LC 2016 (Set A): Paper 1

Question 4 (25 marks)

Question 4 (a)

Step 1: $n = 1$: Prove $8^1 - 1$ is divisible by 7.
Proof: $8^1 - 1 = 7$. Therefore, it is true for $n = 1$.

Step 2: $n = k$: Assume $8^k - 1$ is divisible by 7.
In other words, assume $8^k - 1 = 7m,\ m \in \mathbb{N}$.
$\therefore 8^k = (7m + 1)$

Step 3: $n = (k + 1)$: Prove $8^{k+1} - 1$ is divisible by 7.

Proof:
$$\begin{aligned} 8^{k+1} - 1 &= 8 \times 8^k - 1 \\ &= 8(7m + 1) - 1 \text{ [By Step 2.]} \\ &= 56m + 8 - 1 \\ &= 56m + 7 \\ &= 7(8m + 1) \\ &= 7 \times (\text{whole number}) \end{aligned}$$
$\therefore 8^{k+1} - 1$ is divisible by 7.

Therefore, assuming it is true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$ implies it is true for all $n \in \mathbb{N}$.

Marking Scheme Notes
Question 4 (a) [Scale 15D (0, 4, 7, 11, 15)]
4: • P_1 step
7: • P_k step
• P_{k+1} step
11: • use of P_k step to prove P_{k+1} step
Note: accept P_1 step, P_k step and P_{k+1} step in any order

Question 4 (b)

$\log_a 2 = p,\ \log_a 3 = q,\ a > 0$

(i) $$\begin{aligned} \log_a \frac{8}{3} &= \log_a 8 - \log_a 3 \\ &= \log_a 2^3 - \log_a 3 \\ &= 3\log_a 2 - \log_a 3 \\ &= 3p - q \end{aligned}$$

(ii) $$\begin{aligned} \log_a \frac{9a^2}{16} &= \log_a 9 + \log_a a^2 - \log_a 16 \\ &= \log_a 3^2 + \log_a a^2 - \log_a 2^4 \\ &= 2\log_a 3 + 2\log_a a - 4\log_a 2 \\ &= 2q - 4p + 2 \end{aligned}$$

Formulae and Tables Book
Indices and logs [page 21]
$$\log_a(xy) = \log_a x + \log_a y$$
$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$$\log_a(x^q) = q\log_a x$$
$$\log_a 1 = 0$$
$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

Marking Scheme Notes
Question 4 (b) (i) [Scale 5C (0, 2, 4, 5)]
2: • $\log_a 8 - \log_a 3$
4: • $\log_a 8 = 3\log_a 2$ (and/or $= 3p$)

Question 4 (b) (ii) [Scale 5D (0, 2, 3, 4, 5)]
2: • $\log_a 9a^2 - \log_a 16$
3: • $2\log_a 3$
• $2\log_a a$
• $4\log_a 2$
• $4p$ or $2q$ or 2
4: • $2(\log_a 3 + \log_a a) - 4\log_a 2$ or equivalent