

SAMPLE PAPER 2: PAPER 1

QUESTION 2 (25 MARKS)

Question 2 (a)

$$z = 2 + 2\sqrt{3}i$$

$$|z| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4 \quad \boxed{z = r(\cos \theta + i \sin \theta)}$$

$$\tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3} \quad \boxed{|z| = r, \arg z = \theta}$$

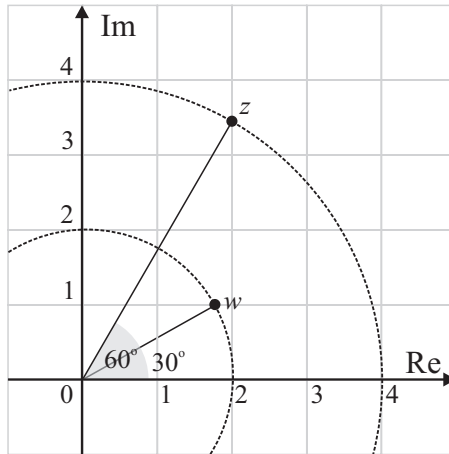
$$\therefore z = 4 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$w = \sqrt{3} + i$$

$$|w| = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ = \frac{\pi}{6}$$

$$\therefore w = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$



Question 2 (b) (i)

$$\begin{aligned} zw &= 4 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \\ &= 8 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) \\ &= 8(0 + i) \\ &= 8i \end{aligned}$$

Question 2 (c) (i)

$$\begin{aligned} \frac{z}{w} &= \frac{4 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}{2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)} = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \\ &= 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \sqrt{3} + i \end{aligned}$$

Question 2 (c) (iii)

$$\begin{aligned} \left(\frac{z}{w} \right)^{14} &= 2^{14} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)^{14} \quad \boxed{[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)} \quad (\text{De Moivre's Theorem}) \\ &= 2^{14} \left(\cos \frac{14\pi}{6} + i \sin \frac{14\pi}{6} \right) \\ &= 2^{14} \left(\cos \frac{7\pi}{3} + i \sin \frac{7\pi}{3} \right) = 2^{14} \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = 2^{13} (1 + \sqrt{3}i) \end{aligned}$$

Question 2 (b) (ii)

‘According to De Moivre’s theorem, when two complex numbers are multiplied together, the arguments are added and the moduli are multiplied to form the new complex number.’

Question 2 (c) (ii)

‘According to De Moivre’s theorem, when two complex numbers are divided, the arguments are subtracted and the moduli are divided to form the new complex number.’