

LC 2016 (SET A): PAPER 2**QUESTION 8 (45 MARKS)****Question 8 (a)**

$$h(t) = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$$

$$a = 1.6, b = 1.5, k = \frac{\pi}{6}$$

$$P = \frac{2\pi}{\frac{\pi}{6}} = 12 \text{ hours}$$

$$\text{Range} = [1.6 - 1.5, 1.6 + 1.5] = [0.1 \text{ m}, 3.1 \text{ m}]$$

$$y = a + b \cos(kx + \alpha)$$

$$\text{Period } P = \frac{2\pi}{k}$$

$$\text{Range } [a - b, a + b], b > 0 \text{ and } [a + b, a - b], b < 0$$

Question 8 (b)

The maximum height is 3.1 m.

Question 8 (c)

$$h = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$$

$$\frac{dh}{dt} = -1.5 \sin\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6} = -\frac{\pi}{4} \sin\left(\frac{\pi}{6}t\right)$$

$$\left(\frac{dh}{dt}\right)_{t=2} = -\frac{\pi}{4} \sin\left(\frac{\pi}{6} \times 2\right) = -\frac{\pi}{4} \sin\left(\frac{\pi}{3}\right)$$

$$= -\frac{\pi}{4} \times \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}\pi}{8}$$

$$= -0.68 \text{ m/h}$$

FORMULAE AND TABLES BOOK
Calculus: Derivatives [page 25]

$$y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x$$

$$y = \cos f(x) \Rightarrow \frac{dy}{dx} = -\sin f(x) \times f'(x)$$

The height of water is decreasing as the tide goes out.

MARKING SCHEME NOTES**Question 8 (a) [Scale 5C (0, 2, 4, 5)]**

- 2: • some use of 2π or $\frac{\pi}{6}$
 • range of cos function

- 4: • period or range correct

NOTE: Accept correct period and/or range without work

Question 8 (b) [Scale 5B (0, 2, 5)]

- 2: • max occurs when $\cos A = 1$ or $t = 0$
 • effort at $h'(t)$

NOTE: Accept correct answer without work

Question 8 (c) [Scale 5C (0, 2, 4, 5)]

- 2: • effort at differentiation
 4: • correct numerical answer but not in context

Question 8 (d) (i)

$$h(t) = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$$

$$h(0) = 1.6 + 1.5 \cos\left(\frac{\pi}{6} \times 0\right) = 1.6 + 1.5 = 3.1 \text{ m}$$

$$h(3) = 1.6 + 1.5 \cos\left(\frac{\pi}{6} \times 3\right) = 1.6 + 0 = 1.6 \text{ m}$$

$$h(6) = 1.6 + 1.5 \cos\left(\frac{\pi}{6} \times 6\right) = 1.6 - 1.5 = 0.1 \text{ m}$$

MARKING SCHEME NOTES

Question 8 (d) (i) [Scale 10C (0, 3, 7, 10)]

3: • one correct height

7: • five correct heights

Question 8 (d) (ii) [Scale 10C (0, 3, 7, 10)]

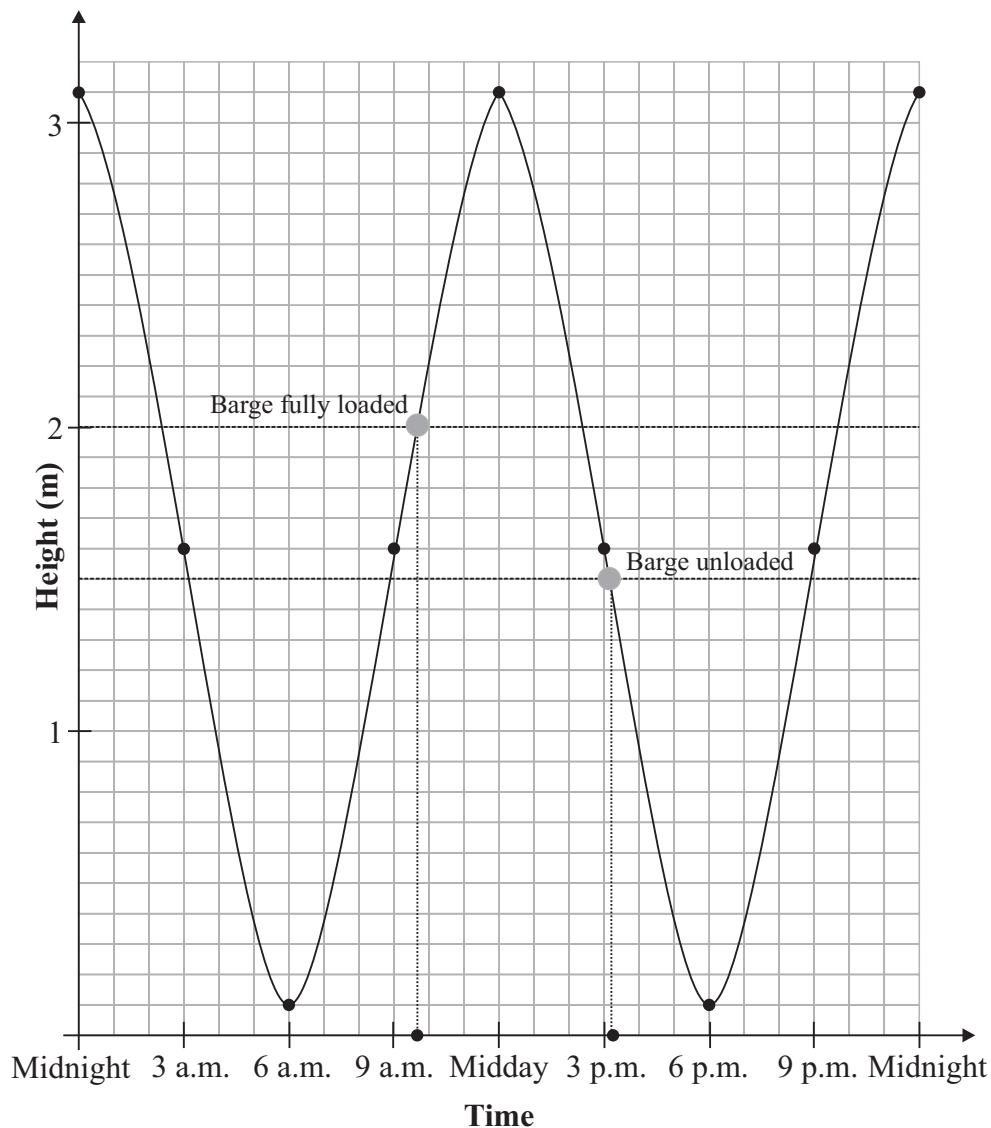
3: • one correct plot

7: • at least 7 correct plots

• plots correct but graph not sketched or sketched incorrectly

$h(t) = 1.6 + 1.5 \cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
t (hours)	0	3	6	9	12	15	18	21	24
$h(t)$ (m)	3.1	1.6	0.1	1.6	3.1	1.6	0.1	1.6	3.1

Question 8 (d) (ii)



Question 8 (e)

High tide = 3.1 m

Low tide = 0.1 m

Difference in water height = $3.1 - 0.1 = 3$ m

Question 8 (f)

Barge enters port when height is 2 m and leaves when height is 1.5 m. This can happen a number of times during the day. One such time is shown on the graph.

Time \approx 5.5 hours

MARKING SCHEME NOTES

Question 8 (e) [Scale 5B (0, 2, 5)]

2: • height of Low tide or High tide correctly identified

NOTES:

- (i) candidates may show work for this section on graph
- (ii) accept values from candidate's graph
- (iii) accept correct answer from graph without work

Question 8 (f) [Scale 5B (0, 2, 5)]

2: • time of entry to port or leave port correctly identified
• value(s) for $h = 2$ and/or $h = 1.5$ on sketch
• time estimated using relevant values other than those required for the maximum time.

NOTES:

- (i) candidates may show relevant work for this section on graph
- (ii) accept values from candidate's graph