p165-p296 : Funções Elementares.

- ► Ler selectivamente.
- ► Conhecer genericamente as funções.
- ► Fazer exercícios se necessário.

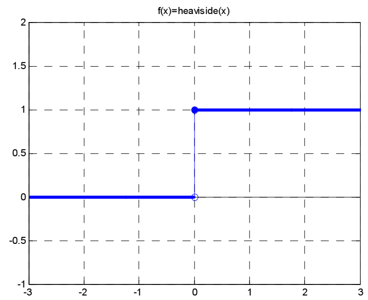
- ► Função de Heaviside.
- ► Função Potência.
- ► Função Exponencial.
- ► Função Logaritmo.
- ► Funções Hiperbólicas.
- ► Funções Trigonométricas.



## Função de Heaviside

$$h(x) = \begin{cases} 0 & , x < 0 \\ 1 & , x \geq 0 \end{cases}$$

No contexto da electrotecnia a função é conhecida por escalão unitário.

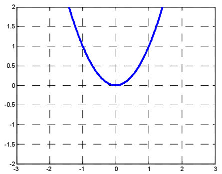




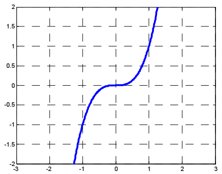

## Função Potência

$$f(x) = x^a$$

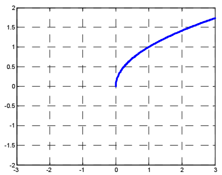
$y = x^2$



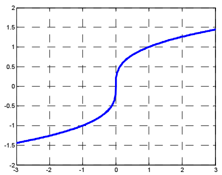
$y = x^3$



$y = \sqrt{x}$

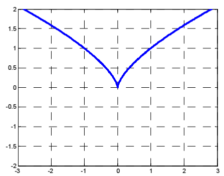


$y = \sqrt[3]{x}$

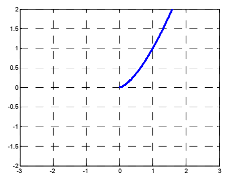




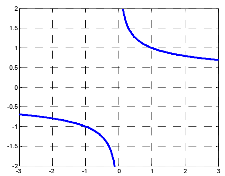
$y = \sqrt[3]{x^2}$



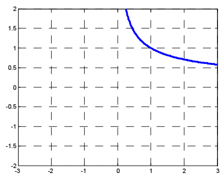
$y = \sqrt[2]{x^3}$



$y = x^{-1/3} = \dfrac{1}{\sqrt[3]{x}}$



$y = x^{-1/2} = \dfrac{1}{\sqrt[2]{x}}$

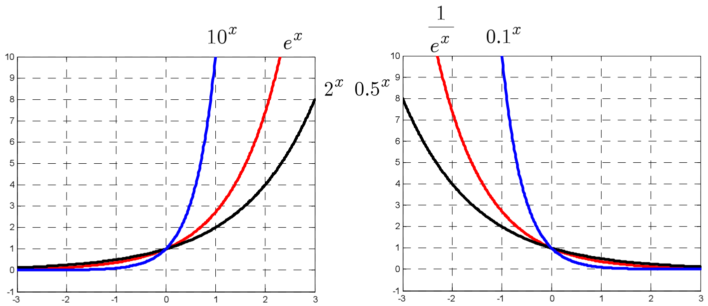


## Função Exponencial

Chama-se função exponencial a uma f.r.v.r cuja expressão analítica seja da forma

$f(x) = a^x \quad , a > 0, a \neq 1$

$a > 1$ $\qquad$ $0 < a < 1$






## Função Logarítmica

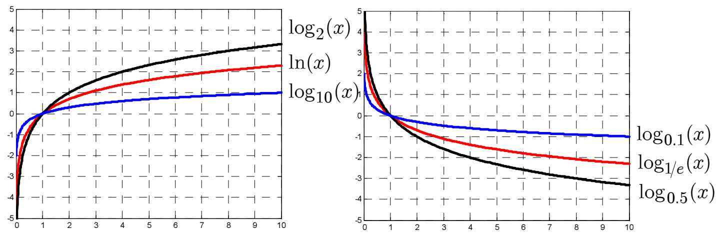
Diz-se que o logaritmo de $x$ na base $a$ é $y$ e escreve-se $y = \log_a x$ se $a^y = x$

$f(x) = \log_a x \quad , a > 0, a \neq 1, e\ x > 0$

Chama-se função logarítmica à função inversa da função exponencial

$f(x) = a^x \qquad f^{-1}(x) = \log_a(x)$

$a > 1$ $\qquad$ $0 < a < 1$






## Propriedades operacionais de exponenciais e logaritmos

| | |
|---|---|
| $\log_a 1 = 0$ | $\log_a u + \log_a v = \log_a (uv)$ |
| $\log_a a = 1$ | $\log_a u - \log_a v = \log_a \left(\frac{u}{v}\right)$ |
| $u = a^{\log_a u}$ | $\alpha \log_a u = \log_a u^\alpha , \alpha \in \mathbb{R}$ |
| $u = \log_a a^u$ | $\log_b u = \frac{\log_a u}{\log_a b}$ |
| $a^u \times a^v = a^{u+v}$ | $\frac{a^u}{a^v} = a^{u-v}$ |
| $(a^u)^v = a^{u \times v}$ | |



## Funções Hiperbólicas

| Símbolo | Expressão analítica | Domínio | Contradomínio |
|---|---|---|---|
| $\mathrm{sh}(x)$ | $\frac{e^x - e^{-x}}{2}$ | $\Re$ | $\Re$ |
| $\mathrm{ch}(x)$ | $\frac{e^x + e^{-x}}{2}$ | $\Re$ | $[1,+\infty[$ |
| $\mathrm{tgh}(x)$ | $\frac{\mathrm{sh}(x)}{\mathrm{ch}(x)}$ | $\Re$ | $]-1,1[$ |
| $\mathrm{cotgh}(x)$ | $\frac{1}{\mathrm{tgh}(x)}$ | $\Re \setminus \{0\}$ | $]-\infty,-1[ \cup ]1,+\infty[$ |
| $\mathrm{sech}(x)$ | $\frac{1}{\mathrm{ch}(x)}$ | $\Re$ | $]0,1]$ |
| $\mathrm{cosech}(x)$ | $\frac{1}{\mathrm{sh}(x)}$ | $\Re \setminus \{0\}$ | $\Re \setminus \{0\}$ |

## Funções Hiperbólicas Inversas

| Símbolo | Expressão analítica | Domínio | Contradomínio |
|---|---|---|---|
| $\operatorname{argsh}(x)$ | $\ln(x+\sqrt{x^2+1})$ | $\Re$ | $\Re$ |
| $\operatorname{argch}(x)$ | $\ln(x+\sqrt{x^2-1})$ | $[1,+\infty[$ | $[0,+\infty[$ |
| $\operatorname{argtgh}(x)$ | $\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ | $]-1,1[$ | $\Re$ |
| $\operatorname{argcotgh}(x)$ | $\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ | $]-\infty,-1[\,\cup\,]1,+\infty[$ | $\Re\setminus\{0\}$ |
| $\operatorname{argsech}(x)$ | $\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ | $]0,1]$ | $[0,+\infty[$ |
| $\operatorname{argcosech}(x)$ | $\frac{\lvert x\rvert}{x}\ln\left(\frac{1+\sqrt{1+x^2}}{\lvert x\rvert}\right)$ | $\Re\setminus\{0\}$ | $\Re\setminus\{0\}$ |



## Funções Seno Hiperbólico e Argumento do Seno Hiperbólico

$$\operatorname{sh}(x)=\frac{e^{x}-e^{-x}}{2}$$

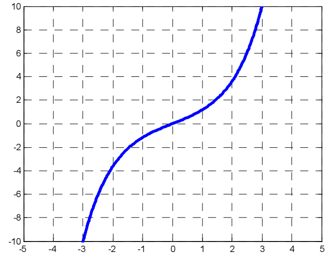
$D=\Re$

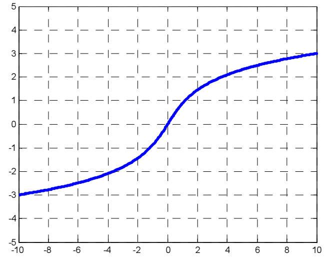
$CD=\Re$

$$\operatorname{argsh}(x)=\ln(x+\sqrt{x^2+1})$$

$D=\Re$

$CD=\Re$






## Funções Coseno Hiperbólico e Argumento do Coseno Hiperbólico

$$\operatorname{ch}(x)=\frac{e^{x}+e^{-x}}{2}$$

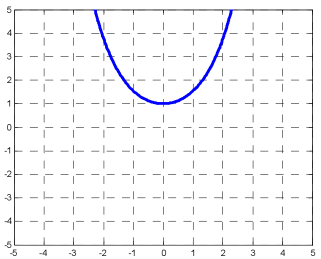
$D=\Re$

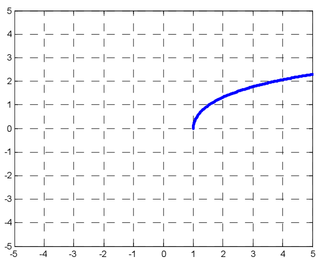
$CD=[1,+\infty[$

$$\operatorname{argch}(x)=\ln(x+\sqrt{x^2-1})$$

$D=[1,+\infty[$

$CD=[0,+\infty[$






Cuidado com o domínio das funções

$$\operatorname{tgh}(x)=\frac{\operatorname{sh}(x)}{\operatorname{ch}(x)}$$

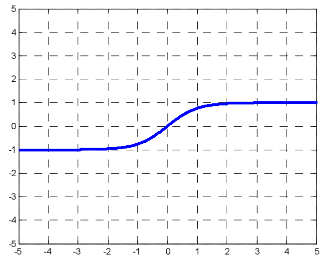
$D=\Re$

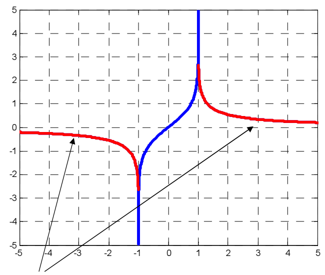
$CD=]-1,1[$

$$\operatorname{argtgh}(x)=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$D=]-1,1[$

$CD=\Re$



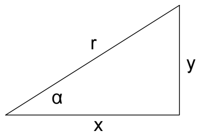




Componente real do valor complexo

## Funções Trigonométricas

Definição das funções trigonométricas com base no triângulo

(funções trigonométricas de um ângulo agudo)




$$\mathrm{sen}(\alpha) = \frac{y}{r} \qquad \mathrm{cosec}(\alpha) = \frac{1}{\mathrm{sen}(\alpha)} = \frac{r}{y}$$

$$\cos(\alpha) = \frac{x}{r} \qquad \sec(\alpha) = \frac{1}{\cos(\alpha)} = \frac{r}{x}$$

$$\mathrm{tg}(\alpha) = \frac{y}{x} \qquad \mathrm{cotg}(\alpha) = \frac{1}{\mathrm{tg}(\alpha)} = \frac{x}{y}$$



## Funções Trigonométricas dos ângulos de 30º, 45º, e 60º

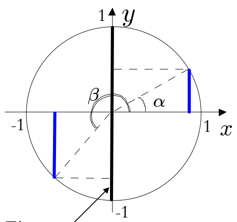
| | $30^\circ$ | $45^\circ$ | $60^\circ$ |
|---|---|---|---|
| $\mathrm{sen}(x)$ | $\frac{\sqrt{1}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos(x)$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2}$ |
| $\mathrm{tg}(x)$ | $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ |

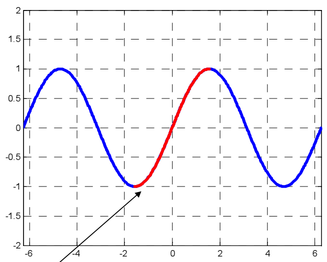


## Funções Trigonométricas

(generalização das funções trigonométricas a ângulos de medida em $\Re$ )

### Função seno




$f(x) = sen(x)$

$D = \Re$

$CD = [-1, 1]$

$T = 2\pi$

Zeros: $x = k\pi$

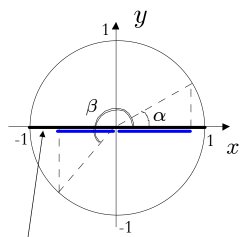
Eixo trigonométrico dos senos

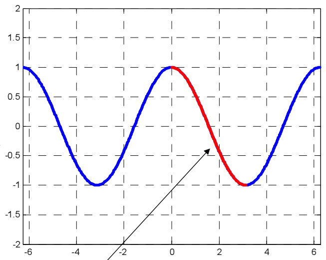
Função injectiva em $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(Restrição principal do seno)



## Função coseno




$f(x) = \cos(x)$

$D = \Re$

$CD = [-1, 1]$

$T = 2\pi$

Zeros: $x = \frac{\pi}{2} + k\pi$

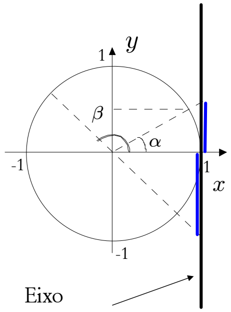
Eixo trigonométrico dos cosenos

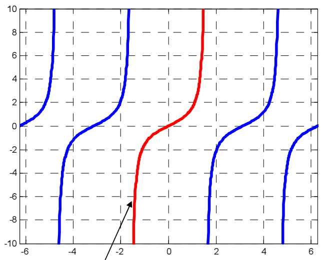
Função injectiva em $[0, \pi]$

(Restrição principal do coseno)

## Função tangente





$f(x) = \mathrm{tg}(x)$

$D = \Re \setminus (2k+1)\dfrac{\pi}{2}$

$CD = \Re$

$T = \pi$

Zeros: $x = k\pi$

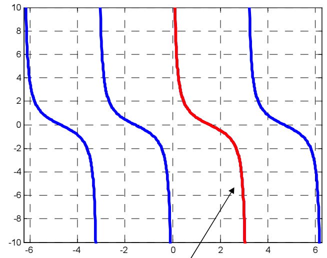
Eixo trigonométrico das tangentes

Função injectiva em $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

(Restrição principal da tangente)

## Função cotangente





$f(x) = \mathrm{cotg}(x)$

$D = \Re \setminus k\pi$

$CD = \Re$

$T = \pi$

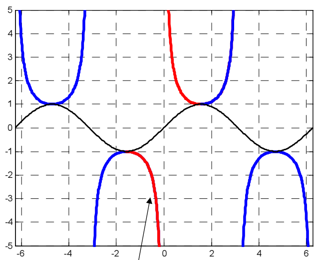
Zeros: $x = (2k+1)\dfrac{\pi}{2}$

Função injectiva em $[0, \pi]$

(Restrição principal da cotangente)

## Função cosecante





$f(x) = \mathrm{cosec}(x)$

$D = \Re \setminus k\pi$

$CD = ]-\infty, -1] \cup [1, +\infty[$

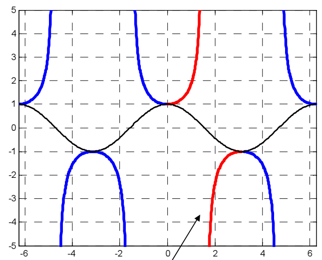
$T = 2\pi$

Função injectiva em $\left[-\dfrac{\pi}{2}, 0\right[ \cup \left]0, \dfrac{\pi}{2}\right]$

(Restrição principal da cosecante)

## Função secante





$f(x) = \sec(x)$

$D = \Re \setminus (2k+1)\dfrac{\pi}{2}$

$CD = ]-\infty, -1] \cup [1, +\infty[$

$T = 2\pi$

Função injectiva em $\left[0, \dfrac{\pi}{2}\right[ \cup \left]\dfrac{\pi}{2}, \pi\right]$

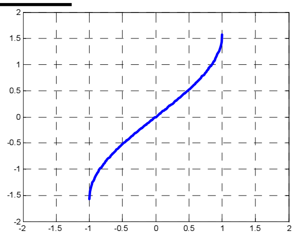
(Restrição principal da cosecante)

## Função arco seno

$f(x) = \mathrm{arcsen}(x)$

$D = [-1, 1]$

$CD = \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

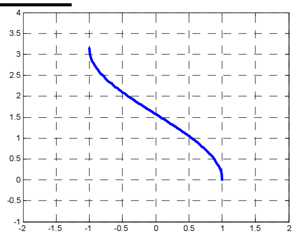


## Função arco coseno

$f(x) = \arccos(x)$

$D = [-1, 1]$

$CD = [0, \pi]$



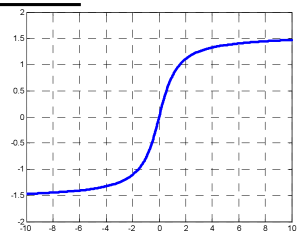


## Função arco tangente

$f(x) = \mathrm{arctg}(x)$

$D = \Re$

$CD = \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[$

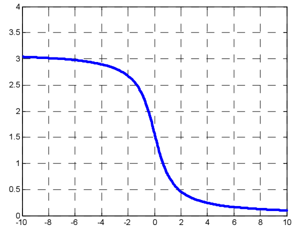


## Função arco cotangente

$f(x) = \mathrm{arccotg}(x)$

$D = \Re$

$CD = ]0, \pi[$



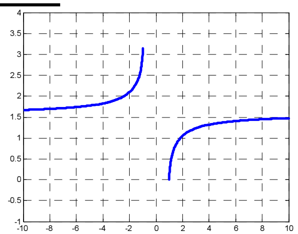


## Função arco secante

$f(x) = \mathrm{arcsec}(x)$

$D = ]-\infty, -1] \cup [1, +\infty[$

$CD = \left[0, \dfrac{\pi}{2}\right[ \cup \left]\dfrac{\pi}{2}, \pi\right]$

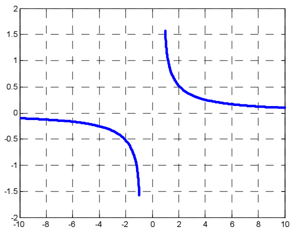


## Função arco cosecante

$f(x) = \mathrm{arccosec}(x)$

$D = ]-\infty, -1] \cup [1, +\infty[$

$CD = \left[-\dfrac{\pi}{2}, 0\right[ \cup \left]0, \dfrac{\pi}{2}\right]$





## Funções de ângulos associados

$$\mathrm{sen}\left(\frac{\pi}{2} + \alpha\right) = \cos(\alpha) \qquad \cos(\pi - \alpha) = -\cos(\alpha)$$

$$\mathrm{cotg}\left(3\frac{\pi}{2} + \alpha\right) = -\mathrm{tg}(\alpha)$$

## Soma e diferença de ângulos

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \mathrm{sen}(\alpha)\mathrm{sen}(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \mathrm{sen}(\alpha)\mathrm{sen}(\beta)$$

$$\mathrm{sen}(\alpha - \beta) = \mathrm{sen}(\alpha)\cos(\beta) - \mathrm{sen}(\beta)\cos(\alpha)$$

$$\mathrm{sen}(\alpha + \beta) = \mathrm{sen}(\alpha)\cos(\beta) + \mathrm{sen}(\beta)\cos(\alpha)$$

$$\mathrm{tg}(\alpha + \beta) = \frac{\mathrm{tg}(\alpha) + \mathrm{tg}(\beta)}{1 - \mathrm{tg}(\alpha)\mathrm{tg}(\beta)} \qquad \mathrm{tg}(\alpha - \beta) = \frac{\mathrm{tg}(\alpha) - \mathrm{tg}(\beta)}{1 + \mathrm{tg}(\alpha)\mathrm{tg}(\beta)}$$

## Transformação de produtos de sen e cos em somas de funções

$$\operatorname{sen}(\alpha)\cos(\beta)=\frac{1}{2}\left[\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)\right]$$

$$\cos(\alpha)\cos(\beta)=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\operatorname{sen}(\alpha)\operatorname{sen}(\beta)=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

## Transformações logarítmicas

$$\operatorname{sen}(\alpha)+\operatorname{sen}(\beta)=2\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\operatorname{sen}(\alpha)-\operatorname{sen}(\beta)=2\operatorname{sen}\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos(\alpha)+\cos(\beta)=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha)-\cos(\beta)=-2\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\operatorname{sen}\left(\frac{\alpha-\beta}{2}\right)$$



## Transformações do ângulo duplo

$$\operatorname{sen}(2\alpha)=2\operatorname{sen}(\alpha)\cos(\alpha)=\frac{2\operatorname{tg}(\alpha)}{1+\operatorname{tg}^2(\alpha)}$$

$$\cos(2\alpha)=\cos^2(\alpha)-\operatorname{sen}^2(\alpha)=\frac{1-\operatorname{tg}^2(\alpha)}{1+\operatorname{tg}^2(\alpha)}$$

$$\operatorname{tg}(2\alpha)=\frac{2\operatorname{tg}(\alpha)}{1-\operatorname{tg}^2(\alpha)}$$

## Transformações de bissecção

$$\operatorname{sen}^2(\alpha)=\frac{1-\cos(2\alpha)}{2}$$

$$\cos^2(\alpha)=\frac{1+\cos(2\alpha)}{2}$$

$$\operatorname{tg}^2(\alpha)=\frac{1-\cos(2\alpha)}{1+\cos(2\alpha)}$$



## Relações fundamentais

$$\operatorname{sen}^2(\alpha)+\cos^2(\alpha)=1$$

$$1+\operatorname{cotg}^2(\alpha)=\operatorname{cosec}^2(\alpha)$$

$$1+\operatorname{tg}^2(\alpha)=\sec^2(\alpha)$$

## Equações trigonométricas

$$\operatorname{sen}(x)=\operatorname{sen}(\alpha)\Leftrightarrow x=\alpha+k2\pi \vee x=(\pi-\alpha)+k2\pi$$

$$\cos(x)=\cos(\alpha)\Leftrightarrow x=\pm\alpha+k2\pi$$

$$\operatorname{tg}(x)=\operatorname{tg}(\alpha)\Leftrightarrow x=\alpha+k\pi$$