

**LC 2016 (SET A): PAPER 1****QUESTION 6 (25 MARKS)****Question 6 (a)**

$$f(x) = (2x + 4)^2 = 4x^2 + 16x + 16$$

$$\begin{aligned} f(x+h) &= 4(x+h)^2 + 16(x+h) + 16 \\ &= 4(x^2 + 2hx + h^2) + 16x + 16h + 16 \\ &= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16 \end{aligned}$$

$$\begin{aligned} f(x+h) - f(x) &= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16 - (4x^2 + 16x + 16) \\ &= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16 - 4x^2 - 16x - 16 \\ &= 8hx + 4h^2 + 16h \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{8hx + 4h^2 + 16h}{h} = 8x + 4h + 16$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right) = \lim_{h \rightarrow 0} (8x + 4h + 16) = 8x + 16$$

**MARKING SCHEME NOTES****Question 6 (a) [Scale 10D (0, 2, 5, 8, 10)]**

2: • any  $f(x+h)$

5: • limit of  $\frac{f(x+h) - f(x)}{h}$

8: • limit of  $\frac{(2x+2h+4)^2 - (2x+4)^2}{h}$

**NOTES:**

- omission of limit sign penalised once only

- answer not from 1st Principles merits 0 marks

**Question 6 (b) (i)**

$$y = x \sin\left(\frac{1}{x}\right) = x \sin(x^{-1})$$

$$\begin{aligned} \frac{dy}{dx} &= x(\cos(x^{-1}) \times -1x^{-2}) + \sin(x^{-1}) \times 1 \\ &= -\frac{x}{x^2}(\cos(x^{-1})) + \sin(x^{-1}) \\ &= -\frac{1}{x} \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right) \end{aligned}$$

**FORMULAE AND TABLES BOOK****Calculus: Derivatives [page 25]**

$$y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$$

**Question 6 (b) (ii)**

$$\begin{aligned}x = \frac{4}{\pi} : \frac{dy}{dx} &= -\frac{\pi}{4} \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right) \\ &= -\frac{\pi}{4} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \left(1 - \frac{\pi}{4}\right) = 0.15\end{aligned}$$

**MARKING SCHEME NOTES****Question 6 (b) (i) & (ii) [Scale 15D (0, 4, 7, 11, 15)]****4:** • any correct differentiation**7:** • product rule applied**11:** • correct differentiation**NOTE:** one penalty for calculator in wrong mode