

**LC 2015 (SET B): PAPER 1****QUESTION 9 (50 MARKS)****Question 9 (a)**

$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) \leftarrow$  Replace  $t$  by 76 in this function and use your calculator in radian mode.

$$f(76) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times 76\right) = 16 \cdot 835 \text{ hours} = 16 \text{ h } 50 \text{ mins}$$

**MARKING SCHEME NOTES****Question 9 (a) [Scale 10C (0, 4, 8, 10)]**

4: • Uses  $t = 76$

8: • Correct substitution

Note: Using  $\pi = 90^\circ \Rightarrow$  one error, but do not penalise again in (b)

**Question 9 (b)**

$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) = 15 \leftarrow$  Put  $f(t)$  equal to 15 and use your calculator to find  $t$ .

$$4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) = 2 \cdot 75$$

$$\sin\left(\frac{2\pi}{365}t\right) = \frac{2 \cdot 75}{4 \cdot 75} = \frac{11}{19}$$

$$t = \frac{365}{2\pi} \sin^{-1}\left(\frac{11}{19}\right) \approx 35 \cdot 87 \text{ days}$$

36 days after 21st March is 26th April

**MARKING SCHEME NOTES****Question 9 (b) [Scale 10C (0, 4, 8, 10)]**

4: • Correct  $f(t)$

•  $f(15)$  substituted

8: • Correct equation with  $t$  only

Note: Accept 35 or 36 substituted correctly and tested

**Question 9 (c)**

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$f'(t) = 4 \cdot 75 \cos\left(\frac{2\pi}{365}t\right) \times \frac{2\pi}{365} = \frac{19\pi}{730} \cos\left(\frac{2\pi}{365}t\right)$$

**FORMULAE AND TABLES BOOK****Calculus: Integrals [page 25]**

$$y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$$

$$y = \sin f(x) \Rightarrow \frac{dy}{dx} = \cos f(x) \times f'(x)$$

**MARKING SCHEME NOTES****Question 9 (c) [Scale 10B (0, 5, 10)]**

5: • Any correct differentiation (note: '0' could be correct differentiation here)

Note: Substituting  $180^\circ$  for  $\pi \Rightarrow$  one error

**Question 9 (d)**

$$f'(t) = 0 \Rightarrow \frac{19\pi}{730} \cos\left(\frac{2\pi}{365}t\right) = 0$$

$$\cos\left(\frac{2\pi}{365}t\right) = 0$$

$$\frac{2\pi}{365}t = \cos^{-1} 0 = \frac{\pi}{2}$$

$$t = \frac{365}{4}$$

$$f(t) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right)$$

$$\therefore f\left(\frac{365}{4}\right) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365} \times \frac{365}{4}\right) = 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{\pi}{2}\right) = 12 \cdot 25 + 4 \cdot 75 = 17 \text{ hours}$$

FIND TURNING POINTS (LOCAL MAXIMUM/MINIMUM)  
Put  $\frac{dy}{dx} = 0$  and solve for  $x$

**MARKING SCHEME NOTES**

**Question 9 (d) [Scale 10D (0, 2, 5, 8, 10)]** – both solutions

2: •  $f'(t) = 0$

5: • Value of  $t$

8: • Value of  $t$  substituted into  $f(t)$

•  $f(t)$  maximum when  $\sin \theta = 1$

**Note:** Accept 91 or 92 substituted and evaluated correctly for full marks

**Question 9 (e)**

$$\bar{L} = \frac{1}{184 - 0} \int_0^{184} \left[ 12 \cdot 25 + 4 \cdot 75 \sin\left(\frac{2\pi}{365}t\right) \right] dt$$

$$= \frac{1}{184} \left[ 12 \cdot 25t - 4 \cdot 75 \cos\left(\frac{2\pi}{365}t\right) \times \frac{365}{2\pi} \right]_0^{184}$$

$$= \frac{1}{184} \left[ 12 \cdot 25t - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365}t\right) \right]_0^{184}$$

$$= \frac{1}{184} \left[ \left( 12 \cdot 25(184) - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365} \times 184\right) \right) - \left( 12 \cdot 25(0) - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365}(0)\right) \right) \right]$$

$$= \frac{1}{184} \left[ \left( 12 \cdot 25(184) - \frac{4 \cdot 75 \times 365}{2\pi} \cos\left(\frac{2\pi}{365} \times 184\right) \right) + \frac{4 \cdot 75 \times 365}{2\pi} \right]$$

$$= 15 \cdot 2488 \text{ hours} = 15 \text{ hours } 15 \text{ minutes}$$

**FORMULAE AND TABLES BOOK**

**Calculus: Integrals** [page 26]

$$\int \sin x \, dx = -\cos x + c$$

$$\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + c$$

**MARKING SCHEME NOTES**

**Question 9 (e) [Scale 10D (0, 2, 5, 8, 10)]**

2: • Correct expression in  $x$  or  $t$

• Correct formula

• Correct limits

5: • Any correct integration

8: • Correct integration and effort at substitution

**Note:** Integration with one error but finished correctly gets High Partial Credit