

SAMPLE PAPER 3: PAPER 1

QUESTION 3 (25 MARKS)

Question 3 (a)

Three consecutive numbers: $n - 1, n, n + 1$

$$\begin{aligned} S &= (n-1)^2 + n^2 + (n+1)^2 \\ &= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1 \\ &= 3n^2 + 2 \end{aligned}$$

$$\frac{3n^2 + 2}{3} = n^2 + \frac{2}{3}$$

Question 3 (b)

STEPS FOR PROOF BY INDUCTION

1. Prove result is true for some starting value of $n \in \mathbb{N}$.
2. Assume result is true for $n = k$.
3. Prove result is true for $n = (k + 1)$.

1. Prove true for $n = 1$: $n = 1: 7^1 + 3(1) + 8 = 18$ [Therefore, true for $n = 1$.]

2. Assume true for $n = k$: Assume $n = k: 7^k + 3k + 8 = 9a, a \in \mathbb{N}$

3. Prove true for $n = k + 1$: Prove $7^{k+1} + 3(k+1) + 8 = 9b, b \in \mathbb{N}$

Proof:

$$\begin{aligned} &7^{k+1} + 3(k+1) + 8 \\ &= 7(7^k) + 3k + 3 + 8 \\ &= 7(9a - 3k - 8) + 3k + 11 \\ &= 63a - 21k - 56 + 3k + 11 \\ &= 63a - 18k - 45 \\ &= 9[7a - 2k - 5] \\ &= 9b, b \in \mathbb{N} \end{aligned}$$

Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n \in \mathbb{N}$.
