

SAMPLE PAPER 5: PAPER 1

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

$$\left(\frac{dN}{dt}\right)_{t=10} = 0.8e^{0.006(10)} = 0.85 \text{ billions of barrels per year}$$

Question 9 (a) (ii)

$$\left(\frac{dN}{dt}\right)_{t=20} = 0.8e^{0.006(20)} = 0.90 \text{ billions of barrels per year}$$

Question 9 (b)

$$\frac{dN}{dt} = 0.8e^{0.006t}$$

$$\int dN = \int 0.8e^{0.006t} dt$$

$$N = \frac{0.8}{0.006} e^{0.006t} + c = \frac{400}{3} e^{0.006t} + c$$

Boundary conditions: At $t = 0$, $N = 0$.

$$N = \frac{400}{3} e^{0.006t} + c$$

$$\therefore 0 = \frac{400}{3} e^0 + c \Rightarrow c = -\frac{400}{3}$$

$$N = \frac{400}{3} e^{0.006t} - \frac{400}{3} = \frac{400}{3} (e^{0.006t} - 1)$$

$$t = 20 : N = \frac{400}{3} (e^{0.006(20)} - 1) = 17 \text{ billion}$$

Question 9 (c)

$$N = 55 : \frac{400}{3} (e^{0.006t} - 1) = 55$$

$$e^{0.006t} = \frac{3 \times 55}{400} + 1 = \frac{113}{80}$$

$$t = \frac{1}{0.006} \ln\left(\frac{113}{80}\right) = 57.56 \approx 58 \text{ years}$$

Question 9 (d)

$$1 \text{ barrel} = 0.159 \text{ m}^3$$

$$4.714 \times 10^{10} \text{ m}^3 = \frac{4.714 \times 10^{10}}{0.159} \text{ barrels}$$

$$= 2.965 \times 10^{11} \text{ barrels}$$

$$= 296.5 \times 10^9 \text{ barrels}$$

$$= 296.5 \text{ billion barrels}$$

Question 9 (e)

$$N = 296.5 : \frac{400}{3} (e^{0.006t} - 1) = 296.5$$

$$e^{0.006t} = \frac{3 \times 296.5}{400} + 1 = \frac{2579}{800}$$

$$t = \frac{1}{0.006} \ln\left(\frac{2579}{800}\right) \approx 195 \text{ years}$$