grid method of multiplication which is supported by their work using arrays.

Children begin to use the grid method of multiplication which is supported by their work using arrays.

TU x U

(Short multiplication – multiplication by a single digit)

$$23 \times 8$$

x	20	3
8	160	24

$$\begin{array}{r} 160 \\ + 24 \\ \hline 184 \end{array}$$

This calculation should be read as 23 x 8. Partition the 23 into 20 and 3 multiply 8 x 20 = 160 and 8 x 3 = 24.

Then do 160 + 24 = 184

HTU x U

(Short multiplication – multiplication by a single digit)

$$346 \times 9$$

x	300	40	6
9	2700	360	54

$$\begin{array}{r} 2700 \\ 360 \\ + 54 \\ \hline 3114 \\ \hline \end{array}$$

This calculation should be read as 346 x 9. Partition the 346 into 300, 40 and 6. Multiply 300 x 9 by multiplying the known multiplication fact first - 3 x 9 = 27 then multiply 27 x 100 (shift the digits 2 places to the left) = 2700

Multiply 40 x 9 (9 x 4 = 36 x 10 = 360)

Multiply 6 x 9 = 54 then add together (2700+360+54 = 3114)

A Common Misconception – Multiplying by 10

Children often think that multiplying by 10 means adding a zero. This appears to be the case BUT it is not mathematically correct.

e.g. $8 \times 10 = 80$

but it doesn't work for decimal fractions.

e.g. $8.8 \times 10 = 8.80$ but

$8.8 \times 10 = 88$ (no added zero!) WE DON'T ADD 0'S, THE DIGITS SHIFT TO THE LEFT – 1 place x10, 2 places x 100 etc. THE DECIMAL POINT NEVER MOVES!

Upper Key Stage 2 (Consolidate previous learning of the Grid Method from Lower Key Stage 2)

ThHTU x U

(Short multiplication – multiplication by a single digit)

$$4346 \times 8$$

x	4000	300	40	6
8	32000	2400	320	48

$$\begin{array}{r}
 32000 \\
 2400 \\
 320 \\
 + 48 \\
 \hline
 34768
 \end{array}$$

Children begin to multiply decimals by a single digit number.

$$4.9 \times 3$$

x	4	0.9
3	12	2.7

$$\begin{array}{r}
 12. \\
 + 2.7 \\
 \hline
 14.7
 \end{array}$$

e.g.

$$\begin{array}{r}
 12.0 \\
 + 2.7 \\
 \hline
 14.7
 \end{array}$$

Useful tips

1. Before calculating bring the decimal point down into the answer line.
2. Ensure that there is the same number of digits after the decimal point by putting in the zeros.

TU x TU

(Long multiplication – multiplication by more than a single digit)

72 x 38

x	70	2
30	2100	60
8	560	16

$$\begin{array}{r}
 2100 \\
 560 \\
 60 \\
 + 16 \\
 \hline
 2736 \\
 \hline
 \end{array}$$

HTU x TU

(Long multiplication – multiplication by more than a single digit)

372 x 24

x	300	70	2
20	6000	1400	40
4	1200	280	8

$$\begin{array}{r}
 6000 \\
 1400 \\
 1200 \\
 280 \\
 40 \\
 + 8 \\
 \hline
 8928 \\
 \hline
 \end{array}$$

Children will be able to multiply decimals with up to two decimal places by a single digit and then two digit numbers.

4.92 x 3

x	4	0.9	0.02
3	12	2.7	0.06

$$\begin{array}{r}
 12 \\
 2.7 \\
 + 0.06 \\
 \hline
 14.76 \\
 \hline
 \end{array}$$

Don't forget your 'useful tips' when working with decimals.

When children are confident using the grid method with a range of short and long multiplication they are able to move onto the column method of multiplication.

Short multiplication (the Partitioning method)

$$\begin{array}{r}
 \text{H T U} \\
 378 \\
 \times \quad 4 \\
 \hline
 32 \quad (4 \times 8) \\
 280 \quad (4 \times 70) \\
 1200 \quad (4 \times 300) \\
 \hline
 1512 \\
 \hline
 1
 \end{array}$$

Progressing to (the Compact method):

$$\begin{array}{r}
 \text{H T U} \\
 378 \\
 \times \quad 4 \\
 \hline
 1512 \\
 \hline
 33
 \end{array}$$

This calculation should be read as 4 Units x 8 Units = 32 put the 2 Units in the units column and carry the 3 Tens over to the Tens column.

4 lots of 7 Tens = 28 Tens + 3 Tens = 31 Tens (equivalent to 310). Put down the 1 Ten into the Tens column and carry the 30 Tens over to the Hundreds column (3 Hundreds).

4 lots of 3 Hundreds = 12 Hundreds + the 3 Hundreds carried = 15 Hundreds (equivalent to 1500) Put down the 5 Hundreds and carry 10 Hundreds over to the Thousands column (1 Thousand) into the answer box.

Multiplying decimals:

4.92 x 3

	U	.	Tths	Hths
4	.	9	2	
x				3
1	4	.	7	6
	2			

Don't forget your Useful tips about decimals!

Long multiplication (Partitioning method):

	4	2	9		
x		5	7		
		6	3	(7 x 9)	
	1	4	0	(7 x 20)	
2	8	0	0	(7 x 400)	
	4	5	0	(50 x 9)	
1	0	0	0	(50 x 20)	
2	0	0	0	(50 x 400)	
	2	4	4	5	3
	1	1			

Progressing to a Long multiplication compact method:

	H	T	U
	4	2	9
x		5	7
	3	0	0
	2	1	6
	2	1	4
	2	1	4
	2	4	4
	5	3	

To multiply by 50 place a '0' into the Units column as a place value holder. (Because we know that when we multiply by 10, the digits move 1 place to the left.)

This line of the calculation reads as 5 tens x 9 Units = 45 tens. Put the 5 in the tens column and carry the 4 Hundreds over to the Hundreds column.

5 tens x 2 tens = 1000 + 4 Hundreds carried = 1400 out the 4 in the Hundreds column and carry the thousand into the Thousands column.

5 tens x 4 hundreds = 20000 plus the 1000 carried over = 21000

Put 1 in the Thousands column and carry 20 Thousand into the Ten Thousand column. Put 2 into the workings.

Add the 2 answers together -

$$3003 + 21450 = 24453$$

This calculation should be read as 7 Units x 9 Units = 63 put the 3 down into the Units column and carry the 6 Tens into the Tens column.

7 Units x 2 Tens = 14 Tens and add the 6 Tens carried = 20 Tens. Put zero in the Tens column and carry the 20 Tens into the Hundreds column (2 Hundreds).

7 Units x 4 Hundreds = 28 Hundreds (equivalent to 2800) + 2 Hundreds carried = 30 Hundreds

Put zero in the Hundreds column and carry 30 Hundreds over to the Thousands column (equivalent to 3 Thousand) Put 3 into the answer box.

TIP

Cross out the carrying as you go!

IMPORTANT

When using written multiplication methods children should only move onto the next stage:

- When they are ready
- When they feel confident

Children should be encouraged to approximate their answers before calculating and check their answers after calculation using an appropriate strategy.

Written division strategies

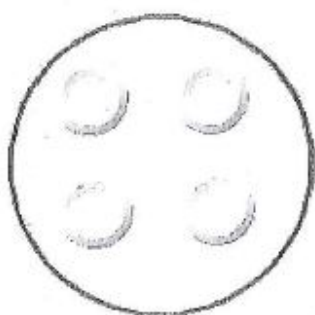
Reception and Yr 1

Children will experience equal groups and share items in play and problem solving. They will start to count in 2's, 5's and 10's.

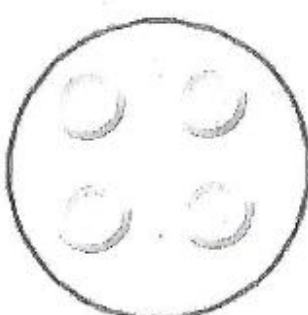
Yr 2

Sharing Equally

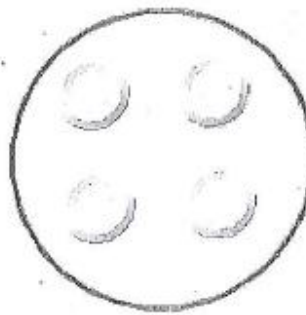
12 shared by 3



12 grouped into 3s



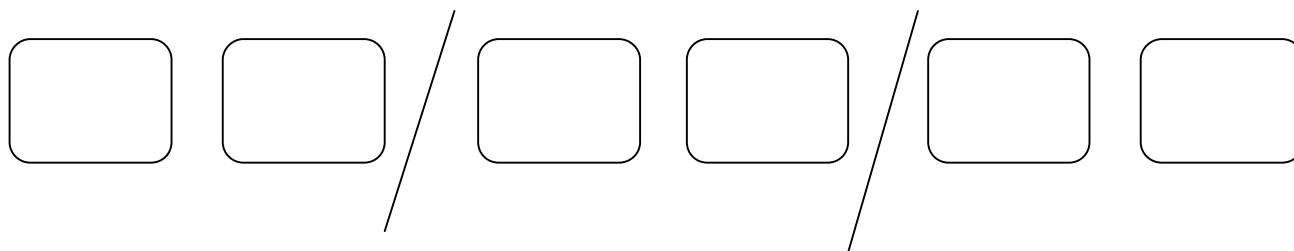
$12 \div 3$



The emphasis is soon placed on **grouping** rather than sharing.

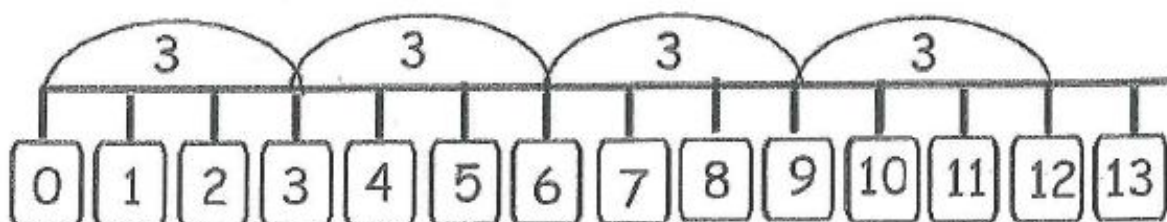
Grouping

6 sweets, how many can have 2 sweets?



Children use landmarked numberlines to develop their understanding of division as repeated subtraction.

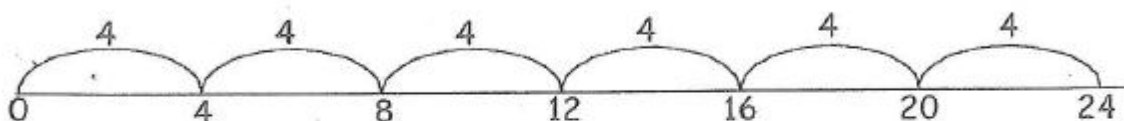
$$12 \div 3 = 4$$



Lower Key Stage 2

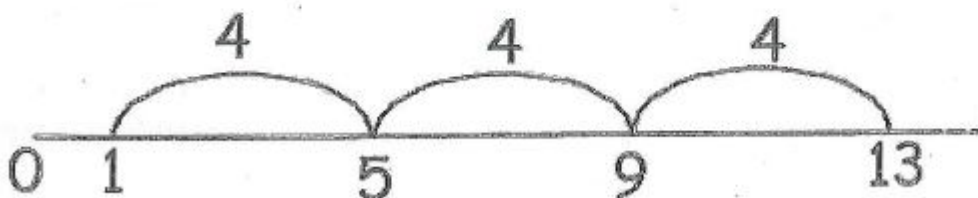
Children will continue to use empty numberlines to support their understanding of division as repeated subtraction.

$$24 \div 4 = 6$$



Children should also move onto division calculations involving remainders.

$$13 \div 4 = 3r1$$



In lower key stage 2 children practise using horizontal methods for division including plenty of practise using the inverse e.g. $24 \div 4 = 6$, $24 \div 6 = 4$ and building on their knowledge of related facts e.g.

$$6 \times 4 = 24$$

$$4 \times 6 = 24$$

$$24 \div 4 = 6$$

$$24 \div 6 = 4$$

We then introduce the term 'Multiple of the Divisor' – Multiples of the number you are dividing by (divisor).

For example:

$$36 \div 6 =$$

'Counting on' in multiples of 6 (the divisor).

When confident, children may move on to:

3 digit \div 1 digit (using informal jottings).

Upper Key stage 2

Chunking method

The first written method taught for doing division calculations is referred to as a chunking method. This means removing chunks that are 'multiples of the divisor'. This is first done without a remainder, then with a remainder.

Always estimate first and check the reasonableness of the answer.

IMPORTANT- Children have to be able to subtract using decomposition to use this method

1. $72 \div 5$

5	72	Multiple
- 50		Divisor
22		(10 x 5)
- 20		(4 x 5)
2		

Answer: $10 + 4 = 14$
 $14 \text{ r } 2$

Children are encouraged to remove the largest multiple they can think of, in this case 50 being 10×5 .

How many multiples of 5 are there in 72?

I know that 10 multiples of 5 = 50

$$72 - 50 = 22$$

They continue to use multiples or chunks until they are left with a remainder.

How many multiples of 5 = 22? 4 or 5?

5 is too many, so 4 multiples of 5 = 20 with 2 left over.

We can't divide 5 into 2. How many multiples in total? $10 + 4 = 14$ so the answer is 14 remainder 2.

2. $256 \div 7$

7	256	
- 70		(10 x 7)
186		
- 140		(20 x 7)
46		
- 42		(6 x 7)
4		

For this question we know that 30 multiples of 7 are 210 which gets us up nearer to our answer, however for less confident children they may find it easier to start with multiples of 10.

NOTE As children become more confident with this method they should be able to get closer to the answer in less steps as their mathematical understanding and knowledge of tables increases.

Or

Answer: $10 + 20 + 6 = 36$
 $36 \text{ r } 4$

7	256	
- 210		(30 x 7)
46		
- 42		(6 x 7)
4		

Answer: $36 \text{ r } 4$

3. Extend to 3 digits \div by 2 digits

$$563 \div 13 =$$

$$\begin{array}{r}
 13 \overline{) 563} \\
 - 130 \quad (10 \times 13) \\
 \hline
 433 \\
 - 390 \quad (30 \times 13) \\
 \hline
 43 \\
 - 39 \quad (3 \times 13) \\
 \hline
 4 \\
 \hline
 \end{array}$$

Answer: 43 r 4

4. Extend to 4 digits \div by 2 digits

$$5634 \div 13 =$$

$$\begin{array}{r}
 13 \overline{) 5634} \\
 - 130 \quad (\underline{10} \times 13) \\
 \hline
 5504 \\
 - 5200 \quad (\underline{400} \times 13) \\
 \hline
 304 \\
 - 260 \quad (\underline{20} \times 13) \\
 \hline
 44 \\
 - 39 \quad (\underline{3} \times 13) \\
 \hline
 5 \text{ remainder}
 \end{array}$$

$$10 + 400 + 20 + 3 = 433 \text{ r } 5$$

5. Extend to simple decimals with up to 2 places

$$\begin{array}{r}
 \hline
 7 \quad 87.5 \\
 - 70.0 \quad (10 \times 7) \\
 \hline
 17.5 \\
 - 14.0 \quad (2 \times 7) \\
 \hline
 3.5 \\
 - 3.5 \quad (0.5 \times 7) \\
 \hline
 0.0 \\
 \hline
 \end{array}$$

Answer: $10 + 2 + 0.5 = 12.5$
12.5

When children have a secure understanding of the chunking method the 'bus stop' method may be used as an alternative to chunking. E.g. to divide three digit numbers by a single digit.

$$549 \div 9 = 61$$

61

$$\begin{array}{r}
 9 \overline{) 549} \\
 \hline
 \end{array}$$

This calculation should be read as 9 into 5 doesn't go, carry over the 5 to the 4. 9 into 54 goes 6 times. 9 into 9 goes once

Extend to division with a remainder.

$$509 \div 7 = 72 \text{ remainder } 5$$

$$\begin{array}{r}
 72 \\
 7 \overline{) 509} \\
 \hline
 \end{array}
 \quad \text{remainder } 5$$

Children are expected to continue consolidating their knowledge of the 'Bus stop' method. Application of the method will take place through the use of larger and more complicated numbers. Particularly able children will be introduced to long division and encouraged to continue applying the 'Bus stop' method e.g.

$$5634 \div 13 = 433 \text{ r } 5$$

$$\begin{array}{r} 433 \text{ r } 5 \\ 13 \overline{) 5634} \\ \underline{56} \\ 34 \\ \underline{35} \\ 5 \end{array}$$

Move towards a remainder as a decimal and rounding to 1 or 2 decimal places. And also move towards a remainder as a fraction.

$$656 \div 5 =$$

$$\begin{array}{r} 131 \text{ r } 1 \\ 5 \overline{) 656} \\ \underline{65} \\ 6 \\ \underline{5} \\ 1 \end{array}$$

$$656 \div 5 =$$

$$\begin{array}{r} 131 \frac{1}{5} \\ 5 \overline{) 656} \\ \underline{65} \\ 6 \\ \underline{5} \\ 1 \end{array}$$

$$656 \div 5 =$$

$$\begin{array}{r} 131.2 \\ 5 \overline{) 656.0} \\ \underline{65} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

By the end of Key Stage 2, children will have a range of mental calculation methods and be secure in their use of the 'Bus stop' method. Children will be encouraged to apply mental calculation methods wherever possible and to choose the method they are most confident in using. Children should not move on to the next stage if they are not ready or not confident.

Summary

- Children should always estimate first.
- Answers should usually be checked, preferably using a different method e.g. The inverse operation, making sure their answer is reasonable.
- Children will always decide first whether a mental method is appropriate.
- As teachers we will pay attention to mathematical vocabulary and the language we use and refer to the actual value of digits
e.g. $56 =$ fifty and six
- Children who make persistent mistakes will return to the method that they can use accurately until ready to move on.
- Children need to know number and multiplication / division facts by heart and will be taught this throughout the school.
- Time will be spent discussing errors and diagnosing problems and then working through the problem, rather than solely re-teaching the method.
- When revising or extending to more complex numbers using written methods, we will refer back to expanded methods when appropriate. This helps reinforce understanding and reminds children that they have an alternative to fall back on if they are having difficulties.
- Written methods will be introduced from Y4 onwards in expanded form as set out above.