

SAMPLE PAPER 1: PAPER 1

QUESTION 7 (50 MARKS)

Question 7 (a)

a is the initial amount of Carbon-14

$$Q = ae^{bt}$$

$t = 0: Q = ae^0 = a$ [At $t = 0$ years the amount of Carbon-14 present is a . After a half-life of
 $t = 5730$ years: $Q = \frac{1}{2}a$ 5730 years, the amount of Carbon-14 present will fall by a half of a .]

$$\therefore \frac{1}{2}a = ae^{b(5730)}$$

$$\frac{1}{2} = e^{b(5730)}$$

$$\ln\left(\frac{1}{2}\right) = b(5730)$$

$$-\ln 2 = 5730b$$

$$\therefore b = -\left(\frac{\ln 2}{5730}\right) \text{ yr}^{-1}$$

Question 7 (b)

$$Q = ae^{bt} \Rightarrow \frac{dQ}{dt} = abe^{bt}$$

$$\frac{\left(\frac{dQ}{dt}\right)_{t=4000}}{\left(\frac{dQ}{dt}\right)_{t=1000}} = \frac{abe^{-\left(\frac{\ln 2}{5730}\right)4000}}{abe^{-\left(\frac{\ln 2}{5730}\right)1000}} = \frac{e^{-\left(\frac{\ln 2}{5730}\right)4000}}{e^{-\left(\frac{\ln 2}{5730}\right)1000}} = e^{-\left(\frac{\ln 2}{5730}\right)3000} = 0.696$$

Question 7 (c) (i)

$$\frac{Q}{a} = e^{-\left(\frac{\ln 2}{5730}\right)2000} = 0.785 = 78.5\% \quad [Q \text{ divided by } a \text{ (the initial amount) will give you the fraction of Carbon-14 present at any time.}]$$

Question 7 (c) (ii)

$$\frac{Q}{a} = 0.93 = e^{-\left(\frac{\ln 2}{5730}\right)t}$$

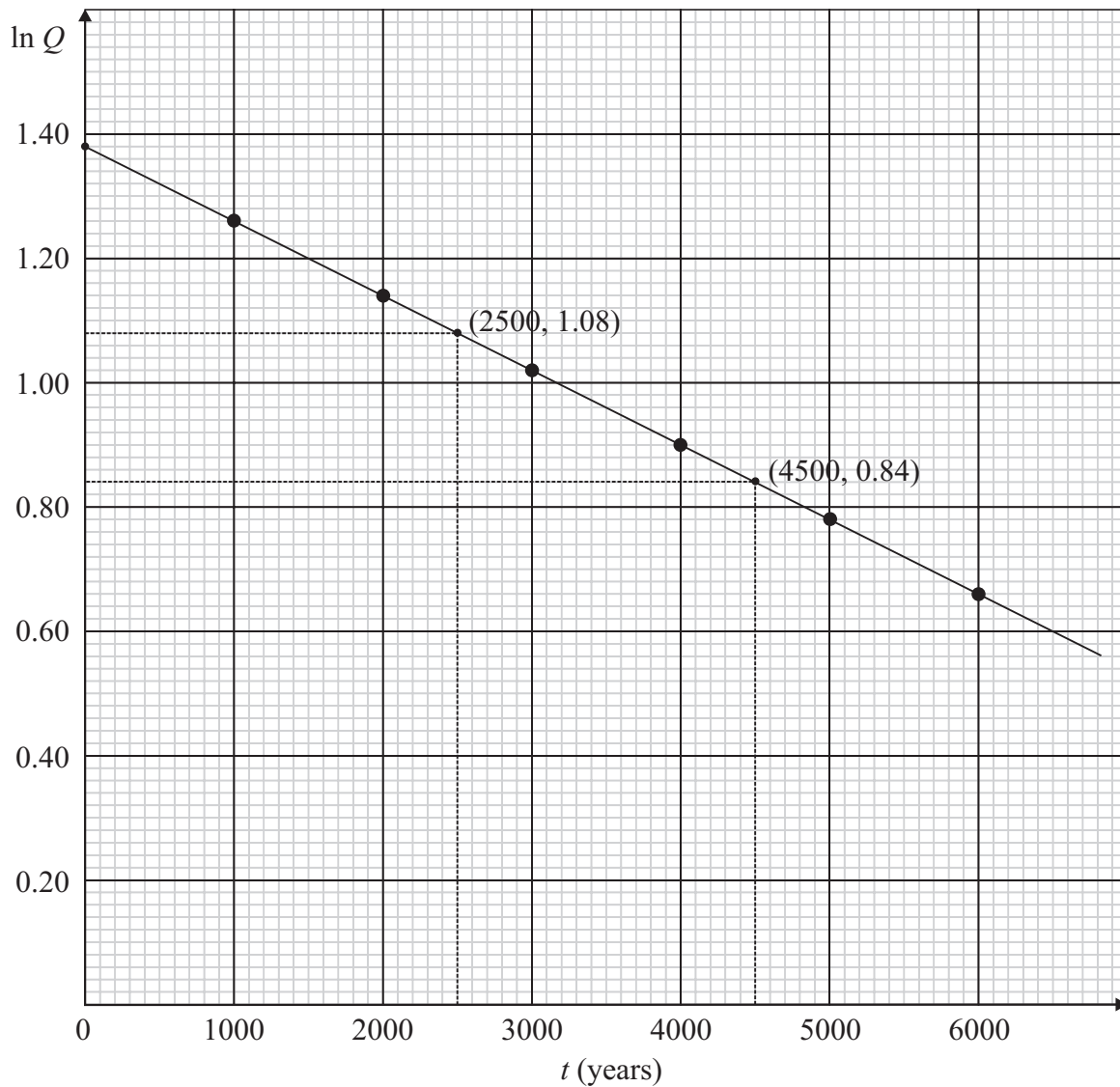
$$\ln(0.93) = -\left(\frac{\ln 2}{5730}\right)t$$

$$\therefore t = -\frac{5730 \ln(0.93)}{\ln 2} = 600 \text{ years}$$

Question 7 (d) (i)

t (years)	1000	2000	3000	4000	5000	6000
Q (mg)	3.54	3.14	2.78	2.47	2.18	1.94
$\ln Q$	1.26	1.14	1.02	0.90	0.78	0.66

Question 7 (d) (ii)



Question 7 (e)

$$Q = ae^{bt}$$

$$\ln Q = \ln(ae^{bt})$$

$$\ln Q = \ln a + \ln(e^{bt})$$

$$\ln Q = bt + \ln a \quad (\rightarrow y = mx + c)$$

b is the slope of the graph and $\ln a$ is the y -intercept.

$$b = \frac{1.08 - 0.84}{2500 - 4500} = -1.2 \times 10^{-4} \text{ year}^{-1}$$

$$b = -\frac{\ln 2}{5730} = -1.21 \times 10^{-4} \text{ year}^{-1}$$

$$\ln a = 1.38 \Rightarrow a = e^{1.38} = 3.975 \text{ mg}$$

Question 7 (f)

$$Q = ae^{bt}$$

$$Q - 0.05Q = ae^{bt_1}$$

$$Q + 0.05Q = ae^{bt_2}$$

$$0.95Q = ae^{bt_1}$$

$$1.05Q = ae^{bt_2}$$

$$\frac{0.95Q}{1.05Q} = \frac{e^{bt_1}}{e^{bt_2}} = e^{b(t_1 - t_2)}$$

$$\therefore \ln\left(\frac{0.95}{1.05}\right) = b(t_1 - t_2)$$

$$(t_1 - t_2) = \frac{\ln\left(\frac{0.95}{1.05}\right)}{b} = \frac{\ln\left(\frac{0.95}{1.05}\right)}{-\left(\frac{\ln 2}{5730}\right)} = 827 \text{ years}$$