

LC 2017 (SET A): PAPER 1

QUESTION 7 (55 MARKS)

Question 7 (a)

$$p(t) = Se^{0.1t} \times 10^6$$

$$t = 0 \text{ (2010): } p(0) = Se^{0.1(0)} \times 10^6 = 1\,100\,000$$

$$S = \frac{1\,100\,000}{10^6} = 1.1$$

Question 7 (b)

$$p(t) = 1.1e^{0.1t} \times 10^6$$

$$t = 5 \text{ (2015): } p(5) = 1.1e^{0.1(5)} \times 10^6 = 1\,813\,593$$

Question 7 (c)

$$\text{Change in population during 2015} = p(6) - p(5)$$

$$= 1.1e^{0.1(6)} \times 10^6 - 1.1e^{0.1(5)} \times 10^6 = 190\,737$$

Question 7 (d)

$$q(t) = 3.9e^{kt} \times 10^6$$

$$t = 1 \text{ (2011): } q(1) = 3.9e^{k(1)} \times 10^6 = 3\,709\,795$$

$$e^k = \frac{3\,709\,795}{3.9 \times 10^6}$$

$$\ln(e^k) = \ln\left(\frac{3\,709\,795}{3.9 \times 10^6}\right)$$

$$k = -0.05$$

Question 7 (e)

$$p(t) = q(t) \Rightarrow 1.1e^{0.1t} \times 10^6 = 3.9e^{-0.05t} \times 10^6$$

$$1.1e^{0.1t} = 3.9e^{-0.05t} \Rightarrow \frac{e^{0.1t}}{e^{-0.05t}} = \frac{3.9}{1.1}$$

$$e^{0.15t} = \frac{3.9}{1.1} \Rightarrow \ln(e^{0.15t}) = \ln\left(\frac{3.9}{1.1}\right)$$

$$0.15t = \ln\left(\frac{3.9}{1.1}\right)$$

$$t = \frac{1}{0.15} \ln\left(\frac{3.9}{1.1}\right) = 8.44$$

Populations are the same during 2018.

Question 7 (f)

The average value \bar{y} of a continuous function $y = g(x)$ on an interval $[a, b]$ is given by:

$$\bar{y} = \frac{\int_a^b g(x) dx}{b-a} = \frac{\int_a^b y dx}{b-a}$$

$$q(t) = 3.9e^{-0.05t} \times 10^6$$

$$\bar{q} = \frac{1}{15-0} \int_0^{15} (3.9e^{-0.05t} \times 10^6) dt$$

$$= \frac{3.9 \times 10^6}{15} \int_0^{15} e^{-0.05t} dt$$

$$= -\frac{3.9 \times 10^6}{15(0.05)} [e^{-0.05t}]_0^{15}$$

$$= -\frac{3.9 \times 10^6}{15(0.05)} [e^{-0.05(15)} - e^{-0.05(0)}]$$

$$= -\frac{3.9 \times 10^6}{15(0.05)} [e^{-0.75} - 1] = 2\,743\,694$$

Question 7 (g)

$$q(t) = 3.9e^{-0.05t} \times 10^6$$

$$\frac{dq}{dt} = 3.9e^{-0.05t} \times 10^6 \times (-0.05) = -195\,000e^{-0.05t}$$

$$\left(\frac{dq}{dt}\right)_{t=8} = -195\,000e^{-0.05(8)}$$

$$= -130\,712 \text{ people/year}$$