Sample Paper 4: Paper 1

Question 1 (25 marks)

Question 1 (a) (i)

$z = -3 - i \Rightarrow \bar{z} = -3 + i$ $\boxed{z = a + bi \Rightarrow \bar{z} = a - bi}$

Question 1 (a) (ii)

$z = -3 - i \Rightarrow (z + 3 + i)$ is a factor

$\bar{z} = -3 + i \Rightarrow (z + 3 - i)$ is a factor

$\therefore (z + 3 + i)(z + 3 - i) = 0$

$z^2 + 3z - iz + 3z + 9 - 3i + iz + 3i - i^2 = 0$

$z^2 + 6z + 9 + 1 = 0$

$z^2 + 6z + 10 = 0$

OR

Roots: $-3 - i, \ -3 + i$

Sum $S = -6$

Product $P = 10$

$z^2 - Sz + P = 0$

$\therefore z^2 + 6z + 10 = 0$

Question 1 (b)

If z is a root of the cubic equation, its conjugate is also a root. This is because the coefficients in the cubic are all real. Therefore, $z^2 + 6z + 10 = 0$ is a factor of the cubic equation.

$az^3 + 22z^2 + bz + 40 = (z^2 + 6z + 10)(az + 4)$

$az^3 + 22z^2 + bz + 40 = az^3 + (6a + 4)z^2 + (10a + 24)z + 40$

$\therefore 22 = 6a + 4 \Rightarrow a = 3$

$10a + 24 = b \Rightarrow 54 = b$

$\therefore 3z^3 + 22z^2 + 54z + 40 = (z^2 + 6z + 10)(3z + 4) = 0$

$z = -3 - i, \ -3 + i, \ -\frac{4}{3}$