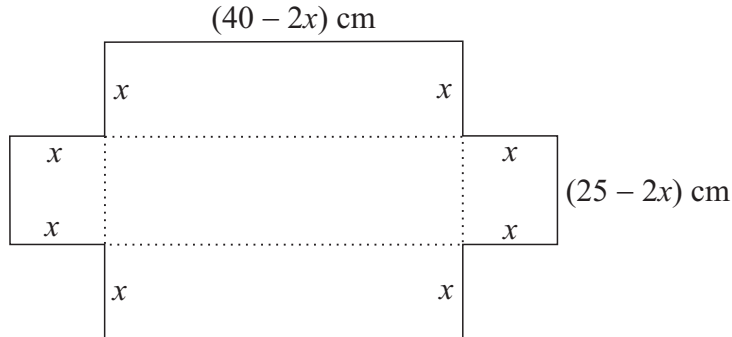


SAMPLE PAPER 2: PAPER 1

QUESTION 7 (50 MARKS)

Question 7 (a) (i)



$$l = 40 - 2x, b = 25 - 2x, h = x$$

Question 7 (b) (i)

$$\begin{aligned} S &= 2(40 - 2x)x + 2(25 - 2x)x + (40 - 2x)(25 - 2x) \\ &= 80x - 4x^2 + 50x - 4x^2 + 1000 - 80x - 50x + 4x^2 \\ &= 1000 - 4x^2 \end{aligned}$$

Question 7 (c)

$$\begin{aligned} V &= 4x^3 - 130x^2 + 1000x \\ 4x^3 - 130x^2 + 1000x &= 1512 \\ 2x^3 - 65x + 500x - 756 &= 0 \\ x = 1: 2(1)^3 - 65(1) + 500(1) - 756 &= -319 \neq 0 \\ x = 2: 2(2)^3 - 65(2) + 500(2) - 756 &= 0 \end{aligned}$$

Question 7 (d)

$$V = 4x^3 - 130x^2 + 1000x$$

$$\frac{dV}{dx} = 12x^2 - 260x + 1000$$

$$\frac{dV}{dx} = 0 \Rightarrow 12x^2 - 260x + 1000 = 0$$

$$3x^2 - 65x + 250 = 0$$

$$(3x - 50)(x - 5) = 0$$

$$x = 5, \frac{50}{3} \text{ cm} \quad [\text{Ignore the second solution as it will cause one of the sides to be negative.}]$$

$$V_{\text{Max.}} = 4(5)^3 - 130(5)^2 + 1000(5) = 500 - 3250 + 5000 = 2250 \text{ cm}^3$$

$$l = 40 - 2(5) = 30 \text{ cm}, b = 25 - 2(5) = 15 \text{ cm}, h = 5 \text{ cm}$$

Question 7 (a) (ii)

$$\begin{aligned} V &= (40 - 2x)(25 - 2x)x \\ &= (1000 - 80x - 50x + 4x^2)x \\ &= (1000 - 130x + 4x^2)x \\ &= 1000x - 130x^2 + 4x^3 \end{aligned}$$

Question 7 (b) (ii)

$$\begin{aligned} 600 &= 1000 - 4x^2 \\ 4x^2 &= 400 \\ x^2 &= 100 \\ \therefore x &= 10 \text{ cm} \end{aligned}$$

Question 7 (e)

$$\frac{dV}{dt} = -(2t + 5)$$

$$V = -\int (2t + 5) dt = -t^2 - 5t + c$$

$$V = 2250 \text{ when } t = 0: 2250 = c$$

$$\therefore V = -t^2 - 5t + 2250$$

$$-t^2 - 5t + 2250 = 0$$

$$t^2 + 5t - 2250 = 0$$

$$(t - 45)(t + 50) = 0$$

$$\therefore t = 45 \text{ s}$$

Question 7 (f) (i)

$$\tan 45^\circ = \frac{r}{h} \Rightarrow 1 = \frac{r}{h}$$

$$\therefore r = h$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi r^3$$

Question 7 (f) (ii)

$$V = \frac{1}{3} \pi r^3$$

$$\frac{dV}{dt} = \pi r^2 \times \frac{dr}{dt} = 3 \Rightarrow \frac{dr}{dt} = \frac{3}{\pi r^2}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt} = 2\pi r \times \frac{3}{\pi r^2} = \frac{6}{r}$$

$$\left(\frac{dA}{dt} \right)_{r=2} = \frac{6}{2} = 3 \text{ cm}^2/\text{s}$$

