



# Calculation Policy

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# Barrow 1618

## Calculation Policy

### Introduction

At Barrow 1618 children are introduced to the processes of calculation through **practical, oral** and **mental** activities. As children begin to understand the underlying ideas they *develop ways of recording to support their thinking* and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved.

Over time children learn how to *use models and images*, such as empty number lines, to **support their mental and informal written methods of calculation**. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly.

**When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.** They will do this by asking themselves:

- Can I do this in my head?
- Can I do this in my head using drawing or jottings?
- Do I need to use a pencil and paper procedure?
- Do I need a calculator?

At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave Barrow 1618 they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient and reliable written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

## Models and Images:

All too often children see Mathematics as compartmentalised and believe they have to learn a different trick for each area of maths and in some cases for each different form of calculation. At Barrow 1618 we use models and images to help children gain an understanding of number as quantities, be able to visualise problems and calculations and help them move from concrete representations to the abstract.

We begin by using actual objects and counting or doing calculations with these. For example, counting how many teddy bears in a row:



From this we begin to represent these objects with blocks and then with drawings or representations of blocks. These are linked to numerals and calculations in order to make these abstractions understandable. The use of models provides an extra step and helps make sure that the child's understanding is secure.

An example 5:

The use of this model helps the child to picture 5 and understand that the numeral '5' represents this quantity.

We can then use shading to split 5 – for example into 2 and 3:

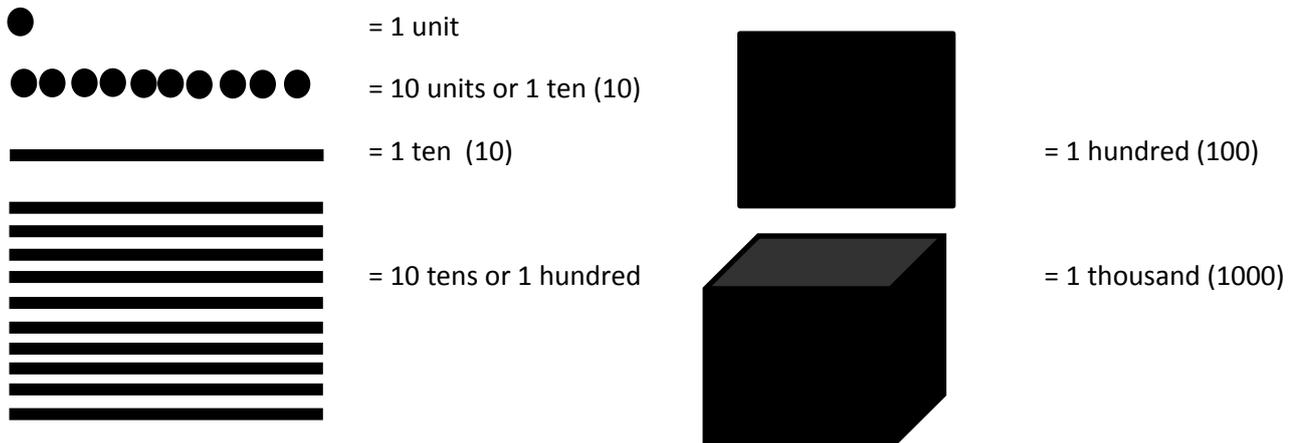


In this way we are creating visualisations of the ways to split this number for the children to use. We can then link this visualisation to number sentences. For example, the above picture represents  $2 + 3 = 5$  and  $5 - 2 = 3$ ,  $5 = 2 + 3$  and  $3 = 5 - 2$ , to name but a few. When we remove one of the numerals from our equation, the visualisation can help children solve this. For example the above model will help children solve  $2 + 3 = [ ]$ , or  $3 = 5 - [ ]$ .

By using a visualisation such as this, the number sentences also become linked in the children's minds and the relationships between addition and subtraction are made clearer.

### The Visualisation Method

Visualisation forms a large and important role in calculating at Barrow 1618. You can see from the example of '5' that 5 dots are used to represent 5 things. This model is extended to include symbols for 10, 100 and 1000.



In the Visualisation Method symbols are used to create a pictorial representation of numbers and used in calculations.

The Visualisation Method for Addition:

$$45 + 36 =$$

$$45 + 36 = 81$$

Keeping the symbols under each number stops them from merging into one another.

If children count the tens (70) and then units (11) they then have complete another addition calculation. By highlighting or crossing out units and adding an extra 10, it is clearer to see the answer 81. This also reinforces the concept that 10 units equals 1 ten.

$$354 + 379 =$$

$$354 + 379 = 733$$

1. As 379 has 9 units it is simple to cross out 1 unit from 354 and make a whole ten.

2. There are enough tens to make an extra hundred.

The Visualisation Method for Subtraction

$$45 - 11 =$$

$$45 - 11 = 34$$

For subtraction draw the larger number then 'take away' (cross out or highlight) the units then tens.

$$354 - 179 =$$

$$354 - 179 =$$

$$354 - 179 = 175$$

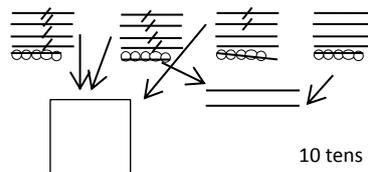
There are not enough units to subtract 9, so a ten has to be broken into 10 units. Similarly there are not enough tens to subtract 70, so a hundred is broken into 10 tens.

Now 9 units can be crossed out, 7 tens and 1 hundred. What is left is the answer.

The Visualisation Method for Multiplication

$$45 \times 4 =$$

$$45 \times 4 = 180$$



Multiplication can be thought of as repeated addition. One number can have 3 or 4 digit but must x by a 1-digit number. If multiplying by a 2 digit number a different method is needed.

Two extra tens can be made by converting the 20 units to tens.

10 tens can be crossed out and turned into one hundred.

Add up the symbols left and you get the answer (180)

## The Visualisation Method for Division

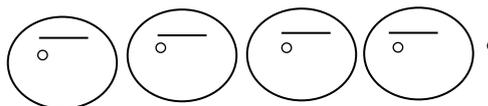
$$45 \div 4 =$$



In this example 45 needs to be shared equally into 4 groups.

One number can have 3 or 4 digits but must be divided by a 1-digit number, as it would be too labour intensive. If dividing by a 2 digit number a different method is needed.

$$45 \div 4 =$$



One unit cannot be shared so is therefore a remainder. It could be broken into tenths and shared to provide a decimal answer.

The symbols are shared into the groups, trying to keep whole hundreds and tens together where possible, but breaking them down when required.

## The Visualisation Method and the Decimal System:

We continue to use models and images for the decimal system. In this case we ask children to imagine we have zoomed in on 1 unit to see what it looks like close up. In doing so we see that we can split one into smaller pieces.

Therefore in this model, our hundred square now represents 1 unit:



Dots represent hundredths



lines represent tenths (10).

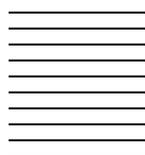


Just like when imagining 100, we use models to help the children imagine 1 unit in a variety of ways:

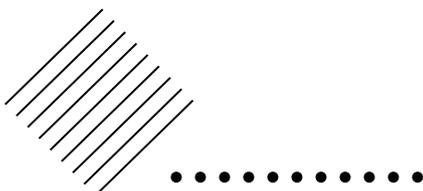
As one whole unit:



As 10 tenths:



Or as 9 tenths and 10 hundredths:



Calculations using decimals can therefore take place using these visualisations and where necessary children and teachers can draw these models to help.

When children are able to visualise decimals in the way outlined above, the relationship between fractions and decimals is easily explained. For example 3 quarters of 1 is found by visualising 1 as 100 hundredths and finding 3 quarters of these hundredths (75 hundredths or 0.75).

By drawing out models using these representations we can explore a variety of calculation strategies, which are discussed in more detail below.

## Written methods for addition and subtraction of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children need to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers.

To add successfully, children need to be able to:

- recall all addition pairs to  $9 + 9$  and complements in 10 e.g.  $6 + 4 = 10$ ;
- add mentally a series of one-digit numbers, such as  $5 + 8 + 4$ ;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as  $160 - 70$ ) using the related subtraction fact,  $16 - 7$ , and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into  $70 + 4$  or  $60 + 14$ ).

**Note:** *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.*

**The Visualisation Method will be the first written method that children learn, as their knowledge of place value becomes cemented and they begin to work with larger numbers children will progress onto other methods of calculation.**

### The Column method for Addition

$$56 + 67 =$$

Numbers are arranged in HTU columns and added vertically from right to left.

	T U	H T U	
	<b>56</b>	<b>56</b>	
+	<b>67</b>	+	<b>67</b>
	<hr style="width: 50%; margin: 0 auto;"/>		<hr style="width: 50%; margin: 0 auto;"/>
	<b>3</b>		<b>123</b>
	<hr style="width: 50%; margin: 0 auto;"/>		<hr style="width: 50%; margin: 0 auto;"/>
	<b>1</b>		<b>1</b>

If the number is greater than ten the ten digit must go into the next column. E.g. the 1 from 13.

Easily extended to decimals by adding a tenths (t) and/or hundredths (h) column.

## The Column method for Subtraction

$$425 - 143 =$$

$$\begin{array}{r} 425 \\ - 143 \\ \hline 282 \end{array}$$

$$\begin{array}{r} 3\cancel{4}25 \\ - 143 \\ \hline 82 \end{array}$$

$$\begin{array}{r} 3\cancel{4}25 \\ - 143 \\ \hline 282 \end{array}$$

If the digit you are subtracting from is smaller than the digit you are subtracting (2 and 4, the tens column of this example) then you need to move a digit from the column on the left in order to make the digit larger. In this case 2 becomes 12. The digits in the column to the left need to be marked as 1 less.

Numbers are arranged in HTU columns with the largest on top and subtracted vertically from right to left.

Easily extended to decimals by adding a tenths (t) and/or hundredths (h) column.

## Written methods for multiplication and division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children need to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication, two-digit by two-digit multiplication, and three-digit by two-digit multiplication.

To multiply successfully, children need to be able to:

- recall all multiplication facts to  $10 \times 10$ ;
- partition number into multiples of one hundred, ten and one;
- work out products such as  $70 \times 5$ ,  $70 \times 50$ ,  $700 \times 5$  or  $700 \times 50$  using the related fact  $7 \times 5$  and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as  $60 + 70$ ) or of 100 (such as  $600 + 700$ ) using the related addition fact,  $6 + 7$ , and their knowledge of place value;
- add combinations of whole numbers

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in  $18 \div 3 = 6$ , the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to  $10 \times 10$ , recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

**Note:** It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication

The Visualisation Method will be the first written method that children learn, as their knowledge of place value becomes cemented and they begin to work with larger numbers children will progress onto other methods of calculation.

The Repeated Addition method for Multiplication

**45 x 4 =**

HTU
45
45
45
45
-----
180
-----
2

TU
45
45
-----
90
-----
1

TU	TU
45	90
45	90
-----	-----
90	180
-----	-----
1	

Multiplication problems are treated as repeated addition. 45 x 4 becomes 45+ 45+45+45.

It can be easier to add a section and add the answer repeatedly. E.g. adding only two 45s to make 90 then add two 90s.

Partitioning Method for Multiplication

**45 x 14 =**

45 x 10 = 450 and 45 x 4 = 180

450 + 180 = **630**

Keep one number whole and partition the other number.

Usually partition into hundreds, tens and units but you could partition further.

**23 x 22 =**

23 x 10 = 230 230 x 2 = 460 23 x 2 = 43

460 + 43 = **503**

The Grid Method for multiplication

Both numbers are partitioned into HTU (thousands and tenths, hundredths if required). They are then multiplied where they cross (vertical/horizontal). E.g. 100x50=5000.

<b>123 x 54</b>				
x	100	20	3	Totals
50	5000	1000	150	6150
4	400	80	12	492
				6642
				1

It is easier to ignore the place holders (0s), multiply the digits then multiply by the place holders. For example 20 x 50. 2 x 5 = 10, multiply by 100 (as there are 2 place holders) = 1000.

Totals of the columns or rows needed to be calculated and then added to produce the final answer.

## Partitioning Method for Division

Partition the number to be divided into smaller chunks, maybe hundred, tens and unit or values of known number facts.

$$172 \div 8 =$$

$$160 \div 8 = 20 \quad 12 \div 8 = 1 \text{ r } 4 \quad 20 + 1\text{r}4 = \mathbf{21\text{r}4}$$

Or

$$100 \div 8 = 12 \text{ r } 4 \quad 72 \div 8 = 9 \quad 12\text{r}4 + 9 = \mathbf{21\text{r}4}$$

## Long Division Method

$$172 \div 8 = 21\text{r}4$$

Essentially this is repeated subtraction, 'chunking' several subtraction steps to quicken the process up.

In this example we need to work out how many 8s are in 172. We could subtract 8 until reached close to zero but it would take ages so we 'chunk', and subtract 10 lots of 8 in one go.

$$\begin{array}{r} 172 \\ - 80 \quad (10 \times 8) \\ \hline 92 \\ - 80 \quad (10 \times 8) \\ \hline 12 \\ - 8 \quad (1 \times 8) \\ \hline 4 \quad \text{r}4 \end{array}$$

Subtract easy values; such as 100x, 50x, 10x, 20x, 5x, 2x.

It is important to underline/circle the 'chunk' (in this example how many 8s) as this forms the answer.

## Adjustment Strategies

Adjustment strategies are those which transform one calculation into another easier one by adjusting the numbers. This is done by moving the amounts from one part of the number or equation to the other.

In its simplest form adjustment strategies may be used as a way to enhance mental calculation strategies.

These are supported with models and images to illustrate what is happening to the totals.

### **Example 1:**

Here we move one from the 26 to the 9 to transform the calculation:

$$26 + 9 \text{ becomes } 25 + 10$$

*move 1*


$$26 + 9 = 25 + 10 = \mathbf{35}$$

Alternatively we can add one to the nine and adjust the total afterwards:

$$26 + 9 =$$

$$26 + 10 (9 + 1) = 36$$

$$36 - 1 = 35$$

### **Example 2:**

In a subtraction calculation we can adjust by adding to both the starting value and the amount to be subtracted:

$$36 - 9 =$$

$$(36 + 1) - (9 + 1) =$$

$$37 - 10 = \mathbf{27}$$

Or we can adjust the final total:

$$36 - 9 =$$

$$36 - 10 (9 + 1) = 26$$

$$26 + 1 = \mathbf{27}$$

## Calculation Progression

The calculation method that a child is taught will depend upon their numeracy level and not their year group. The tables below provide a rough outline of the relationship between numeracy assessment level and calculation method for each of the four operations.

### **Addition**

<b><u>Level</u></b>	<b><u>Numbers working with</u></b>	<b><u>Method</u></b>
EYFS	$U + U$	Practical
1	$TU + U < 20$	Practical/visualisation
2	$TU + TU$	Visualisation
3	$HTU + HTU$	Column
4	$ThHTU.t + ThHTU.t$	Column
5	$ThHTU.th + ThHTU.th$	Column

### **Subtraction**

<b><u>Level</u></b>	<b><u>Numbers working with</u></b>	<b><u>Method</u></b>
EYFS	$U - U$	Practical
1	$TU - U < 20$	Practical/visualisation
2	$TU - TU$	Visualisation
3	$HTU - HTU$	Column
4	$ThHTU.t - ThHTU.t$	Column
5	$ThHTU.th - ThHTU.th$	Column

### **Multiplication**

<b><u>Level</u></b>	<b><u>Numbers working with</u></b>	<b><u>Method</u></b>
EYFS	$U \times U$	Practical
1	$TU \times U < 20$	Practical/Visualisation
2	$TU \times U$	Visualisation
3	$HTU \times U$	Repeated Addition/ Partitioning
4	$HTU \times TU$	Grid
5	$HTU.th \times HTU.th$	Grid

### **Division**

<b><u>Level</u></b>	<b><u>Numbers working with</u></b>	<b><u>Method</u></b>
EYFS	$U \div U$	Practical
1	$TU \div U < 20$	Practical/Visualisation
2	$TU \div U$	Visualisation
3	$HTU \div U$	Visualisation/ Partitioning
4	$HTU \div TU$	Long Division
5	$HTU.th \div HTU$	Long Division