Just Sort It!

A Simple and Effective Approach to Active Preference Learning

Lucas Maystre, Matthias Grossglauser
School of Computer and Communication Sciences, EPFL

ICML — August 8th, 2017
Goal

Learning a ranking from **noisy pairwise comparisons**.

some outcomes are **inconsistent** with the ranking

Recover the ranking **accurately**, but sample **sparingly**.

choose pairs of items **adaptively**

based on previous observations
Which better expresses *amusement*?
Main Idea

Use a **sorting algorithm**!

Ground-truth ranking

Random ranking

Outputs of Quicksort with noise

ML estimate
Why Sorting-based AL?

Prior work: greedy active learning strategies [Houlsby et al. NIPS 2012, Chen et al. WSDM 2013, Wang et al. KDD 2014, ...]

Time (in seconds) to select the n+1-st pair among n items

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$T [s]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 10^2$</td>
</tr>
<tr>
<td>uncertainty</td>
<td>0.05</td>
</tr>
<tr>
<td>entropy</td>
<td>0.3</td>
</tr>
<tr>
<td>KL-divergence</td>
<td>0.9</td>
</tr>
<tr>
<td>Mergesort</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>Quicksort</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>random</td>
<td>$\varepsilon$</td>
</tr>
</tbody>
</table>

$\varepsilon < 10^{-5}$ orders of magnitude faster

... and simpler to implement
This Work

Theory
Accuracy of output of **single call** to Quicksort

Practice
**Empirical evaluation** of sorting-based active learning on real data
Noise Model

Bradley-Terry model [Zermelo 1928, Bradley & Terry 1952]

\[ p(i > j) = \frac{1}{1 + e^{-(\theta_i - \theta_j)}} \]

- Error is **likely** if \( \theta_i \approx \theta_j \)
- Error is **unlikely** if \( \theta_i \gg \theta_j \) or \( \theta_i \ll \theta_j \)
We analyze Quicksort.
Model Parameters

**Difficulty of ranking the items**
depends on $|\theta_2 - \theta_1|$, $|\theta_3 - \theta_2|$, ...

**Our approach:** postulate a distribution over the parameters s.t.

$$\mathbb{E}[|\theta_{i+1} - \theta_i|] = \lambda^{-1}$$

controls the average amount of noise

We assume that parameters are drawn from a **Poisson point process**.

$$p(i > j) = \frac{1}{1 + e^{-(\theta_i - \theta_j)}}$$

i.i.d. $\theta_{i+1} - \theta_i \sim \text{Exp}(\lambda)$

n = 20
Main Result

We measure the **rank displacement** of Quicksort's output $\sigma$.

$$\Delta(\sigma) = \sum_{i=1}^{n} |\sigma(i) - i|$$

**Theorem:** if noise is Bradley-Terry and i.i.d. $\theta_{i+1} - \theta_i \sim \text{Exp}(\lambda)$

- with high probability,
  $$\Delta(\sigma) = O(\lambda^2 n)$$

- with high probability,
  $$\max_i |\sigma(i) - i| = O(\lambda \log n)$$
Sketch of Proof

**Lemma**: Displacement of Quicksort's output can be bounded by a sum over “individual errors”.

\[ \Delta(\sigma) \leq 2 \sum_{(i,j) \in E} |i - j| \]

pairs sampled by Quicksort that resulted in an error

Does not assume anything about the noise generating process.

w.h.p., \( \Delta(\sigma) = O(\lambda^2 n) \)

\[ Z_{ij} = \begin{cases} 1 & \text{if outcome is incorrect} \\ 0 & \text{otherwise} \end{cases} \]

comparison pair

\[ E[\Delta(\sigma)] \leq 2 \sum_{i<j} |i - j| E[Z_{ij}] \]

\[ \leq 2n \sum_{k=1}^{\infty} kE[Z_{i,i+k}] = O(\lambda^2 n) \]

decreases exponentially fast in \( k \)
Experimental Results

Matches the performance of alternatives at a fraction of computational cost.
Conclusions

- Use **sorting algorithms** to learn a ranking from noisy comparisons!
  - **works well** in practice
  - computationally **inexpensive**
- Some **theoretical results** under assumptions on the noise

**GIFGIF dataset**

- 6170 animated GIF images
- 2.7+ million pairwise comparisons
# How To Select Comparison Pairs?

## Batch setting

Role of the spectral gap of the comparison graph.

- [Negahban et al. NIPS 2012](#)
- [Hajek et al. NIPS 2014](#)
- [Vojnovic et al. ICML 2016](#)

## Sequential setting

Greedy active learning strategies (EIG, uncertainty sampling, ...)

- [Houlsby et al. NIPS 2012](#)
- [Chen et al. WSDM 2013](#)
- [Ailon et al. NIPS 2011](#)

## Bandit approaches

**Dueling bandits** [Yue et al. COLT 2009](#)

- [Szörényi et al. NIPS 2015](#)
- [Heckel et al. arXiv 2016](#)
Practical AL Strategy

In practice: comparison budget is more than that of a single call to Quicksort.