Sample Paper 6: Paper 1

Question 5 (25 marks)

Question 5 (a)

$$y = f(x) = \frac{2}{x} - 1 + \ln\left(\frac{x}{2}\right) = 2x^{-1} - 1 + \ln\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = -2x^{-2} + \frac{1}{\left(\frac{x}{2}\right)} \times \frac{1}{2} = -\frac{2}{x^2} + \frac{1}{x}$$

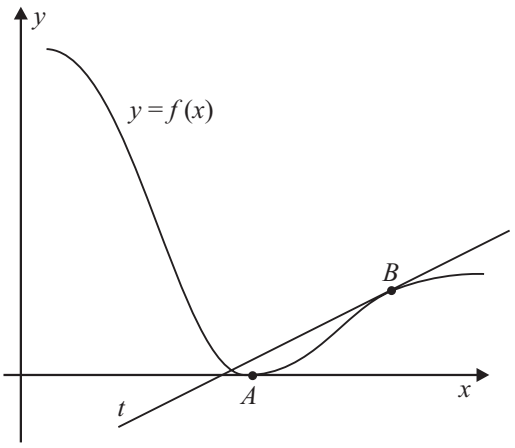
$$\frac{dy}{dx} = 0 \Rightarrow -\frac{2}{x^2} + \frac{1}{x} = 0$$

$$-2 + x = 0$$

$$\therefore x = 2$$

$$y = f(2) = \frac{2}{2} - 1 + \ln\left(\frac{2}{2}\right) = 0$$

Local minimum $A(2,\ 0)$



Question 5 (b)

$$\frac{dy}{dx} = -\frac{2}{x^2} + \frac{1}{x} = -2x^{-2} + x^{-1}$$

$$\frac{d^2y}{dx^2} = 4x^{-3} - 1x^{-2} = \frac{4}{x^3} - \frac{1}{x^2}$$

$$\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{4}{x^3} - \frac{1}{x^2} = 0$$

$$4 - x = 0$$

$$\therefore x = 4$$

$$f(4) = \frac{2}{4} - 1 + \ln\left(\frac{4}{2}\right) = \tfrac{1}{2} - 1 + \ln 2 = \ln 2 - \tfrac{1}{2}$$

Point of inflection $B(4, \ln 2 - \frac{1}{2})$

Question 5 (c)

$$\frac{dy}{dx} = -\frac{2}{x^2} + \frac{1}{x} = \frac{x-2}{x^2}$$

$$\frac{dy}{dx} > 0 \Rightarrow \frac{x-2}{x^2} > 0$$

$$x - 2 > 0$$

$$\therefore x > 2,\ x \in \mathbb{R}$$

Question 5 (d)

$$\frac{dy}{dx} = -\frac{2}{x^2} + \frac{1}{x}$$

$$\left(\frac{dy}{dx}\right)_{x=4} = -\frac{2}{4^2} + \frac{1}{4} = -\frac{1}{8} + \frac{1}{4} = \frac{1}{8}$$

Equation of t : $m = \frac{1}{8}$, $B(4,\ \ln 2 - \frac{1}{2})$

$$x - 8y + k = 0$$

$$4 - 8\ln 2 + 4 + k = 0 \Rightarrow k = 8\ln 2 - 8$$

Equation of t : $x - 8y + 8(\ln 2 - 1) = 0$