# Guidance paper – Calculation

## Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

# Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10 (Year 2), sums and differences of multiples of 10 (Year 3) and multiplication facts up to 10 × 10 (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1), partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine (Year 2), when applying mental methods in special cases (Year 5);
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

# Written methods of calculation

The 1999 Framework sets out progression in written methods of calculation that highlights how children would move from informal methods of recording to expanded methods that are staging posts to a compact written method for each of the four operations.

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as ‘standard’ written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that schools adopt greater consistency in their approach to calculation that all teachers understand and towards which they work. There has been some confusion as to the progression to written methods and for too many children the staging posts along the way to the more compact method have instead become end points. While this may represent a significant achievement for some children, the great majority are entitled to learn how to use the most efficient methods. The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

The incidence of children moving between schools and localities is very high in some parts of the country. Moving to a school where the written method of calculation is unfamiliar and does not relate to that used in the previous school can slow the progress a child makes in mathematics. There will be differences in practices and approaches which can be beneficial to children. However, if the long-term aim is shared across all schools and if expectations are consistent then children’s progress will be enhanced rather than limited. The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the renewed objectives. Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

# Objectives

The objectives in the revised Framework show the progression in children's use of written methods of calculation in the strands 'Using and applying mathematics' and 'Calculating'.

| Using and applying mathematics | Calculating |
|---|---|
| **Year 1**<br>• Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change'<br>• Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context | **Year 1**<br>• Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number<br>• Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number<br>• Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences |
| **Year 2**<br>• Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence<br>• Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem | **Year 2**<br>• Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders<br>• Use the symbols +, –, ×, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. ☐ ÷ 2 = 6, 30 – ☐ = 24) |
| **Year 3**<br>• Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations<br>• Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure | **Year 3**<br>• Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers<br>• Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 × 3, 50 ÷ 4); round remainders up or down, depending on the context<br>• Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences |

| Using and applying mathematics | Calculating |
| --- | --- |
| **Year 4**<br>• Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate<br>• Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem | **Year 4**<br>• Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p<br>• Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 × 9, 98 ÷ 6) |
| **Year 5**<br>• Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use<br>• Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem | **Year 5**<br>• Use efficient written methods to add and subtract whole numbers and decimals with up to two places<br>• Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000<br>• Refine and use efficient written methods to multiply and divide HTU × U, TU × TU, U.t × U and HTU ÷ U |
| **Year 6**<br>• Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use<br>• Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of *c* pens at 15 pence each is 15*c* pence) | **Year 6**<br>• Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer |

# Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

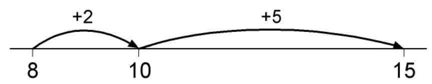
***Note:*** *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.*

## Stage 1: The empty number line

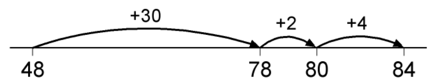
- The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

8 + 7 = 15




48 + 36 = 84

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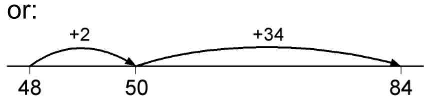


## Stage 2: Partitioning

- The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums.
- Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

Record steps in addition using partitioning:

47 + 76 = 47 + 70 + 6 = 117 + 6 = 123

47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123

Partitioned numbers are then written under one another:

$$\begin{array}{rcrl} 47 & = & 40 + & 7 \\ \underline{+76} & & \underline{70 +} & \underline{6} \\ & & 110 + & 13 \; = 123 \end{array}$$

<table>
<tr><th>Stage 3: Expanded method in columns</th><th></th></tr>
<tr><td>

- Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums either the tens or the ones can be added first, and the total of the partial sums can be found by adding them in any order. As children gain confidence, ask them to start by adding the ones digits first always.
- The addition of the tens in the calculation 47 + 76 is described in the words ‘forty plus seventy equals one hundred and ten’, stressing the link to the related fact ‘four plus seven equals eleven’.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

</td><td>

Write the numbers in columns.

Adding the tens first:

$\begin{array}{r} 47 \\ +\ 76 \\ \hline 110 \\ 13 \\ \hline 123 \end{array}$

Adding the ones first:

$\begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$

Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to adding the ones digits first consistently.

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<tr><th>Stage 4: Column method</th><th></th></tr>
<tr><td>

- In this method, recording is reduced further. Carry digits are recorded below the line, using the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’.
- Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

</td><td>

$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \hline {\scriptstyle 1\,1} \end{array} \qquad \begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ \hline {\scriptstyle 1\,1} \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ \hline {\scriptstyle 1\,1} \end{array}$

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

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# Written methods for subtraction of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

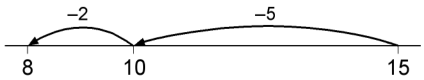
***Note:*** *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.*

## Stage 1: Using the empty number line

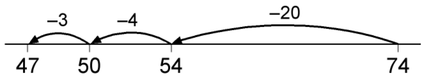
- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.
- The notes below give more detail on the counting-up method using an empty number line.

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

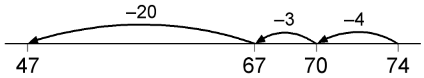
15 – 7 = 8



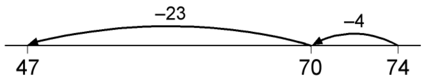

74 – 27 = 47 worked by counting back:




The steps may be recorded in a different order:

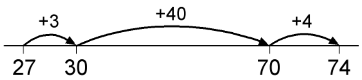



or combined:



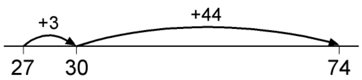

## The counting-up method

- The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally.



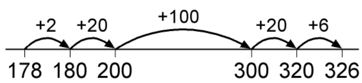

$$\begin{array}{rl} & 74 \\ - & \underline{27} \\ & 3 \quad \to 30 \\ & 40 \quad \to 70 \\ & \underline{4} \quad \to 74 \\ & 47 \end{array}$$

or:



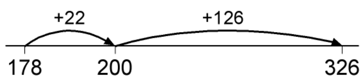

$$\begin{array}{rl} & 74 \\ - & \underline{27} \\ & 3 \quad \to 30 \\ & \underline{44} \quad \to 74 \\ & 47 \end{array}$$

- With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally.
- The most compact form of recording remains reasonably efficient.



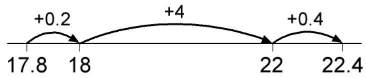

$$\begin{array}{rl} & 326 \\ - & \underline{178} \\ & 2 \quad \to 180 \\ & 20 \quad \to 200 \\ & 100 \quad \to 300 \\ & \underline{26} \quad \to 326 \\ & 148 \end{array}$$

or:



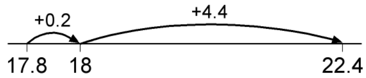

$$\begin{array}{rl} & 326 \\ - & \underline{178} \\ & 22 \quad \to 200 \\ & \underline{126} \quad \to 326 \\ & 148 \end{array}$$

- The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.
- This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.




$$\begin{array}{rl} & 22.4 \\ - & \underline{17.8} \\ & 0.2 \to 18 \\ & 4.0 \to 22 \\ & \underline{0.4} \to 22.4 \\ & 4.6 \end{array}$$

or:




$$\begin{array}{rl} & 22.4 \\ - & \underline{17.8} \\ & 0.2 \to 18 \\ & \underline{4.4} \to 22.4 \\ & 4.6 \end{array}$$

## Stage 2: Partitioning

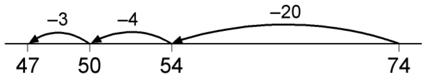
- Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.

Subtraction can be recorded using partitioning:

74 – 27 = 74 – 20 – 7 = 54 – 7 = 47

74 – 27 = 70 + 4 – 20 – 7 = 60 + 14 – 20 – 7 = 40 + 7

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.




## Stage 3: Expanded layout, leading to column method

- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Partitioned numbers are then written under one another:

Example: 74 − 27

$$\begin{array}{rr} & 70 + 4 \\ - & \underline{20 + 7} \end{array} \qquad \begin{array}{rr} & \overset{60}{\cancel{70}} + \overset{14}{\cancel{4}} \\ - & \underline{20 + 7} \\ & 40 + 7 \end{array} \qquad \begin{array}{rr} & \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ - & \underline{2\ 7} \\ & 4\ 7 \end{array}$$

Example: 741 − 367

$$\begin{array}{rr} & 700 + 40 + 1 \\ - & \underline{300 + 60 + 7} \end{array} \qquad \begin{array}{rr} & \overset{600}{\cancel{700}} + \overset{130}{\cancel{40}} + \overset{11}{\cancel{1}} \\ - & \underline{300 + 60 + 7} \\ & 300 + 70 + 4 \end{array} \qquad \begin{array}{rr} & \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ - & \underline{3\ 6\ 7} \\ & 3\ 7\ 4 \end{array}$$

## The expanded method for three-digit numbers

Example: 563 − 241, no adjustment or decomposition needed

Expanded method leading to

$$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array} \qquad\qquad \begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying ‘sixty take away forty’, not ‘six take away four’.

Example: 563 − 271, adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+1 \\ \hline \end{array} \qquad \begin{array}{r} 400+160+3 \\ -\ 200+\ \ 70+1 \\ \hline 200+\ \ 90+2 \end{array} \qquad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{160}{\cancel{60}}+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{16}{\cancel{6}}\ 3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: ‘five hundred and sixty-three’. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes ‘160 minus 70’, an application of subtraction of multiples of ten.

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+150+13 \\ -\ 200+\ \ 70+\ \ 8 \\ \hline 200+\ \ 80+\ \ 5 \end{array} \qquad \begin{array}{r} \overset{400}{\overset{\cancel{500}}{\cancel{500}}}+\overset{150}{\overset{\cancel{50}}{\cancel{60}}}+\overset{13}{\overset{13}{\cancel{3}}} \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{15}{\cancel{6}}\ \overset{13}{\cancel{3}} \\ -\ 2\ 7\ 8 \\ \hline 2\ 8\ 5 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

$$\begin{array}{r} 500+\ \ 0+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\ 200+70+\ \ 8 \\ \hline 200+20+\ \ 5 \end{array} \qquad \begin{array}{r} \overset{400}{\overset{\cancel{400}}{\cancel{500}}}+\overset{90}{\overset{\cancel{100}}{\cancel{0}}}+\overset{13}{\overset{3}{\cancel{3}}} \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \qquad \begin{array}{r} \overset{4}{\cancel{5}}\ \overset{9}{\cancel{0}}\ \overset{13}{\cancel{3}} \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

# Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

***Note:*** *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.*

<table>
<tr><th colspan="2">Stage 1: Mental multiplication using partitioning</th></tr>
<tr>
<td>• Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.</td>
<td>Informal recording in Year 4 might be:<br>
$\begin{array}{cccc} & 43 & & \\ 40 & + & 3 & \\ \downarrow & & \downarrow & \times 6 \\ 240 & + & 18 & = 258 \end{array}$<br>
Also record mental multiplication using partitioning:<br>
$14 \times 3 = (10 + 4) \times 3$<br>
$= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$<br>
$43 \times 6 = (40 + 3) \times 6$<br>
$= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$<br>
Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:<br>
○○○○○○○ ○○○○○…○○<br>
○○○○○○○ ○○○○○…○○<br>
○○○○○○○ ○○○○○…○○<br>
$7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$</td>
</tr>
</table>

## Stage 2: The grid method

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

$$\begin{array}{c|r}
\times & 7 \\ \hline
30 & 210 \\ \hline
8 & 56 \\ \hline
 & 266
\end{array}$$

- The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.
- The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.

$$\begin{array}{c|r}
 & 30 + 8 \\ \hline
\times & 7 \\ \hline
 & 210 \\ \hline
 & 56 \\ \hline
 & 266
\end{array}$$

## Stage 3: Expanded short multiplication

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.
- Most children should be able to use this expanded method for TU × U by the end of Year 4.

$$\begin{array}{r}
30 + 8 \\
\times \underline{\quad 7} \\
210 \\
\underline{56} \\
\underline{266}
\end{array}
\qquad
\begin{array}{r}
\\
\\
30 \times 7 = 210 \\
8 \times 7 = \ \ 56 \\
\\
\end{array}
\qquad
\begin{array}{r}
38 \\
\times \underline{\quad 7} \\
210 \\
\underline{56} \\
\underline{266}
\end{array}$$

## Stage 4: Short multiplication

- The recording is reduced further, with carry digits recorded below the line.
- If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

$$\begin{array}{r}
38 \\
\times \underline{\quad 7} \\
\underline{266} \\
{\scriptstyle 5\ }
\end{array}$$

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

<table>
<tr><th colspan="2">Stage 5: Two-digit by two-digit products</th></tr>
<tr>
<td>

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
- As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

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56 × 27 is approximately 60 × 30 = 1800.

<table>
<tr><td>×</td><td>20</td><td>7</td><td></td></tr>
<tr><td>50</td><td>1000</td><td>350</td><td>1350</td></tr>
<tr><td>6</td><td>120</td><td>42</td><td>162</td></tr>
<tr><td></td><td></td><td></td><td>1512</td></tr>
<tr><td></td><td></td><td></td><td>1</td></tr>
</table>

<table>
<tr><td></td><td>50</td><td>6</td><td></td></tr>
<tr><td>×</td><td>20</td><td>7</td><td></td></tr>
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<tr><td></td><td>120</td><td>42</td><td>162</td></tr>
<tr><td></td><td></td><td></td><td>1512</td></tr>
<tr><td></td><td></td><td></td><td>1</td></tr>
</table>

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<tr>
<td>

- Reduce the recording, showing the links to the grid method above.

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<td>

56 × 27 is approximately 60 × 30 = 1800.

$$\begin{array}{rr} & 56 \\ \times & \underline{27} \\ & 1000 \\ & 120 \\ & 350 \\ & \underline{42} \\ & \underline{1512} \\ & 1\phantom{000} \end{array} \qquad \begin{array}{rl} \\ \\ 50\times20 = & 1000 \\ 6\times20 = & \phantom{0}120 \\ 50\times7 = & \phantom{0}350 \\ 6\times7 = & \phantom{00}42 \\ \\ \\ \end{array}$$

</td>
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<td>

- Reduce the recording further.
- The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally.
- The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.

</td>
<td>

56 × 27 is approximately 60 × 30 = 1800.

$$\begin{array}{rr} & 56 \\ \times & \underline{27} \\ & 1120 \\ & \underline{392} \\ & \underline{1512} \\ & 1\phantom{000} \end{array} \qquad \begin{array}{l} \\ \\ 56\times20 \\ 56\times\phantom{0}7 \\ \\ \\ \end{array}$$

</td>
</tr>
</table>

<table>
<tr><th colspan="2">Stage 6: Three-digit by two-digit products</th></tr>
<tr><td>

- Extend to HTU × TU asking children to estimate first. Start with the grid method.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

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286 × 29 is approximately 300 × 30 = 9000.

<table>
<tr><td>×</td><td>20</td><td>9</td><td></td></tr>
<tr><td>200</td><td>4000</td><td>1800</td><td>5800</td></tr>
<tr><td>80</td><td>1600</td><td>720</td><td>2320</td></tr>
<tr><td>6</td><td>120</td><td>54</td><td>174</td></tr>
<tr><td></td><td></td><td></td><td>8294</td></tr>
<tr><td></td><td></td><td></td><td>1</td></tr>
</table>

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<tr><td>

- Reduce the recording, showing the links to the grid method above.
- This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.

</td><td>

$$\begin{array}{rr}
 & 286 \\
\times & \underline{\phantom{0}29} \\
 & 4000 \quad 200 \times 20 = 4000 \\
 & 1600 \quad 80 \times 20 = 1600 \\
 & 120 \quad 6 \times 20 = \phantom{0}120 \\
 & 1800 \quad 200 \times \phantom{0}9 = 1800 \\
 & 720 \quad 80 \times \phantom{0}9 = \phantom{0}720 \\
 & \underline{\phantom{00}54} \quad 6 \times \phantom{0}9 = \phantom{00}54 \\
 & \underline{8294} \\
 & 1\phantom{000}
\end{array}$$

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- Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.
- Again, the carry digits in the partial products are usually carried mentally.

</td><td>

286 × 29 is approximately 300 × 30 = 9000.

$$\begin{array}{rr}
 & 286 \\
\times & \underline{\phantom{0}29} \\
 & 5720 \quad 286 \times 20 \\
 & \underline{2574} \quad 286 \times \phantom{0}9 \\
 & \underline{8294} \\
 & 1\phantom{000}
\end{array}$$

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# Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

***Note:*** *It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.*

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

## Stage 1: Mental division using partitioning

- Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.
- Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.
- Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.

One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately.

Informal recording in Year 4 for 84 ÷ 7 might be:

84

70 + 14

÷ 7

10 + 2 = 12

In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

Another way to record is in a grid, with links to the grid method of multiplication.

| × | | |
|---|---|---|
| 7 | 70 | 14 |

→

| × | 10 | 2 |
|---|---|---|
| 7 | 70 | 14 |

10 + 2 = 12

As the mental method is recorded, ask: ‘How many sevens in seventy?’ and: ‘How many sevens in fourteen?’

Also record mental division using partitioning:

64 ÷ 4 = (40 + 24) ÷ 4
= (40 ÷ 4) + (24 ÷ 4)
= 10 + 6 = 16

87 ÷ 3 = (60 + 27) ÷ 3
= (60 ÷ 3) + (27 ÷ 3)
= 20 + 9 = 29

Remainders after division can be recorded similarly.

96 ÷ 7 = (70 + 26) ÷ 7
= (70 ÷ 7) + (26 ÷ 7)
= 10 + 3 R 5 = 13 R 5

| Stage 2: Short division of TU ÷ U | |
|---|---|
| • 'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning.<br>• Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.<br>• For most children this will be at the end of Year 4 or the beginning of Year 5.<br>• The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21divided by three?' which gives the answer 7. | For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21. Each number is then divided by 3.<br>$81 \div 3 = (60 + 21) \div 3$<br>$= (60 \div 3) + (21 \div 3)$<br>$= 20 + 7$<br>$= 27$<br>The short division method is recorded like this:<br>$3\overline{)60+21}$ with $20+7$ above<br>This is then shortened to:<br>$3\overline{)8\,{}^{2}1}$ with $2\ 7$ above<br>The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript.<br>The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones. |

| Stage 3: 'Expanded' method for HTU ÷ U | |
|---|---|
| • This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.<br>• For TU ÷ U there is a link to the mental method.<br>• As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?'<br>• Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.<br>• This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.<br>• Chunking is useful for reminding children of the link between division and repeated subtraction.<br>• However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples. | 97 ÷ 9<br>$\begin{array}{rl} 9\overline{)97} & \\ -\underline{90} & 9\times10 \\ 7 & \end{array}$<br>Answer: 10 R 7<br><br>$\begin{array}{rl} 6\overline{)196} & \\ -\underline{\ 60} & 6\times10 \\ 136 & \\ -\underline{\ 60} & 6\times10 \\ 76 & \\ -\underline{\ 60} & 6\times10 \\ 16 & \\ -\underline{\ 12} & 6\times\underline{\ 2} \\ 4 & \quad 32 \end{array}$<br>Answer: 32 R 4 |
| • The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend.<br>• Estimating has two purposes when doing a division:<br>  – to help to choose a starting point for the division;<br>  – to check the answer after the calculation.<br>• Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right. | To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40.<br>Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.<br>$\begin{array}{rl} 6\overline{)196} & \\ -\underline{180} & 6\times30 \\ 16 & \\ -\underline{\ 12} & 6\times\underline{\ 2} \\ 4 & \quad 32 \end{array}$<br>Answer: 32 R 4<br>The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted. |

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## Stage 4: Short division of HTU ÷ U

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- ‘Short’ division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction.
- The accompanying patter is ‘How many threes in 290?’ (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: ’How many threes in 21?’ which has the answer 7.
- Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.
- For most children this will be at the end of Year 5 or the beginning of Year 6.

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For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3.

$$\begin{aligned} 291 \div 3 &= (270 + 21) \div 3 \\ &= (270 \div 3) + (21 \div 3) \\ &= 90 + 7 \\ &= 97 \end{aligned}$$

The short division method is recorded like this:

$$3\overline{)290+1} = \begin{array}{r} 90+\ 7 \\ 3\overline{)270+21} \end{array}$$

This is then shortened to:

$$\begin{array}{r} 9\ 7 \\ 3\overline{)2\ 9\,{}^{2}1} \end{array}$$

The carry digit ‘2’ represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3.

The 97 written above the line represents the answer: 90 + 7, or 9 tens and 7 ones.

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## Stage 5: Long division

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- The next step is to tackle HTU ÷ TU, which for most children will be in Year 6.
- The layout on the right, which links to chunking, is in essence the ‘long division’ method. Recording the build-up to the quotient on the left of the calculation keeps the links with ‘chunking’ and reduces the errors that tend to occur with the positioning of the first digit of the quotient.
- Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

</td><td>

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$\begin{array}{rrl} 24 & \overline{)\ 560} & \\ 20 & -\underline{480} & 24 \times 20 \\ & 80 & \\ 3 & \underline{72} & 24 \times 3 \\ & 8 & \end{array}$$

Answer: 23 R 8

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$\begin{array}{r} 23 \\ 24\overline{)\ 560} \\ -\underline{480} \\ 80 \\ -\underline{72} \\ 8 \end{array}$$

Answer: 23 R 8

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