

SAMPLE PAPER 5: PAPER 1

QUESTION 7 (50 MARKS)

Question 7 (a)

$$p = \frac{3000ke^{0.25t}}{1 + ke^{0.25t}}$$

$$t = 0 : 500 = \frac{3000ke^0}{1 + ke^0} = \frac{3000k}{1 + k}$$

$$500(1 + k) = 3000k$$

$$500 + 500k = 3000k$$

$$500 = 2500k$$

$$\therefore k = \frac{500}{2500} = 0.2$$

Question 7 (b)

$$p = \frac{3000ke^{0.25t}}{1 + ke^{0.25t}}$$

$$1233 = \frac{3000(0.2)e^{0.25t}}{1 + 0.2e^{0.25t}}$$

$$1233(1 + 0.2e^{0.25t}) = 600e^{0.25t}$$

$$1233 + 246.6e^{0.25t} = 600e^{0.25t}$$

$$1233 = 353.4e^{0.25t}$$

$$\frac{1233}{353.4} = e^{0.25t}$$

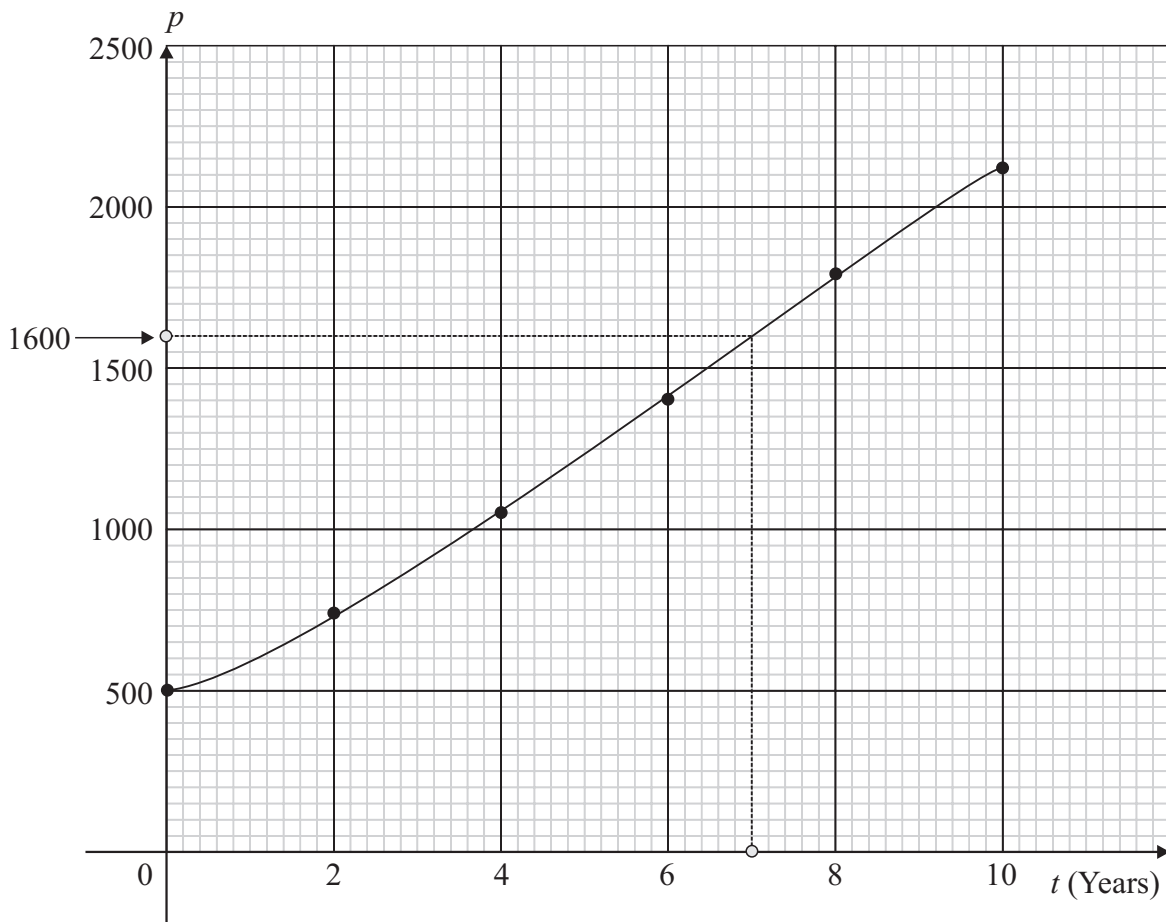
$$\ln\left(\frac{1233}{353.4}\right) = 0.25t$$

$$\therefore t = 4 \ln\left(\frac{1233}{353.4}\right) \approx 5 \text{ years}$$

Question 7 (c) (i)

t (years)	0	2	4	6	8	10
p	500	744	1057	1418	1789	2127

Question 7 (c) (ii)



Question 7 (d) (i)

$p = 1600$ after seven years

Question 7 (d) (ii)

$$p = \frac{3000(0.2)e^{0.25(7)}}{1 + (0.2)e^{0.25(7)}} \approx 1605$$

Question 7 (e)

$$\begin{aligned} p &= \frac{3000(0.2)e^{0.25t}}{1 + (0.2)e^{0.25t}} = \frac{600e^{0.25t}}{1 + 0.2e^{0.25t}} \\ &= \frac{600e^{0.25t}}{e^{0.25t} + \frac{0.2e^{0.25t}}{e^{0.25t}}} = \frac{600}{0.2 + e^{-0.25t}} \end{aligned}$$

Question 7 (f) (i)

$$\lim_{n \rightarrow \infty} r^n = \infty, |r| > 1$$

Question 7 (f) (ii)

$$\lim_{t \rightarrow \infty} p = \lim_{t \rightarrow \infty} \left(\frac{600}{0.2 + e^{-0.25t}} \right) = \lim_{t \rightarrow \infty} \left(\frac{600}{0.2 + \frac{1}{e^{0.25t}}} \right) = \frac{600}{0.2 + \frac{1}{\infty}} = \frac{600}{0.2 + 0} = 3000$$

Question 7 (g)

$$p = \frac{600}{0.2 + e^{-0.25t}} = 600(0.2 + e^{-0.25t})^{-1}$$

$$\frac{dp}{dt} = -600(0.2 + e^{-0.25t})^{-2} (e^{-0.25t} \times (-0.25))$$

$$= \frac{150}{e^{0.25t} (0.2 + e^{-0.25t})^2}$$

$$\left(\frac{dp}{dt} \right)_{t=1} = \frac{150}{e^{0.25} (0.2 + e^{-0.25})^2} \approx 122 \text{ flowers/year}$$
