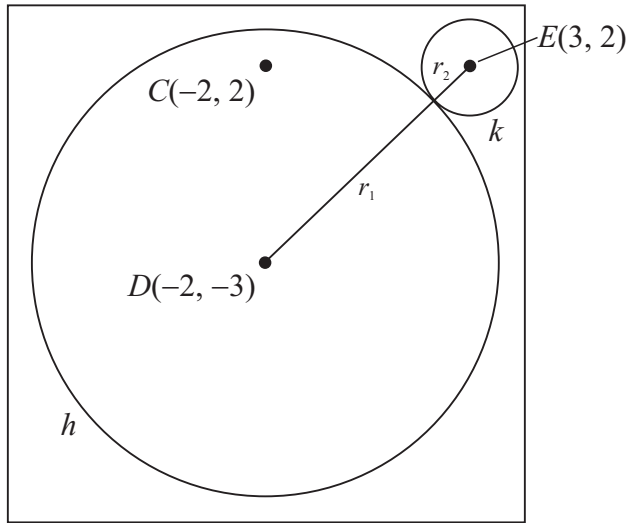


LC 2014 (SET C): PAPER 2

QUESTION 9 (60 MARKS)



FORMULAE AND TABLES BOOK Co-ordinate geometry: Circle [page 19]

Given centre (h, k) and radius r

$$(x - h)^2 + (y - k)^2 = r^2$$

Given equation

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Centre $(-g, -f)$

Radius $\sqrt{g^2 + f^2 - c}$

Question 9 (a) (i)

$$h: x^2 + y^2 + 4x + 6y - 19 = 0$$

Centre $D(-2, -3)$

$$\begin{aligned} r_1 &= \sqrt{(-2)^2 + (-3)^2 - (-19)} \\ &= \sqrt{4 + 9 + 19} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

Question 9 (a) (ii)

$$E(3, 2), D(-2, -3), r_1 = 4\sqrt{2}$$

$$|ED| = r_1 + r_2$$

$$|ED| = \sqrt{(-2 - 3)^2 + (-3 - 2)^2} = \sqrt{50} = 5\sqrt{2}$$

$$\therefore 5\sqrt{2} = 4\sqrt{2} + r_2$$

$$r_2 = \sqrt{2}$$

MARKING SCHEME NOTES

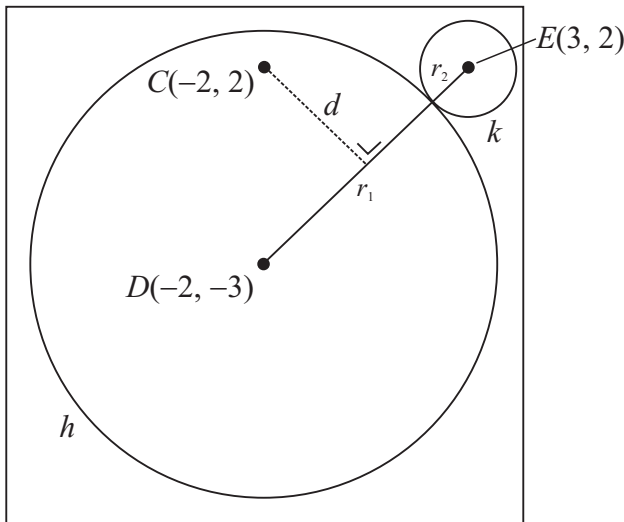
Question 9 (a) (i) [Scale 15C (0, 5, 10, 15)]

- 5:**
- Effort at relating one or more coefficients of given equation to general equation of circle
 - Effort at completing square(s)
- 10:**
- Either radius or centre correct
 - Substantive work with one critical error

Question 9 (a) (ii) [Scale 10C (0, 3, 7, 10)]

- 3:**
- Effort at finding $|DE|$
 - Length of line segment formula
 - Indicates some understanding of
- $$r_1 + r_2 = |DE|$$
- 7:**
- $r_1 + r_2 = |DE|$ or equivalent with known values substituted

Question 9 (a) (iii)



$$D(-2, -3), E(3, 2)$$

$$m = \frac{2 - (-3)}{3 - (-2)} = \frac{5}{5} = 1$$

$$y - 2 = 1(x - 3)$$

$$y - 2 = x - 3$$

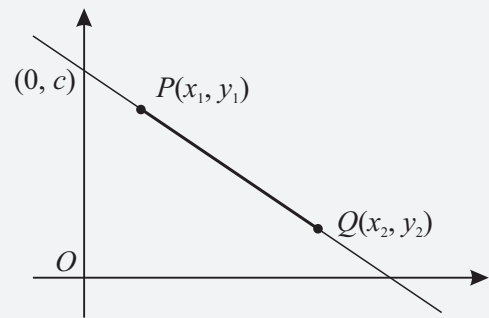
$$x - y - 1 = 0$$

$$d = \frac{|(-2) - (-2) - 1|}{\sqrt{1^2 + (-1)^2}} = \frac{|-5|}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$|DE| = 5\sqrt{2}$$

$$\therefore d = \frac{1}{2}|DE|$$

FORMULAE AND TABLES BOOK
Co-ordinate geometry: Line



Slope of PQ [page 18]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Length of PQ [page 18]

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of PQ [page 18]

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Equation of PQ [page 18]

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

Distance from (x_1, y_1) to the line
 $ax + by + c = 0$ [page 19]

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

MARKING SCHEME NOTES

Question 9 (a) (iii) [Scale 10C (0, 3, 7, 10)]

- 3: • Slope DE
• Equation DE and stops
• Formula for slope and/or equation of DE
• Perpendicular distance formula
- 7: • Values inserted into perpendicular distance formula
• No conclusion stated or implied

Question 9 (a) (iv)

$$D(-2, -3), E(3, 2)$$

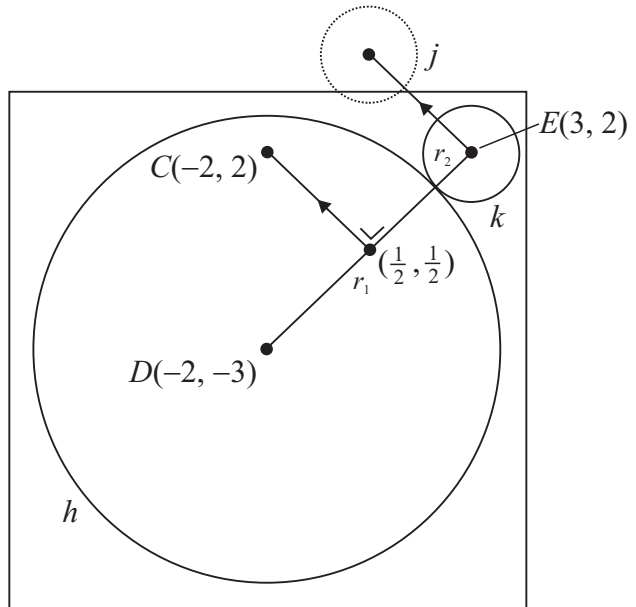
$$\text{Midpoint} = \left(\frac{-2+3}{2}, \frac{-3+2}{2} \right) = \left(\frac{1}{2}, -\frac{1}{2} \right)$$

$$\left(\frac{1}{2}, -\frac{1}{2}\right) \rightarrow C(-2, 2) \left[-\frac{5}{2}, +\frac{5}{2}\right]$$

$$E(3, 2) \rightarrow \left(3 - \frac{5}{2}, 2 + \frac{5}{2}\right) = \left(\frac{1}{2}, \frac{9}{2}\right)$$

Equation of j : $(x - \frac{1}{2})^2 + (y - \frac{9}{2})^2 = (\sqrt{2})^2$

$$(x - \frac{1}{2})^2 + (y - \frac{9}{2})^2 = 2$$



MARKING SCHEME NOTES

Question 9 (a) (iv) [Scale 5C (0, 2, 3, 5)]

- 2: • Effort to find midpoint of DE
 • Centre found from scaled drawing
- 3: • Centre of j found and inserted into equation of circle i.e radius omitted

Question 9 (a) (v)

$$x^2 + x^2 = (5\sqrt{2})^2$$

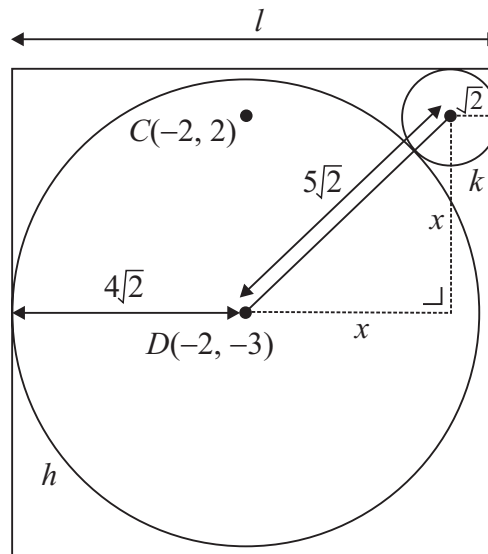
$$2x^2 = 50$$

$$x^2 = 25$$

$$\therefore x = 5$$

$$\text{Minimum } l = 4\sqrt{2} + 5 + \sqrt{2} = 12.07$$

$$\therefore l = 13$$

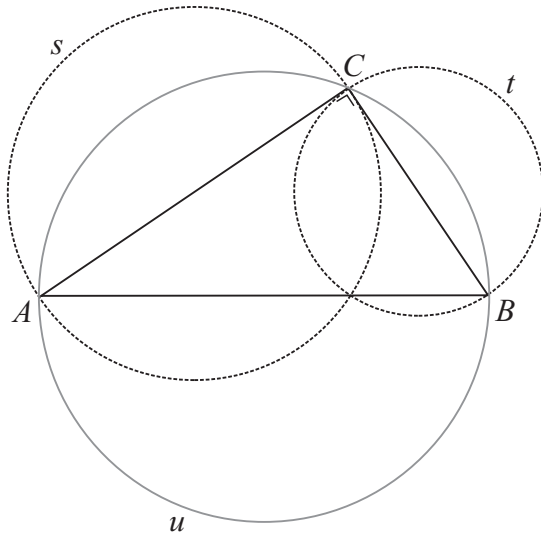


MARKING SCHEME NOTES

Question 9 (a) (v) [Scale 5C (0, 2, 3, 5)]

- 2: • Effort to find F
 • Indication length $r_1 + r_2 + |DE|$ (or equivalent)
- 3: • F found

Question 9 (b) (i)

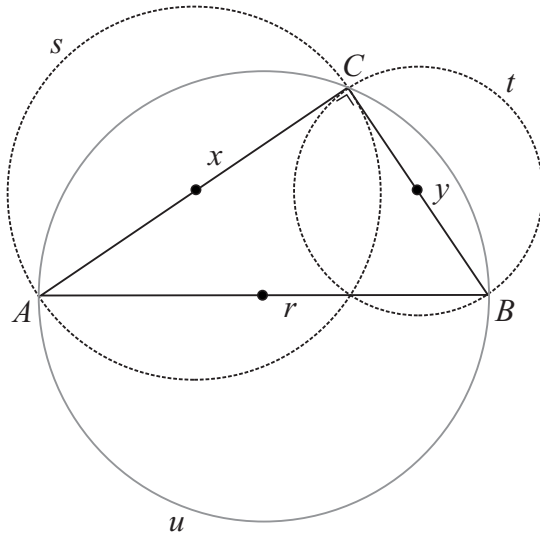


MARKING SCHEME NOTES

Question 9 (b) (i) [Scale 5B (0, 2, 5)]

- 2: • Circle containing A and B but lacking in accuracy

Question 9 (b) (ii)



Let $|AB| = r$, $|AC| = x$ and $|BC| = y$.

$$x^2 + y^2 = r^2 \text{ [Pythagoras]}$$

$$\text{Area of } s = \pi \left(\frac{1}{2}x\right)^2 = \frac{1}{4}\pi x^2$$

$$\text{Area} = \pi r^2$$

$$\text{Area of } t = \pi \left(\frac{1}{2}y\right)^2 = \frac{1}{4}\pi y^2$$

$$\text{Area of } u = \pi \left(\frac{1}{2}r\right)^2 = \frac{1}{4}\pi r^2$$

Area of s + Area of t

$$= \frac{1}{4}\pi x^2 + \frac{1}{4}\pi y^2$$

$$= \frac{1}{4}\pi(x^2 + y^2)$$

$$= \frac{1}{4}\pi r^2 = \text{Area of } u$$

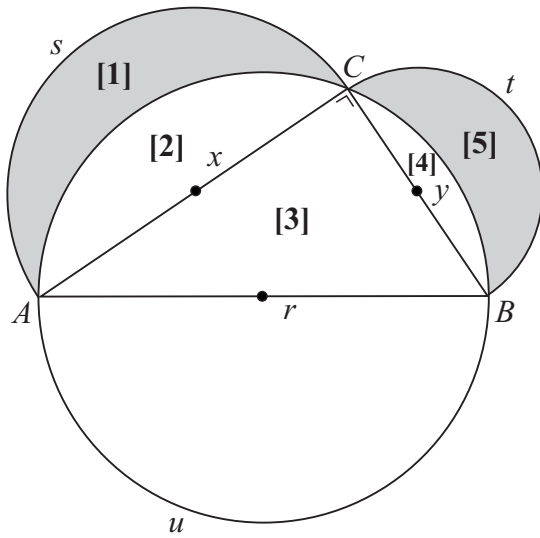
MARKING SCHEME NOTES

Question 9 (b) (ii) [Scale 5C (0, 2, 3, 5)]

- 2: • Pythagoras stated or implied
• Effort at finding area of s or t or u

- 3: • Correct expression for area of any circle e.g. $\frac{\pi}{4} \times |AB|^2$ or $\pi \times \frac{|AB|^2}{4}$

Question 9 (b) (iii)



Area of s + Area of t = Area of u

$$\therefore \frac{1}{2}(\text{Area of } s + \text{Area of } t) = \frac{1}{2}(\text{Area of } u)$$

$$\text{Area [1]} + \text{Area [2]} + \text{Area [4]} + \text{Area [5]} = \text{Area [2]} + \text{Area [3]} + \text{Area [4]}$$

$$\therefore \text{Area [1]} + \text{Area [5]} = \text{Area [3]}$$

MARKING SCHEME NOTES

Question 9 (b) (iii) [Scale 5C (0, 2, 3, 5)]

- 2:**
- Statement using result from (b)(ii)
 - Recognising half the area of s or half the area t can be expressed in terms of two component areas
 - Recognising half area of u can be expressed in terms of three components
- 3:**
- Correct expression for two of the relevant areas