

LC 2013 (SET D): PAPER 1

QUESTION 1 (25 MARKS)

Question 1 (a)

$$\begin{aligned} & \frac{4}{1+\sqrt{3}i} \leftarrow \text{Multiply above and below by} \\ & \quad \text{the conjugate of the denominator.} \\ & = \frac{4}{(1+\sqrt{3}i)} \times \frac{(1-\sqrt{3}i)}{(1-\sqrt{3}i)} \\ & = \frac{4(1-\sqrt{3}i)}{1-3i^2} \\ & = \frac{4(1-\sqrt{3}i)}{4} \\ & = 1-\sqrt{3}i \end{aligned}$$

FORMULA: Complex Numbers

Conjugates \bar{z}

$$z = a + bi \Rightarrow \bar{z} = a - bi$$

Multiplying a complex number by its conjugate:

$$z\bar{z} = (a + bi)(a - bi) = a^2 + b^2$$

MARKING SCHEME NOTES

Question 1 (a) [Scale 10C (0, 3, 7, 10)]

- 3: • Does not multiply by conjugate
 • Drops i , or $i^2 \neq -1$
 • Incomplete cross-multiplication
- 7: • Work not simplified

Question 1 (b)

Plotting z : Find the modulus of z

$$z = 1 - \sqrt{3}i$$

$$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

FORMULA: Complex Numbers

Modulus $|z|$

$$|z| = |a + bi| = \sqrt{a^2 + b^2}$$

Put the point of the compass at the origin and stretch the pencil out to 2 on the Real axis. Draw an arc as shown. The point $(1, -\sqrt{3})$ occurs at $x = 1$ **or** use your protractor to draw a 60° angle as shown.

FORMULA: COMPLEX NUMBERS

$$z = a + bi \text{ (Cartesian form)}$$

$$z = r(\cos \theta + i \sin \theta) \text{ (Polar form)}$$

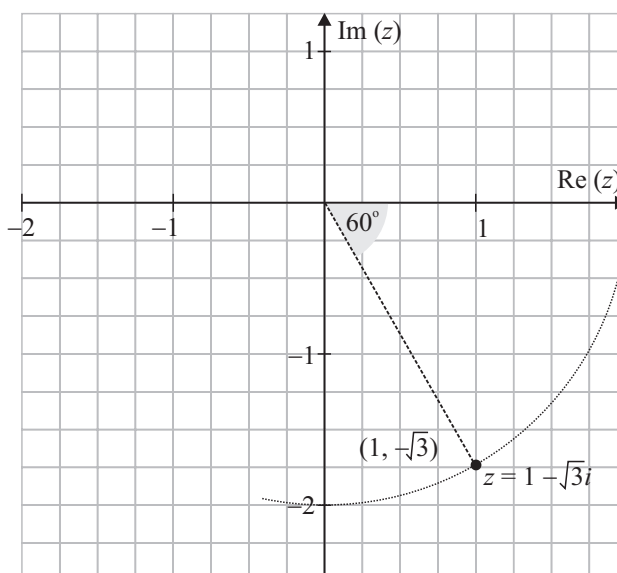
α is the reference angle in the first quadrant.

$$|\tan \theta| = \left| \frac{-\sqrt{3}}{1} \right| = \sqrt{3} = \tan \alpha$$

$$\therefore \alpha = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\therefore \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = 300^\circ$$

$$\therefore z = 2 \left\{ \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right\}$$



MARKING SCHEME NOTES

Question 1 (b) [Scale 10C (0, 3, 7, 10)]

- 3: • Work with α
 • Work with θ
 • Work with modulus
 • Plotting z
- 7: • Correct z but incorrect or no plotting
NOTE: Accept r , θ and plot for full marks.

Question 1 (c)

$$z = 2 \left\{ \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right\}$$

$$\therefore z^{10} = 2^{10} \left\{ \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right\}^{10}$$

$$= 2^{10} \left\{ \cos \frac{50\pi}{3} + i \sin \frac{50\pi}{3} \right\}$$

$$= 2^{10} \left\{ \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right\}$$

$$= 2^{10} \left\{ -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right\}$$

$$= -2^9 (1 - \sqrt{3}i)$$

FORMULAE AND TABLES BOOK:

Algebra (page 20)

DE MOIVRE'S THEOREM

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

MARKING SCHEME NOTES

Question 1 (c) [Scale 5C (0, 2, 4, 5)]

- 2: • Some work with De Moivre
 • De Moivre not used correctly
- 4: • Answer not simplified
 • n included in answer

NOTE: Allow for full marks candidates incorrect angle from (b), with correct conclusion.

0: no use of De Moivre.