

LC 2013 (SET D): PAPER 1

QUESTION 8 (50 MARKS)

Question 8 (a)

$$s(t) = 6t + 0.3t^2 - 0.01t^3 \leftarrow \text{Replace } t \text{ by } 10 \text{ in } s(t).$$

$$\begin{aligned} s(10) &= 6(10) + 0.3(10)^2 - 0.01(10)^3 \\ &= 60 + 30 - 10 \\ &= 80 \text{ m} \end{aligned}$$

MARKING SCHEME NOTES

Question 8 (a) [Scale 10C* (0, 3, 7, 10)]

3: • Some correct substitution into $s(t)$

7: • Correct substitution.

• $10s = 80 \Rightarrow s = 8$

Full Marks: Correct answer without work

Question 8 (b)

$$s = 6t + 0.3t^2 - 0.01t^3$$

$$v = \frac{ds}{dt} = 6 + 0.6t - 0.03t^2$$

$$v = 8.25 \text{ m s}^{-1}: 8.25 = 6 + 0.6t - 0.03t^2$$

$$0.03t^2 - 0.6t + 2.25 = 0$$

$$t^2 - 20t + 75 = 0$$

$$(t - 5)(t - 15) = 0$$

$$\therefore t = 5 \text{ s, } 15 \text{ s}$$

ANSWER: $t = 5 \text{ s}$

Note: Ignore the other solution as the function used is specified for the first 10 s.

FORMULAE: Speed

$$\text{Velocity } v = \frac{\text{Change } (d) \text{ in distance } s}{\text{Change } (d) \text{ in time } t}$$

$$v = \frac{ds}{dt}$$

FORMULAE AND TABLES BOOK Calculus: Derivatives [page 25]

$$y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$$

MARKING SCHEME NOTES

Question 8 (b) [Scale 10D* (0, 3, 5, 8, 10)]

3: • Any correct differentiation

5: • $\frac{ds}{dt}$ found correctly

8: • Fails to select correct value for t

Question 8 (c)

$$v = 6 + 0.6t - 0.03t^2$$

$$a = \frac{dv}{dt} = 0.6 - 0.06t$$

$$a = 0.006 \text{ m s}^{-2}: 0.006 = 0.6 - 0.06t$$

$$0.06t = 0.6 - 0.006$$

$$t = \frac{0.6 - 0.006}{0.06} = 9.9 \text{ s}$$

FORMULAE: Acceleration

$$\text{Acceleration } a = \frac{\text{Change } (d) \text{ in velocity } v}{\text{Change } (d) \text{ in time } t}$$

$$a = \frac{dv}{dt}$$

MARKING SCHEME NOTES

Question 8 (c) [Scale 10C* (0, 3, 7, 10)]

- 3: • Some correct differentiation of the first derivative
 7: • Sets up correct equation.

Question 8 (d)

You discovered in **part (a)** that the drop falls 80 m in the first 10 s. Therefore, it falls the remaining 540 m at constant velocity. Calculate this velocity.

$$t = 10 \text{ s: } v = 6 + 0.6t - 0.03t^2$$

$$v = 6 + 0.6(10) - 0.03(10)^2 = 9 \text{ m s}^{-1}$$

$$v = \frac{s}{t} \Rightarrow t = \frac{s}{v} = \frac{540}{9} = 60 \text{ s}$$

ANSWER: Total time = 10 + 60 = 70 s

MARKING SCHEME NOTES

Question 8 (d) [Scale 10D* (0, 3, 5, 8, 10)]

- 3: • Some correct substitution
 • Finds distance travelled after initial 10 seconds i.e. 540m
 • Correct differentiation (relevant for terminal velocity)
 5: • Finds k
 8: • Finds time travelling at terminal velocity and stops

Question 8 (e)

$$\frac{dV}{dt} = 6 \text{ mm s}^{-1}$$

$$\left(\frac{dr}{dt}\right)_{r=1.5} = ?$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$$

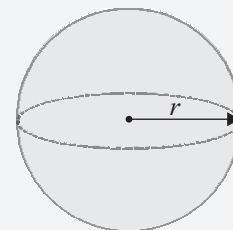
$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} \Rightarrow 6 = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{3}{2\pi r^2}$$

$$\therefore \left(\frac{dr}{dt}\right)_{r=1.5} = \frac{3}{2\pi(1.5)^2} = 0.2122 \text{ mm s}^{-1}$$

FORMULAE AND TABLES BOOK

Surface area and volume:
Sphere [page 8]



$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

MARKING SCHEME NOTES

Question 8 (e) [Scale 10D* (0, 3, 5, 8, 10)]

- 3: • $V = \frac{4}{3}\pi r^3$
 • Any relevant step
 5: • Any two of $\frac{dv}{dt} = 6$, $\frac{dv}{dr} = 4\pi r^2$, $\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$
 8: • Correct substitution into chain rule
 • Fails to substitute for r