Sample Paper 4: Paper 2

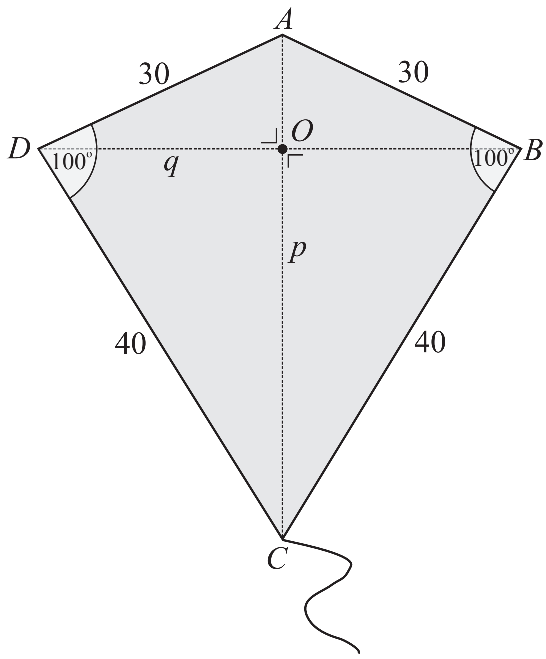
Question 9 (75 marks)

Question 9 (a)

(i) A **rhombus** is a simple quadilateral whose sides are equal.

(ii) The angles in a quadrilateral add up to 360°. If each angle is equal, each angle will be 90° and hence a **square** is the shape formed.

Question 9 (b)



(i) Perimeter $P = 2a + 2b$

(ii)

$$\text{Area of } \Delta ADC = \tfrac{1}{2}(p)(\tfrac{1}{2}q) = \tfrac{1}{4}pq$$
$$\text{Area of } \Delta ABC = \tfrac{1}{2}(p)(\tfrac{1}{2}q) = \tfrac{1}{4}pq$$
$$\text{Area of } ABCD = \tfrac{1}{4}pq + \tfrac{1}{4}pq = \tfrac{1}{2}pq$$

$$\text{Area of } \Delta ADC = \tfrac{1}{2}ab\sin\theta$$
$$\text{Area of } \Delta ABC = \tfrac{1}{2}ab\sin\theta$$
$$\text{Area of } ABCD = ab\sin\theta$$

Question 9 (c)

(i)

$$|AC|^2 = 30^2 + 40^2 - 2(30)(40)\cos 100^\circ$$
$$\therefore |AC| = 54 \text{ cm}$$

(ii)

$$\frac{\sin(|\angle CAD|)}{40} = \frac{\sin 100^\circ}{54}$$
$$\sin(|\angle CAD|) = \frac{40\sin 100^\circ}{54}$$
$$|\angle CAD| = \sin^{-1}\left(\frac{40\sin 100^\circ}{54}\right) = 46.8^\circ$$

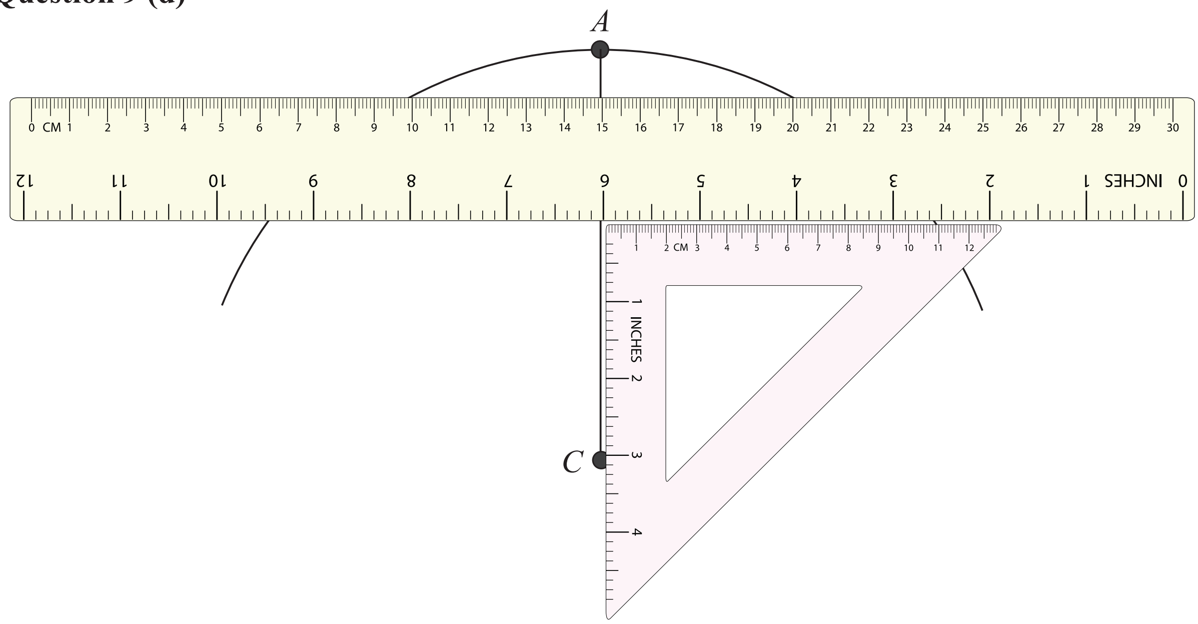
(iii)

$$\sin(|\angle CAD|) = \frac{\frac{1}{2}|BD|}{30}$$
$$\therefore \sin 46.8^\circ = \frac{|BD|}{60}$$
$$|BD| = 60\sin 46.8^\circ = 43.7 \text{ m}$$

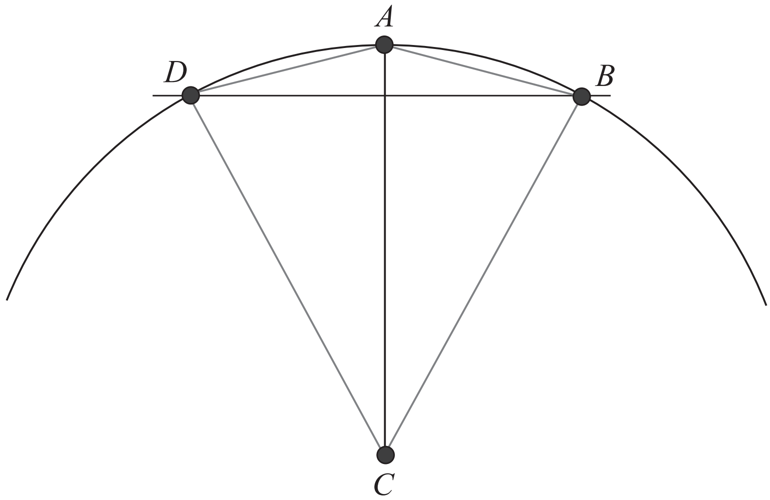
(iv)

$$\text{Area} = (30)(40)\sin 100^\circ = 1182 \text{ cm}^2$$

Question 9 (d)

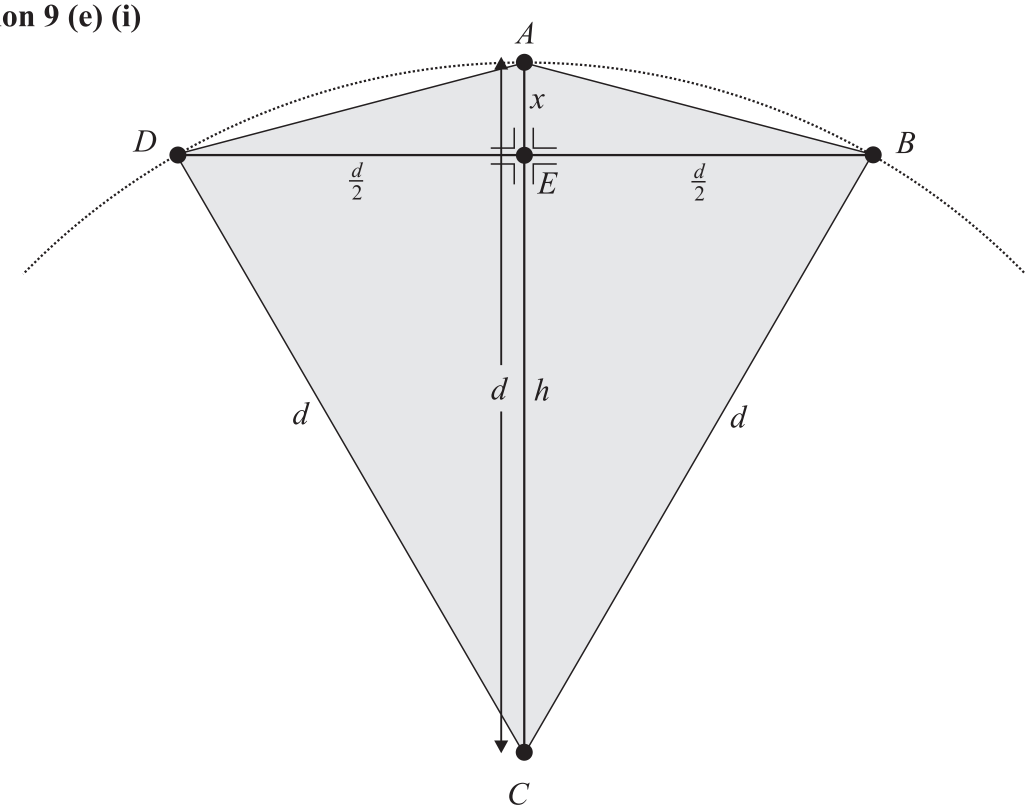


Draw a line segment [AC] of length 10 cm.
Using a compass draw an arc of radius 10 cm with C as centre.
Using a set square place a rule perpendicular to AC and move the ruler into position such that it measures a chord of distance of 10 cm. Draw a line along the ruler in this position.



Draw the kite $ABCD$.

Question 9 (e) (i)



Consider the right-angled triangle BEC:

$$d^2 = h^2 + \left(\frac{d}{2}\right)^2$$

$$d^2 = h^2 + \frac{d^2}{4}$$

$$\therefore h^2 = d^2 - \frac{d^2}{4} = \frac{3d^2}{4}$$

$$h = \sqrt{\frac{3d^2}{4}} = \frac{\sqrt{3}}{2}d$$

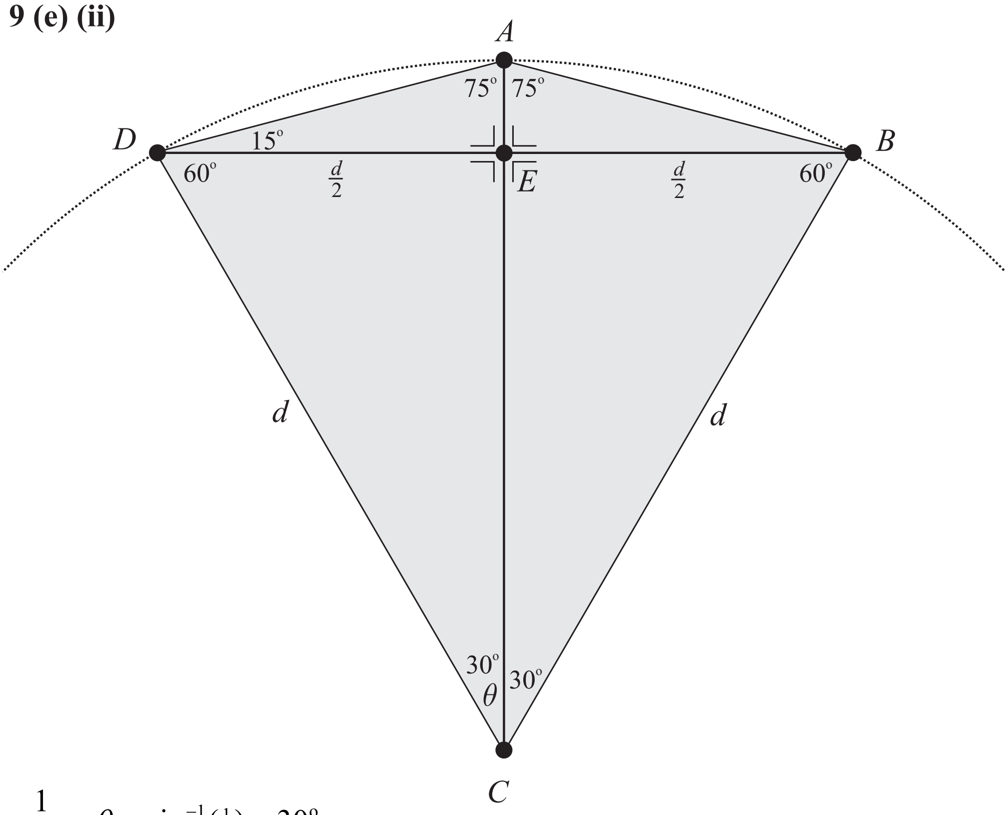
Consider the right-angled triangle BEA:

$$x = d - h = d - \frac{\sqrt{3}}{2}d = d\left(1 - \frac{\sqrt{3}}{2}\right) = d\left(\frac{2-\sqrt{3}}{2}\right)$$

$$\tan(|\angle ABD|) = \frac{d\left(\frac{2-\sqrt{3}}{2}\right)}{\frac{d}{2}} = (2-\sqrt{3})$$

$$\therefore |\angle ABD| = \tan^{-1}(2-\sqrt{3}) = 15^\circ$$

Question 9 (e) (ii)



$$\sin\theta = \frac{\frac{d}{2}}{d} = \frac{1}{2} \Rightarrow \theta = \sin^{-1}(\tfrac{1}{2}) = 30^\circ$$

$$|\angle ADC| = 75^\circ,\ |\angle ABC| = 75^\circ,\ |\angle BAD| = 150^\circ,\ |\angle BCD| = 60^\circ$$

Question 9 (f)

$$\tan 15^\circ = \tan(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\left(1 - \frac{1}{\sqrt{3}}\right)}{\left(1 + \frac{1}{\sqrt{3}}\right)} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$