

LC 2012 (SET E): PAPER 1**QUESTION 3 (25 MARKS)****Question 3 (a)**

$$r = \frac{81}{16}, \theta = \left(\frac{4\pi}{9} + 2n\pi \right) \leftarrow \text{Write } z \text{ in polar form.}$$

$$z = r(\cos \theta + i \sin \theta)$$

$$= \frac{81}{16} \left[\cos \left(\frac{4\pi}{9} + 2n\pi \right) + i \sin \left(\frac{4\pi}{9} + 2n\pi \right) \right]$$

$$w^4 = z \Rightarrow w = z^{\frac{1}{4}}$$

$$\therefore w = \left(\frac{81}{16} \right)^{\frac{1}{4}} \left[\cos \left(\frac{4\pi}{9} + 2n\pi \right) + i \sin \left(\frac{4\pi}{9} + 2n\pi \right) \right]^{\frac{1}{4}} \leftarrow \text{Apply De Moivre's theorem.}$$

$$= \left(\frac{\square}{2} \right) \left[\cos \left(\frac{\pi}{9} + \frac{n\pi}{2} \right) + i \sin \left(\frac{\pi}{9} + \frac{n\pi}{2} \right) \right]$$

The fourth root of z has 4 solutions which you find by putting $n = 0, 1, 2, 3$.

$$n = 0: w_1 = \left(\frac{3}{2} \right) \left[\cos \left(\frac{\pi}{9} \right) + i \sin \left(\frac{\pi}{9} \right) \right]$$

$$n = 1: w_2 = \left(\frac{3}{2} \right) \left[\cos \left(\frac{11\pi}{18} \right) + i \sin \left(\frac{11\pi}{18} \right) \right]$$

$$n = 2: w_3 = \left(\frac{3}{2} \right) \left[\cos \left(\frac{10\pi}{9} \right) + i \sin \left(\frac{10\pi}{9} \right) \right]$$

$$n = 3: w_4 = \left(\frac{3}{2} \right) \left[\cos \left(\frac{29\pi}{18} \right) + i \sin \left(\frac{29\pi}{18} \right) \right]$$

MARKING SCHEME NOTES**Question 3 (a) [Scale 20C (0, 14, 18, 20)]**

14: • Any reasonable first step

- $\cos \frac{4\pi}{9} + i \sin \frac{4\pi}{9}$ or $\cos 80^\circ + i \sin 80^\circ$ appears without any further work

18: • Correct solution with one or two minor errors

- One fully correct value

- $\left(5 \frac{1}{16} \right)^{\frac{1}{4}} \left(\cos \frac{1}{4} \left(\frac{4\pi}{9} \right) + i \sin \frac{1}{4} \left(\frac{4\pi}{9} \right) \right)$

- Not in polar form

FORMULAE AND TABLES BOOK:**Algebra (page 20)****DE MOIVRE'S THEOREM**

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

Question 3 (b)

Draw a circle with centre at the origin and radius 1.5. All 4 roots lie on this circle.

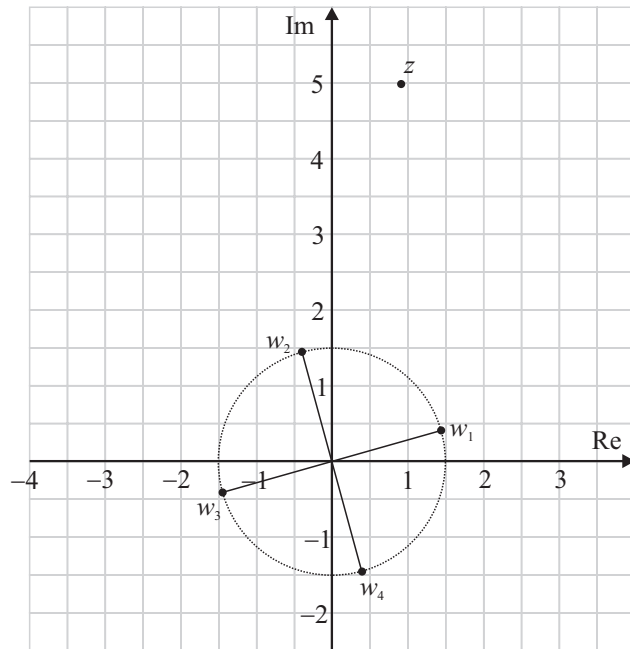
I changed the polar form of the roots to degrees to help me choose the angle the roots make with the positive x -axis.

$$w_1 = \frac{3}{2}(\cos 20^\circ + i \sin 20^\circ)$$

$$w_2 = \frac{3}{2}(\cos 110^\circ + i \sin 110^\circ)$$

$$w_3 = \frac{3}{2}(\cos 200^\circ + i \sin 200^\circ)$$

$$w_4 = \frac{3}{2}(\cos 290^\circ + i \sin 290^\circ)$$

**MARKING SCHEME NOTES****Question 3 (b) [Scale 5B (0, 3, 5)]**

- 3: • Some work of merit.
• If moduli are not the same, then partial credit only

NOTE: Need four reasonable solutions to part (a) in order to gain full credit in part (b).